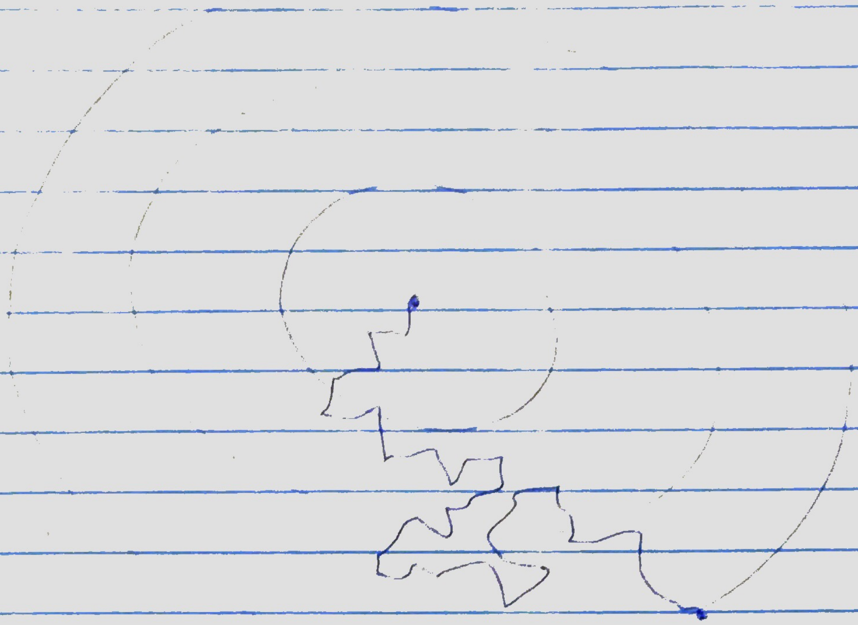
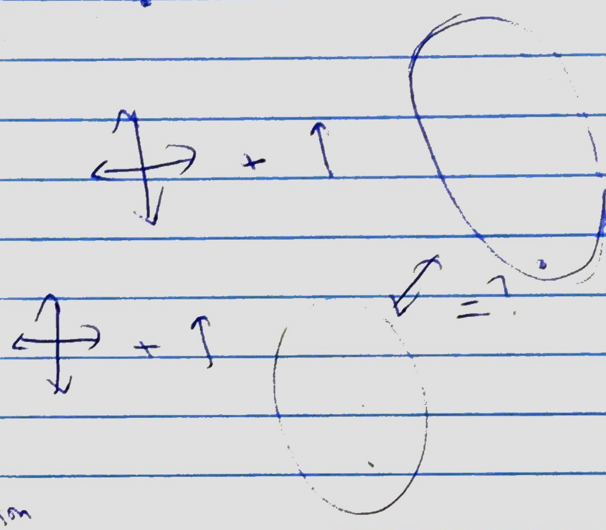
IDLA with drift

10/16/18



IDLA (standard) with drift

rotor-router:



advection-diffusion

$$\frac{\partial\phi}{\partial t} + U\partial_y\phi = D\nabla^2\phi$$

$$\frac{\partial\hat{\phi}}{\partial t} + iUk_y\hat{\phi} = -D|\underline{k}|^2\hat{\phi}$$

$\underline{k} = (k_x, k_y)$

$$\frac{\partial\hat{\phi}}{\partial t} = -(D|k|^2 + ik_yU)\hat{\phi}$$

$$\hat{\phi}(\underline{k},t) = \hat{f}(\underline{k})\, e^{-(D|\underline{k}|^2 + i k_y U)t}$$

$$\phi(\underline{x},t) = \frac{1}{(2\pi)^2}\int_{\mathbb{R}^2} \hat{\phi}(\underline{k},t)\, e^{i\underline{k}\cdot\underline{x}}\, \underbrace{d^2k}_{dk_x\, dk_y}$$

$$e^{ik_x x}\; e^{ik_y y}$$

$$= \frac{1}{2\pi}\underbrace{\left(\int_{-\infty}^{\infty} e^{-Dk_x^2 t}\, e^{ik_x x}\, dk_x\right)}_{S_1(x,t)}\frac{\left(\int_{-\infty}^{\infty} e^{-(Dk_y^2 + ik_y U)t}\, e^{ik_y y}\, dk_y\right)}{2\pi}$$

Find: $$\underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-(Dk_y^2)}\, e^{ik_y(Ut+y)}\, dk_y}$$

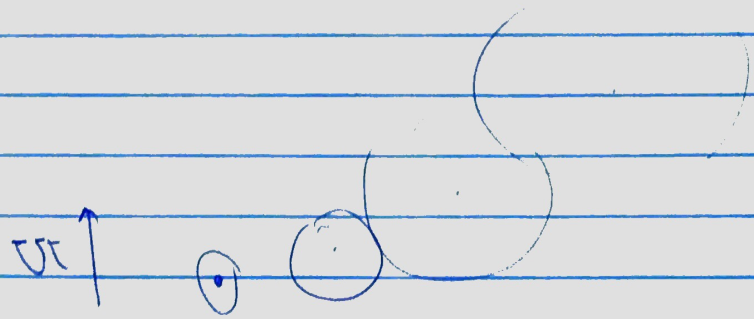
$$= S_1(y+Ut,t)$$

$$\phi(\underline{x},t) = S_1(x,t)\, S_1(y+Ut,t)$$

$$\underline{x} = (x,y)$$

$-$

$$\phi(\underline{x}, t) = \frac{1}{4\pi D t} \exp\left(\frac{-(x^2 + (y + Ut)^2)}{4Dt}\right)$$



With source: $\partial_t \phi + U \partial_y \phi - D \nabla^2 \phi = \delta(\underline{x})$

$$\phi(x, t) = \int_0^t \frac{1}{4\pi(t-\tau)} \exp\left(\frac{-(x^2 + (y + U(t-\tau))^2)}{4D(t-\tau)}\right) d\tau$$

$$= \int_0^t \frac{1}{4\pi\tau} \exp\left(\frac{-(x^2 + (y + U\tau)^2)}{4D\tau}\right) d\tau$$

$$x^2 + \cancel{y^2} + 2U\tau y + \cancel{U^2\tau^2}$$

$$x^2 + 2U\tau y = c$$