1) Show that the components of the momentum operator commute: $[\hat{p}_x, \hat{p}_y] = 0$.

2) Let $|\boldsymbol{p}\rangle$ be the eigenvectors of the momentum operator, with eigenvalues $\hbar\,\boldsymbol{k}$, and $|\boldsymbol{r}\rangle$ the eigenvectors of the position operator. Find the position representation $\langle\boldsymbol{r}|\boldsymbol{p}\rangle$ of the momentum eigenstates.

3) Show that in a region of space with $V(\boldsymbol{r}) = \text{const}$, $E = \frac{\hbar^2 k^2}{2m} + V$ are the eigenvalues of the Schrodinger equation, and $|\boldsymbol{p}\rangle$ are the eigenvectors.

4) Snell's law for matter waves: suppose $V(\boldsymbol{r}) = \begin{cases} V_0 & z < 0 \\ V_1 & z > 0 \end{cases}$. A beam of particles with momentum $\hbar\,\boldsymbol{k}_i$ impinges upon the plane $z = 0$, with $\hat{z}\cdot\boldsymbol{k}_i = k_i\cos(\theta_i)$. At $z = 0$, the beam is partially reflected and partially transmitted. Show that the reflected beam has $\hat{z}\cdot\boldsymbol{k}_r = -k_i\cos(\theta_i)$, and find the value of $\hat{z}\cdot\boldsymbol{k}_t = k_t\cos(\theta_t)$ for the transmitted beam.

5) Fresnel coefficients for matter waves: Use boundary conditions at $z = 0$ to find the amplitudes of the reflected and transmitted beams.

6) Let $V_0 = 0$, $V_1 = \frac{\hbar^2 Q^2}{2m}$. Plot the transmitted probability as a function of k_i, for $\theta_i = 45^o$.

7) Why is the beam totally reflected for $Q < k < 1.4\,Q$?

8) Suppose that in 2 dimensions the potential energy is $V(x, y) = V_x(x) + V_y(y)$, so that $H = H_x(x) + H_y(y)$. Calculate $[H_x, H_y]$ and use your result to argue that the eigenstates must be of the form $\psi(x, y) = \psi_x(x)\,\psi_y(y)$. Find the Schrödinger equations obeyed by ψ_x and ψ_y.

9) Find the energies and eigenstates for motion in the potential $V(x, y) = \begin{cases} \frac{1}{2}\kappa x^2 & 0 < y < a \\ \infty & \text{Elsewhere} \end{cases}$.

Name your two quantum numbers n_x and n_y. Plot the lowest 10 energy levels for the case $\frac{m\,a^4\,\kappa}{\hbar^2} = \frac{\pi^4}{4}$ and be sure to note any degeneracies.