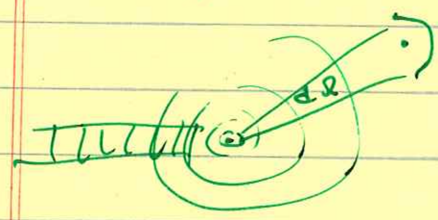
Scattering

$$\psi = A\left(e^{ikz} + \frac{f}{r} e^{ikr}\right)$$

$d\Omega = \frac{\text{area}}{r^2}$



measure incoming current to figure out $|A|^2$

measure outgoing current $= |A|^2 \frac{|f|^2}{r^2}$ area $= |A|^2 |f|^2 d\Omega$

ratio $= |f|^2 d\Omega = \frac{d\sigma}{d\Omega} d\Omega$

differential cross section

$$\psi = \psi^0 + \psi^{sc}$$

$$\underbrace{(k^2+\nabla^2)\psi^0}_{=0} \quad (k^2+\nabla^2)(\psi^0 + \psi^{sc}) = \frac{2\mu V}{\hbar^2}\psi \Rightarrow \psi^{sc} = G^0 V \psi \frac{2\mu}{\hbar}$$

so $\psi = \psi^0 + G^0 V \psi \, 2\mu/\hbar^2$

need G^0: $(\nabla^2 + k^2) G^0 = \delta(\vec{r} - \vec{r}\,')$

recall Poisson $\nabla^2 \phi = -4\pi\rho = -4\pi q \, \delta(\vec{r} - \vec{r}\,')$

$\Rightarrow \phi = \frac{q}{|\vec{r} - \vec{r}\,'|}$

$\therefore$ as $k \to 0$, $G^0 = -\frac{1}{4\pi|\vec{r} - \vec{r}\,'|}$

Try $G^0 = \dfrac{e^{ikr}}{-4\pi r}$

$r \neq 0$ $\quad \nabla^2 G^0 + k^2 G^0 = \underbrace{\frac{1}{r}\frac{d^2}{dr^2}(rG^0)}_{-k^2 r G^0} + k^2 G^0 = 0$ ✓

integrate around $r=0$

$$\int dV \left(\nabla^2 G^0 + k^2 G^0\right) = \int dV\, \nabla^2 G^0 = \int dV\, \nabla^2\left(\frac{-1}{4\pi r}\right) = 1 \checkmark$$

$$\therefore \quad \psi(\bar{r}) = e^{ikz} + \int d^3 r' \frac{e^{ik|\bar{r}-\bar{r}'|}}{-4\pi|r-r'|} V(\bar{r}')\psi(\bar{r}')\, 2\mu/\hbar^2$$

large $|\bar{r}|$ $\quad |\bar{r}-\bar{r}'| \approx \sqrt{r^2 - 2\bar{r}\cdot\bar{r}'} = r - \frac{\bar{r}\cdot\bar{r}'}{r}$

$$\text{large } r \quad \psi(r) = e^{ikz} - \frac{\mu e^{ikr}}{2\pi\hbar^2 r}\int d^3 r'\, e^{-i\bar{k}_f\cdot\bar{r}'} V(r')\psi(r')$$

$$\therefore \quad f = -\frac{\mu}{2\pi\hbar^2}\int d^3\bar{r}'\, e^{-i\bar{k}_f\cdot\bar{r}'} V(r')\psi(r')$$

Born Approx $\quad \psi(r') \approx e^{ikz'} = e^{i\bar{k}_i\cdot\bar{r}'}$

$$f = -\frac{\mu}{2\pi\hbar^2}\int d^3 r'\, e^{-i(\bar{k}_f-\bar{k}_i)\cdot\bar{r}'} V(r')$$

Yukawa $V = g\dfrac{e^{-m_0 r}}{r}$

Spherically symmetric

$$\int d^3r'\, e^{i\bar{q}\cdot\bar{r}'} V(\bar{r}') = \int \underbrace{d^3r'}_{r^2\sin\theta\, d\theta}\, e^{iqr'\cos\theta}\, V(r')$$

$$= \int \frac{r'^2\,dr'}{iqr'}\left(e^{iqr'} - e^{-iqr'}\right) V(r')$$

$$\Rightarrow \quad f = \frac{-2\mu g}{\hbar^2(m_0^2 + q^2)}$$ for Yukawa $V = g\dfrac{e^{-m_0 r}}{r}$

$$q^2 = (\bar{k}_i - \bar{k}_f)^2 = 2k^2(1-\cos\theta) = 4k^2\sin^2\theta/2$$

$$f = \frac{-2\mu g}{\hbar^2(m_0^2 + 4k^2\sin^2\theta/2)}$$

$$\frac{d\sigma}{d\Omega} = \frac{4\mu^2 g^2}{\hbar^4(m_0^2 + 4k^2\sin^2\theta/2)^2}$$