Physics 449 hw#3

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Due: 2018/2/16 W4F

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<< "http://www.physics.wisc.edu/~tgwalker/448defs.m"
```

1) Townsend 10.7

```
Integrate[r^2 e^(-3 r/a0), {r, 0, ∞}]
```

$$\text{ConditionalExpression}\left[\frac{2\,a0^3}{27}, \text{Re}[a0] > 0\right]$$

```
8 * 64 / 27^2
```

$$\frac{512}{729}$$

```
N[512/729]
```

$$0.702332$$

5) What is the degeneracy and parity of the 17/2 $\hbar\omega$ energy levels of the 3 d isotropic harmonic oscillator? Calculate the wavefunction of the $l = 5$ state.

Given by (10.90~10.93), $u = \rho^{l+1} e^{-\rho^2/2} f(\rho)$, $f(\rho) \simeq \sum_{k=0}^{\infty} c_k \rho^k \simeq e^{\rho^2}$,

$$\rho = \sqrt{\frac{\mu\omega}{\hbar}}\, r \to \psi = \frac{u(r)}{r} Y_{l,m}(\theta, \phi) = \frac{1}{r}\left(\sqrt{\frac{\mu\omega}{\hbar}}\, r\right)^{5+1} e^{-\left(\sqrt{\frac{\mu\omega}{\hbar}}\, r\right)^2/2} e^{\left(\sqrt{\frac{\mu\omega}{\hbar}}\, r\right)^2} Y_{5,m}(\theta, \phi)$$

```
Table[1/r (Sqrt[μ ω/ħ] r)^(5+1) e^(-(Sqrt[μ ω/ħ] r)^2/2) e^(Sqrt[μ ω/ħ] r)^2 SphericalHarmonicY[5, m, θ, ϕ], {m, -5, 5}] // Simplify
```

$$\left\{\frac{3\, e^{-5 i \phi+\frac{r^2 \mu \omega}{2 \hbar}} \sqrt{\frac{77}{\pi}}\, r^5 \mu^3 \omega^3 \text{Sin}[\theta]^5}{32 \hbar^3}, \frac{3\, e^{-4 i \phi+\frac{r^2 \mu \omega}{2 \hbar}} \sqrt{\frac{385}{2\pi}}\, r^5 \mu^3 \omega^3 \text{Cos}[\theta]\, \text{Sin}[\theta]^4}{16 \hbar^3},\right.$$

$$\frac{1}{64 \hbar^3} e^{-3 i \phi+\frac{r^2 \mu \omega}{2 \hbar}} \sqrt{\frac{385}{\pi}}\, r^5 \mu^3 \omega^3 \left(7+9 \text{Cos}[2\theta]\right) \text{Sin}[\theta]^3, \frac{1}{16 \hbar^3}$$

$$e^{-2 i \phi+\frac{r^2 \mu \omega}{2 \hbar}} \sqrt{\frac{1155}{2\pi}}\, r^5 \mu^3 \omega^3 \text{Cos}[\theta] \left(1+3 \text{Cos}[2\theta]\right) \text{Sin}[\theta]^2, \frac{1}{16 \hbar^3}$$

$$e^{-i \phi+\frac{r^2 \mu \omega}{2 \hbar}} \sqrt{\frac{165}{2\pi}}\, r^5 \mu^3 \omega^3 \left(1-14 \text{Cos}[\theta]^2+21 \text{Cos}[\theta]^4\right) \text{Sin}[\theta],$$

$$\frac{1}{16 \hbar^3} e^{\frac{r^2 \mu \omega}{2 \hbar}} \sqrt{\frac{11}{\pi}}\, r^5 \mu^3 \omega^3 \text{Cos}[\theta] \left(15-70 \text{Cos}[\theta]^2+63 \text{Cos}[\theta]^4\right),$$

$$-\frac{1}{16 \hbar^3} e^{i \phi+\frac{r^2 \mu \omega}{2 \hbar}} \sqrt{\frac{165}{2\pi}}\, r^5 \mu^3 \omega^3 \left(1-14 \text{Cos}[\theta]^2+21 \text{Cos}[\theta]^4\right) \text{Sin}[\theta],$$

$$\frac{1}{16 \hbar^3} e^{2 i \phi+\frac{r^2 \mu \omega}{2 \hbar}} \sqrt{\frac{1155}{2\pi}}\, r^5 \mu^3 \omega^3 \text{Cos}[\theta] \left(1+3 \text{Cos}[2\theta]\right) \text{Sin}[\theta]^2,$$

$$-\frac{1}{64 \hbar^3} e^{3 i \phi+\frac{r^2 \mu \omega}{2 \hbar}} \sqrt{\frac{385}{\pi}}\, r^5 \mu^3 \omega^3 \left(7+9 \text{Cos}[2\theta]\right) \text{Sin}[\theta]^3,$$

$$\left.\frac{3\, e^{4 i \phi+\frac{r^2 \mu \omega}{2 \hbar}} \sqrt{\frac{385}{2\pi}}\, r^5 \mu^3 \omega^3 \text{Cos}[\theta]\, \text{Sin}[\theta]^4}{16 \hbar^3}, -\frac{3\, e^{5 i \phi+\frac{r^2 \mu \omega}{2 \hbar}} \sqrt{\frac{77}{\pi}}\, r^5 \mu^3 \omega^3 \text{Sin}[\theta]^5}{32 \hbar^3}\right\}$$

7) Townsend 12.4

```
Integrate[(N e^(-α x^2))^2, {x, -∞, ∞}]
```

```
ConditionalExpression[N^2 Sqrt[π/2]/Sqrt[α], Re[α] > 0]
```

```
Solve[N^2 Sqrt[π/2]/Sqrt[α] == 1, N]
```

$$\left\{\left\{N \to -\left(\frac{2}{\pi}\right)^{1/4} \alpha^{1/4}\right\}, \left\{N \to \left(\frac{2}{\pi}\right)^{1/4} \alpha^{1/4}\right\}\right\}$$

```
ψT2 = (2/π)^(1/4) α^(1/4) E^(-α x^2);
KE = -ħ^2/(2 m) D[ψT2, {x, 2}] // Simplify
```

$$-\frac{e^{-x^2 \alpha} \left(\frac{2}{\pi}\right)^{1/4} \alpha^{5/4} \left(-1 + 2 x^2 \alpha\right) \hbar^2}{m}$$

```
Energy7 = Integrate[ψT2 (KE + b x^4 ψT2), {x, -∞, ∞}]
```

$$\text{ConditionalExpression}\left[\frac{3 b m + 8 \alpha^3 \hbar^2}{16 m \alpha^2}, \text{Re}[\alpha] > 0\right]$$

```
DEnergy7 = D[Energy7, {α, 1}] // Simplify
```

$$\text{ConditionalExpression}\left[-\frac{3 b}{8 \alpha^3} + \frac{\hbar^2}{2 m}, \text{Re}[\alpha] > 0\right]$$

```
Solve[-3 b/(8 α^3) + ħ^2/(2 m) == 0, α]
```

$$\left\{\left\{\alpha \to -\frac{(-3)^{1/3} b^{1/3} m^{1/3}}{2^{2/3} \hbar^{2/3}}\right\}, \left\{\alpha \to \frac{3^{1/3} b^{1/3} m^{1/3}}{2^{2/3} \hbar^{2/3}}\right\}, \left\{\alpha \to \frac{(-1)^{2/3} 3^{1/3} b^{1/3} m^{1/3}}{2^{2/3} \hbar^{2/3}}\right\}\right\}$$

```
Energy7b = 3 b/(16 (3^(1/3) b^(1/3) m^(1/3)/(2^(2/3) ħ^(2/3)))^2) + (3^(1/3) b^(1/3) m^(1/3)/(2^(2/3) ħ^(2/3)) ħ^2)/(2 m) // Simplify // N
```

$$\frac{0.68142\, b^{1/3} \hbar^{4/3}}{m^{2/3}}$$

```
4^(1/3) 0.6814202223120523` b^(1/3) ħ^(4/3)/m^(2/3)
```

$$\frac{1.08169\, b^{1/3} \hbar^{4/3}}{m^{2/3}}$$

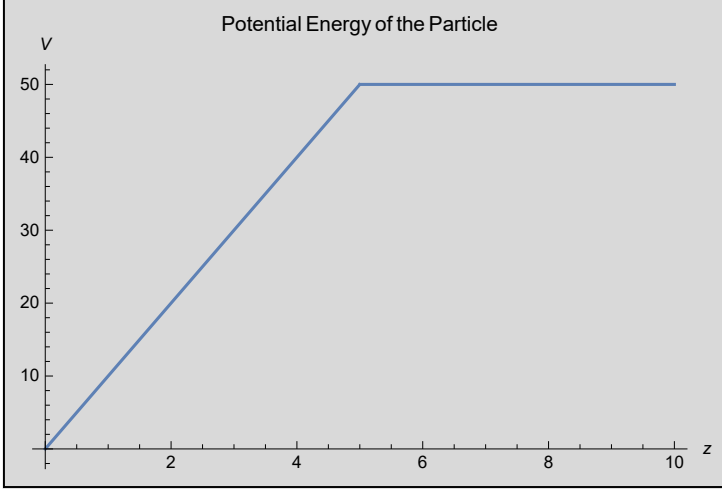
8) Townsend 12.6

a)

In[30]:=
```
VPlot = Plot[Piecewise[{{10 z, z < 5}, {50, z > 5}}], {z, 0, 10}]

Show[VPlot, AxesLabel → {HoldForm[z], HoldForm[V]},
 PlotLabel → HoldForm[Potential Energy of the Particle], LabelStyle → {GrayLevel[0]}]
```

Out[31]=



In[34]:=
```
ψ8 = C z ℯ^(-α z);
Integrate[ψ8^2, {z, 0, ∞}]
```

Out[35]=
$$\text{ConditionalExpression}\left[\frac{C^2}{4\,\alpha^3},\ \text{Re}[\alpha] > 0\right]$$

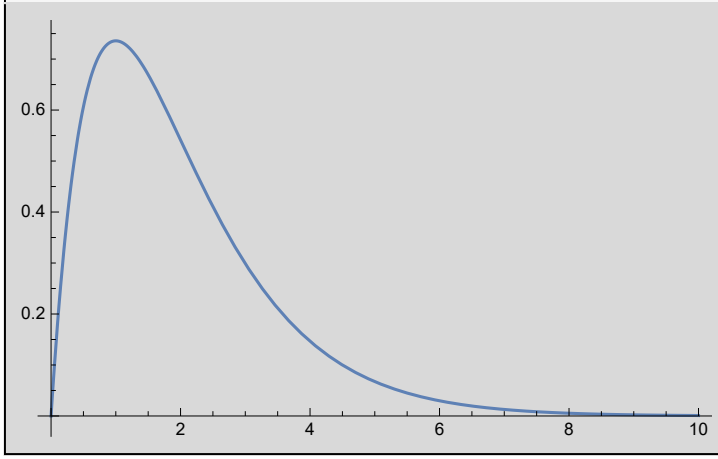
In[37]:=
```
Solve[C^2/(4 α^3) == 1, C]
```

Out[37]=
$$\left\{\left\{C \to -2\,\alpha^{3/2}\right\},\ \left\{C \to 2\,\alpha^{3/2}\right\}\right\}$$

In[51]:=
```
ψ8b = 2 α^(3/2) z ℯ^(-α z);
```

In[52]:=
```
Plot[2 z e^-z, {z, 0, 10}]
```

Out[52]=



b)

In[53]:=
```
D2ψ8b = D[ψ8b, {z, 2}]
```

Out[53]=
$$-4\, e^{-z\alpha}\, \alpha^{5/2} + 2\, e^{-z\alpha}\, z\, \alpha^{7/2}$$

In[74]:=
```
ExpH8 = Integrate[ψ8b * (-ħ^2/(2 m) D2ψ8b - m g z ψ8b), {z, 0, ∞}]
```

Out[74]=
$$\text{ConditionalExpression}\left[-\frac{3\, g\, m}{2\,\alpha} + \frac{\alpha^2\, \hbar^2}{2\, m},\ \text{Re}[\alpha] > 0\right]$$

In[75]:=
```
D1ExpH8 = D[-(3 g m)/(2 α) + (α^2 ħ^2)/(2 m), {α, 1}]
```

Out[75]=
$$\frac{3\, g\, m}{2\,\alpha^2} + \frac{\alpha\, \hbar^2}{m}$$

In[80]:=
```
Solve[(3 g m)/(2 α^2) + (α ħ^2)/m == 0, α]
```

Out[80]=
$$\left\{\left\{\alpha \to \frac{\left(-\frac{3}{2}\right)^{1/3} g^{1/3}\, m^{2/3}}{\hbar^{2/3}}\right\},\ \left\{\alpha \to -\frac{\left(\frac{3}{2}\right)^{1/3} g^{1/3}\, m^{2/3}}{\hbar^{2/3}}\right\},\ \left\{\alpha \to -\frac{(-1)^{2/3} \left(\frac{3}{2}\right)^{1/3} g^{1/3}\, m^{2/3}}{\hbar^{2/3}}\right\}\right\}$$

In[82]:=
$$\varepsilon 1 = -\frac{3\,g\,m}{2\left(-\frac{\left(\frac{3}{2}\right)^{1/3} g^{1/3} m^{2/3}}{\hbar^{2/3}}\right)} + \frac{\left(-\frac{\left(\frac{3}{2}\right)^{1/3} g^{1/3} m^{2/3}}{\hbar^{2/3}}\right)^2 \hbar^2}{2\,m} \;//\; \text{Simplify} \;//\; \text{N}$$

Out[82]=
$$1.96556\, g^{2/3}\, m^{1/3}\, \hbar^{2/3}$$

c)

In[71]:=
```
ExpZ = Integrate[ψ8b * (-z ψ8b), {z, 0, ∞}]
```

Out[71]=
$$\text{ConditionalExpression}\left[-\frac{3}{2\,\alpha}, \text{Re}[\alpha] > 0\right]$$

In[84]:=
$$\frac{3}{2\,\frac{\left(\frac{3}{2}\right)^{1/3} g^{1/3} m^{2/3}}{\hbar^{2/3}}} \;//\; \text{N}$$

Out[84]=
$$\frac{1.31037\, \hbar^{2/3}}{g^{1/3}\, m^{2/3}}$$