Physics 449 hw#7 Due 5/4

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W14W 5/2

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<< "http://www.physics.wisc.edu/~tgwalker/448defs.m"
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1) T 4.9

$$i\begin{pmatrix} c'[t] \\ d'[t] \end{pmatrix} = \frac{\omega_1}{2}\text{Cos}[\omega t]\begin{pmatrix} d[t]\, e^{i\omega_0 t} \\ c[t]\, e^{-i\omega_0 t} \end{pmatrix} \rightarrow d(t) = \frac{i\, c'[t]}{\frac{\omega_1}{2}\text{Cos}[\omega t]\, e^{i\omega_0 t}},\ c[t] = \frac{i\, d'[t]}{\frac{\omega_1}{2}\text{Cos}[\omega t]\, e^{-i\omega_0 t}}$$

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D[i d'[t] == ω1/2 Cos[ω t] c[t] e^(-i ω0 t), {t, 1}] // Simplify
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$$i\, d''[t] == \frac{1}{2} e^{-i\, t\, \omega 0}\, \omega 1 \left(c[t] \left(-i\, \omega 0 \text{Cos}[t\, \omega] - \omega \text{Sin}[t\, \omega]\right) + \text{Cos}[t\, \omega]\, c'[t]\right)$$

$$c[t] = \frac{i\, d'[t]}{\frac{\omega 1}{2}\text{Cos}[\omega t]\, e^{-i\,\omega 1\, t}};\ c'[t] = \frac{\omega 1}{2\, i}\text{Cos}[\omega t]\, d[t]\, e^{i\,\omega 1\, t};$$

$$i\, d''[t] == \frac{1}{2} e^{-i\, t\, \omega 0}\, \omega 1 \left(c[t] = \frac{i\, d'[t]}{\frac{\omega 1}{2}\text{Cos}[\omega t]\, e^{-i\,\omega 0\, t}} \left(-i\, \omega 0\, \text{Cos}[t\, \omega] - \omega\, \text{Sin}[t\, \omega]\right) + \text{Cos}[t\, \omega]\, \frac{\omega 1}{2\, i}\text{Cos}[\omega t]\, d[t]\, e^{i\,\omega 1\, t}\right) // \text{Simplify}$$

$$i\, d''[t] == -\frac{1}{4} i\, e^{-i\, t\, (\omega 0 - \omega 1)}\, \omega 1^2\, \text{Cos}[t\, \omega]^2\, d[t] + \left(\omega 0 - i\, \omega\, \text{Tan}[t\, \omega]\right) d'[t]$$

```
D[i c'[t] == ω1/2 Cos[ω t] d[t] e^(i ω0 t), {t, 1}] // Simplify
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$$i\, c''[t] == \frac{1}{2} e^{i\, t\, \omega 0}\, \omega 1 \left(d[t] \left(i\, \omega 0\, \text{Cos}[t\, \omega] - \omega\, \text{Sin}[t\, \omega]\right) + \text{Cos}[t\, \omega]\, d'[t]\right)$$

$$i\, c''[t] == \frac{1}{2} e^{i\, t\, \omega 0}\, \omega 1 \left(d[t] \left(i\, \omega 0\, \text{Cos}[t\, \omega] - \omega\, \text{Sin}[t\, \omega]\right) + \text{Cos}[t\, \omega]\, d'[t]\right)$$

6)

$$\langle \hat{n} \rangle = \langle n | \hat{n} | n \rangle = n \langle n | n \rangle$$
$$\langle \hat{n}^2 \rangle = \langle n | \hat{n}^2 | n \rangle = n^2 \langle n | n \rangle$$

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ψt = MatrixExp[-i ( 0  √n Ω/2 ; √n Ω/2  -Δ ) t].{{1, 0}} // ExpToTrig // Simplify
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$$\left\{\left\{\frac{1}{\sqrt{\Delta^2+n\,\Omega^2}}\left(\text{Cos}\left[\frac{t\,\Delta}{2}\right]+i\,\text{Sin}\left[\frac{t\,\Delta}{2}\right]\right)\left(\sqrt{\Delta^2+n\,\Omega^2}\ \text{Cos}\left[\frac{1}{2}t\sqrt{\Delta^2+n\,\Omega^2}\right]-i\,\Delta\,\text{Sin}\left[\frac{1}{2}t\sqrt{\Delta^2+n\,\Omega^2}\right]\right)\right\},\left\{\frac{\sqrt{n}\ \Omega\left(-i\,\text{Cos}\left[\frac{t\,\Delta}{2}\right]+\text{Sin}\left[\frac{t\,\Delta}{2}\right]\right)\text{Sin}\left[\frac{1}{2}t\sqrt{\Delta^2+n\,\Omega^2}\right]}{\sqrt{\Delta^2+n\,\Omega^2}}\right\}\right\}$$

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n ⟨ψt | ψt⟩ // Simplify
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n

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n^2 ⟨ψt | ψt⟩ // Simplify
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n^2