Physics 449
Li Project

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1)

$$\hat{H}^{++} = -\frac{1}{2}\nabla^2 - \frac{Z}{r} \quad , \text{where } \nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)$$

$\hat{H}^{++}\psi = E\psi$ Given by the derivation in 9.6 in Townsend the radial differential equation can be obtained:

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right) + \frac{l(l+1)\hbar^2}{2\mu r^2} - \frac{Z}{r}\right] u_{E,l}(r) = E u_{E,l}(r)$$

plug in the dimensionless variable $\rho = \sqrt{\frac{8\mu|E|}{\hbar^2}}\, r$

$$\frac{d^2 u}{d\rho^2} - \frac{l(l+1)}{\rho^2}u + \left(\frac{\lambda}{4} - \frac{1}{4}\right)u = 0 \quad \text{where } \lambda = \frac{Ze^2}{\hbar}\sqrt{\frac{\mu}{2|E|}}$$

a solution with the form $u(\rho) = \rho^{l+1} e^{-\rho/2} F(\rho)$ can be obtained.

where

$$\rho = \sqrt{\frac{8\mu|E|}{\hbar^2}}\, r = \frac{2Z\mu c\alpha}{\hbar n} r = \frac{2Z}{n}\frac{r}{a_0} \quad (10.40) \qquad a_0 = \frac{\hbar}{\mu c\alpha}$$

Hence, the radial eigen functions of $\hat{H}^{++}$ can be considered as functions with both $r$ and $Zr$ as variables.

$$\Rightarrow P_{nl}^{++}(r) = N(Z)\, P_{n,l}(Zr)$$

normalization factor $N(Z)$.

2) $\hat{H}^{+} = \hat{H}^{++}(1) + \hat{H}^{++}(2) + \frac{1}{r_{12}} = \hat{H}_0 + V_{12} = -\frac{1}{2}\nabla_1^2 - \frac{Z}{r} - \frac{1}{2}\nabla_2^2 - \frac{Z}{r} + \frac{1}{r_{12}}$

$\langle n_1; n_2 | \hat{H}^{+} | n_3; n_4 \rangle = \langle n_1 n_2 | H^{++}(1) + H^{++}(2) | n_3 n_4 \rangle + \langle n_1 n_2 | V | n_3 n_4 \rangle$

$= \langle n_1 n_2 | H^{++}(1) | n_3 n_4 \rangle + \langle n_1 n_2 | H^{++}(2) | n_3 n_4 \rangle + \langle n_1 n_2 | V | n_3 n_4 \rangle$

$H^{+}(1)$ operates on the 1st particle; $H^{+}(2)$ operates on the 2nd particle.

$\Rightarrow$

$$= \boxed{\underbrace{\langle n_1 | \hat{H}^{++}(1) | n_3 \rangle}_{E_{n_1}\delta_{n_1 n_3}} \delta_{n_2, n_4} + \underbrace{\langle n_2 | \hat{H}^{++}(2) | n_4 \rangle}_{E_{n_2}\delta_{n_2 n_4}} \delta_{n_1 n_3} + \langle n_1 n_2 | V | n_3 n_4 \rangle}$$

$$\Rightarrow \hat{H}^{+} = \begin{pmatrix} \langle 1 | \hat{H}^{+} | 1 \rangle & \langle 1 | \hat{H}^{+} | 2 \rangle \\ \langle 2 | \hat{H}^{+} | 1 \rangle & \langle 2 | \hat{H}^{+} | 2 \rangle \end{pmatrix} = \begin{pmatrix} \langle n_1 | \hat{H}^{++}(1) | n_3 \rangle & 0 \\ 0 & \langle n_2 | \hat{H}^{+}(2) | n_4 \rangle \end{pmatrix}$$