Physics 449 hw#6 Due 4/20 W12F

Name: Ruojun Wang

W12R 4/19

In[3]:=
```
<< "http://www.physics.wisc.edu/~tgwalker/448defs.m"
```

Syntax::sntx:
Invalid syntax in or before
"<!DOCTYPE HTML PUBLIC "-//IETF//DTD HTML 2.0//EN">"
^
(line 1 of
"http://www.physics.wisc.edu/~tgwalker/448defs.m").

2) T 13.7

In[1]:=

$$\frac{-\mu}{2\pi\hbar^2}\,\mathtt{Integrate}\left[\mathtt{Sin}[\theta]\, r^2\, e^{-r/a}\, e^{i\,q\,r\,\mathtt{Cos}[\theta]},\ \{r, 0, \infty\},\ \{\theta, 0, \pi\},\ \{\phi, 0, 2\pi\}\right]$$

Out[1]=

$$\mathtt{ConditionalExpression}\left[-\frac{4\,a^3\,\mu}{\left(1+a^2 q^2\right)^2\hbar^2},\ \mathtt{Re}\left[\frac{1}{a}\right] > \mathtt{Abs}[\mathtt{Im}[q]]\right]$$

5) Plot your results from 0 to 10 eV, on a log scale. From your zero energy wavefunction, how many bound states are there in this potential?

It follows from the handout “SwaveScattering,”

$$d_r^2 P + k^2 P = \frac{2\mu}{\hbar^2} V P$$

pick scaling length $r = s\,\Lambda$, and plug in $V(r) = \frac{-V_0}{1+\left(\frac{r}{b}\right)^6} = \frac{-9.5\,\text{eV}}{1+\left(\frac{r}{5\,a_0}\right)^6}$

$$d_s^2 P + (k\,\Lambda)^2 P = \frac{2\,\mu\,\Lambda^2}{\hbar^2}\,\frac{V_0}{1+\left(\frac{s\,\Lambda}{5\,a_0}\right)^6} P$$

pick $k_s = k\,\Lambda$, $\Lambda = a_0$

$$d_s^2 P + {k_s}^2 P = \frac{2\,\mu\,{a_0}^2}{\hbar^2}\,\frac{V_0}{1+\left(\frac{s}{5}\right)^6} P$$

With $1\,E_h = \frac{\hbar^2}{m\,a_0} = 27.2\,\text{eV} \to \frac{2\,\mu\,{a_0}^2}{\hbar^2}\,\frac{V_0}{1+\left(\frac{s}{5}\right)^6} P = \frac{2}{27.2}\,\frac{9.5}{1+\left(\frac{s}{5}\right)^6} P$

In[115]:=

$$\texttt{SE}[\epsilon\_] := \frac{-1}{2}\,\texttt{p''}[s] - \frac{9.5}{27.2\left(1+\left(\frac{s}{5}\right)^5\right)}\,\texttt{p}[s] == \epsilon * \texttt{p}[s];$$

Calculate the scattering length: at large r $\quad P = A(s - a_s),\ P' = A \rightarrow a_s = s - \frac{P}{P'}$

In[116]:=

```
sm = 40;
p0[s_] = NDSolve[{SE[0], p[0] == 0, p'[0] == 1}, p[s], {s, 0, sm}][[1, 1, 2]]
```

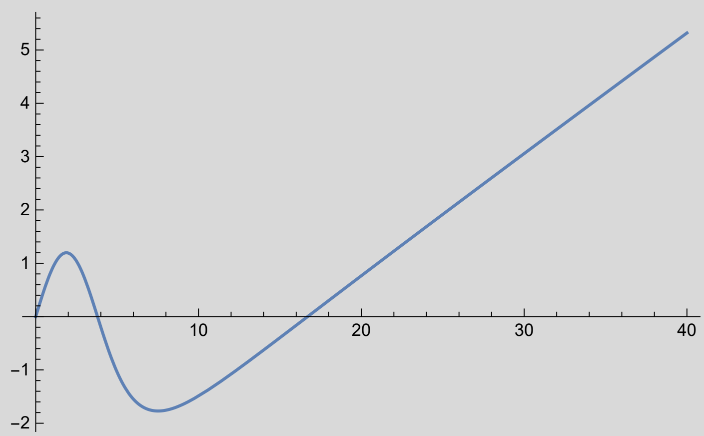
Out[116]=

```
InterpolatingFunction[ Domain: {{0., 40.}} Output: scalar ][s]
```

In[117]:=

```
Plot[p0[s], {s, 0, sm}]
```

Out[117]=



In[118]:=

$$\texttt{sm} - \frac{\texttt{p0[sm]}}{\texttt{p0'[sm]}}$$

Out[118]=

```
16.407
```

In[119]:=

$$\sigma 0 = 4\,\pi\,\%^2$$

Out[119]=

```
3382.75
```

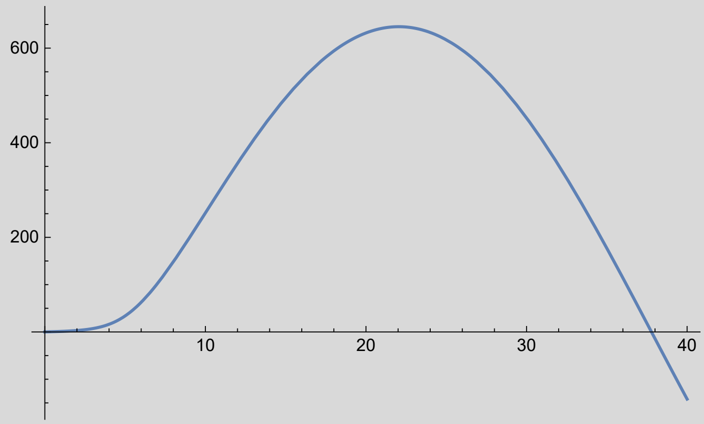
so the scattering length is $a = 16.407\,\Lambda$ and the cross section is $\sigma = 3382.75\,\Lambda^2$. There are 3 bounded states b/c 3 crossings.

In[120]:=

$$\delta[k\_] := \texttt{Module}\Big[\{\},$$
$$\quad \texttt{pk}[s\_] = \texttt{NDSolve}\Big[\Big\{k^2\,\texttt{p}[s] + \texttt{p}''[s] == \frac{2 * 9.5}{27.2\left(1+\left(\frac{s}{5}\right)^6\right)}\,\texttt{p}[s],\ \texttt{p}[0] == 0,\ \texttt{p}'[0] == 1\Big\},$$
$$\qquad \texttt{p}[s], \{s, 0, \texttt{sm}\}\Big][[1, 1, 2]];$$
$$\quad \texttt{Mod}\Big[\texttt{ArcCot}\Big[\frac{\texttt{pk}'[\texttt{sm}]}{k\ \texttt{pk}[\texttt{sm}]}\Big] - k\ \texttt{sm},\ \pi\Big]\Big]$$

In[121]:=
```
δ[.1]; Plot[pk[s], {s, 0, sm}]
```

Out[121]=



In[122]:=

$$ks = 10^{\text{Range}[-3,2,.03]};$$

In[123]:=

$$\sigma s = \text{Table}\left[\left\{k^2, \text{Sin}[\delta[k]]^2 \frac{4\pi}{k^2}\right\}, \{k, ks\}\right];$$

In[124]:=

$$\text{Show}\left[\text{ListLogLogPlot}[\sigma s], \text{LogLogPlot}\left[\frac{\sigma 0}{1 + \sigma 0 \frac{ksq}{4\pi}}, \left\{ksq, .0001, 10^4\right\}, \text{PlotStyle} \to \{\text{Thick}, \text{Darker}[\text{Green}]\}\right]\right]$$

Out[124]=