Physics 449 hw#7 Due 5/14

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1) T 4.9

$\omega \neq \omega_0$

from (4.41)

$$i\begin{bmatrix} \dot{c}(t) \\ \dot{d}(t) \end{bmatrix} = \frac{\omega_1}{2}\cos\omega t \begin{pmatrix} d(t)\, e^{i\omega_0 t} \\ c(t)\, e^{-i\omega_0 t} \end{pmatrix} \quad \begin{matrix} ① \\ ② \end{matrix}$$

Take derivative on both sides of ② →

$$i d''(t) = \frac{1}{2} e^{-it\omega_0}\, \omega_1 \{ c(t)[-i\omega_0 \cos(t\omega) - \omega \sin(t\omega)] + \cos(t\omega)\, c'(t) \} \quad ③$$

$$② \to c(t) = \frac{i d'(t)}{\frac{\omega_1}{2}\cos(\omega t)\, e^{-i\omega_0 t}} \;; \quad ① \to c'(t) = \frac{\omega_1}{2i}\cos\omega t\; d(t)\, e^{i\omega_0 t}$$

Plug into ③

$$\to \cancel{i} d''(t) = -\frac{1}{4}\cancel{i}\, e^{-it(\omega_0-\omega_1)}\, \omega_1^2 \cos^2(t\omega)\, d(t) + [\omega_0 - i\omega \tan(t\omega)]\, d'(t)$$

Guess a form for $d(t) = c_1 \sin(c_2 t)$

$$d'(t) = c_1 c_2 \cos(c_2 t) \;; \quad d''(t) = -c_1^2 c_2 \sin(c_2 t)$$

$$-\cancel{i}\, c_1^2 c_2 \sin(c_2 t) = -\frac{1}{4}\cancel{i}\, e^{-it(\omega_0-\omega_1)}\, \omega_1^2 \cos(t\omega)\, [c_1 \sin(c_2 t)] + \left[\omega_0 - i\omega \frac{\sin(t\omega)}{\cos(t\omega)}\right] c_1 c_2 \cos(c_2 t)$$

$$= -\frac{c_1}{4}\{\cos[t(\omega_0-\omega_1)] - i\sin[t(\omega_0-\omega_1)]\}\, \omega_1^2 \cos(t\omega)\sin(c_2 t) + \omega_0 c_1 c_2 \cos(c_2 t) - i\omega c_1 c_2 \frac{\sin(t\omega)\cos(c_2 t)}{\cos(t\omega)}$$

$$-c_1^2 c_2 \sin(c_2 t)\cos(t\omega) = -\frac{c_1}{4}\{\cos[t(\omega_0-\omega_1)] - i\sin[t(\omega_0-\omega_1)]\}\, \omega_1^2\cos^2(t\omega)\sin(c_2 t) + \omega_0 c_1 c_2 \cos(c_2 t)\cos(t\omega) - i\omega c_1 c_2 \sin(t\omega)\cos(c_2 t)$$

$$\to \frac{c_1^2 c_2}{2}\{\sin[(c_2+\omega)t] + \sin[(c_2-\omega)t]\}$$

$$\to \omega_1^2 \cos(t\omega)\frac{1}{2}\{\sin[(c_2+\omega)t] + \sin[(c_2-\omega)t]\}$$

$$\to \frac{\omega_0 c_1 c_2}{2}\{\cos[(c_2-\omega)t] + \cos[(c_2+\omega)t]\}$$

$$\to -\frac{i\omega c_1 c_2}{2}\{\sin[(c_2+\omega)t] - \sin[(c_2-\omega)t]\}$$

$$-\frac{c_1^2 c_2}{2}\left\{\sin[(c_2+w)t]+\sin[(c_2-w)t]\right\}$$

$$= -\frac{c_1}{4}\left\{\cos[t(w_0-w_1)] - i\sin[t(w_0-w_1)]\right\}$$

$$\frac{w_1^2\cos(tw)}{2}\left\{\sin[(c_2+w)t]+\sin[(c_2-w)t]\right\}$$

$$\tfrac{1}{2}\{\sin[(c_2+2w)t]+2\sin(c_2t)\} + \tfrac{1}{2}\{\cancel{\sin(c_2t)}+\sin[(c_2-2w)t]\}$$

$$+\frac{w_0 c_1 c_2}{2}\left\{\cos[(c_2-w)t]+\cos[(c_2+w)t]\right\}$$

$$-\frac{iw c_1 c_2}{2}\ \sin[(c_2+w)t]-\sin[(c_2-w)t]$$

$$= -\frac{c_1 w_1^2}{8}\Big\{\tfrac{1}{2}\cos[t(w_0-w_1)]\sin[(c_2+2w)t] + \cos[t(w_0-w_1)]\sin(c_2t)$$
$$- i\sin[t(w_0-w_1)]\sin[(c_2+2w)t]$$
$$+\tfrac{1}{2}\cos[t(w_0-w_1)]\sin[(c_2-2w)t] - \tfrac{1}{2}\sin[t(w_0-w_1)]\sin[(c_2-2w)t]\Big\}$$
$$-\sin[t(w_0-w_1)]\sin(c_2t)$$

$$+\frac{w_0 c_1 c_2}{2}\left\{\cos[(c_2-w)t]+\cos[(c_2+w)t]\right\}$$

$$-\frac{iw c_1 c_2}{2}\left\{\sin[(c_2+w)t]-\sin[(c_2-w)t]\right\}$$

$$= -\frac{c_1 w_1^2}{8}\Big\{\frac{1}{2}\frac{1}{2}\Big[\sin[(c_2+2w+w_0-w_1)t]+\sin[(c_2+2w-w_0+w_1)t]\Big]$$
$$+\sin[(c_2-w_0+w_1)t]\Big]$$
$$+\frac{1}{2}\Big[\sin[(c_2+w_0-w_1)t] + \sin[(c_2-2w-w_0+w_1)t]\Big]$$
$$+\frac{1}{2}\frac{1}{2}\Big[\sin[(c_2-2w+w_0-w_1)t]\Big]$$
$$-\frac{i}{2}\Big[\cos[(c_2+2w-w_0+w_1)t]-\cos[(c_2+2w+w_0-w_1)t]\Big]$$
$$-\frac{1}{2}\Big[\cos[(c_2-w_0+w_1)t]-\cos[(c_2+w_0-w_1)t]\Big]$$
$$-\frac{1}{2}\Big[\cos[(c_2-2w-w_0+w_1)t]-\cos[(c_2-2w+w_0-w_1)t]\Big]\Big]\Big\}$$

$$+\frac{w_0 c_1 c_2}{2}\left\{\cos[(c_2-w)t]+\cos[(c_2+w)t]\right\}$$

$$-\frac{iw c_1 c_2}{2}\left\{\sin[(c_2+w)t]-\sin[(c_2-w)t]\right\}$$

Taylor expansion:

$$-\frac{C_1^2 C_2}{2}\left\{(C_2+w)t + (C_2-w)t\right\}$$

$$= -\frac{C_1 w_1^2}{8}\left\{\frac{1}{4}\left[(C_2+2w+w_0-w_1)t + (C_2+2w-w_0+w_1)t\right]\right.$$

$$+\frac{1}{2}\left[(C_2-2w+w_0-w_1)t + (C_2- \quad w_0+w_1)t\right]$$

$$+\frac{1}{4}\left[(C_2-2w+w_0-w_1)t + (C_2-2w-w_0+w_1)t\right]$$

$$+\frac{w_0 C_1 C_2}{\cancel{2}}\cdot\cancel{t} - \frac{i w C_1 C_2}{2}\left[(C_2+w)t - (C_2-w)t\right]$$

$$\Rightarrow 4C_2^2 = (w_0-w)^2 + \frac{w_1^2}{4} \Rightarrow C_2 = \sqrt{\frac{(w_0-w)^2 + w_1^2/4}{4}}$$

$$C_1^2\left((w_0-w)^2 + w_1^2/4\right) = \frac{w_1^2}{4} \Rightarrow C_1 = \frac{w_1/2}{\sqrt{(w_0-w)^2 + w_1^2/4}}$$

$$\Rightarrow d(t) = \frac{w_1/2}{\sqrt{(w_0-w)^2 + w_1^2/4}}\sin\left[\sqrt{\frac{(w_0-w)^2 + w_1^2/4}{4}}\,t\right]$$

By(4.44a)

$$\boxed{\begin{aligned}|\langle -z|\psi(t)\rangle|^2 &= b^*(t)b(t) = d^*(t)d(t)\\ &= \frac{w_1^2/4}{(w_0-w)^2 + w_1^2/4}\sin^2\left[\sqrt{\frac{(w_0-w)^2 + w_1^2/4}{2}}\,t\right]\end{aligned}}$$

2) T 4.10

By (4.54) $|1\rangle = \frac{1}{\sqrt{2}}|I\rangle + \frac{1}{\sqrt{2}}|II\rangle$, $|2\rangle = \frac{1}{\sqrt{2}}|I\rangle - \frac{1}{\sqrt{2}}|II\rangle$

(4.47) $\langle 1|\hat{H}|1\rangle = \langle 2|\hat{H}|2\rangle = E_0$

$$\langle 1|\hat{H}|1\rangle = \frac{1}{\sqrt{2}}(\langle I| \pm \langle II|)\hat{H}\frac{1}{\sqrt{2}}(|I\rangle \pm |II\rangle)$$

$$= \frac{1}{2}(\langle I|\hat{H}|I\rangle + \langle I|\hat{H}|II\rangle + \langle II|\hat{H}|I\rangle + \langle II|\hat{H}|II\rangle) = E_0 + \mu_e|E|$$

$$\langle 2|\hat{H}|2\rangle = \frac{1}{2}(\langle I|\hat{H}|I\rangle - \langle I|\hat{H}|II\rangle - \langle II|\hat{H}|I\rangle + \langle II|\hat{H}|II\rangle) = E_0 - \mu_e|E| \quad ②$$

By (4.48)

$$\langle 1|\hat{H}|2\rangle = \frac{1}{2}(\langle I| + \langle II|)\hat{H}(|I\rangle - |II\rangle)$$

$$= \frac{1}{2}(\langle I|\hat{H}|I\rangle - \langle I|\hat{H}|II\rangle + \langle II|\hat{H}|I\rangle - \langle II|\hat{H}|II\rangle) = -A \quad ③$$

$$\langle 2|\hat{H}|1\rangle = \frac{1}{2}(\langle I| - \langle II|)\hat{H}(|I\rangle + |II\rangle)$$

$$= \frac{1}{2}(\langle I|\hat{H}|I\rangle + \langle I|\hat{H}|II\rangle - \langle II|\hat{H}|I\rangle - \langle II|\hat{H}|II\rangle) = -A \quad ④$$

① + ②: $\cancel{\frac{1}{2}}(\cancel{2}\langle I|\hat{H}|I\rangle + \cancel{2}\langle II|\hat{H}|II\rangle) = 2E_0 \quad ⑤$

③ + ④: $\cancel{\frac{1}{2}}(\cancel{2}\langle I|\hat{H}|I\rangle - \cancel{2}\langle II|\hat{H}|II\rangle) = -2A \quad ⑥$

⑤ + ⑥: $\cancel{2}\langle I|\hat{H}|I\rangle = \cancel{2}E_0 - \cancel{2}A \Rightarrow \langle I|\hat{H}|I\rangle = E_0 - A$

The same process follows $\Rightarrow \langle II|\hat{H}|II\rangle = E_0 + A$

$\langle I|\hat{H}|II\rangle = \langle II|\hat{H}|I\rangle = \mu_e|E| = \mu_e E_0 \cos\omega t$ (p.133)

$\Rightarrow$ The Hamiltonian in $|I\rangle$-$|II\rangle$ basis is

$$\boxed{\hat{H} = \begin{pmatrix} E_0 - A & \mu_e|E_0|\cos\omega t \\ \mu_e|E_0|\cos\omega t & E_0 + A \end{pmatrix} \quad (4.61)}$$

(4.38) $\hat{H} = \frac{\hbar}{2}\begin{pmatrix} \omega_0 & \omega_1 \cos\omega t \\ \omega_1 \cos\omega t & -\omega_0 \end{pmatrix}$

Compare: This Hamiltonian is in the $|\pm z\rangle$ basis ; correspondingly the matrix elements $\langle +z|\hat{H}|-z\rangle$, $\langle -z|\hat{H}|+z\rangle$ and $\langle I|\hat{H}|II\rangle$, $\langle II|\hat{H}|I\rangle$ have oscillation terms archieved by $\cos\omega t$.

To find the P of finding the molecule in state $|I\rangle$ at time t, initially at state $|II\rangle$.

Let $|\psi(0)\rangle = |II\rangle = (1, 0)^T$

$|\psi(0)\rangle = (a, b)^T$

$$\hat{H}|\psi(t)\rangle = i\hbar \frac{d|\psi(t)\rangle}{dt}$$

$$\begin{pmatrix} E_0 - A & \mu_e|E_0|\cos\omega t \\ \mu_e|E_0|\cos\omega t & E_0 + A \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = i\hbar \begin{pmatrix} \dot{a}(t) \\ \dot{b}(t) \end{pmatrix}$$

Based on the comparison of the Hamiltonians, a one to one correspondence can be drawn: (p 133-134)

$E_+ = \frac{\hbar\omega_0}{2} \rightarrow E_0 - A \Rightarrow \omega_0 = \frac{2(E_0 - A)}{\hbar}$ $\quad \frac{\hbar\omega_1}{2} \rightarrow \mu_e|E_0| \Rightarrow \omega_1 = \frac{2\mu_e|E_0|}{\hbar}$

$E_- = -\frac{\hbar\omega_0}{2} \rightarrow E_0 + A$

Plug in (4.45)

$$|\langle I|\psi(t)\rangle|^2 = \frac{\frac{1}{4}\left(\frac{2\mu_e|E_0|}{\hbar}\right)^2}{\left(\frac{2(E_0-A)}{\hbar} - \omega\right)^2 + \frac{1}{4}\left(\frac{2\mu_e|E_0|}{\hbar}\right)^2} \sin^2\left[\frac{\sqrt{\left(\frac{2(E_0-A)}{\hbar} - \omega\right)^2 + \frac{1}{4}\left(\frac{2\mu_e|E_0|}{\hbar}\right)^2}}{2} t\right]$$

$$= \boxed{\frac{(2\mu_e|E_0|)^2}{4(2(E_0-A) - \omega\hbar)^2 + (2\mu_e|E_0|)^2} \sin^2\left[\frac{\sqrt{4(2(E_0-\omega) - \hbar\omega)^2 + (2\mu_e|E_0|)^2}}{8\hbar^2} t\right]}$$

3) T14.11

$E(t) = E_0 e^{-t/\tau}$ suppose the electric field is classical and aligned along some direction $\hat{x}$.

$$\hat{H} = \hat{H}^{(0)} + \hat{H}^{(1)} = \frac{\hat{p}^2}{2m} - e\hat{x} E_0 e^{-t/\tau}$$

By (14.96) $$C_n(\infty) = \delta_{n,0} - \frac{i}{\hbar}\int_0^t dt' \, e^{i[E_n^{(0)} - E_0^{(0)}]t'/\hbar} \langle E_n^{(0)} | \hat{H}_1(t') \cancel{+} E_0^{(0)} \rangle + \cdots$$

$$= \frac{ieE_0}{\hbar}\int_0^\infty dt' \, e^{in\omega t'} e^{-t'/\tau} \langle n | \hat{x} | 0 \rangle \quad \text{where } n \neq 0.$$

$$\hat{x} = \hat{a} + \hat{a}^+$$

$$\Rightarrow C_n(\infty) = \frac{ieE_0}{\hbar}\sqrt{\frac{\hbar}{2m\omega}} \int_0^\infty dt' \, e^{in\omega t'} e^{-t'/\tau} \langle n | \hat{a} + \hat{a}^+ | 0 \rangle$$

$$= \frac{ieE_0}{\hbar}\sqrt{\frac{\hbar}{2m\omega}} \int_0^\infty dt' \, e^{in\omega t'} e^{-t'/\tau} \langle n | 1 \rangle$$

$$C_1(\infty) = \frac{ieE_0}{\hbar}\sqrt{\frac{\hbar}{2m\omega}} \int_0^\infty dt' \, e^{i\omega t'} e^{-t/\tau}$$

$$= \frac{ieE_0}{\hbar}\sqrt{\frac{\hbar}{2m\omega}} \frac{\tau}{1 - i\omega\tau}$$

The 1st excited state contains 3 different orbitals: s, p, f

$$\boxed{\text{Probability} = \frac{1}{3}|C_1(\infty)|^2 = \frac{1}{3}\frac{(eE_0\tau)^2}{2m\hbar\omega}\frac{1}{1+\omega^2\tau^2}}$$

4)

By (14.167) $\hbar\omega = E_{np} - E_{1s} = 1\,eV.$

$$\omega = \frac{1eV}{\hbar} = \frac{1\,Hartree}{27.2114} = \frac{\hbar c\alpha}{a_0}\frac{1}{27.2114}$$

By (14.166) $R_{np\to 1s} = \sum_s \int d\Omega \frac{\alpha\omega^3}{2\pi c^2}\frac{2^{15}}{3^{11}}a_0^2$

$$= \frac{\alpha\omega^3}{c^2}\frac{2^{17}}{3^{11}}a_0^2$$

$$= \alpha\frac{\left(\frac{1eV}{\hbar}\right)^3}{c^3}\frac{2^{17}}{3^{11}}a_0^2$$

$$= \frac{\alpha\left(\frac{\hbar c\alpha}{a_0}\frac{1}{27.2114}\right)^3}{c^3}\frac{2^{17}}{3^{11}}a_0^2$$

$$= \frac{\hbar^3 \cancel{c^3}\alpha^3}{a_0^{\cancel{3}}}\left(\frac{1}{27.2114}\right)^3\frac{\alpha}{\cancel{c^3}}\frac{2^{17}}{3^{11}}\cancel{a_0}$$

$$= \frac{\hbar^3\alpha^4}{a_0}\left(\frac{1}{27.2114}\right)^3\frac{2^{17}}{3^{11}}$$

$$m_e c^2\alpha^2 = \frac{\hbar c\alpha}{a_0}$$

$$a_0 = \frac{\hbar}{m_e c\alpha}$$

$$= \frac{\cancel{\hbar^3}\hbar^2\,\cancel{\alpha^4}\alpha^5}{\frac{\cancel{\hbar}}{m_e c\cancel{\alpha}}}\left(\frac{1}{27.2114}\right)^3\frac{2^{17}}{3^{11}}$$

$$= \hbar^2 m_e c\alpha^5\left(\frac{1}{27.2114}\right)^3\frac{2^{17}}{3^{11}}$$

$$= \frac{m_e c^2}{\hbar}\alpha^5\left(\frac{1}{27.2114}\right)^3\frac{2^{17}}{3^{11}} \leftarrow (14.168)$$

$$= \frac{0.6\times 10^9\,s^{-1}}{\left(\frac{2}{3}\right)^8}\left(\frac{1}{27.2114}\right)^3\frac{2^{17}}{3^{11}}$$

$$= 1.399 \times 10^7\,s^{-1}$$

$$\Rightarrow \boxed{\tau_{n,p\to 1s} = \frac{1}{R_{n,p\to 1s}} = 7.146\times 10^{-8}\,s}$$

5) 2-level system:

$|g\rangle \xrightarrow{\pi\text{-pulse}} |e\rangle$

Rabi freq: $\chi_{i,j} = \frac{\vec{d}_{i,j} \cdot \vec{E}_0}{\hbar}$; $\Omega_{i,j} = \sqrt{|\chi_{i,j}|^2 + \Delta^2}$.

$\chi_{i,j} = r$. (Δ = detuning)

$\psi(r,t) = c_g(t)|g\rangle + c_e(t)|e\rangle e^{-i\omega_0 t}$

initially $c_g = 1$ $c_e = 0$

Solve for $i\hbar \frac{\partial \psi(r,t)}{\partial t} = \hat{H}\psi(r,t)$, where $\hat{H} = \hat{H}_0 + \hat{H}_1 = \frac{\hat{p}^2}{2m} + \hat{d}\cdot\hat{E}$

The Rabi frequency $\Omega = \frac{E_0}{\hbar} \langle e|\hat{d}\cdot\hat{\epsilon}|g\rangle$

$$\Rightarrow i\hbar \frac{d\,c_g(t)}{dt} = c_e \hbar \Omega^* \frac{e^{i(\omega-\omega_0)t} + e^{-i(\omega-\omega_0)t}}{2}$$

$$i\hbar \frac{d\,c_e(t)}{dt} = \underbrace{\cancel{c_g}}_{0} \hbar \Omega \frac{e^{i(\omega-\omega_0)t} + e^{-i(\omega-\omega_0)t}}{2}$$

The same process in the problem 1) follows to solve:

$$c_e(t) = \frac{\Omega}{\Omega'^2} \sin\left[\frac{\Omega' t}{2}\right], \text{ where } \Omega' = \sqrt{\Omega^2 + \Delta^2} = \sqrt{2}\, r$$

($\Omega = r$, $\Delta = r$)

$$c_g(t) = \sqrt{1 - \frac{\Omega^2}{\Omega'^2} \sin^2\left[\frac{\Omega' t}{2}\right]}$$

$$T = \pi/\Omega = \pi/r$$

The probability of being in the state $e = |\langle e|\psi\rangle|^2$

$$= \langle\psi|\underbrace{e\rangle\langle e}_{\hat{P}_e}|\psi\rangle = |c_e(t)|^2 ; \quad \psi(t) = \underbrace{\sqrt{1 - \frac{\Omega^2}{\Omega'^2}\sin\left[\frac{\Omega' t}{2}\right]}}_{c_g(t)}\, |g\rangle + \underbrace{\frac{\Omega}{\Omega'^2}\sin\left[\frac{\Omega' t}{2}\right]}_{c_e(t)} e^{-i\omega_0 t}|e\rangle$$

The uncertainty $\langle \Delta \hat{P}_e \rangle = \langle \hat{P}_e^2 \rangle - \langle \hat{P}_e \rangle^2 = \langle\psi|\hat{P}_e^2|\psi\rangle - (\langle\psi|\hat{P}_e|\psi\rangle)^2$

$$= (c_g^*(t)\langle g| + c_e^* e^{+i\omega_0 t}\langle e|)\, |e\rangle\langle e|\, (c_g(t)|g\rangle + c_e e^{-i\omega_0 t}|e\rangle)$$

$$- \left[(c_g^*(t)\langle g| + c_e^* e^{i\omega_0 t}\langle e|)|e\rangle\langle e|(c_g(t)|g\rangle + c_e e^{-i\omega_0 t}|e\rangle)\right]^2$$

The uncertainty

$= c_e^* e^{i\omega_0 t} \cdot c_e e^{-i\omega_0 t} - [c_e^* e^{i\omega_0 t} c_e e^{-i\omega_0 t}]^2$

$= c_e^2 - [c_e^2]^2 = c_e^2 - c_e^4$

$= \frac{\Omega^2}{\Omega'^2} \sin^2\left[\frac{\Omega' t}{2}\right] - \frac{\Omega^4}{\Omega'^4} \sin^4\left[\frac{\Omega'}{2} t\right]$, where $\Omega' = \sqrt{2}\, r$.

For N identical such systems. $\hat{P}_e = \frac{1}{N} \sum_{i}^{N} \hat{P}_{e,i} = \frac{1}{N}(\hat{P}_{e,1} + \hat{P}_{e,2} + \hat{P}_{e,3} \cdots \hat{P}_{e,N})$

$\hat{P}_e^2 = \frac{1}{N^2} (\hat{P}_{e,1}\hat{P}_{e,1} + \hat{P}_{e,1}\hat{P}_{e,2} + \cdots \hat{P}_{e,1}\hat{P}_{e,N}) \rightarrow = \frac{1}{N^2}(|e\rangle\langle e| + \cdots |e\rangle\langle e|)^N$

$(\hat{P}_{e,2}\hat{P}_{e,1} \cdots \hat{P}_{e,2}\hat{P}_{e,N})$

$= \frac{1}{N^2}(N|e\rangle\langle e|)^N$

$\vdots$

$(\hat{P}_{e,N}\hat{P}_{e,1} \cdots \hat{P}_{e,N}\hat{P}_{e,N})$

$= N^{N-2} |e\rangle\langle e|$

$= \frac{1}{N^2} \hat{P}_{e,1} \sum_{i}^{N} \hat{P}_{e,i} \; \hat{P}_{e,2} \sum_{i}^{N} \hat{P}_{e,i} \cdots \hat{P}_{e,N} \sum_{i}^{N} \hat{P}_{e,i}$

$= \frac{1}{N^2} \left(\sum_{i}^{N} \hat{P}_{e,i}\right)^N \prod_{i}^{N} \hat{P}_{e,i} = N^{N-2} |e\rangle\langle e|$

$\Rightarrow$ For N identical such systems. the uncertainty =

$$\boxed{N^{N-2} \frac{\Omega^2}{\Omega'^2} \sin^2\left[\frac{\Omega' t}{2}\right] - N^{2(N-2)} \frac{\Omega^4}{\Omega'^4} \sin^4\left[\frac{\Omega'}{2} t\right], \text{ where } \Omega' = \sqrt{2}\, r}$$

$P_e(t) = |\langle e|\psi\rangle|^2 = \langle\psi|e\rangle\langle e|\psi\rangle = \langle\psi| \frac{1}{N} \sum_{i}^{N} \hat{P}_{e,i} |\psi\rangle$

$= \left(c_g^*(t)\langle g| + c_e^*(t) e^{+i\omega_0 t} \langle e|\right) \frac{1}{\cancel{N}} \cancel{N} |e\rangle\langle e| \left(c_g(t)|g\rangle + c_e(t) e^{-i\omega_0 t}|e\rangle\right)$

$= |c_e(t)|^2 = \boxed{\frac{\Omega^2}{\Omega'^2} \sin^2\left[\frac{\Omega' t}{2}\right]}$

Suppose originally there are n atoms; then $\# = \left(\frac{\Omega^2}{\Omega'^2} \sin^2\left[\frac{\Omega' t}{2}\right]\right) n$ would be in the state e.