Suggestion for a 2-Qubit Experiment, Bob J, 5/23/17

Mark E suggested that since there is a 2-qubit system up and running it makes sense to see what we can do in that context. Here is a sketch of a simple experiment for 2 qubits that could test some of our ideas about the charge noise and its mitigation in multiqubit systems. It's clear that we cannot do anything meaningful as far as creating entanglement and mitigating noise at the same time with this many qubits, but we can think about experiments that test lifetimes of various states.
The current sample in Delft has 2 spin qubits in an inhomogeneous magnetic field. The total Hamiltonian is

$$H = H_0 + H_n(t) + H_g(t).$$

It consists of

$$H_0 = b_1 Z_1 + b_2 Z_2,$$

a static Hamiltonian that provides the qubit splittings; b_1 and b_2 differ by about 5%. X, Y, Z are the Pauli matrices. Also

$$H_n(t) = \delta b_1(t) Z_1 + \delta b_2(t) Z_2$$

is the noise Hamiltonian. The interesting question about the noise is whether $\delta b_1(t) \approx \delta b_2(t)$; if so, then we can hope to design some mitigation strategies based on the fact that H_n is approximately proportional to $Z_1 + Z_2 = Z_{tot}$. This becomes more interesting in multiqubit systems where the $\langle Z_{tot} \rangle = 0$ subspace is a large decoherence-free subspace, but at least the hypothesis about H_n can be tested even in the 2-qubit case. Finally $H_g(t)$ is the gate Hamiltonian that turns on and off. We can do 1-qubit gates: schematically this is $H_g(t) = \vec{B}_1(t) \cdot \vec{S}_1$ or $H_g(t) = \vec{B}_2(t) \cdot \vec{S}_2$ and we can do 2-qubit gates: $H_g(t) = J(t) \vec{S}_1 \cdot \vec{S}_2$. The qubits are coupled by the exchange interaction rather than an electrostatic one. We're not going to use H_g here, except to assume that we can use it to create arbitrary states of 2 qubits. I have absorbed the gyromagnetic ratios into the b's, which have dimensions of energy.
The basis states we'll use are

$$|00\rangle, |01\rangle, |10\rangle, |11\rangle,$$

and they are all eigenfunctions of H_0:

$$\begin{aligned} H_0 |00\rangle &= (b_1 + b_2) |00\rangle \\ H_0 |01\rangle &= (b_1 - b_2) |01\rangle \\ H_0 |10\rangle &= (-b_1 + b_2) |10\rangle \\ H_0 |11\rangle &= (-b_1 - b_2) |11\rangle. \end{aligned}$$

Experiment 1. Ramsey in the $\{|00\rangle, |11\rangle\}$ basis.
We start in the state $|00\rangle$ and then use H_g to make a $\pi/2$ rotation about the y-axis in the $\{|00\rangle, |11\rangle\}$ subspace preparing the state

$$\Psi_1(t=0) = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle),$$

and then let it evolve under the influence of H_0 alone. Then we have

$$\Psi_1(t) = \frac{1}{\sqrt{2}} \exp\left[-i(b_1+b_2)t\right] |00\rangle + \frac{1}{\sqrt{2}} \exp\left[i(b_1+b_2)t\right] |11\rangle ,$$

and if we make another $\pi/2$ rotation about the y-axis and then measure the probability of being in the state $|11\rangle$ after a time t we get

$$P_1 = \cos^2\left[(b_1+b_2)t\right] = \frac{1}{2} + \frac{1}{2}\cos\left[2(b_1+b_2)t\right],$$

so the period is $\tau_1 = \pi/(b_1+b_2)$. If we now add in H_n, the noise, we should have

$$P_1 = \frac{1}{2} + \frac{1}{2} e^{-t/T_2^{(1)}} \cos\left[2(b_1+b_2)t\right],$$

where $1/T_2^{(1)}$ is given by the integral of the Fourier transform of $\left\langle (2\delta b_1 + 2\delta b_2)^2 \right\rangle$ and a windowing function and some other factors involving the temperature, $\hbar$, etc. Omitting these we have that

$$\begin{aligned} 1/T_2^{(1)} &\sim \left\langle (2\delta b_1 + 2\delta b_2)^2 \right\rangle \\ &= 4\left\langle (\delta b_1 + \delta b_2)^2 \right\rangle . \end{aligned}$$

The number of oscillations observed will be N_1, which is

$$N_1 = \frac{T_2^{(1)}}{\tau_1} \sim \frac{b_1+b_2}{4\pi \left\langle (\delta b_1 + \delta b_2)^2 \right\rangle}.$$

Experiment 2. Ramsey in the $\{|01\rangle, |10\rangle\}$ basis.
We start in the state $|01\rangle$ and then use H_g to make a $\pi/2$ rotation about the y-axis in the $\{|01\rangle, |10\rangle\}$ subspace preparing the state

$$\Psi_2(t=0) = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle),$$

and then let it evolve under the influence of H_0 alone. Then we have

$$\Psi_2(t) = \frac{1}{\sqrt{2}} \exp\left[-i(b_1-b_2)t\right] |01\rangle + \frac{1}{\sqrt{2}} \exp\left[i(b_1-b_2)t\right] |10\rangle ,$$

and if we make another $\pi/2$ rotation about the y-axis and then measure the probability of being in the state $|10\rangle$ after a time t we get

$$P_2 = \cos^2\left[(b_1-b_2)t\right] = \frac{1}{2} + \frac{1}{2}\cos\left[2(b_1-b_2)t\right],$$

so the period is $\tau_2 = \pi/|b_1-b_2|$. If we now add in H_n, the noise, we should have

$$P_2 = \frac{1}{2} + \frac{1}{2} e^{-t/T_2^{(2)}} \cos\left[2(b_1-b_2)t\right],$$

where $1/T_2^{(2)}$ is given by the integral of the Fourier transform of $\left\langle (2\delta b_1 - 2\delta b_2)^2 \right\rangle$ and a windowing function and some other factors involving the temperature. Omitting these we have that

$$\begin{aligned} 1/T_2^{(2)} &\sim \left\langle (2\delta b_1 - 2\delta b_2)^2 \right\rangle \\ &= 4\left\langle (\delta b_1 - \delta b_2)^2 \right\rangle . \end{aligned}$$

The number of oscillations observed will be N_2, which is

$$N_2 = \frac{T_2^{(2)}}{\tau_2} \sim \frac{|b_1 - b_2|}{4\pi \left\langle (\delta b_1 - \delta b_2)^2 \right\rangle}.$$

Comparison

We see that the periods of the oscillations are very different, since one depends on the sum of the local fields and one depends on the difference. If the local fields differ by only 5%, then the ratio of the periods would be

$$\tau_2/\tau_1 = |b_1 + b_2| \,/\, |b_1 - b_2| \approx 40$$

which would be 40. However, the T_2's could differ even more drastically, since they are proportional to the square of the field. If we take the ratio, then all the factors cancel except the δb terms. It is not easy to estimate $\delta b_1 - \delta b_2$, but if we arbitrarily choose them to be 5% apart, then we get

$$T_2^{(2)}/T_2^{(1)} = \left\langle (\delta b_1 + \delta b_2)^2 \right\rangle / \left\langle (\delta b_1 - \delta b_2)^2 \right\rangle \sim 2 \times 10^3.$$

This is a pretty dramatic effect. Using the same estimates, we get

$$\frac{N_2}{N_1} \sim 40.$$

I have assumed that all these gate operations and measurements can be done. How practical this is I do not know.