449 Exam 1 Spring 2018

Each problem is worth the same. The usual take-home rules apply, in particular no discussing this with any other humans. Due 8 am Friday April 6. Hand in electronically, maximum of one .nb and one pdf file. I reserve the right to deduct points for answers that are difficult to read.

1) Sr has 2 valence electrons. Some of its important low-lying excited states are denoted $5s5p\,{}^3P_j$.

a) What are the possible values of j?

b) Let your largest answer to a) be $j_{\max}$. Assuming the radial wavefunctions are $P_{5\,s}(r)$ and $P_{5\,p}(r)$, write down the total wavefunction for the state with $j = m = j_{\max}$.

c) Simplify: $\left\langle 5s5p\,{}^3P_{j_{\max}} \right| \frac{1}{r_{12}} \left| 5s5p\,{}^3P_{j_{\max}} \right\rangle =$

2) A spin-0 particle with charge $q = |e|$ and mass M moves in the spherically symmetric potential $V(r) = \begin{cases} 0 & r < a \\ \infty & r > a \end{cases}$. The energy levels can be parameterized by the equation $E_{nl} = \frac{\pi^2 \hbar^2}{2 M a^2} (n + s_l)^2$, where n is the number of radial zero crossings and s_l depends, to a good approximation, mostly on l but only slightly on n.

a) For s-states, the equation is exact. What is the value of δ for s-states?

b) Put δ_0, δ_1, and δ_2 in numerical order. Briefly explain your reasoning.

c) A magnetic field is applied, of strength B. Find the first-order correction to the energy levels.

d) Two more identical particles are added. What is the energy and degeneracy of the first excited state for $B = 0$?

3) A Rb atom in its ground (5s) state is place in an electric field, adding a term $V = e\,x\,\mathcal{E}$ to the Rb Hamiltonian. Ignore spin.

a) With the field on, the $|5\,s\rangle$ state is changed to $\left|5\,s\right\rangle + \sum_m \epsilon_{lm} \left|5\,l\,m\right\rangle$. What are the values of l and m and why?

b) Use perturbation theory to give a formula for ϵ_{lm} in terms of relevant matrix elements of V and unperturbed Rb energy levels.

c) Calculate the induced dipole moment $-e\,\langle x\rangle$.

4) What is $\left\langle \|2\,s\,d;\ 1\,s\,u;\ 2\,s\,u\| \right| \hat{S}^2 \left| \|2\,s\,d;\ 1\,s\,u;\ 2\,s\,u\| \right\rangle$, where $\overline{S} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3$ is the total spin operator of the 3 electrons in the Li atom? You can probably figure out the answer from the NIST tables, but use your knowledge of Li wavefunctions and spin operators to prove it.