Decoherence Free Subspaces for 2 Qubits

Motivation

Interested in decoherence free subspaces (DFS) as a way to mitigate charge noise in multiqubit systems

Mark E suggested we come up with 2 qubit experiments to do on a current device

2 Qubit Hamiltonian

If $\delta b_1 \approx \delta b_2$ then we can hope to design a strategy to mitigate charge noise using a DFS.

$$H = H_0 + H_n(t)$$

$$H_0 = b_1 Z_1 + b_2 Z_2 \quad H_n(t) = \delta b_1(t) Z_1 + \delta b_2(t) Z_2$$

$$\begin{aligned} H_0 \left|00\right\rangle &= (b_1 + b_2) \left|00\right\rangle \\ H_0 \left|01\right\rangle &= (b_1 - b_2) \left|01\right\rangle \\ H_0 \left|10\right\rangle &= (-b_1 + b_2) \left|10\right\rangle \\ H_0 \left|11\right\rangle &= (-b_1 - b_2) \left|11\right\rangle . \end{aligned}$$

To explore how useful this can be, we want to divide the Hilbert space into two 2-dimensional subspaces (DFS and its complement) and do a Ramsey experiment to extract T_2 for each. We picture the two subspaces as two-level systems with associated Bloch spheres.

Two Ramsey Experiments

Prepare two states $\psi_1(0), \psi_2(0)$ and allow them to evolve under H_0 for a time t.

$$\psi_1(0) = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \rightarrow \psi_1(t) = \frac{e^{-i(b_1+b_2)t}\,|00\rangle + e^{i(b_1+b_2)t}\,|11\rangle}{\sqrt{2}}$$

$$\psi_2(0) = \frac{|01\rangle + |10\rangle}{\sqrt{2}} \rightarrow \psi_2(t) = \frac{e^{-i(b_1-b_2)t}\,|01\rangle + e^{i(b_1-b_2)t}\,|10\rangle}{\sqrt{2}}$$

Rotate $\psi_1(t)$ and $\psi_2(t)$ by $\frac{\pi}{2}$ around Y on the Bloch sphere and measure the probability of being in |11> or |10>.

$$P_1(t) = \frac{1}{2} + \frac{1}{2}\cos\left(2(b_1 + b_2)t\right)$$

$$P_1(t) = \frac{1}{2} + \frac{1}{2}\cos\left(2(b_1 - b_2)t\right)$$

Effect of noise

Including the effect of $H_n(t)$ will change these probabilities to

$$P_1(t) = \frac{1}{2} + \frac{1}{2} e^{-t/T_2^{(1)}} \cos\left(2(b_1 + b_2)t\right)$$
$$P_2(t) = \frac{1}{2} + \frac{1}{2} e^{-t/T_2^{(2)}} \cos\left(2(b_1 - b_2)t\right)$$

$$\begin{aligned} 1/T_2^{(1)} &\sim \left\langle (2\delta b_1 + 2\delta b_2)^2 \right\rangle \\ &= 4 \left\langle (\delta b_1 + \delta b_2)^2 \right\rangle . \end{aligned}$$

$$\begin{aligned} 1/T_2^{(2)} &\sim \left\langle (2\delta b_1 - 2\delta b_2)^2 \right\rangle \\ &= 4 \left\langle (\delta b_1 - \delta b_2)^2 \right\rangle . \end{aligned}$$

Comparison

We find a pretty stark difference between the dephasing times for these two experiments. The experiments indicate that $\delta b_1 \; and \; \delta b_2$ are within 5% of each other. In that case we have

$$T_2^{(2)}/T_2^{(1)} = \left\langle (\delta b_1 + \delta b_2)^2 \right\rangle / \left\langle (\delta b_1 - \delta b_2)^2 \right\rangle \sim 2 \times 10^3.$$

In addition, the number of coherent oscillations before phase information is lost is enhanced in the DFS

$$\frac{N_2}{N_1} \sim 40.$$

Experimental Protocol

To actually do this experiment we need to prepare $\psi_1(0), \psi_2(0)$, rotate them to |11> and |10> and measure the state.

So what can we control?

- $V(z)$ is the confining potential for the QD.
- We take the two lowest lying states to be our qubit states.

$$H = \frac{p^2}{2m} + V(z) - g\mu_B B_0 S_z - g\mu_B \frac{dB}{dz} z S_x$$

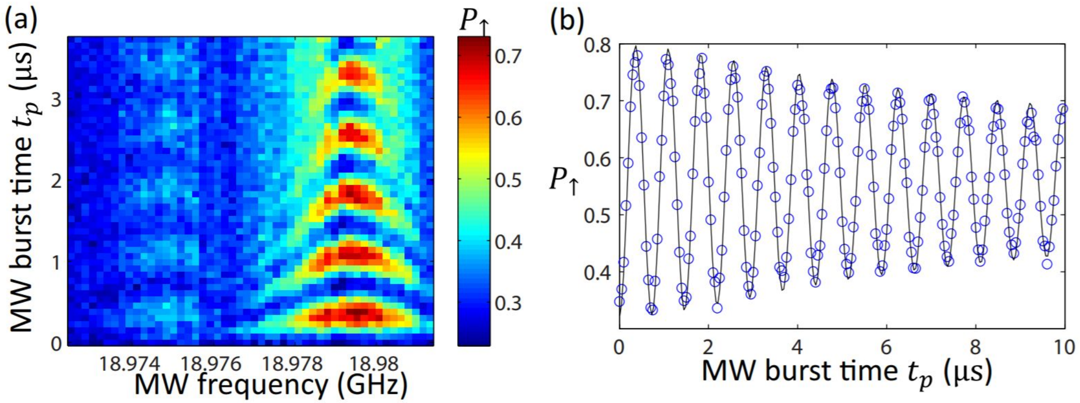
- The device uses this magnetic field gradient to control spins through an AC E field oscillating on resonance with the qubit.

Electrical Control

The AC field is included as $H_1(t) = e\, V_0\, \mathrm{z} \cos(\omega t)$. If the confining potential is symmetric, then this time-dependent Hamiltonian has 0 along the diagonal in the basis of the two qubit states we have chosen.

We are left with the effective qubit Hamiltonian

$$H_e = \frac{1}{2}\varepsilon_z Z + \frac{1}{2}\varepsilon_x cos(\omega t) X$$



ε_x and ε_z depend on many things, including the confining potential, static B field, etc. The thrust of this is that we can do Z and X rotations.

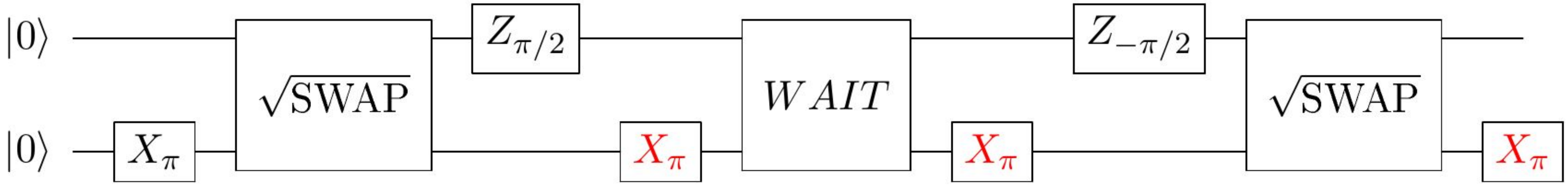
Two qubit gates are done by allowing neighboring wavefunctions to overlap, resulting in an interaction like $H_{2q} = J(t)\, S_1 \cdot S_2$

Gate Sequence for Ramsey Experiments

As it turns out, we can do the experiments using only Z rotations on qubit 1, X rotations on qubit 2, and two $\sqrt{SWAP}$ gates.

The red gates should be included for the |00>, |11> experiment and excluded for |01>, |10> case.

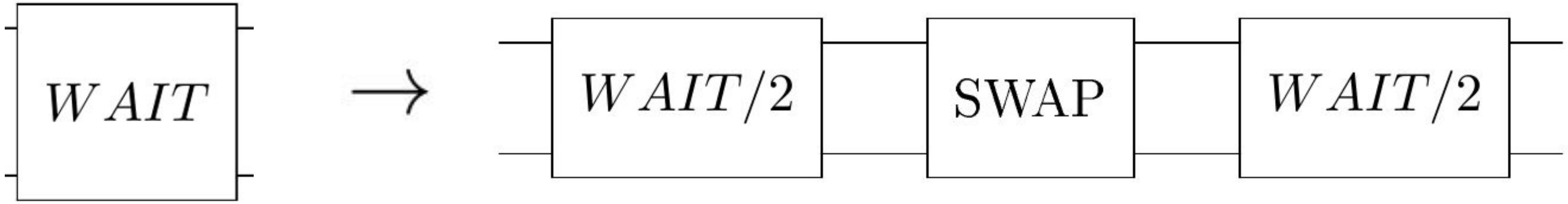
In both cases, the system is mostly in the DFS. The red gates take the state out of the DFS right before the wait period. F = .99 and .9 for one- and two- qubit gates respectively.



Extending T_2 with an Echo

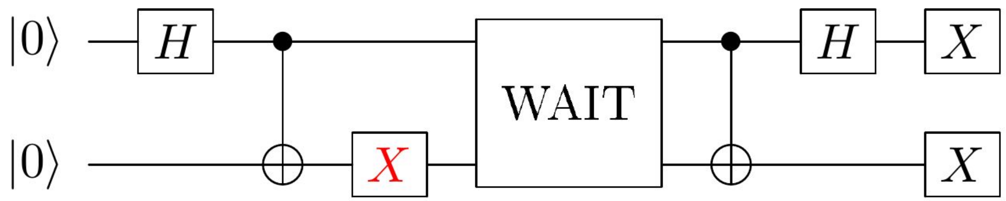
Performing a SWAP halfway through the wait period will result in an echo, and a longer dephasing time

This comes at the cost of introducing another 2 qubit gate.



Gates outside the DFS

The gate sequence was designed to do the entire |01>, |10> experiment inside the DFS. We could also try to measure T_2 without insisting that the state never leaves the DFS. A potential gate sequence for this would be



A direct comparison of this with the previous circuit would be unwise, since this would use two CNOT gates which amounts to 4 $\sqrt{SWAP}$ gates.