

CROSS RIVER UNIVERSITY OF TECHNOLOGY, CALABAR

DEPARTMENT OF MATHEMATICS

SECOND SEMESTER 2018/2019 EXAMINATION

MTH 1201: GENERAL MATHEMATICS II TIME: 2½ HOURS

INSTRUCTIONS: Read the questions carefully; **QUESTION ONE** is compulsory and any other **THREE**. Only duly registered students should take the exams. No mobile phone is allowed. Only scientific calculator is allowed. Any form of malpractice will be punished accordingly.

1. In each of the following functions: (a) $h(x) = \begin{cases} x, & \text{if } x > 1 \\ 2 - x^2, & \text{if } x < 1 \end{cases}$

(b) $f(x) = |2x - 5|$,

- i) Find the domains of h and f .
- ii) Compute the table of values for each of the function for all values $-4 \leq x \leq 6$.
- iii) On the same graph, plot the graphs of each function using the scale $2cm$ to 2 unit on x – axis and $2cm$ to 4 units on y – axis.
- iv) Find the roots of the equations for $h(x)$ and $f(x)$.
- v) Deduce the roots of the equations at which the functions $h(x) = f(x)$.

Note: Both tables and graph must be on graph sheet and only pencil should be used.

2. a) i) Find the value of $\lim_{x \rightarrow \sqrt{3}} \frac{\sqrt{3+x} - \sqrt{3}}{x}$

ii) Let $(x) = 2x^2 + 1$. Use the limit definition of the derivative to find $f'(x)$.

a) i) Find the derivative of $f(x) = \left[1 + x^4 - \frac{1}{x}\right]^{\frac{5}{3}}$

ii) Let $f(x) = \frac{5}{2}x^2 - e^x$. Find the value of x for which the second derivative equals zero.

b) Show that the curve $y = x^2$ is symmetric about the **y-axis** and sketch the graph.

3. a) Evaluate the following integrals

$\int x^2 \ln x dx$, (ii) $\int \sqrt{1-x} dx$.

a) Integrate by partial fraction, the integral $\int \frac{3x}{1+x-2x^2} dx$.

b) determine the area bounded by the curve $y = x^2 - 4$ on the **x-axis** and having ordinates $x = -1$ and $x = 4$.

4. a) Find the equation of the line that passes through the point $P(4, -6)$ and perpendicular to the line with shape $\frac{2}{5}$.

b) Determine the coordinates of the point \mathcal{A} that divides internally the line segment joining $P_1(5,-4)$ and $P_2(-7,5)$ in such a way that $P_1P:PP_2 = 5:7$.

c) Find the equation of a circle which passes through the points $(1,0)$, $(0,1)$ and $(0,0)$.

5. a) Find the acute angle between the straight lines.

$$2x - 3y + 1 = 0 \text{ and } 3x - 4y + 11 = 0.$$

b) Find the centre and radius of the circle $2x^2 - 2y^2 - 12x - 16y + 10 = 0$

c) Find the equation of the tangent to the circle $x^2 + y^2 - 3x - 4y - 8 = 0$ at the point $(1, -2)$.

6. a) If the equation of a parabola is $x^2 = -8y$, then find the following:

i) Using the standard form, draw the graph of the parabola and determine the direction of the opening.

ii) The coordinates of the focus, (iii) The equation of directrix.

iv) The length of the latus rectum.

b) If the general equation of an ellipse is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Using $b^2 = a^2(1 - e^2)$, find i) the value for b^2 (ii) the equation of this particular ellipse, where $a^2 = \frac{32}{3}$ and $e^2 = \frac{2}{5}$.

c) Given the hyperbola with equation $9x^2 - 4y^2 = 36$. Find the following:

i) the eccentricity, (ii) equation of the latus rectum,

iii) the asymptotes of the hyperbola.

←===== **Good-luck** =====→

CROSS RIVER UNIVERSITY OF TECHNOLOGY, CALABAR

DEPARTMENT OF MATHEMATICS/STATISTICS

SECOND SEMESTER 2016/2017 EXAMINATION

MTH 1201: GENERAL MATHEMATICS 11

TIME: 2½ HOURS

INSTRUCTION: Read the questions carefully; answer QUESTION ONE and any other THREE QUESTIONS. Only duly registered students should take the exams. No mobile phone is allowed & any form of malpractice will be punished accordingly. Any scientific calculator is allowed.

1 (a). A curve has equation $4y + 80 = 20x^2 (3 - x)$, copy and complete the table as shown below for the interval $-3 \leq x \leq 5$ on your graph sheet with equation defined as $y = f(x)$.

x	-3	-2	-1	0	1	2	3	4	5
y									

(b) Draw the graph of the relation $y = f(x)$ in Q.1(a) above using a scale of 2cm to 1 unit on the x-axis and 1cm to 20 units on the y-axis.

(c) Use the graph to find; (i) The co-ordinates of the critical points on the curve $y = f(x)$.

(ii) The inflection points.

(d) Identify (i) Local maxima (ii) Global maxima (iii) Global minima, in the interval.

NOTE: NO BIRO PEN IS ALLOWED ON THE GRAPH SHEET

2. (a) Draw a mapping diagram of the function for $-3 \leq x \leq 5$ and indicate whether the function is one – to – one.

(b) Evaluate the limits (i) $\lim_{x \rightarrow 0} \left[\frac{1 + \cos x}{(x^2 - 1^2)^2} \right]$ (ii) $\lim_{x \rightarrow 0} \left[\frac{5x^2 - x + 2}{3x^2 - 4} \right]$

(c) **If** $f: x \mapsto \frac{x+1}{x+2}$ and $g: y \mapsto 3y + 2$, determine:-

(i) f^{-1} (ii) $f^{-1}[g(1)]$ (iii) $g^{-1}[f^{-1}(2)]$

3. (a) Show that the tangent to the curves $y^3 - x^2y + 5y - 2x = 6$ and $x^3y^2 - 2y - 8x$ at the point (0, 1) are perpendicular.

(b) Find the value of the derivative $f(x) = x^2 - 5$ at $x = \alpha$, by first principles. Hence write down the tangent to the curve $y = x^2 - 5$ at the point (3, 4).

4. (a) Using Partial Fractions, evaluate the integral $\int \frac{x+1}{x(x-2)(x+3)} dx$

(b) Using integration by parts formula, evaluate (i) $\int_0^1 x^2 e^{2x} dx$ (ii) $\int x^2 \ln^{2x} x dx$

(c) Find the area under the curve $y = 6x - x^2$, the x -axis and the ordinates $x = 0$ and $x = 9$.

5. (a) Find the equation of a straight line which passes through (2,3) and which makes an angle of 135° ($\frac{\pi}{4}$) and $4x - 3y + 5 = 0$

(b) Given the vertices of triangle (1, 3), (-2, -4) and (1, -2);

(i) Find the length of the midpoint P(x, y) of (1, -2) and (1, 3) from a point Q(x, y) which divides the line joining (1, -2) and (1, 3).

(ii) Find the equation of the line joining P and Q.

6. (a) Find the centre and radius of the circle whose equation is given by $x^2 + y^2 - 8x + 10y - 8 = 0$

(b) Find the equation of the ellipse which passes through the point (-3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$, with x -axis as its major axis and centre at origin.

(c) Given the equation of the hyperbola $36x^2 - 9y^2 = 144$, find (i) the eccentricity

(ii) The co-ordinates of the foci

(iii) the equation of the directrices

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INSTRUCTIONS: Read the questions carefully; answer QUESTION ONE and any other THREE QUESTIONS. Only duly registered students should take the exams. No mobile phone is allowed & any form of malpractice will be punished accordingly. Any scientific calculator is allowed.

1.(a) (i) Evaluate the function $6y = 2x^3 - 3x^2 - 12x + 30$ into the form $y = f(x)$

(ii) Copy and complete the table below ON YOUR GRAPH SHEET for the relation $y = f(x)$ in 1 (a) (i).

x	-4.0	-3.0	-2.0	-1.0	0.0	1.0	2.0	3.0	4.0	5.0
y										

(b) Draw the graph of the relation in (a) above using a scale of 2cm to 1 unit on X axis and 2cm to 4 units on Y axis for the interval $-4.0 \leq x \leq 5.0$

(c) Use the graph to find; **(i)** The co-ordinates of the stationary points on the curve.

(ii) The maximum point of the function.

(iii) The minimum point of the function.

(b) Use the derivative method to find the points of inflection.

NOTE: NO BIRO PEN IS ALLOWED ON THE GRAPH SHEET.

2. (a) Use implicit differentiation to:- **(i)** Find the gradient of the tangent line to the general point P(x,y) on the ellipse $y^2 + xy + xy + x^2 = 4$ **(ii)** Find the equation of the tangent line at (2, -2) in Q.2(a) (i) above.

(b) Use Quotient Rule to differentiate $\sqrt{\frac{x-8}{x^2+5}}$ with respect to x.

(c) Differentiate $f(x) = \frac{x+7}{x+5}$ using first principles.

3. (a) Find the equation of the line that passes through the point $P(-1, -5)$ and which is perpendicular to the line $5y - 2x = 4$.

(b) Determine the coordinates of the point Q which divides the line joining $P(3, -5)$ and $R(-8, 7)$ in the ratio $2:3$.

(c) Find the acute angle between the straight lines $3y + x - 1 = 0$ and $y = \frac{2x+3}{4}$

4. (a) Using Partial Fractions, evaluate the integral $\int \frac{2}{x^2 - 5x + 6} dx$

(b) Using Integration by Parts, evaluate (i) $\int x^2 \ln(4x) dx$ (ii) $\int x^2 e^x dx$

5. (a) Evaluate the limits (i) $\lim_{x \rightarrow 3} \frac{x-3}{x^2+x-12}$ (ii) $\lim_{x \rightarrow 1} \left[\frac{x^2+5}{x+1} \right]$

b) (i) By means of a sketch diagram, find the area of a region formed when it is bounded by the curve $y = x^2 + 1$ and the straight line $y = x + 3$.

(ii) What volume of solid is formed when the region is rotated through 360° about the x-axis?

6. An ellipse and a hyperbola are given respectively as $\frac{x^2}{144} + \frac{y^2}{81} = 1$ and $\frac{x^2}{144} - \frac{y^2}{169} = 1$

(a) Find for each of the curves above separately:=
(i) The eccentricity
ii) The co-ordinates of the foci.
iii) The equations of the directrices.

(b) Give the equation of a circle which passes through the points $(1,0)$, $(0,1)$ and $(0,0)$.