CROSS RIVER UNIVERSITY OF TECHNOLOGY, CALABAR
DEPARTMENT OF MATHEMATICS AND STATISTICS
2017/2018 FIRST SEMESTER EXAMINATION
MTH 2104: ELEMENTARY DIFFERENTIAL EQUATIONS. TIME ALLOWED: 2HOURS

INSTRUCTIONS: Only those who are qualified should take the examination. Any form of examination malpractices will be punished accordingly. No calculator, table, micro-chip or phone is allowed in the exam venue.

ANSWER ANY FOUR QUESTIONS

QUESTION ONE

a. (i) What is an ordinary differential equation? (ii) How is an ordinary differential equation different from a Partial differential equation?

b. (i) Show that $y = e^{xy}$ satisfies the differential equation $(1 - xy)y^1 - y^2 = 0$ (ii) Form a differential equation from $y = Ax^2 + Bx$

c. Solve the equations $(i)y' = \sqrt{4x + 2y - 1}$ $(ii)y' = \cos(y - x)$

QUESTION TWO

a. In each of the following equations, state the order of the equation and determine whether the equation is linear or non-linear:

(i) $\frac{dy}{dx} + x^2 y = x\ell^x$ (ii) $\frac{d^2y}{dx^2} + x\,sin\,y = 0$ (iii) $\frac{d^2y}{dx^2} + y\,sin\,x = 0$

(iv) $\frac{d^2y}{dx^2} + 5y\frac{dy}{dx} + 6y = 0$

b. In each of the following equations, find the general solution:

(i) $\frac{dy}{dx} + y\,cost = 0$ (ii) $\frac{dy}{dx} + t^2 y = 1$

c. Find the solution of the initial value problem; $\frac{dy}{dx} + 2ty = t,\ y\,(1) = 2$

QUESTION THREE

a. (i) Solve the initial value problem, $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 4y - 0\ \ y(0) = 1, y(0) - 0$

(ii) Find the general solution of the equation $\frac{d^2y}{dt^2} - y = 0$

b. Find the solution of the initial value problem $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 4y = 0;\ y(0) = 1, y(0) = 1$

QUESTION FOUR

a. Define the term, Laplace transform of a function, f (t), and state its existence theorem

b. Using Laplace transform, show that $y^{11} - 3y^{1} + 2y = 4\ell^{2}$ $y(0) = -3$ $y'(0) = 5$ has the solution, $y(t) = -7\ell^{1} + 4\ell^{2t} + 4t\ell^{2t}$

c. Solve $y^{11} + y = t, y(0) = 1, y^{1}(0) = -2$, using Laplace transform.

QUESTION FIVE

a. Show that (i) $x^2 + y^2 = 1$ $is\ a\ solution\ of$ $\frac{dy}{dx} = -\frac{x}{y}$

(ii) $y = \ell^{2x}$ $is\ a\ solution\ of$ $\frac{dy}{dx} = 2y$ (iii) $Y = A\cos x + B\ sinx$ $is\ a\ solution\ of$ $\frac{d^2y}{dx^2} + y = 0$

b. Find a general solution of $9y\frac{dy}{dx} + 4x = 0$ by a separable variable method.

c. By applying Laplace transform, obtain the subsidiary equation (or transformed equation) of $y^{11}(t) + w^2y(t) = r(t)$. Hence, describe how you will get the solution, $y(t)$.

QUESTION SIX

a. Solve the initial value problem $y^{11} - 2y^{1} + 5y = 0,\ y(0) = 2,\ \ y^{1}(0) = -4,$ using Laplace transform technique.

b. State the necessary and sufficient condition for the equation, $M(x, y)dx + N(x, y)dy = 0$, to be exact.

c. Show that $\frac{1}{xy}$ is an integrating of $ydx - xdy = 0$

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MTH 2104: DIFFERENTIAL EQUATIONS. TIME ALLOWED: 2 ½ HOURS
USE OF CALCULATORS AND PHONES ARE HIGHLY PROHIBITED.

ANSWER ANY FOUR QUESTIONS

1a. Form the differential equations for the functions (i) $y = c_1e^{2x} + c_2e^{-2x}$ (ii) $y = c_1e^x + c_2$

b. Solve (i) $xe^{2y}y' + e^{2y} = \frac{Inx}{x}$ (ii) $\frac{dy}{dx} = e^{3x+2y}$

c. Find the orthogonal trajectories of the family of curves $x^2 + y^2 = cx$

2a. Let $M(x, y) and\ N(x, y)$ be continuous and have continuous first partial derivatives in a region R of the xy – plane. Prove that a necessary and sufficient condition that $M(x, y) + N(x, y)$ be an exact differential is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

b. Show that the following equations are exact and solve each

(i) $(y^2 + 2xy + 1)\, dx + (2xy + x^2 + 2)dy = 0$

(ii) $(1 - \sin x\, tan\, y)\, dx + (\cos x sec^2 y)\, dy = 0)$

c. Solve $(x^2 + y^2)\, dx + (x^2 - xy)\, dy = 0$ (ii) $\frac{dy}{dx} + \frac{1}{x}y = xy^2$

3a. Use the method of reduction of order to determine a second solution of the associated homogeneous equation and a particular solution of the nonhomogeneous equation $y'' - 4y = 2$ given that $y_1 = e^{-2x}$ is one solution.

b. Show by computing the Wronskian that the functions e^x, e^{-x}, e^{4x} are linearly independent on the interval on $-\infty < x < \infty$.

c. Let $y_1, y_2, \dots \dots, y_k$ be solutions of the homogenous linear nth-order differential equation on an interval I. prove that the linear combination $y = c_1y_1(x) + c_2y_2(x) + c_ky_k(x) where\ the\ c_i, i = 1,2 \dots, k$ are arbitrary constants is also a solution on the interval (Hint: prove for $n = k = 2$.

4a. Solve the following differential questions (i) $3y'' + y' - 2y = 0$ (ii) $y'' - 6y' + 9y = 0$

(iii) $y'' + y' + y = 0$

b. By finding the complimentary function and particular integral of the differential equation $\frac{d^2y}{dx^2} + 8y = 5x + 2e^{-x}$, find the general solution.

c. A 12 voltbattery is connected to a simple series circuit in which the inductance is ½ henry and the resistance is 10 ohms. Determine the current I if the initial current is zero.

5a. Let f be piecewise continuous on $t \geq 0$ and satisfy the condition $/f(t)/ \leq \mathrm{Me}^{\mathrm{at}}$ for $t \geq T$, where $a, M, and\ T$ are fixed nonnegative constants. Prove that $L\{f(t)\}$ exists for all constants.

b. If $L\{f(t)\}$ and $L\{g(t)\}$ exists, prove that $L\{af(t) + bg(t)\}$ exists and $L\{af(t) + bg(t)\} = aL\{f(t)\} + bL\{g(t)\}$.

c. Solve $y'' + 3y' + 2y = 4x^2$

6a. Evaluate (i) $L\{t^2 \sin kt\}$ (ii) $L\{te^{-t}cost\}$

b. Evaluate (i) $L^{-1}\left\{\frac{3s-2}{s^3(S^2+4)}\right\}$ (ii) $L^{-1}\left\{\frac{s+9}{S^2+6s+13}\right\}$

c. Solve the pair of simultaneous equations by Laplace transforms

$2x' + y' - y = t$

$x' + y' = t^2$

$subject\ to\ x(0) = 1, Y(0) = 0$