CROSS RIVER UNIVERSITY OF TECHNOLOGY, CALABAR
DEPARTMENT OF MATHEMATICS/STATISTICS
2017/2018 FIRST SEMESTER EXAMINATION
MTH 2103: LINEAR ALGEBRA I. TIME ALLOWED: 2HOURS

INSTRUCTIONS: Only those who are qualified should take the examination. Any form of examination malpractices will be punished accordingly. No calculator, table, micro-chip or phone is allowed in the exam venue.

ANSWER ANY FOUR QUESTIONS

QUESTION ONE

a. Let A and B be matrices and k a scalar. Then whenever the products are defined. Prove that (i) $(AB)^T = B^T A^T$ (ii) $(KA)^T = KA^T$

b. Using block multiplication, find M^2 and M^3 for

$$\begin{bmatrix} 2 & 0\,0 & 0 \\ 0 & 1\,4 & 0 \\ 0 & 2\,1 & 0 \\ 0 & 0\,0 & 3 \end{bmatrix}$$

Find M, find f (M) where $f(X) = x^2 + 4x - 5$

QUESTION TWO

a. Let A, B, C be matrices. Then whenever the products and sums are defined, prove that (i) A (B + C) = AB + AC, (ii) (AB) C = A (BC)

b. (i) Using block multiplication, find M^2 and M^3 for

$$M = \begin{bmatrix} 1\,1 & 0\,0 \\ 2\,3 & 0\,0 \\ 0\,0 & 1\,2 \\ 0\,0 & 4\,5 \end{bmatrix}$$

(ii) Find M, find f (M) where $f(X) = x^2 + 4x - 5$

QUESTION THREE

a. Let A, B and C be matrices of same order and k a scalar. Prove that (i) (A + B)+ C = A + (B + C) (ii) k (A + B) = kA + kB

b. Let M = diag (A, B, C), where A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, B = [5], C = $\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$, find M^2

QUESTION FOUR

a. (i) When a matrix A said to be orthogonal? (ii) Discuss the characteristics of orthogonal matrices.

b. Given that A = $\begin{pmatrix} 1/9 & 8/9 & -4/9 \\ 4/9 & -4/9 & -7/9 \\ 8/9 & 1/9 & 4/9 \end{pmatrix}$, Show that A is orthogonal.

QUESTION FIVE

a. Let A = $\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$ and let $f(x) = 2x^2 - 4x + 5$ and $g(x) = x^2 + 2x + 11$

(i) A^2 (ii) A^3 (iii) f(A) (iv) g (A), what do you notice?

b. Find real numbers x, y, z such A is Harmitian where

A = $\begin{pmatrix} 3 & x+2i & yi \\ 3-2i & 0 & 1+zi \\ yi & 1-xi & -1 \end{pmatrix}$

QUESTION SIX

a. Solve the following equation, Ax = b by Gaussian elimination where

A = $\begin{pmatrix} 1 & 2 & 4 \\ 4 & -1 & 1 \\ 2 & 5 & 2 \end{pmatrix}$, b = $[11 \quad 8 \quad 3]^T$

b. Let A = $\begin{pmatrix} \alpha & 1 & 0 \\ \beta & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$, Find all values of α and β which A is

(i) Singular (ii) Symmetric

CROSS RIVER UNIVERSITY OF TECHNOLOGY, CALABAR
DEPARTMENT OF MATHEMATICS/STATISTICS
2014/2015 FIRST SEMESTER EXAMINATION
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ANSWER ANY FOUR QUESTIONS

QUESTION ONE

a. Write A as the sum of a symmetric and skew-symmetric given that

$$A = \begin{pmatrix} 3/2 & -1 & 1/2 \\ 5/3 & 2/3 & 3/2 \\ -1/6 & 3/2 & 1/2 \end{pmatrix}$$

b. Use the gauss-Jordan method to solve

$$\begin{aligned} 2x_1 - 4x_2 + x_3 + x_4 &= 0 \\ x_1 - 5x_2 + 2x_3 &= 0 \\ -2x_2 - 2x_3 - x_4 &= 0 \\ x_1 - 2x_2 - x_3 + x_4 &= 0 \end{aligned}$$

QUESTION TWO

a. Let A and B be (m x n) matrices and k a scalar, prove that
(i) $(A^T)^T = A$ (Transportation is reflexive)
(ii) $(A \pm B)^T = A^T + B^T$
(iii) $(KA)^T = KA^T$

b. Let A be an (m x n) matrix and B an (n x p) matrix, prove that $(AB)^T = B^T A^T$ and show how this proof works by using

$$A = \begin{pmatrix} 1/3 & 2/5 & 3/5 \\ 2/3 & 1/5 & 4/4 \\ 3/5 & 6/7 & 5/7 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2/5 & 3/7 \\ 1/3 & 5/7 \\ 3/5 & 2/3 \end{pmatrix}$$

QUESTION THREE

a. Prove the following:
(i) If the inverse of a matrix M exists, then it is unique.
(ii) If M and N are invertible matrices of the same order than MN is invertible and $(MN)^{-1} = N^{-1}M^{-1}$

b. Solve by using Crammer's Rule, the following system of equations

$$x + 2y + 2z = 4$$

$$-25 - y + 3x = -4z$$
$$4 - z + 3x = -2y$$

QUESTION FOUR

a. Prove that is M is any n-square matrix and M^1 is:
(i) the matrix obtained from M by interchanging two rows of M, then det $M^1 = -\det(M)$
(ii) the matrix obtained when a single row of M is multiplied by a constant r, then det $M^1 = r\det(M)$
(iii) the matrix obtained when a multiple of one row of M is added to another row, then $det\ (M^1) = \det(M)$

QUESTION FIVE

a. Find the adjoint and inverse of A given that A = $\left\langle \begin{array}{c|c|c} -1 & 2 & 4 \\ 2 & 1 & -2 \\ -3 & 0 & 5 \end{array} \right\rangle$ hence or otherwise solve the system

$2x_2 + 4x_3 = 7 + x_1$
$2x_1 - x_2 - 2x_3 = -2$
$-3x_1 + 5x_3 = 7$

b. For what values of K does the following system of equations have non-trivial solution?
$(k-4)x_1 + x_2 = 0$
$x_1 + (k-4)x_2 = 0$

c. Prove the cumulative law of matrix addition.

QUESTION SIX

a. Prove that id A is an (m x n) matrix and B and C are (n x p) matrices, then
$A(B + C) = AB + AC$

b. Find by elementary row operations the inverse of A given that A = $\left\langle \begin{array}{c|c|c} 1 & 2 & 3 \\ 4 & -5 & 6 \\ 7 & 8 & 9 \end{array} \right\rangle$

Hence show that $AA^{-1} = A^1A = I_n$

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2018/2019 FIRST SEMESTER EXAMINATION
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QUESTION ONE

a. Let A and B be matrices and k a scalar. Then whenever the products are prove that
(i) $(AB)^T = B^T A^T$
(ii) $(KA)^T = KA^T$

b. Using block multiplication, find M^2 and M^3 for
M = $\left[\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix}\right]$
For M, find $f\ (M) where f\ (x) = x^2 + 4x - 5$

QUESTION TWO

a. (i) When a matrix said to be orthogonal?
(ii) Discuss the characteristics of orthogonal matrices.

b. Given that A = $\begin{pmatrix} 1/9 & 8/9 & -4/9 \\ 4/5 & -4/9 & -7/9 \\ 8/9 & 1/9 & 4/9 \end{pmatrix}$
Show that A is orthogonal.

QUESTION THREE

a. Let A, B and C be matrices of same order and k, a scalar. Prove that
(i) $(A + B) + C = A + (B + C)$
(ii) $k(A + B) = kA\_kB$

b. Let $M = diag\ (A, B, C) where\ A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} B = [5] and\ C = \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix}$. Find M^3

QUESTION FOUR

a. Let $A = \left\langle \begin{array}{c|c} 1 & 2 \\ 4 & -3 \end{array} \right\rangle$ $and\ let\ f(x) = 2x^3 - 4x + 5\ and\ g(x) = x^2 + 2x + 11$

Find (i) A^2 (ii) A^3 (iii) $f(A)$ (iv) $g(A)$ What do you notice?

b. Find real numbers x, y, z such that A is Harmintain where A = $\left\langle \begin{array}{c|c|c} 3 & x+2i & yi \\ 3-2i & 0 & 1+zi \\ yi & 1-xi & -1 \end{array} \right\rangle$

QUESTION FIVE

a. Let A, B, C be matrices. Then whenever the products and sums are defined, prove that

(i) $A(B + C) = AB + AC$ (ii) $(AB)C = A(BC)$

b. (i) Using block multiplication, find M^2 and M^3 for M = $\left[\left\langle \begin{array}{c|c|c} 1 & 1 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 4 \end{array} \right\rangle \begin{bmatrix} 0 \\ 0 \\ 2 \\ 5 \end{bmatrix} \right]$

(ii) For M, find $f(M) where\ f(x) = x^2 + 4x - 5$

QUESTION SIX

a. Solve the equation $Ax = b$ by Gaussian elimination, where

A = $\left\langle \begin{array}{c|c|c} 1 & 2 & 4 \\ 4 & -1 & 1 \\ 2 & 5 & 2 \end{array} \right\rangle$ $b = \langle 11|8|3 \rangle^T$

b. Let $A = \left\langle \begin{array}{c|c|c} \alpha & 2 & 4 \\ \beta & -1 & 1 \\ 0 & 5 & 2 \end{array} \right\rangle$, find all values of α and β for which A is (i) singular (ii) symmetric

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b. Let $M = diag\ (A, B, C) where\ A = \left\langle\begin{array}{c|c}1 & 2\\ 3 & 4\end{array}\right\rangle B = [5] and\ C = \left\langle\begin{array}{c|c}1 & 3\\ 5 & 7\end{array}\right\rangle$. Find M^2

QUESTION TWO

a. Let $A = \left\langle\begin{array}{c|c}1 & 2\\ 4 & -3\end{array}\right\rangle and\ let\ f(x) = 2x^3 - 4x + 5\ and\ g(x) = x^2 + 2x + 11$
Find (i) A^2 (ii) A^3 (iii) $f(A)$ (iv) $g(A)$ What do you notice?

b. Find real numbers x, y, z such that A is Harmintain where A = $\left\langle\begin{array}{c|c|c}3 & x+2i & yi\\ 3-2i & 0 & 1+zi\\ yi & 1-xi & -1\end{array}\right\rangle$

QUESTION THREE

a. Let A, B, C be matrices. Then whenever the products and sums are defined. Prove that
(i) $A(B+C) = AB + AC$

b. (i) Using block multiplication, find M^2 and M^3 for M = $\left[\left\langle\begin{array}{c|c|c}1 & 1 & 0\\ 2 & 3 & 0\\ 0 & 0 & 1\\ 0 & 0 & 4\end{array}\right\rangle \begin{bmatrix}0\\ 0\\ 2\\ 5\end{bmatrix}\right]$

(ii) For M, find $f(M) where\ f(x) = x^2 + 4x - 5$

QUESTION FOUR

a. Solve the equation $Ax = b$ by Gaussian elimination where
A = $\begin{pmatrix} 1 & 2 & 4 \\ 4 & -1 & 1 \\ 2 & 5 & 2 \end{pmatrix}$ $b = \langle 11|8|3\rangle^{T}$

a. Let $A = \begin{pmatrix} \alpha & 1 & 0 \\ \beta & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$, find all values of α and β for which A is (i) singular (ii) symmetric

QUESTION FIVE

a. Let A and B be matrices and k a scalar. Then whenever the products are defined. Prove that
(i) $(AB)^T = B^T A^T$ (ii) $(kA)^T = kA^T$

b. Using block multiplication, find M^2 and M^3 for M = $\begin{bmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \end{bmatrix} \end{bmatrix}$

For M, find $f(M) where\ f(x) = x^2 + 4x - 5$

QUESTION SIX

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b. Given that A = $\begin{pmatrix} 1/9 & 8/9 & -4/9 \\ 4/9 & -4/9 & -7/9 \\ 8/9 & 1/9 & 4/9 \end{pmatrix}$
Show that A is orthogonal