

CROSS RIVER UNIVERSITY OF TECHNOLOGY, CALABAR
DEPARTMENT OF MATHEMATICS/STATISTICS
2015/2016 FIRST SEMESTER EXAMINATION
MTH 1101: GENERAL MATHEMATICS I. TIME: 2 ½ HOURS

INSTRUCTIONS: Read the questions carefully, answer **question one** and any other **three questions**. Only duly registered students should take the exams. No calculator, table, micro-chip or phone is allowed. Any form of examination malpractices will be punished accordingly.

1a. Copy and complete the table below for relation; $y = 10x(2x - 3) + 15$ **on your graph sheet**

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
$20x^2$				5							
$-30x$											
15					15					15	
y					15					215	

- b. Using a scale of 2cm to 1unit on the x-axis and 1cm to 10units on the y-axis, draw the graph of $y = 20x^2 - 30x + 15$ for $-2 \leq x \leq 3$
- c. Use the graph to find; (i) The minimum value of y (ii) The two values of x when $y = 35$ (iii) The gradient of the curve $20x^2 - 30x + 15$ at $x = 1$ (iv) The solution set of the equation $20x^2 - 30x - 40$

Note: No biro pen is allowed on the graph sheet

- 2a. (i) Find the values of $\cos 75^\circ$ leaving your answer in surd form (ii) Express $\sin 5x$ as a polynomial in $\sin x$.
- b. Expand $(\cos \theta + i \sin \theta)^n$ for $n = 2$ & 3 and obtain expression for $\cos 2\theta$, $\sin 2\theta$ & $\cos 3\theta$, $\sin 3\theta$ in terms of $\cos \theta$ and $\sin \theta$.
- c. An arithmetic progression whose first term is three (3) and whose nth term is 48 has the sum of its first n terms equal to 225.
- (i) Find n and d (ii) Give the first three terms of the arithmetic progression.
- 3a. Given that $\mu = \{0, 1, 2, 3, \dots, 12\}$, $P = \{p: 0 < p < 8 \text{ is integer}\}$, $Q = \{q: q \text{ is even}\}$ and $R = \{r: 2 \leq r \leq 7, \text{ is prime}\}$, where P, Q and R are subsets of μ .

Prove the identities (i) $P \cup (P' \cap Q) = P \cup Q$ (ii) $(P \cap Q) \cup (Q \cap R) = (P' \cap R')' \cap Q$

- b. All the 50 science student in a college were asked their subject combination, 18 of the students offered further mathematics, 21 offered Chemistry while 16 offered biology, 7 students offered further math and chemistry, 8 students offered biology while 9 offered chemistry and biology while 5 offered the three subject combinations using the Venn Diagram. Find

$$(i) n(M \cap C' \cap B') \quad (ii) n(M \cup C \cup B)'$$

- c. If A, B and C are a non-empty subset of a universal set μ , show by means of Venn diagram

$$(i) (A \cup B) \cup C \quad (ii) (A \cap B) \cap C \quad (iii) A \cap (B \cup C)$$

- 4a. Solve the system using crammer's Rule

$$2x + y - z = 3$$

$$x + y + z = 1$$

$$x - 2y - 3z = 4$$

- b. Find the values of a and b which make the expression $x^4 + 6x^3 + 13x^2 + ax + b$ a perfect square for all values of x .

- c. If α and β are the roots of the $2x^2 + 3x - 2 = 0$, form the equation whose roots are α^2, β^2

- 5a. Find polynomial expression $p(x)$, when divided by $(x - 1)$ it leaves the remainder 3 and divided by $(x - 1)(x - 2)$ it leaves the remainder $-2x + 5$

- b. i. The 4th term & 8th term of GP are 24 and $8/27$. Find the two possible values of a and r (ii) Find the true set of the inequality $8x - 2x^2 - 1 > 5$

- c. Prove by induction $1^3 + 2^3 + \dots + n^3 = \frac{1}{4}n^2(n + 1)^2$

- 6a. Simplify (i) $(12^{3/2} \times 16^{1/8}) \div (27^{1/6} \times 18^{1/2})$ (ii) $\left(\frac{a+b}{a-b}\right)^{1/2} + \left(\frac{a-b}{a+b}\right)^{1/2}$ when $a = \sqrt{5}$ and $b = \sqrt{2}$ (iii) $\log_{10}(3x^2 + 8) = 1 + \log_{10}\left(\frac{x}{2} + 1\right)$

- b. Given that $\log_7 2 = \alpha, \log_7 3 = \beta$ and $\log_7 5 = \gamma$, express the following

$$(i) \log_7 6 \quad (ii) \log_7 75 \quad (iii) \log_7 \frac{15}{2} \text{ in terms of } \alpha, \beta, \gamma$$

- c. Given that $\tan \theta = 5/12$, where θ is an angle between 180° and 270° . Calculate without using table, the value of (i) $\sin 2\theta$ (ii) $\cos 2\theta$ (iii) $\cos 3\theta$

CROSS RIVER UNIVERSITY OF TECHNOLOGY, CALABAR
DEPARTMENT OF MATHEMATICS/STATISTICS
2017/2018 FIRST SEMESTER EXAMINATION
MTH 1101: GENERAL MATHEMATICS I. TIME: 2HOURS 30 MINS

INSTRUCTIONS: Read the questions carefully, answer **question one** and any other **three questions**. Only duly registered students should take the exams. No calculator, table, micro-chip or phone is allowed. Any form of examination malpractices will be punished accordingly.

1a. Rewrite the quadratic equation $\frac{1}{x+1} = \frac{2x-9}{2x-4}$ as the quadratic function $f(x)$.

b. Tabulate the values of $f(x)$ using the table shown below **ON YOUR GRAPH SHEET**

x	-3.0	-1.5	0	1.5	3.0	4.5	6.0
$f(x) = y$							

c. Graph the points using a scale of 2cm to 1 unit on the x-axis and 4cm to 10 units on the y-axis and draw a smooth line through the points for $-3 \leq x \leq 6$

d. Use the graph to find; (i) The co-ordinates of the x intercepts (ii) The solution set of the equation $y = 2x^2 - 9x - 14$ (iii) What are the co-ordinates of the vertex? Is it a minimum or a maximum?

Note: No biro pen is allowed on the graph sheet

2a. 120 First year CRUTECH students sat for an examination in Mathematics involving three papers, Geometry, Analysis and Calculus. 34 students passed all three papers; 52 students passed Geometry and Analysis papers; 20 students passed Calculus and Geometry but failed Analysis; 60 students passed only two of the three papers; 80 students passed the Calculus paper; 78 students passed Geometry paper; and 12 students passed only one paper

(i) Represent the information in a Venn diagram

(ii) How many students failed all the three papers?

(iii) How many students failed the Geometry paper?

b. Solve the equation $\log_4(3x + 4) = \log_2(2x + 1)$

c. For what values of a and b are $x - 1$ and $x + 2$ factors of $x^3 + ax^2 + bx + 4$?

3a. (i) List the four basics steps required for the application of mathematical induction.

(ii) Prove by induction that $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6}n(n+1)(2n+1) \forall n \in N$

b. (i) In an A.P, the 13th term is 27 and the 7th term is three times the 2nd term. Find the first term, the common difference and the sum of the first ten terms.

(ii) The sum of the 2nd and 3rd term of G.P. is 6 and the 3rd and 4th terms is -12. Find the 1st term and common ratio.

c. (i) Find the expansion of $\left(2x - \frac{1}{2}\right)^4$ in descending powers of x.

(ii) Find the coefficient of y^{10} in the expansion of $(2y - 3)^4$

4a. Simplify the following complex numbers; (i) $\frac{3+i}{1+2i} - \frac{2}{3+i}$ (ii) $\frac{1}{\cos \theta + i \sin \theta}$ (iii) $(1+i)^2 + (1-i)^2$

b. Express the complex number $\frac{i\sqrt{2}}{4+4i}$ in trigonometric form.

c. Apply De Moivre's theorem to simplify $(1+i)^8 + (1-i)^8$

5a. Define with examples, the following matrices (i) Square matrix (ii) Diagonal matrix (iii) Identity matrix (iv) Symmetric matrix (v) Zero matrix

b. Solve the system below, using Cramer's Rule;

$$-x + 3y - 2z = 5$$

$$4x - y - 3z = -8$$

$$2x + 2y - 5z = 7$$

6a. Evaluate without using tables $\tan\left(\frac{\pi}{12}\right)$

b. If $\tan A = \frac{4}{3}$ and $\cos B = \frac{12}{13}$ find the values of (i) $\sin(A - B)$ (ii) $\cos(A + B)$ if A is acute

c. A chord of a circle of radius 9cm subtends an angle of 75° at the centre. Find the length of the chord and the perimeter of the major segment formed.

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- 1a. Consider the relation $2(y - 25x) = 3(13 - 7x^2)$ and make y the subject of the equation.
- b. Use the “ y ” equation obtained in (a) to complete the table of values shown below for the interval $-3 \leq x \leq 5$ **on your graph sheet**

x	-3	-2	-1	0	1	2	3	4	5
Y									

- c. Using a scale of 2cm to 1unit on the x-axis and 1cm to 10units on the y-axis, plot the points for $-3 \leq x \leq 5$.
- d. Use the graph to find; (i) The maximum value of y (ii) The two values of x when $y = -16$ (iii) The solution set of the equation $2y = -21x^2 + 50x + 199$ (iv) Find algebraically the equation of the axis of symmetry and the co-ordinates of the vertex of the parabola.

Note: No biro pen is allowed on the graph sheet

- 2a. If $\log 2 = x$ and $\log 3 = y$, find in terms of x and y the value of $\log_{27} \left(\frac{1}{72} \right)^{\frac{1}{2}}$
- b. Solve the equation $2(9^x) - 3^{\frac{2x+1}{2}} - 3 = 0$
- c. Simplify without mathematical tables; (i) $\frac{8^{1/3} \times 5^{2/3}}{10^{2/3}}$ (ii) $\sqrt{\frac{8.1 \times 10^{-16}}{2.25 \times 10^{17}}}$
- 3a. (i) Write in surd form $\tan 15^\circ$ and simplify as much as possible
- (ii) Verify and identity $\frac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \operatorname{Cosec} \theta$
- b. Prove that $3^{4n+2} + 2(4^{3n+1}), n \in N$ is an integer multiple of 17.

- 4a. If α and β are the roots of the equation $3x^2 + 2x - 6 = 0$, find the equation whose roots are $\alpha + 1$ and $\beta + 1$.
- b. Solve the system below, using Cramer's Rule;
- $$3y + 2x = z + 1$$
- $$3x + 2z = 8 - 5y$$
- $$3z - 1 = x - 2y$$
- 5a. Express $\frac{(1+i)^4}{(2-2i)^3}$ in the form of $x + yi$
- b. (i) State and prove De Moivre's Theorem (i) Compute $(1 - \sqrt{3}i)^6$
- c. Find the values of p and q if $x - 2$ and $x + 1$ are both factors of $Px^3 + 3x^2 - 9x + q$ and hence obtain the third factor.
- 6a. The arithmetic progression with first term a , common difference d and sum S , satisfy the equation $n^2 + \left(\frac{2a}{d} - 1\right)n - \frac{2s}{d} = 0$. Find n when $a=3$, $d = \frac{1}{2}$ and $S = 2828$
- b. The sum of the first n terms of an AP is given by $S_n = pn + qn^2$. Given also that $S_3 = 6$ and $S_5 = 11$
- (i) Find the values of p and q
- (ii) Deduce an expression for the n^{th} term and the value of the common difference.
- c. If A, B, C are a non-empty subset of a Universal Set " U ", show by means of Venn diagram
- (i) $(A \cup B) \cup C$ (ii) $(A \cap B) \cap C$ (iii) $A \cap (B \cup C)$ (iv) $(A \cap B \cap C)^c$ (v) $(A \cap B) \cup (A \cap C)$