CROSS RIVER UNIVERSITY OF TECHNOLOGY, CALABAR
DEPARTMENT OF MATHEMATICS AND STATISTICS
2015/2016 SECOND SEMESTER EXAMINATION
MTH 2201 LINEAR ALGEBRA II

INSTRUCTIONS: Only those who are qualified should take the examination. Read the questions carefully and answer only the questions asked. Credit will be given for clarity of expression and logical flow. NO CALCULATOR, TABLE OR PHONE IS ALLOWED. Any form of examination malpractice will be punished accordingly. ANSWER ANY FOUR QUESTIONS IN TWO HRS.

QUESTION ONE:

1(a) Solve the system by first finding the inverse of the coefficient matrix

$x_1 + x_2 - 2x_3 = 3$
$2x_1 - x_2 + x_3 = 0$
$3x_1 + x_2 - x_3 = 8$

(b) Define with examples the basis of a vector space V over a field F.

(c) Prove that if a vector space V (over F) has one basis with a finite number of elements, then all other basis of V are finite and have the same number of elements.

QUESTION TWO:

2(a) Prove that in a finite dimensional vector space V any linearly independent set of vectors can be extended to a basis.

(b) Show that the set A = {(1, 1, 0), (1, 0, 1), (0, 1, 1)} is a basis of R^3 by showing that;

(i) It is linearly independent

(ii) L (A) contains the set B = {(1, 0, 0)(0, 1, 0), (0, 0, 1)} where L (A) is a linear combination of vectors in A.

Why does it suffice to show (ii)

(c) Prove that in a finite dimensional vector space, every spanning set contains a basis.

QUESTION THREE:

3(a) Let {Ui}n i=1 be a basis of a vector space V (over F)

Define the coordinate vector of V relative to the basis $\{U_i\}$ of V

(b) Find the coordinate vector of V = (4, - 3, 2) relative to the basis

$\{U_1 = (1, 1, 1), U_2 = (1, 1, 0)$ and $U_3 = (1, 0, 0)$ of R^3

(c) Prove that similarity of metrices is an equivalence relation.

QUESTION FOUR:

(a) Let T: U – V be a linear transformation;

Prove that (i) m T is a subspace of V

(ii) Ker T is a subspace of U

(b) Let T: R^3 – R be defined by (1, 1, 1) T = 3, (0, 1, - 2) T = 1 and (0, 0, 1) T = - 2. Find (a, b, c) T

(c) If T: R^3 – R^3 be defined by T (x, y, 2) = (x + 2y, y – z, x + 2z)

Find a basis for each of Im T and ker T.

QUESTION FIVE:

(a) Let T be a linear transformation on R^2 defined by

$(x_1\, x_1)$ T = $(x_1 - 2x_2\, ,3x_1 + x_2)$ for every vector $(x_1\, x_2)$ in R^2

Find (i) the matrix A of T relative to the standard basis of R^2

(ii) the transition matrix P from the standard basis to the basis $\{(f_1 = (1,1),\ f_2 = (1-1)\}$

(b) State and prove Cayley – Hamiltan theorem

(c) (i) if A = $\begin{bmatrix} 5 & 10 \\ 2 & 4 \end{bmatrix}$ Determine the characteristics polynomial, the characteristics equation and hence verify Cayley – Hamilton theorem.

(ii) Let $f(t) = 3^{t2} + 6t + 7. Find\ f\ (A) given\ that$

A = $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ and hence find the characteristics polynomial having f (A) as a root.

QUESTION SIX:

(a) Prove that a symmetric matrix A = $\begin{bmatrix} a & c \\ c & b \end{bmatrix}$ is diagonalizable

(b) Prove that if A be an $n\ X\ n$ matrix, then A is Orthogonally diagonalizable and has real eigen values if and only if A is symmetric.

(c) Consider the following two basis of R^3

$\{E = e_1,\ e_2, e_3\} = \{(1,0,0), (0,1,0), (0,0,1)\}$

and$\{S = \{u_1,\ u_2, u_3\} = \{(1,0,1), (2,1,2), (1,2,2)\}$

(i) Find the change –of-basis matrix P from the “old” basis E to the “new” basis S.

(ii) Find the change –of-basis matrix Q from the “new” basis S back to the “old” basis E.

CROSS RIVER UNIVERSITY OF TECHNOLOGY, CALABAR
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2018/2019 SECOND SEMESTER EXAMINATION
MTH 2201 LINEAR ALGEBRA II

INSTRUCTIONS: Answer any FOUR questions. No calculator or cell phone is allowed. Any form of exam malpractice will be punished accordingly.

TIME: 2 ½ Hours

Q1 (a) Let $T: U - W$ be a linear transformation, prove that

(i) Ker T is a subspace of U

(ii) Im T is a subspace of W

(b) Let A = $\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$ Show that A is diagonalizable

Q2 (a) (i) Show that P is orthogonal (i.e $PP^T = I$) given that P = $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ \frac{-2}{3\sqrt{5}} & \frac{-4}{3\sqrt{5}} & \frac{5}{3\sqrt{5}} \end{pmatrix}$

(ii) Show also that column vectors of P form an orthonormal set.

(b) (i) Find the inverse of the matrix A if it exits given that A = $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 2 \end{pmatrix}$

(ii) Determine A + B, AB and trace of A if A = $\begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & -7 \\ 1 & 0 & 0 \end{pmatrix}$, B = $\begin{pmatrix} -1 & -1 & -1 \\ -2 & -2 & -2 \\ 3 & 1 & 7 \end{pmatrix}$

Q3 (a) Solve the non-homogenous system $-2 + x + 2y = z$

$$3y + 8x = 4 + 7z$$
$$4y - 12z = -8$$

(b) Find the eigen values and associated eigen vectors of M = $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Q4 (a) Prove that if $S = \{v_1, v_2, \dots, v_n\}$ be a basis for a vector space V and v£V, then the coefficients in the representation $V = a_1v_1 + a_2v_2 + \dots + a_nv_n$ are unique.

(b) Determine the linear independence or otherwise of the following set of vectors

(i) $S = \{v_1, v_2, v_3\}, where\ v_1 = (4, -2, 0, 6), v_2 = (2, 4, 10, -2), v_3 = (14, -2, 10, 16)$ in R^4

(ii) $S = \{v_1, v_2, v_3\}, where\ v_1 = (1,0,0), v_2 = (0,1,0), v_3 = (0,0,1)$ in R^3

Q5 (a) Express M as a linear combination of A, B,= and C where M = $\begin{pmatrix} 4 & 7 \\ 7 & 9 \end{pmatrix}$, A = $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, B = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ And C = $\begin{pmatrix} 1 & 1 \\ 4 & 5 \end{pmatrix}$

(b) (i) Show that the matrix A is diagonalizable where A = $\begin{pmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & 1 & -1 \end{pmatrix}$

(ii) Find a matrix P such that $P^{-1}AP = D$ where D is a diagonal matrix of the same order as A.

Q6 (a) Prove that if a vector space V has one basis with a finite number of elements, then all other base of V are finite and have the same number of elements.

(b) Determine the basis and dimension for the solution space of the homogenous system of equations

$$x + 2y - 8z = 0$$
$$2x + 3y - 5z = 0$$
$$3x + 2y - 12z = 0$$

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2017/2018 SECOND SEMESTER EXAMINATION
MTH 2201 LINEAR ALGEBRA II

INSTRUCTIONS: Answer any FOUR questions. No calculator or cell phone is allowed. Any form of exam malpractice will be punished accordingly.

TIME: 2 ½ Hours

Q1 (a) State and prove Cayley-Hamilton theorem.
(b) (i) Find the inverse of the matrix A if it exists given that

$$A = = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 2 \end{pmatrix}$$

(ii) Determine A + B, AB and trace of A for

$$A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & -7 \\ 1 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & -1 & -1 \\ -2 & -2 & -2 \\ 3 & 1 & 7 \end{pmatrix}$$

Q2 (a) Prove that if $S = \{v_1, v_2, \dots, v_n\}$ be a basis for a vector space V and v£V, then the coefficients in the representation $V = a_1v_1 + a_2v_2 + \dots + a_nv_n$ are unique.
(b) Determine whether S is linearly independent in the following set
(i) $S = \{v_1, v_2, v_3\}, where\ v_1 = (4, -2, 0, 6), v_2 = (2, 4, 10, -2), v_3 = (14, -2, 10, 16)$ in R^4
(ii) $S = \{v_1, v_2, v_3\}, where\ v_1 = (1,0,0), v_2 = (0,1,0), v_3 = (0,0,1)$ in R^3

Q3 (a) a) Let *T: U – W* be a linear transformation, prove that
(i) Ker T is a subspace of U
(ii) Im T is a subspace of W
(b) Let $A = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$ Show that A is diagonalizable

Q4 (a) Solve the non-homogenous system

$$-2 + x + 2y = z$$
$$3y + 8x = 4 + 7z$$
$$4y - 12z = -8$$

(b) Find the eigenvalues and associated eigenvectors of A given that

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Q5 Prove that if a vector space V has one basis with a finite number of elements, then all other base of V are finite and have the same number of elements.
(b) Determine the basis and dimension for the solution space of the homogenous system of equations

$$x + 2y - 8z = 0$$
$$2x + 3y - 5z = 0$$
$$3x + 2y - 12z = 0$$

Q6 Express M as a linear combination of A, B,= and C where $M = \begin{pmatrix} 4 & 7 \\ 7 & 9 \end{pmatrix}$, $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 1 \\ 4 & 5 \end{pmatrix}$

(b) Consider the following two bases of R^3:
$E = \{e_1, e_2, e_3\} = \{(1,0,0), (0,1,0), (0,0,1)\}$ and
$S = \{u_1, u_2, u_3\} = \{(1,0,1), (2,1,2), (1,2,2)\}$

(i) Find the change of basis matrix P from the basis E to the basis S.
(ii) Find the change of basis matrix Q from the basis S to the basis E.

CROSS RIVER UNIVERSITY OF TECHNOLOGY, CALABAR
DEPARTMENT OF MATHEMATICS/STATISTICS
2012/2013 SECOND SEMESTER EXAMINATION
MTH 2201 LINEAR ALGEBRA II

INSTRUCTIONS: Answer any FOUR questions. TIME: 2 ½ Hours

Q1 (a) Let u, v and w be vectors in R^2 and c and d be scalars. Prove that (i) (u+v) + w = u + (v-w) (ii) (c + d) u = cu + du

(b) Let u = (2, -1, 5, 0), v = (4, 3, 1, -1) and w = (-6, 2, 0, 3) be scalars in R^4. Solve for x in each of the following (i) x = 2u – (v + 3u) (ii) 3 (x + w) = 2u - v + x

(c) Let P^2 be the set of all polynomials of the form $p(x) = a_2x^2 + a_1x + a_a$ where a_0, a_1 and a_2 are real numbers. The sum of two polynomials $p(x) = a_2x^2 + a_1x + a_a$ and $q(x) = b_2x^2 + b_1x + b_a$ is defined in the usual way by $p(x) + q(x) = (a_2+b_2)x^2 + (a_1 + b_1)x$, and the calculate multiple of $p(x)$ by a scalar c is defined by $cp(x) = ca_2x^2 + ca_2$. Show that p_2 is a vector space.

Q2 (a) Let W be the of all 2 x 2 symmetric matrices. Show that W is a subspace of the vector space $M_{2.2}$ with the standard operations of matrix addition and scalar multiplication.

(b) Which of the following subsets is a subspace of R^3?

(i) W = $\{(x_1, x_2, 1): x_1 \text{ } and \text{ } x_2 \text{ } are \text{ } real \text{ } numbers\}$

(ii) W = $\{(x_1, x_1 + x_3, x_3): x_1 \text{ } and \text{ } x_3 \text{ } are \text{ } real \text{ } numbers\}$

(c) Define a spanning set and show that the set $s = \{(1,2,3), (0,1,2), (-2,0,1)\}$ spans R^3

Q3 (a) (i) Prove that if $s = (v_1, v_2 \dots v_k)$ is a set of vectors in a vectors space V, then span(s) is a subspace.

(ii) When is a set of vectors $s = (v_1, v_2 \dots v_k)$ in a vector space V said to be linearly dependent (or independent)?

(b) (i) Determine whether the set of vectors in R^3 defined by $s = \{(1,2,3), (0,1,2), (-2,0)\}$ is linearly dependent or independent.

(ii) Prove that if a vector space V has one basis with n vectors, then every basis for V has a vectors.

(c) Using the above theorem explain why each of the following theorems is true

(i) The set $s_1 = \{(3,2,1), (7, -1, 4)$ is not a basis for R^3