# Ligo Formal Description

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## **Syntax**

```
variables(x)
   label (1)
   constructor (c)
                             declaration(d) =
                                               | type x is te
                                                                                                     (Type declaration)
                                               | const x (: te) = e
                                                                                                     (Const declaration)
                              typeexpression(te) =
                                                      | (* te_i)
                                                                                                            (Type tuple)
                                                      | (| l_i \ of \ te_i)|
                                                                                                             (Type sum)
                                                      |\{l_i:te_i\}
                                                                                                           (Type record)
                                                      |te1 \rightarrow te2
                                                                                                               (function)
                                                      |l|
                                                                                                                (variable)
                                                      | l (te_i)
                                                                                             (type of built in function)
                                     expression(e) =
                                                       |value|
                                                                                                                  (values)
                                                       \mid built_i n
                                                                                                      (built-in function)
                                                                                                               (variables)
                                                       \mid x
                                                       | \lambda x \cdot expr
                                                                                                                (lambda)
                                                       |e1|e2
                                                                                                            (application)
                                                       | let x = e1 in e2
                                                                                                                   (let in)
                                                       |(e_i)|
                                                                                                                   (tuple)
                                                       |c|e
                                                                                                            (constructor)
                                                       |\{l_i = e_i\}|
                                                                                                                 (record)
                                                       |[e1_i = e2_i]
                                                                                                                    (map)
                                                       | [[e1_i = e2_i]]
                                                                                                               (big map)
                                                       |[e_i]
                                                                                                                     (list)
                                                       \mid \{ e_i \}
                                                                                                                     (set)
                                                       | e(.ai)
                                                                                                               (accessor)
                                                       |e1[e2]
                                                                                                                (look up)
                                                                                                              (matching)
                                                       | match e with matching
```

| e1; e2

|SKIP|

 $\mid e \ as \ T$ 

| while e1 do e2

 $|x.(a_i)| = e$ 

(sequence)

(ascription)

(loop)

(skip)

(assign)

```
access(a) =
                                           \mid int
                                                                                       (for tuples)
                                           string
                                                                                       (for record)
                                                                                         (for map)
                                           \mid e
                 value(v) =
                            literal
                                                                         (values of built-in types)
                            | const v |
                                                                       (values of construct types)
                            | \lambda x \cdot expr
                                                                                          (lambda)
                              literal =
                                       unit
                                                                                                 ()
                                       bool
                                                                                                 ()
                                                                                                 ()
                                       \mid int
                                       \mid nat
                                                                                                 ()
                                       \mid mutez
                                                                                                 ()
                                       string
                                                                                                 ()
                                       | bytes
                                                                                                 ()
                                       \mid address
                                                                                                 ()
                                       | timestamp
                                                                                                 ()
                                       | operation
                                                                                                 ()
             matching(m) =
                             |\{true => e; false => e;\}
                                                                                      (match bool)
                             |\{nil => e; cons(hd :: tl) => e;\}
                                                                                       (match list)
                             |\{ none => e; some(x) => e; \}
                                                                                    (match option)
                             |(x_i)| => e
                                                                                     (match tuple)
                             |(const_i(x_i) => e_i)|
                                                                                   (match variant)
matchingvalue(mv) =
                       |\{true => v; false => v;\}
                                                                               (match bool value)
                       \mid \{ \ nil \ => \ v; \ cons(hd :: tl) \ => \ v; \}
                                                                                 (match list value)
                       | \{ none => v; some(x) => v; \}
                                                                             (match option value)
                       |(x_i)| => v
                                                                               (match tuple value)
                       |(const_i(x_i) => v_i)|
                                                                             (match variant value)
```

## Evaluation of expression

#### base

Values and variables are not evaluted

$$\frac{\overline{built_i n} (e_i) \rightarrow built_i n_r esult (* evaluated depending on each case *)}{(\lambda x. e) v \rightarrow [x \rightarrow v] e}$$
( E-BUILTIN)

$$\frac{e1 \rightarrow e1'}{e1 \ e2 \rightarrow e1' \ e2} \qquad \qquad (E-APP1)$$

$$\frac{e2 \rightarrow e2'}{v1 \ e2 \rightarrow v1 \ e2'} \qquad \qquad (E-APP2)$$

$$\frac{e1 \rightarrow e1'}{let \ x = e1 \ in \ e2 \rightarrow let \ x = e1' \ in \ e2} \qquad \qquad (E-LET)$$

$$\frac{e1 \rightarrow e1'}{let \ x = v1 \ in \ e2 \rightarrow [x \rightarrow v1] \ e2} \qquad \qquad (E-LET)$$

$$\frac{e1 \rightarrow e1'}{e1; \ e2 \rightarrow e1'; \ e2} \qquad \qquad (E-SEQ)$$

$$\frac{unit; \ e2 \rightarrow e2}{unit; \ e2 \rightarrow e2} \qquad \qquad (E-SEQNEXT)$$

$$\frac{e1 \rightarrow e1'}{while \ e1 \ then \ e2 \rightarrow while \ e1' \ then \ e2} \qquad \qquad (E-LOOP)$$

$$\frac{while \ true(=e1) \ then \ e2 \rightarrow e2; \ while \ e1 \ then \ e2}{while \ false \ then \ e2 \rightarrow unit} \qquad (E-LOOPFALSE)$$

$$\frac{E-SKIP}{e \ as \ T \rightarrow e' \ as \ T} \qquad (E-ASCR1)$$

$$\frac{e \rightarrow e'}{e \ as \ T \rightarrow e' \ as \ T} \qquad (E-ASCR2)$$

#### data structure

$$\frac{e \to e'}{c \ e \to c \ e'} \qquad (E-CONST)$$

$$\frac{e_j \to e'_j}{(v_i, \ e_j, \ e_k) \to (v_i, \ e'_j, \ e_k)} \qquad (E-TUPLES)$$

$$\frac{e_j \to e'_j}{\{l_i = v_i, \ l_j = e_j, \ l_k = e_k\} \to \{l_i = v_i, \ l_j = e'_j, \ l_k = e_k\}} \qquad (E-RECORDS)$$

$$\frac{e_{2j} \to e_{2j}'}{[e_{1i} = v_i, \ e_{1j} = e_{2j}, \ e_{1k} = e_{2k}] \to [e_{1i} = v_i, \ e_{1j} = e_{2j}', \ e_{1k} = e_{2k}]} \qquad (E-MAP)$$

$$\frac{e_{2j} \to e_{2j}'}{[e_{1i} = v_i, \ e_{1j} = e_{2j}, \ e_{1k} = e_{2k}]] \to [[e_{1i} = v_i, \ e_{1j} = e_{2j}', \ e_{1k} = e_{2k}]]} \qquad (E-BIGMAP)$$

$$\frac{e_j \to e'_j}{[v_i, \ e_j, \ e_k] \to [v_i, \ e'_j, \ e_k]} \qquad (E-LIST)$$

$$\frac{e_j \to e'_j}{\{v_i, \ e_j, \ e_k\} \to \{v_i, \ e'_j, \ e_k\}} \qquad (E-SET)$$

$$\frac{e_j \to e'_j}{e(a_i) \to e'(a_i)} \qquad (E-SET)$$

### look up

$$\frac{\overline{(v_i)[j] \to v_j}}{\{l_i = v_i\}[lj] \to v_j} \qquad \qquad \text{(E-LUPTUPLE)}$$

$$\frac{\overline{[l_i = v_i][l_j] \to v_j}}{\overline{[l_i = v_i][l_j] \to v_j}} \qquad \qquad \text{(E-LUPMAP)}$$

$$\frac{\overline{[v_i][j] \to v_j}}{\overline{[v_i][j] \to v_j}} \qquad \qquad \text{(E-LUPLIST)}$$

$$\frac{e \to e'}{x(.a_i) = e \to x(.a_i) = e'} \qquad \qquad \text{(E-ASSIGN)}$$

$$\overline{x(.a_i) = v \to x'(.a_i) \text{ with } x' \text{ as } x \text{ with } field \text{ (.a_i) } replace \text{ by } v}} \qquad \qquad \text{(E-ASSIGN2)}$$

#### matching

$$\frac{e \rightarrow e'}{match \ e \ with \ m \rightarrow match \ e' \ with \ m} \qquad (E-MATCH1)$$

$$\frac{m \rightarrow m'}{match \ v \ with \ m \rightarrow match \ v \ with \ m'} \qquad (E-MATCH2)$$

$$\frac{e1 \rightarrow e1'}{\{ \ true \ = > \ e1; \ false \ = > \ e2; \} \rightarrow \{ \ true \ = > \ e1'; \ false \ = > \ e2; \}} \qquad (E-MATTHBOOL1)$$

$$\frac{e2 \rightarrow e2'}{\{ \ true \ = > \ v1; \ false \ = > \ e2; \} \rightarrow \{ \ true \ = > \ v1; \ false \ = > \ e2'; \}} \qquad (E-MATTHBOOL2)$$

$$\frac{e1 \rightarrow e1'}{\{ \ nil \ = > \ e1; \ cons(hd :: tl) \ = > \ e2; \} \rightarrow \{ \ nil \ = > \ e1'; \ cons(hd :: tl) \ = > \ e2; \}} \qquad (E-MATCHLIST1)$$

$$\frac{e2 \rightarrow e2'}{\{ \ nil \ = > \ v1; \ cons(hd :: tl) \ = > \ e2; \} \rightarrow \{ \ nil \ = > \ v1; \ cons(hd :: tl) \ = > \ e2'; \}} \qquad (E-MATCHLIST2)$$

$$\frac{e1 \rightarrow e1'}{\{ \ none \ = > \ e1; \ some(x) \ = > \ e2; \} \rightarrow \{ \ none \ = > \ e1'; \ some(x) \ = > \ e2; \}} \qquad (E-MATCHOPT1)$$

$$\frac{e2 \rightarrow e2'}{\{ \ none \ = > \ v1; \ some(x) \ = > \ e2; \} \rightarrow \{ \ none \ = > \ v1'; \ some(x) \ = > \ e2; \}} \qquad (E-MATCHOPT2)$$

$$\frac{e2 \rightarrow e2'}{\{ \ none \ = > \ v1; \ some(x) \ = > \ e2; \} \rightarrow \{ \ none \ = > \ v1'; \ some(x) \ = > \ e2'; \}} \qquad (E-MATCHOPT2)$$

$$\frac{e \rightarrow e'}{\{ \ (x_i) \ = > \ e \rightarrow \ e'_i \ (x_i) \ = > \ e_i, \ c_i(x_i) \ = > \ e_i, \ (E-MATCHVARIANT)$$

## Derive form

e1; e2 is 
$$(\lambda x : Unit.e1)$$
 e2 with x not a free variable in e1 (1)

$$let \ x = e1 \ in \ e2 \ is \ (\lambda x : T1.e2) \ e1$$
 (2)