# Ligo Formal Description

# **Syntax**

```
variables(x)
label(l)
constructor(c)
declaration(d) =
                    | type x is te
                                                                                               (Type declaration)
                    | const x (: te) = e
                                                                                              (Const declaration)
type \ expression \ (te) =
                          |(*te_i)|
                                                                                                      (Type tuple)
                          | (| l_i \ of \ te_i) |
                                                                                                       (Type sum)
                          |\{l_i:te_i\}|
                                                                                                     (Type record)
                          |te1 \rightarrow te2|
                                                                                                         (function)
                          \mid l
                                                                                                         (variable)
                          | l (te_i)
                                                                                       (type of built in function)
expression(e) =
                    |value|
                                                                                                           (values)
                    | built_i n
                                                                                                (built-in function)
                                                                                                        (variables)
                    \mid x
                    | \lambda x \cdot expr
                                                                                                          (lambda)
                    |e1|e2
                                                                                                      (application)
                    | let x = e1 in e2
                                                                                                            (let in)
                    |(e_i)|
                                                                                                            (tuple)
                    |c|e
                                                                                                     (constructor)
                    |\{ l_i = e_i \}
                                                                                                           (record)
                    |[e1_i = e2_i]
                                                                                                             (map)
                    |[e1_i = e2_i]|
                                                                                                         (big map)
                    |[e_i]
                                                                                                              (list)
                    |\{e_i\}
                                                                                                               (set)
                    \mid e(.ai)
                                                                                                         (accessor)
                    |e1[e2]
                                                                                                          (look up)
                                                                                                        (matching)
                    | match e with matching
                    | e1; e2
                                                                                                        (sequence)
                    | while e1 do e2
                                                                                                             (loop)
                    |x.(a_i)| = e
                                                                                                           (assign)
                    |SKIP|
                                                                                                              (skip)
                    \mid e \ as \ T
                                                                                                       (ascription)
```

```
access(a) =
             \mid int
                                                                                                 (for tuples)
             string
                                                                                                 (for record)
             \mid e \mid
                                                                                                   (for map)
value(v) =
            | literal
                                                                                   (values of built-in types)
            | const v |
                                                                                 (values of construct types)
            | \lambda x \cdot expr
                                                                                                   (lambda)
literal =
          unit
                                                                                                           ()
          bool
                                                                                                           ()
          \mid int
                                                                                                           ()
          | nat
                                                                                                           ()
                                                                                                           ()
          \mid mutez
                                                                                                           ()
          string
          | bytes
                                                                                                           ()
                                                                                                           ()
          \mid address
                                                                                                           ()
          | timestamp
                                                                                                           ()
          | operation
matching(m) =
                  |\{ true => e; false => e; \}|
                                                                                               (match bool)
                  |\{ nil => e; cons(hd :: tl) => e; \}
                                                                                                 (match list)
                  |\{ none => e; some(x) => e; \}
                                                                                             (match option)
                  |(x_i)| => e
                                                                                              (match tuple)
                  |(const_i(x_i) => e_i)|
                                                                                             (match variant)
matching\ value\ (mv)\ =
                         |\{ true => v; false => v; \}
                                                                                         (match bool value)
                         |\{ nil => v; cons(hd :: tl) => v; \}
                                                                                           (match list value)
                         |\{ none => v; some(x) => v; \}
                                                                                       (match option value)
                         |(x_i)| => v
                                                                                        (match tuple value)
                         |(const_i(x_i) => v_i)|
                                                                                       (match variant value)
```

# Evaluation of expression

#### base

Values are not evaluted

$$x \rightarrow v \ (corresponding \ value \ in \ the \ environment)$$
 (E-VARIABLE)  
 $built \ in \ (e_i) \rightarrow built \ in \ result \ (* \ evaluated \ depending \ on \ each \ case *)$  (E-BUILTIN)  
 $(\lambda x.e) \ v \rightarrow [x \rightarrow v] \ e$  (E-LAMBDA)  
 $\frac{e1 \rightarrow e1'}{e1 \ e2 \rightarrow e1' \ e2}$  (E-APP1)  
 $\frac{e2 \rightarrow e2'}{v1 \ e2 \rightarrow v1 \ e2'}$ 

$$\frac{e1 \rightarrow e1'}{let \ x = e1 \ in \ e2 \rightarrow let \ x = e1' \ in \ e2} \qquad \qquad \text{(E-LET)}$$

$$let \ x = v1 \ in \ e2 \rightarrow [x \rightarrow v1] \ e2 \qquad \qquad \text{(E-LETIN)}$$

$$\frac{e1 \rightarrow e1'}{e1; \ e2 \rightarrow e1'; \ e2} \qquad \qquad \text{(E-SEQ)}$$

$$unit; \ e2 \rightarrow e2 \qquad \qquad \text{(E-SEQNEXT)}$$

$$\frac{e1 \rightarrow e1'}{while \ e1 \ then \ e2 \rightarrow while \ e1' \ then \ e2} \qquad \qquad \text{(E-LOOP)}$$

$$while \ true(= e1) \ then \ e2 \rightarrow e2; \ while \ e1 \ then \ e2 \qquad \qquad \text{(E-LOOPTRUE)}$$

$$while \ false \ then \ e2 \rightarrow unit \qquad \qquad \text{(E-SKIP)}$$

$$SKIP \rightarrow unit \qquad \qquad \text{(E-SKIP)}$$

$$\frac{e \rightarrow e'}{e \ as \ T \rightarrow e' \ as \ T} \qquad \qquad \text{(E-ASCR1)}$$

#### data structure

 $v \ as \ T \rightarrow v$ 

$$\frac{e \rightarrow e'}{c \ e \rightarrow c \ e'}$$
 ( E-CONST)

(E-ASCR2)

$$\frac{e_j \rightarrow e'_j}{(v_i, e_j, e_k) \rightarrow (v_i, e'_j, e_k)}$$
 (E-TUPLES)

$$\frac{e_j \to e'_j}{\{l_i = v_i, \ l_j = e_j, \ l_k = e_k\}} \to \{l_i = v_i, \ l_j = e'_j, \ l_k = e_k\}$$
 (E-RECORDS)

$$\frac{e2_j \ \to \ e2'_j}{[e1_i = v_i, \ e1_j = e2_j, \ e1_k = e2_k] \ \to \ [e1_i = v_i, \ e1_j = e2'_j, \ e1_k = e2_k]} \ ( \text{E-MAP})$$

$$\frac{e2_j \rightarrow e2'_j}{[[e1_i = v_i, \ e1_j = e2_j, \ e1_k = e2_k]] \rightarrow [[e1_i = v_i, \ e1_j = e2'_j, \ e1_k = e2_k]]} \tag{E-BIGMAP}$$

$$\frac{e_j \rightarrow e'_j}{[v_i, e_j, e_k] \rightarrow [v_i, e'_j, e_k]}$$
 (E-LIST)

$$\frac{e_j \rightarrow e'_j}{\{v_i, e_j, e_k\} \rightarrow \{v_i, e'_j, e_k\}}$$
 (E-SET)

$$\frac{e \to e'}{e(.a_i) \to e'(.a_i)}$$
 ( E-ACCESS)

## look up

$$\begin{aligned} &(v_i)[j] \to v_j & & \text{(E-LUPTUPLE)} \\ &\{l_i = v_i\}[lj] \to v_j & & \text{(E-LUPRECORD)} \\ &[e_i = v_i][ej] \to v_j & & \text{(E-LUPMAP)} \\ &[[e_i = v_i]][ej] \to v_j & & \text{(E-LUPBIGMAP)} \\ &[v_i][j] \to v_j & & \text{(E-LUPLIST)} \\ &\{v_i\}[j] \to v_j & & \text{(E-LUPSET)} \\ &\frac{e \to e'}{x(.a_i) = e \to x(.a_i) = e'} & & \text{(E-ASSIGN)} \\ &x(.a_i) = v \to x'(.a_i) \text{ with } x' \text{ as } x \text{ with } field (.a_i) \text{ replace by } v & \text{(E-ASSIGN2)} \end{aligned}$$

### matching

$$\frac{e \rightarrow e'}{match\ e\ with\ m \rightarrow match\ e'\ with\ m} \qquad (E-MATCH1)$$

$$\frac{m \rightarrow m'}{match\ v\ with\ m \rightarrow match\ v\ with\ m'} \qquad (E-MATCH2)$$

$$match\ v\ with\ m \rightarrow match\ v\ with\ m'$$

$$\frac{e1 \rightarrow e1'}{\{true \Rightarrow e1;\ false \Rightarrow e2;\}} \rightarrow \{true \Rightarrow e1';\ false \Rightarrow e2;\} \qquad (E-MACTHBOOL1)$$

$$\frac{e2 \rightarrow e2'}{\{true \Rightarrow v1;\ false \Rightarrow e2;\}} \rightarrow \{true \Rightarrow v1;\ false \Rightarrow e2';\} \qquad (E-MACTHBOOL2)$$

$$\frac{e1 \rightarrow e1'}{\{nil \Rightarrow e1;\ cons(hd::tl) \Rightarrow e2;\}} \rightarrow \{nil \Rightarrow e1';\ cons(hd::tl) \Rightarrow e2;\} \qquad (E-MATCHLIST1)$$

$$\frac{e2 \rightarrow e2'}{\{nil \Rightarrow v1;\ cons(hd::tl) \Rightarrow e2;\}} \rightarrow \{nil \Rightarrow v1;\ cons(hd::tl) \Rightarrow e2';\} \qquad (E-MATCHLIST2)$$

$$\frac{e1 \rightarrow e1'}{\{none \Rightarrow v1;\ some(x) \Rightarrow e2;\}} \rightarrow \{none \Rightarrow e1';\ some(x) \Rightarrow e2;\} \qquad (E-MATCHOPT1)$$

$$\frac{e2 \rightarrow e2'}{\{none \Rightarrow v1;\ some(x) \Rightarrow e2;\}} \rightarrow \{none \Rightarrow v1';\ some(x) \Rightarrow e2';\} \qquad (E-MATCHOPT2)$$

$$\frac{e2 \rightarrow e2'}{\{none \Rightarrow v1;\ some(x) \Rightarrow e2;\}} \rightarrow \{none \Rightarrow v1';\ some(x) \Rightarrow e2';\} \qquad (E-MATCHOPT2)$$

$$\frac{e3 \rightarrow e'}{\{(x_i) \Rightarrow e \rightarrow (x_i) \Rightarrow e'} \qquad (E-MATCHOPT2)$$

## Derive form

e1; e2 is 
$$(\lambda x : Unit.e1)$$
 e2 with x not a free variable in e1 (1)

$$let x = e1 in e2 is (\lambda x : T1.e2) e1$$

$$(2)$$