The Heat Topology Project
Topology optimization of a heat conduction problem
Academic year 2018 – 2019

Diabatix is a young company focusing on high-performant cooling component designs. Cooling components are used on lasers, electronics, engines etc. in order to limit the operating temperature. The performance of a cooling component is context-dependent, but is mostly achieved by having a uniform temperature on a surface (which increases product lifetime) or the most compact design possible. Diabatix achieves the design of cooling components using a state-of-the-art numerical optimization procedure.

A heat spreader is a device that conducts heat from a heat source to an outlet. The heat spreader is physically put on top of the heat source. The heat spreader itself is made of high conductivity metal embedded in low conductivity plastic. Figure 1 is from Diabatix' website.

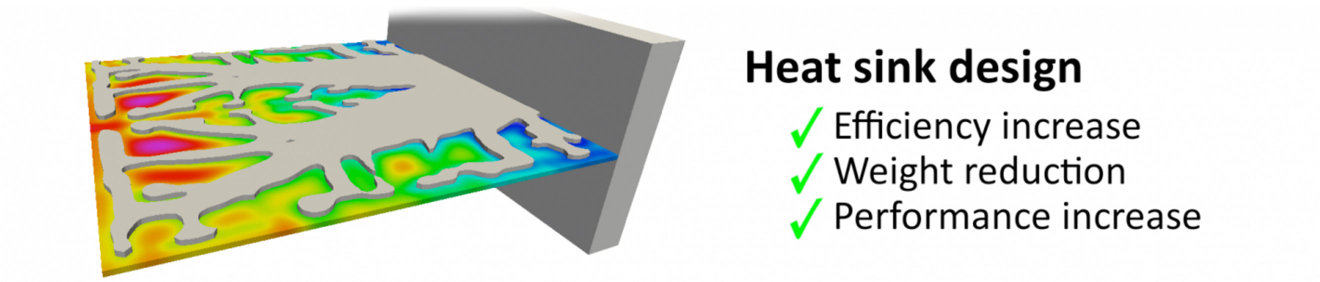
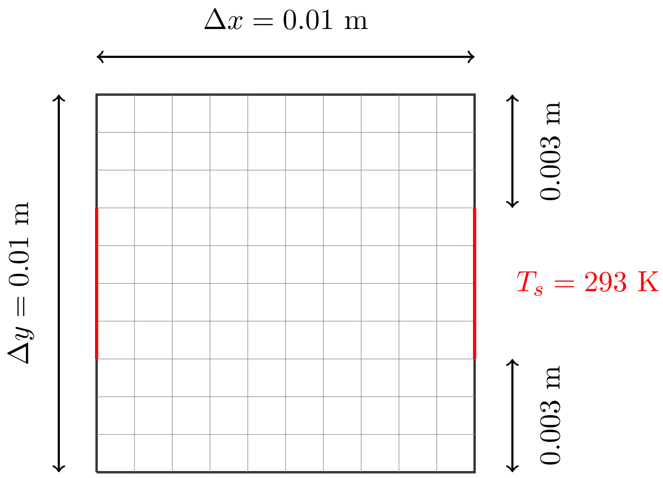


Figure 1: Illustration of a cooling component

In the assignment, we will design a heat spreader based on a simple heat conductivity problem on a rectangular domain. The heat source Q is uniformly distributed over the domain. The heat spreader has a thickness Δz, see Figure, and the heat conductivity coefficients are given in $Wm^{-1}K^{-1}$, in which the length dimension is this thickness. The outlet of the heat spreader is situated on the left and on the right side of the domain where a given temperature T_s is imposed. The design question now is to minimize the difference in temperature on the device with the temperature of the outlet, or, in other words: where should metal be put on the domain to maximize heat transport towards the outlets, with a given maximum amount of metal to be used. The relevant physical details are given in the scheme below with total heat Q.



$k_{\text{metal}} = 65\ \text{Wm}^{-1}\text{K}^{-1}$

$k_{\text{plastic}} = 0.2\ \text{Wm}^{-1}\text{K}^{-1}$

$Q = 2$ W

$\Delta z = 0.001$ m

The heat conduction on a domain $\Omega \subset \mathbb{R}^2$ is governed by the partial differential equation

$$-\nabla \cdot (k(x,y)\,\nabla T(x,y)) = q(x,y)$$

with suitable conditions on the boundary $\partial\Omega$ and where $k(x,y)$ denotes the thermal conductivity coefficient at location (x,y) and $q(x,y)$ is the volumetric heat source. Confirm the dimensions of the equation to get the correct conversion from the total generated heat Q to the volumetric heat source $q(x,y)$. For the specified problem, this coefficient depends on the chosen material, implying that $k(x,y) \in [k_{\text{plastic}}, k_{\text{metal}}]$ for $(x,y) \in \Omega$. We assume that all edges of the domain are insulated except for the ones with the fixed temperature T_s. Denoting by $\Gamma_1 \subset \partial\Omega$ the part of the boundary with given temperature T_s and setting $\Gamma_2 := \partial\Omega \setminus \Gamma_1$, we obtain the following boundary conditions

$$T(x,y) = T_s \quad \text{on } \Gamma_1 \qquad \text{and} \qquad \vec{n} \cdot \nabla T(x,y) = 0 \quad \text{on } \Gamma_2,$$

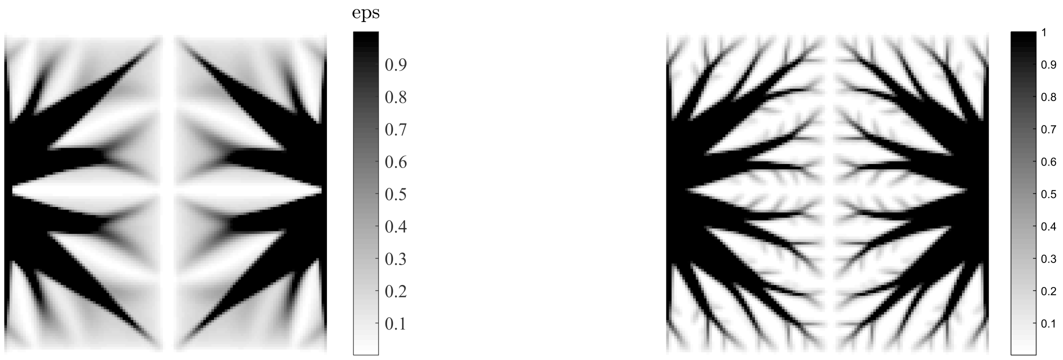
where $\vec{n} \in \mathbb{R}^2$ is the unit normal vector on the boundary part Γ_2.

The problem at hand is an optimization problem, where, after discretization, it is decided how much metal to put in a particular cell. The presence of metal is represented by a real variable, which allows for the use of numerical optimization. More formally, consider a partition of the domain Ω into subdomains Ω_j with $j = 1, \ldots, n$. For each of these subdomains let $v_j \in [0,1]$ denote the fraction of Ω_j which is coated with metal. This way, e.g., $v_j = 1$ corresponds to a subdomain which is entirely filled with metal, while $v_j = 0.5$ means that the subdomain is filled to equal parts with plastic and metal. This parametrization directly influences the constant value of $k(x,y)$ on each subdomain Ω_j; this value can be modelled as a combination of k_{plastic} and k_{metal}. Additionally we constrain the total amount of metal to be used. The general optimization problem can then be formulated as follows.

$$\begin{aligned}
\underset{\mathbf{v} \in \mathbb{R}^n}{\text{minimize}} \quad & J(T(x,y), \mathbf{v}) \\
\text{subject to} \quad & -\nabla \cdot (k(x,y,\mathbf{v})\,\nabla T(x,y)) = q(x,y), \\
& \frac{1}{n}\sum_{j=1}^{n} v_j \leq M, \\
& 0 \leq v_j \leq 1 \quad \forall j = 1, \ldots, n.
\end{aligned}$$

Here J is some cost functional depending on the solution $T(x,y)$ of the given PDE and $k(x,y,\mathbf{v})$ denotes the conductivity coefficient depending on the parameter vector $\mathbf{v} = (v_1, \ldots, v_n)$. Furthermore, M is the constraint on the maximum relative volume of metal in the domain. We choose $M = 0.4$.

The distribution of metallic (black) and plastic (white) material is the desired optimization result. Figure 2 below show design iterations of the optimization procedure. Illustration 2a shows an intermediate result, whereas 2b shows the final design. Note that a black and white design is achieved by the use of “continuation”, i.e., making it more expensive for the design to use intermediate v_j, as detailed in [2].



(a) Illustration of an intermediate design

(b) Illustration of the final design

Figure 2: Display of two cooling designs

Key problems in this assignment

In this assignment you are required to study the outlined optimization problem and implement an algorithm for its solution. Hints for the representation of the presence of metal using a real variable can be found in [2]. Furthermore, a similar problem is discussed in [3]. For a given distribution of metal, the temperature can be computed by solving the above heat equation with appropriate boundary conditions. Which method you choose to solve this PDE is up to you, but you have to justify your choice. You can either implement your own software or combine existing software packages (some are listed in [1]).

Hints for the assignment

When solving the problem, you will have to choose a sensible cost functional J. The choice of J will have direct consequences for the optimization problem. You should therefore consider the following questions.

- Which good properties has a particular cost functional with respect to the optimization problem?
- What quantities do you have to pass to the numerical optimizer? Can you calculate these?

References

[1] M.M. Gregersen, A. Evgrafov, M.P. Sørensen. Topology optimization of heat conduction problems. *Workshop on industrial design optimization for fluid flow*, September 2010. See presentation slides

[2] C.F. Hvejsel, E. Lund. Material interpolation schemes for unified topology and multi-material optimization. *Structural and multidisciplinary optimization*, 2011.

[3] Y. Zhang, S. Liu. Design of conducting paths based on topology optimization. *Heat and mass transfer*, 2008.

[4] S.G. Johnson. Notes on Adjoint Methods, 2012. Available here