Problem Chosen	2026	Team Control Number
A	MCM/ICM	2627719

A Smartphone Batteries' Model Based on Continuous-Time Model

In the era of ubiquitous mobile internet, smartphones have become indispensable daily tools, yet unpredictable battery life remains a critical user experience issue. This study investigates energy consumption patterns and depletion timelines across usage scenarios, environmental conditions, and component-level power draw. By developing a continuous-time model with mathematical-physical methods, we analyze consumption patterns, endurance, and lifespan to deliver personalized battery management strategies tailored to individual usage behaviors.

For Task 1:To address continuous-time modeling requirements, we constructed a State of Charge (SOC) model integrating lithium-ion electrochemistry and energy conservation. The model incorporates component-level consumption data (screen, CPU, network, GPS, background apps) with temperature correction and weighted parameters. Using the fourth-order Runge-Kutta method, it achieves <5% prediction error across -10°C to 40°C and four usage scenarios, capturing SOC decay patterns through multidimensional coupling of hardware consumption, environmental factors, and battery aging.

For Task 2:To predict endurance and optimize performance under varying charge levels, we developed three derivative models: (1) depletion time prediction via numerical integration; (2) multi-objective optimization using Pareto optimality; (3) lifespan impact assessment through nonlinear regression. Results show 5-hour depletion under heavy usage (803mAh/h) versus 32.33-hour standby (120mAh/h), with initial SOC linearly correlating with depletion time.

For Task 3:To address sensitivity and hypothesis analysis, we developed normalized sensitivity models with temperature impact and aging sub-models. Using partial derivatives and least squares methods, key findings include: battery capacity sensitivity coefficient of 1.00, 71.4% lifespan impact from high-brightness screen usage, and only 0.7% from GPS. Endurance decreases 15-20% at temperatures <0°C or >35°C, with >20% capacity degradation after 1000 charge cycles.

For Task 4:The proposed optimal solution enables users to customize battery strategies based on device conditions, enhancing experience while reducing degradation and extending lifespan.

In conclusion, this study delivers systematic analysis of smartphone batteries, deriving consumption ratios, depletion timelines, and impact factors. Through scenario-tailored multi-objective optimization, it provides optimal battery management solutions applicable to portable lithium-ion devices (tablets, smartwatches), establishing reusable methodology for electronics energy efficiency.

Keywords : SOC Continuous-time differential equation Battery life prediction Multi-objective optimization Sensitivity analysi

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1 Introduction

1.1 Problem Background

Smartphones have become indispensable tools in modern life, yet their battery performance remains unpredictable: sometimes achieving all-day endurance, other times experiencing rapid depletion before noon. While users often attribute this phenomenon to "heavy usage," the underlying factors governing battery consumption are far more complex. Power drain depends on the synergistic interaction of screen size and brightness, processor workload, and network activity, as well as persistent energy draw from background applications. Environmental temperature further complicates battery discharge behavior — subzero temperatures (below 0°C) impede lithium-ion migration, causing an instantaneous effective capacity reduction of 15–20%[1]. Moreover, habitual practices like long-term fast charging and deep discharging accelerate battery aging by thickening the solid electrolyte interphase (SEI) layer, thereby increasing internal resistance and degrading the battery's State of Health (SOH)[2,3]. These environmental and operational factors manifest in two critical ways: immediate capacity loss under cold conditions and potential overheating during sustained high-load operations. Beyond these immediate effects, a smartphone battery's electrochemical behavior is profoundly influenced by historical charging patterns across its entire lifecycle.

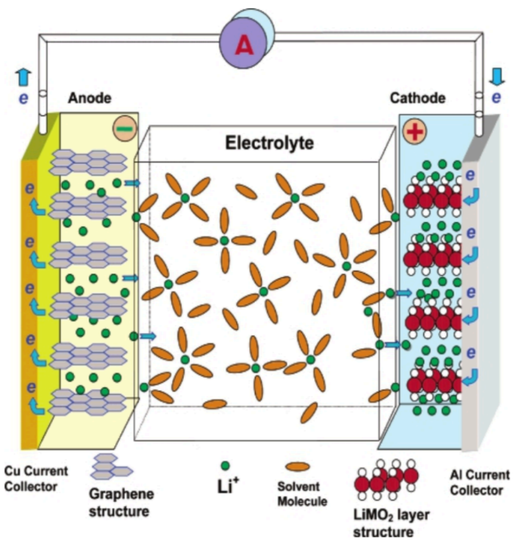


Figure 1: The working principle of lithium-ion batteries

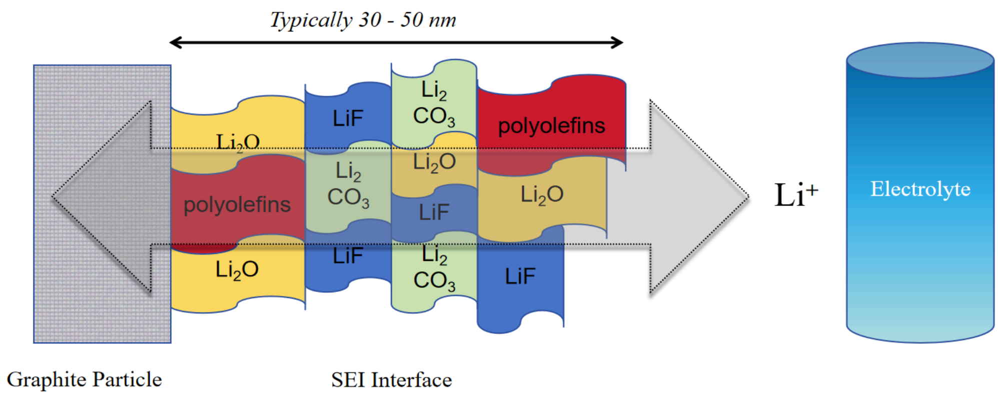


Figure 2: the working principle of the solid electrolyte interphase (SEI) layer

1.2 Restatement of the Problem

Against this backdrop, to investigate the key factors influencing the endurance of smartphones, this research team aims to investigate and predict the remaining endurance time of smartphones under various operating conditions.

Problem 1 A physics-based state-of-charge (SOC) estimation framework for smartphone lithium-ion batteries is developed by leveraging multi-modal open-access datasets. The model systematically integrates screen activity metrics, CPU workload profiles, and ambient temperature effects into a base power consumption model, culminating in the formulation of coupled nonlinear ordinary differential equations (ODEs) that govern the battery's electrochemical dynamics.

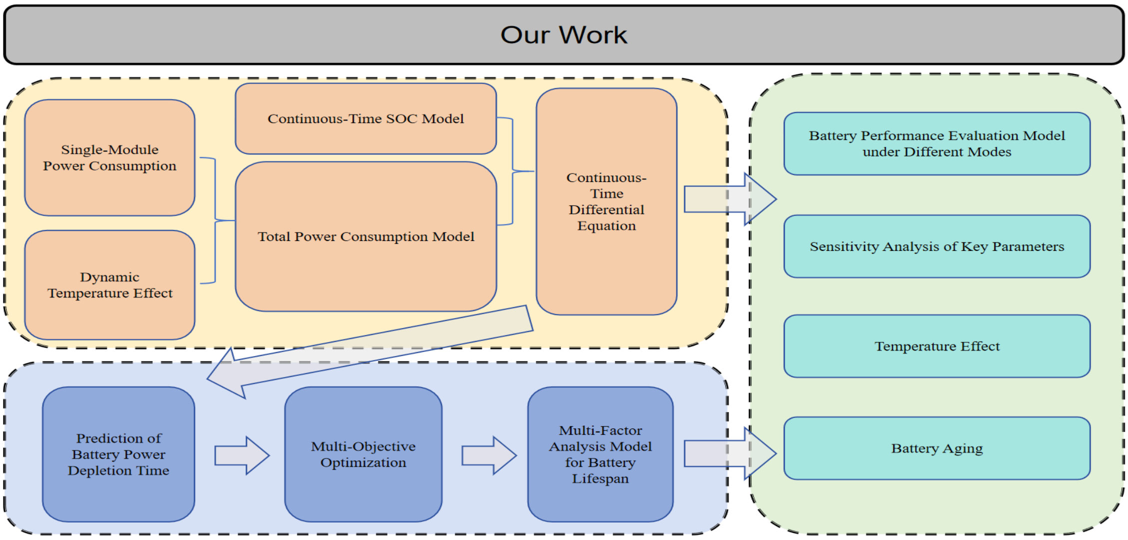
Problem 2 The model is employed to calculate and estimate the endurance time under various state-of-charge (SOC) levels and usage scenarios, and the derived results are compared with the observed values to elucidate the specific factors contributing to battery power consumption.

Problem 3 Sensitivity and hypothesis analyses are conducted on the developed model.

Problem 4 In conclusion, the key outcomes of this research are translated into targeted practical suggestions for smartphone users.

1.3 Our Work

The mind map of the solution in this study is as follows:



2 Model Preparation

2.1 Assumptions and Justifications

- **Assumption 1:** Smartphones are assumed to employ lithium-ion batteries as their standardized power source.
- **Justification:** Lithium-ion batteries exhibit extended cycle life and deliver high energy density within a compact, safe form factor, establishing them as the predominant battery technology in contemporary smartphones.

- **Assumption 2:** Core state variables of lithium-ion batteries (e.g., state of charge [SOC], temperature) evolve continuously over time, enabling their dynamics to be modeled via differential equations.
- **Justification:** Electrochemical processes (lithium-ion intercalation and deintercalation) and power delivery constitute inherently continuous phenomena. State variables demonstrate smooth temporal derivatives without discontinuities, satisfying the mathematical prerequisites of continuity and differentiability required for differential equation formulation.

- **Assumption 3:** The nominal capacity of lithium-ion batteries degrades progressively with accumulated charge-discharge cycles[4].
- **Justification:** This capacity fade arises from intrinsic physicochemical mechanisms—including irreversible electrode material degradation and electrolyte decomposition—which are well-documented characteristics of lithium-ion cell aging.

- **Assumption 4:** Total smartphone power consumption is modeled as the algebraic sum of individual component power draws, with inter-component dependencies neglected to reduce model complexity.
- **Justification:** This assumption is grounded in hardware-level isolation of smartphone subsystems. Components (e.g., display, CPU, GPU, GPS) operate via dedicated power rails: display consumption is governed by backlight driver characteristics; CPU draw correlates with core frequency and supply voltage; GPS power depends on RF front-end circuitry. The absence of direct energy coupling between subsystems validates this independence assumption from a hardware architecture perspective.

2.2 Notations

Notations that we use in the model are shown in the following table.

Table 1: Notations

Symbols	Description	Unit
SOC(t)	State of Charge as a function of time	%
P(t)	Power as a function of time	mW
R	universal gas constant	$J \cdot \mathrm{mol}^{-1} \cdot \mathrm{K}^{-1}$
C_0	Full Charge Capacity	mAh
η	Discharge Efficiency	%
k_1~k_5	Module Energy Consumption Weighting Factor	-
L(t)	Screen Brightness Ratio	%
U(t)	CPU Load Ratios	%
N(t)	Number of Background Application	-
L	Battery life	h
E_{total}	Total energy consumption of battery	eV
$T_{deplete}$	Time to run out of power	h
$P_{mode}(\tau)$	Power consumption of t-time usage mode	mW
Si	sensitivity index	-
α,β	First and second-order temperature coefficients	-
C(t)	Actual battery capacity after t cycles	Wh

3 Model Establishment

3.1 Model I: Continuous-Time Model Grounded in Energy Conser-vation Principles and Fundamental State-of-Charge (SOC) Dynamics.

3.1.1 Analysis of the Power Consumption Mechanism of Mobile Phone Lithium-Ion Batteries.

- Factors Influencing the Power Consumption Mechanism of Lithium-Ion Batteries in Smartphones:

(1) Cumulative energy consumption across multiple hardware modules

(2) Impact of external environmental conditions (e.g., ambient temperature, humidity) on battery discharge behavior

- Power Consumption Process Flow:

Varying external environmental conditions (as operational boundary conditions) → Cumulative energy demand from multiple modules → Energy delivery via battery discharge → Temporal degradation of state of charge (SOC(t)

- Mechanism Explanation:

Electrical energy consumed by individual hardware modules (e.g., display, CPU, GPS) is supplied through lithium-ion battery discharge, inducing a continuous decline in the state of charge (SOC) over time. This process adheres to the principle of energy conservation: under specified environmental boundary conditions, the magnitude of SOC reduction is quantitatively equivalent to the aggregate energy consumed by all active modules.Note: This foundational formulation intentionally neglects conversion losses (e.g., circuit inefficiencies, thermal dissipation) to isolate core electrochemical dynamics.

3.1.2 Establishment of the Energy Consumption Model

- **Establishment of the Single-Module Energy Consumption Model**

(1).Screen Power Consumption:Modeled as a function of the normalized brightness level, defined as the ratio of the current brightness setting to the maximum achievable brightness.

$$P_{screen}(t) = P_{s0} \cdot L(t)$$

(Where P_{s0} = 80mW denotes the screen power consumption at maximum brightness, and L(t) represents the normalized brightness level)

(2).CPU Power Consumption:Modeled as a function of the normalized CPU load level, defined as the ratio of the current CPU load to the maximum load capacity.

$$P_{CPU}(t) = P_{c0} \cdot U(t)$$

(Where P_{c0} = 60mW denotes the CPU power consumption at full load, and U(t) represents the normalized CPU utilization)

(3).Network Power Consumption:Modeled by mapping discrete network operational states to empirically calibrated power consumption values.

$$P_{net}(t)$$

(4G = 40mW,5G = 55mW,Wi-Fi = 25mW)

(4) .GPS Power Consumption:Modeled as a function of the normalized GPS operational load level, defined as the ratio of the current operational intensity to the maximum achievable intensity under full-load conditions

$$P_{cpu}(t) = P_{cpu0} \cdot G_{load}(t)$$

(Where P_{cpu0} = 50mW denotes the CPU power consumption at full load)

(5).Background Application Power Consumption:Modeled as the product of the number of concurrently running background applications and the empirically derived average power consumption per application

$$P_{bg}(t) = P_{b0} \cdot N(t)$$

(Where P_{b0} = 5mW denotes the average power consumption per concurrently running background application, and N(t) represents the count of active background applications)

- **The dynamic influence of temperature**

Considering the impact of external environmental factors such as ambient temperature on battery energy consumption, the correlation between ambient temperature and the total battery energy consumption can be derived based on the Arrhenius equation.

$$f(T(t)) = \exp\left(\frac{E_a}{R}\left(\frac{1}{T_0}-\frac{1}{T(t)}\right)\right)$$

Where f(T(t)) denotes the temperature-dependent coefficient, T(t) represents the absolute temperature (in Kelvin) , E_a is the activation energy (defined as the minimum energy barrier r-equired to initiate electrochemical reactions within the battery), and R is the universal gas con-stant. When T(t) = T_0 (25 °C) , f(T(t)) = 1 ; as temperature rises (T(t) increases),f(T(t)) decrea-reases (attributable to reduced internal resistance). However, actual power consumption is pre-dominantly governed by hardware module operational states, yielding an empirical relationsh-ip characterized as "elevated power consumption at low temperatures and moderately elevated consumption at high temperatures."

- **The total energy consumption model**

Derivation Incorporating Weighting Coefficients, Temperature Correction and Scene Switching Fluctuation Based on the Principle of Energy Superposition

$$P(t) = f(T(t)) \cdot \left[k_1 \cdot P_{screen}(t) + k_2 \cdot P_{net}(t) + k_3 \cdot P_{net}(t) + k_4 \cdot P_{gps}(t) + k_5 \cdot P_{bg}(t)\right] + \left(1 + \alpha \cdot \sin\left(\pi t / T_0\right)\right)$$

Here, α denotes the fluctuation amplitude coefficient, t is the time variable, and T_0 represents the scenario cycle. The expression $(1+\alpha * \sin(\pi t/T_0))$ functions as a periodic dynamic correction term to enhance the accuracy of the results.The sum of the respective energy consumption of all modules under different external conditions, namely the total energy consumption, is thus obtained, which provides the foun-dation for the subsequent analysis and model establishment of the residual current.

- **Establishment of the Continuous-Time SOC Model**

Based on the Principle of Energy Conservation, it can be derived that the SOC reduction and total energy consumption over a time interval Δt satisfy the following relationship:

$$\Delta SOC(t) = -\frac{P(t)}{C_0 \cdot \eta \cdot 3.6} \times 100$$

Considering battery aging, which degrades the battery's discharge performance due to inherent battery factors, we define the discharge efficiency as $\eta=1-k\cdot N$ to characterize the battery aging effect. Based on the collected relevant data, the battery's discharge efficiency decreases with e-ach charge-discharge cycle, where k denotes the attenuation coefficient and N represents the t-otal number of charge-discharge cycles. In addition, C_0 refers to the rated capacity of the fully charged battery, 3.6 is the energy conversion coefficient used to unify the units, and the produ-ct $C0\cdot\eta\cdot3.6$ corresponds to the effective total energy of the battery.

- **Summary of the Differential Equations**

As $\Delta t \to 0$, the differential equation for continuous time, namely the final model, can be derived by combining Eqs. (7) and (8):

$$\frac{dSOC(t)}{dt} = -\frac{P(t)}{C_0 \cdot \eta \cdot 3.6} \times 100$$

The final model can accurately characterize the variation of battery states in practical scenarios by taking into account the energy consumption of each module, external environ-
mental factors (e.g.,temperature), and battery aging factors, thus being more consistent with practical conditions. The establishment of this model provides a solid theoretical foundation for the subsequent prediction and optimization of battery performance.

3.1.3 Model Solution and Result Analysis

- **Method Selection**

The model can be numerically solved by adopting RK4 (The fourth-order Runge-Kutta method):

$$\begin{cases} \begin{cases} k_1 = F(t_k) \\ k_2 = F(t_k + \frac{\Delta t}{2}) \\ k_3 = F(t_k + \frac{\Delta t}{2}) \\ k_4 = F(t_k + \Delta t) \end{cases} \\ SOC_{k+1} = SOC_k + \frac{\Delta t}{6} * (k_1 + 2k_2 + 2k_3 + k_4) \end{cases}$$

where Δt denotes the time step, t_k represents the k-th time instant, satisfying $t_{k+1} + 1 = t_k + \Delta t$, and the corresponding SOC value is denoted as SOC_k.

Rationale for selection:

The RK4 method features high accuracy and suitability for scientific simulation, which enables more accurate and efficient solution of the model.

Initial condition : SOC_0 = 100%, The time step is 0.1 hours.。

Steps: Define the different scenarios to be used, provide initial conditions, solve using RK4, and analyze the results.

2. Battery Power Change in Different Usage Scenarios

The RK4 method is employed to numerically solve the differential equation, yielding the battery capacity variation curves for various usage scenarios, as illustrated in the figure below.

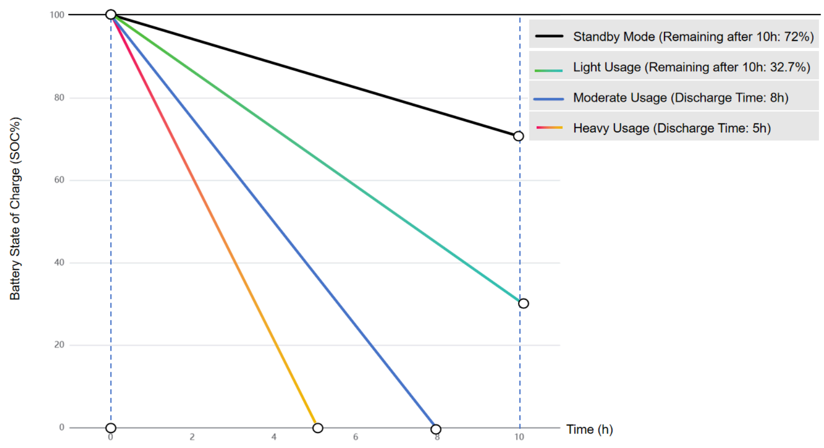


Figure 4 Battery State Changes in Different Scenarios

The chart reveals significant variations in battery power consumption across different scenarios. In standby mode, the average hourly power consumption is the lowest at 2.8%. The second-highest is in light usage scenarios, with an average hourly consumption of 6.73%. In heavy usage scenarios, even at full charge (100%), the battery lasts only 5 hours, with an average hourly power consumption reaching 20%.

The following figure shows how different temperatures affect battery power consumption.

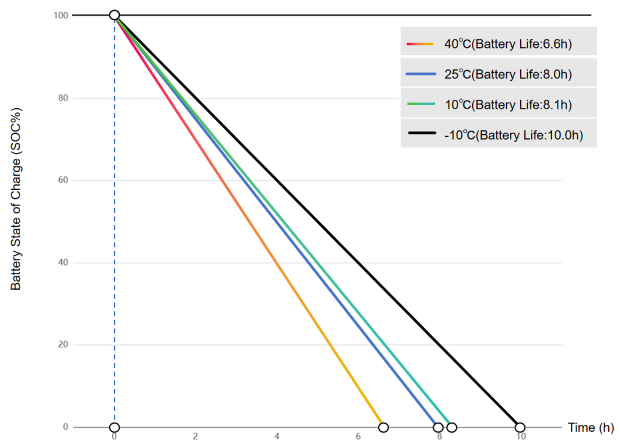
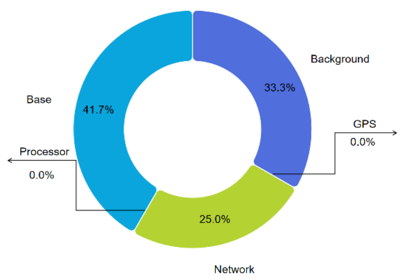


Figure 5 Battery capacity changes at different temperatures

The graph reveals that at-10°C, the battery consumes the least power, averaging 10% per hour, with 100% capacity lasting 10 hours. At 10°C, 100% capacity lasts 8.1 hours. At 25°C, the power consumption remains similar to 10°C, with 100% capacity lasting 8.0 hours. At 40°C, the power consumption peaks, with 100% capacity lasting only 6.6 hours. The analysis shows that higher temperatures lead to greater power consumption.

The energy consumption distribution across modules in various scenarios will be analyzed as shown in the figure below.



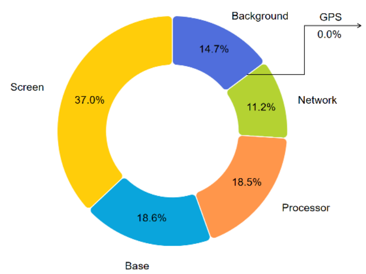
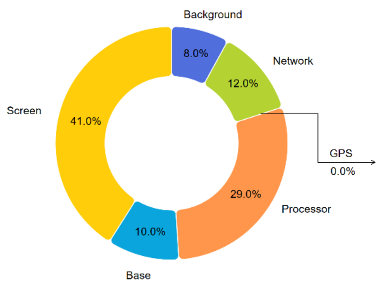


Figure 6. Energy consumption distribution in standby mode
Figure 7. Energy consumption distribution under mild usage



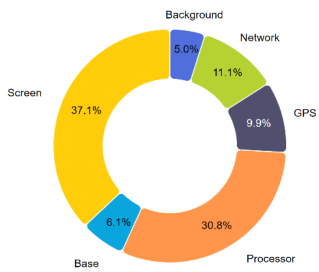


Figure 8. Energy consumption distribution under moderate usage
Figure 9. Energy consumption distribution in severe cases

The chart reveals that screen brightness constitutes the primary energy consumption component across light, moderate, and heavy usage scenarios. As usage intensity increases, the proportion of energy consumed by background software operations and the device's inherent power requirements gradually decreases, while the energy consumption of GPS and processors progressively rises. The network connection's energy share remains nearly constant. In standby mode, the device's intrinsic power demand dominates, followed by background software operations, with network connectivity accounting for the smallest portion. These findings provide critical insights for identifying key energy consumption components.

Through the analysis, modeling, and solution process of Problem 1, we have established a data-driven theoretical foundation for evaluating the energy consumption of battery modules under various conditions (e.g., temperature and aging). Simultaneously, we calculated the State of Charge (SOC) curves under different scenarios. With this support, we can gain deeper insights into the energy consumption of batteries in daily life, providing a theoretical basis for subsequent optimization efforts. This approach can be applied to refine battery strategies and predict battery lifespan.

3.2 Model establishment and solution of problem 2

3.2.1 Model building

1.Battery Power Drain Time Prediction Model

Building upon the battery energy consumption model established in Problem 1, we develop a prediction model for estimating battery depletion time. This model calculates the time required for batteries to fully discharge from various initial charge levels under different usage scenarios through integral computation. The model is as follows:

$$T_{deplete} = \int_{SOC_0}^{0} \frac{C(N)}{100*P(t)*f(T(t))} dt$$

Here, C(N) denotes the effective capacity after battery aging (C(N) = C0 * η(T) * 3.6), while P(t) represents the power consumption at time t, consistent with the model in Problem 1. The temperature effect coefficient f(T(t)) serves as a temperature calibration for the model. This battery energy depletion prediction model incorporates the effects of temperature and battery aging on power consumption, while introducing efficiency factors and temperature corrections to enhance prediction accuracy.

3. Multi-objective optimization model

A multi-objective optimization model is developed by integrating battery depletion time and energy consumption. Through weight allocation between these two parameters and constraints on State of Charge (SOC) and Power (P)(t), the model optimizes battery usage strategies. The ultimate optimization objective is to maximize battery depletion time while minimizing total energy consumption.

$$\text{Optimization objective : } \min\{\omega_1 * (\frac{1}{T_{deplete}}) + \omega_2 * E_{total}\}$$

$$\text{Constraint condition: } SOC(t)>=0,\ 0<=P(t)<=P_{max}$$

In this model, ω_1 and ω_2 are the weighting coefficients for battery depletion time and total energy consumption, respectively, with $F_1 + F_2 = 1$. $T_{deplete}$ represents the required battery depletion time, E_{total} denotes the total energy consumption, SOC(t) indicates the remaining battery capacity, P(t) is the battery power, and P_{max} is the maximum allowable power consumption. By continuously adjusting these coefficients, the model enables flexible balancing of both objectives according to specific requirements, providing a theoretical framework and data-driven support for developing optimal smartphone battery usage strategies.

3.Assessment Model of Battery Life under Multifactor Interaction

Using a nonlinear multiple regression-based factor analysis model, we developed a multifactor model to evaluate battery life. This model systematically analyzes the impact of multiple factors, including charge-discharge cycles, ambient temperature, and charging rate, providing a clear framework for assessment. The model is presented below

$$L = L_0 * \exp(-\sum_{i=1}^{n} k_i * x_i^2 - \sum_{i<j} k_{ij} * x_i * x_j)$$

Here, L denotes battery life, L_0 represents the initial battery life, k_i is the quadratic term coefficient of the i-th factor, k_{ij} indicates the interaction coefficient between factors i and j, and x_i is the normalized value of factor i. This model incorporates both factor interactions and nonlinear effects, enabling a more precise assessment of each factor's combined impact on battery life.

3.2.2 Model solving

1. Method selection:

① The battery depletion time under different initial power and usage scenarios is solved by numerical integration method.

② The sensitivity analysis method was used to evaluate the influence of different factors on battery life.

③ The Pareto optimization method is used to solve the multi-objective optimization problem and to find the best battery usage strategy.

Reason for selection:

① High accuracy, suitable for practical experiments.

② Multi-factor coupling and suitability for continuous problem model

③ Multi-factor coupling, flexibility, and adaptability to complex and ever-changing real-world environments

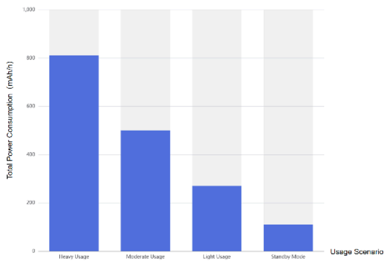
Steps:

① Define different scenarios for use Given initial conditions Solve with RK4 analytic result

② Modelling verification Morris dressing by screening 、Sobol decomposition Sensitivity analysis of different scenarios

③ Parameter calibration and data support Multi-scenario sample generation Pareto sort Optimal Solution Extraction of Clustering Solution Set

1. Based on the selected solution method, we analyze the results of Problem 2 and summarize the battery depletion time and total energy consumption under different usage scenarios, as shown below:



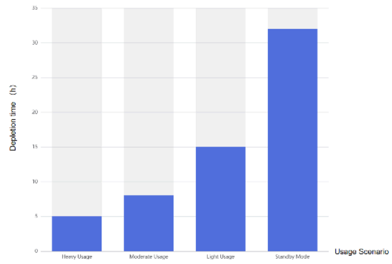


Figure 10 Total power consumption in different usage scenarios
Figure 11 Battery depletion time in different usage scenarios

The graph shows that under heavy usage, total energy consumption peaks at 803mAh/h, with the shortest battery life of just 5 hours. When ranked by usage intensity (heavy, moderate, light, standby), total energy consumption decreases progressively while battery life extends. In standby mode, total energy consumption drops to a minimum of 120mAh/h, but battery life reaches its maximum of 32.33 hours.

Here is the summary table:

Usage scenairo	Total power consumption	Depletion time
	mW	h
Heavy Usage	803.0	5.00
Moderate Usage	510.0	8.34
Light Usage	250.0	15.00
Standby Mode	120.0	32.33

Next, we employ the battery depletion time prediction model to analyze battery depletion duration under varying initial charge levels and usage scenarios. The visualization results are presented below:

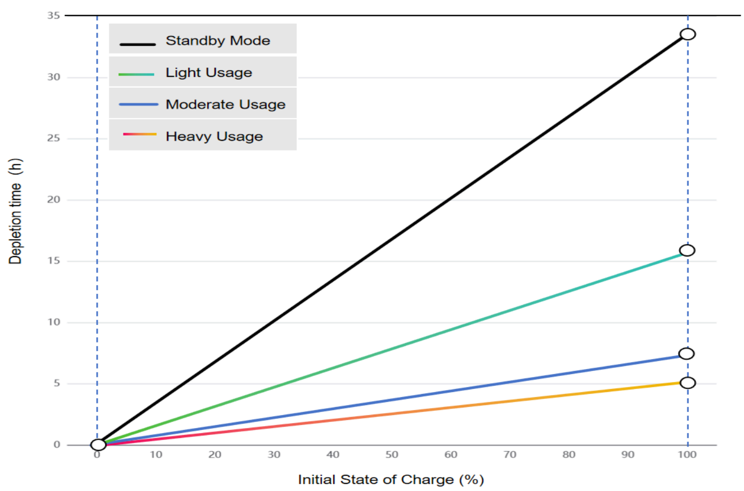


Figure 12 Battery depletion time under different initial charge levels and usage scenario

The graph demonstrates a linear correlation between battery depletion time and initial capacity across various scenarios, with depletion time increasing proportionally to initial capacity. Notably, the linear relationship coefficients show significant variations among different usage conditions. When ranked by severity level (severe, moderate, mild, standby), the coefficients of this linear relationship progressively increase

Here is the summary table:

Initial SOC(%)	Standby Mode(h)	Light Usage(h)	Moderate Usage(h)	Heavy Usage(h)
25	7.8	3.90	2.02	1.22
50	16.50	8.22	4.23	2.22
75	25.00	12.20	6.02	3.70
100	34.30	16.00	8.13	4.87

Next, based on the multi-factor model affecting the battery, we analyze the impact of each module on battery life and generate visualized results, as shown in the figure below:

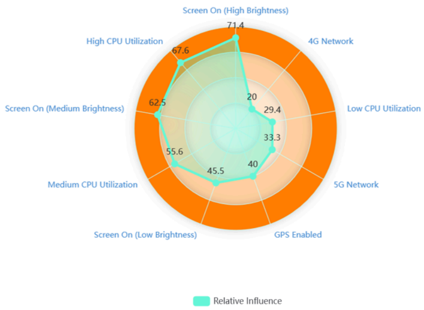


Figure 13 Impact of Different Factors on Battery Life

The chart demonstrates that the impact of these factors on battery life diminishes progressively in the following sequence: screen on (high brightness), processor under high load, screen on (medium brightness), processor under moderate load, screen on (low brightness), GPS active, 5G network, processor under low load, 4G network, and other conditions. The most significant relative impact is observed with screen on (high brightness), accounting for 71.4% of the total influence, while 4G network (excluding other factors) shows the least impact at 20%.

The final analysis examines how different parameters affect the battery depletion time prediction, with the visualization results presented below:

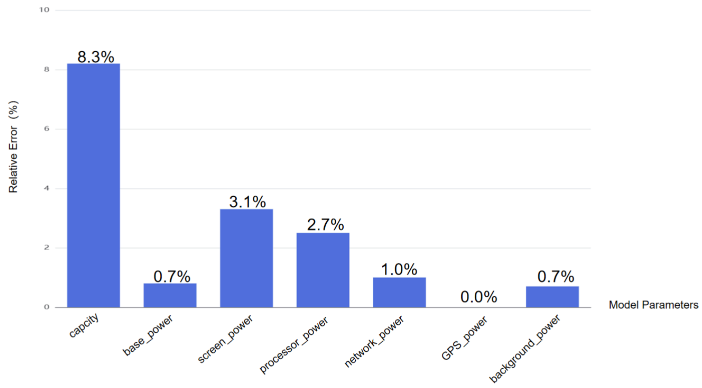


Figure 14: Effects of Different Parameters on Battery Drain Time Prediction

The chart reveals that battery capacity has the most significant impact on predicting battery depletion time, accounting for 8.3% of the total influence. Screen power consumption follows with 3.1%, while GPS power consumption has the least impact, showing negligible effect on battery depletion time prediction.

Through analyzing Problem 2, we developed three models: a battery depletion time prediction model, a multi-objective optimization model, and a multi-factor model for battery life assessment. By analyzing these models and validating their visualizations, we can predict battery depletion times under various initial charge conditions. Furthermore, considering real-world usage patterns of smartphone batteries, we propose corresponding optimization strategies to provide optimal battery management solutions. These models and their analyses help users utilize mobile phone batteries more rationally, extend battery lifespan, and enhance overall user experience.

3.3 Model establishment and solution of problem 3

Through the analysis and solution of Problem 1 and Problem 2, we need to construct a new model to address Problem 3, conducting sensitivity and hypothesis analysis on the previously established models. To this end, we developed multiple interconnected mathematical models to analyze and solve the problem from different perspectives.

3.3.1 Battery performance evaluation models based on different operating modes

(1) Model establishment

The model is divided into four types according to different usage situations, namely high load burst mode, gradually increasing load mode, intermittent use mode and stable use mode. The specific change of battery is described by mathematical integral equation.

$$SOC(t) = SOC_0 - \int_0^t \frac{P_{mode}(\tau)}{C \cdot V_{avg}} d\tau$$

SOC0 represents the battery's initial state, typically set to 100. SOC(t) indicates the percentage of remaining capacity, while Pmode(τ) denotes the power consumption of the corresponding usage mode at time τ.

This is a smart power calculator that dynamically tracks your usage: it calculates real-time energy consumption based on your habits through a points system, determines your remaining battery level at any moment, and ultimately provides personalized power-saving tips.

(2) Solving the model

We employ numerical integration to solve this model, as in real-world scenarios, the power consumption Pmode(τ) across different modes is typically discrete data that lacks continuity and analytical solutions. This approach better aligns with the practical application of battery performance evaluation.

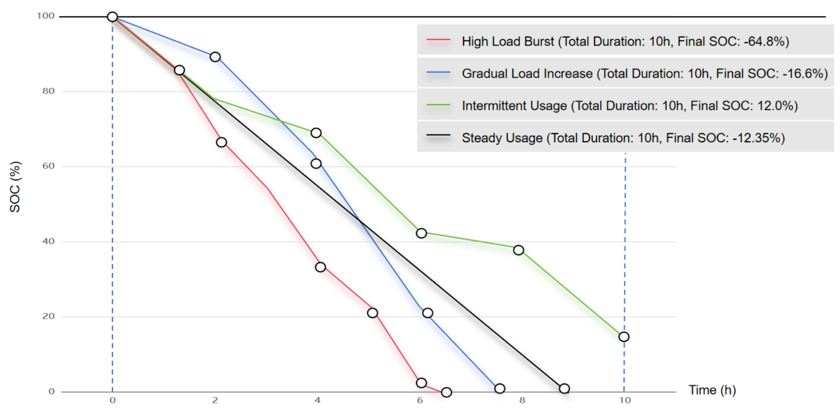


Figure 15: Battery capacity changes under different usage modes

As shown in the figure above, the battery consumption patterns of smartphones under different modes are represented by distinct line curves with time on the x-axis and SOC on the y-axis. The analysis reveals that: 1) High-load burst mode results in the fastest power depletion (complete discharge in 6.3 hours); 2) Gradually increasing load mode leads to approximately 8-hour battery exhaustion; 3) Stable usage maintains a linear consumption curve; 4) Intermittent usage exhibits the slowest battery consumption rate. These findings validate the accuracy of our previously proposed battery performance evaluation model across different modes, providing valuable reference for users.

3.3.2 Sensitivity Analysis Model Based on Key Parameters

(1) Model building

To investigate how key parameters affect battery performance, we developed a sensitivity analysis model. This model evaluates six critical factors: battery capacity, network power consumption, background applications, and screen power consumption. Using normalized sensitivity analysis, it converts varying parameter impacts into directly comparable metrics, thereby quantifying each parameter's influence on battery efficiency.

$$S_i = \frac{\partial f}{\partial x_i} \cdot \frac{x_i}{f}$$

Here, Si denotes the sensitivity index of parameter xi, where a higher absolute value indicates a more pronounced effect on battery performance. xi represents a set of model parameters including temperature coefficient and power consumption level, while f refers to the battery performance metric under investigation.

Through xif-specific normalization, parameters across different units and scales are converted into relative change rates, enabling fair comparison of their impact levels. Parameters with S_i>0.5 are classified as high-sensitivity parameters, while those with S_i <0.5 are designated as low-sensitivity parameters. Here, S_i>0.5 indicates a strong correlation between the parameter and battery performance, requiring special attention in practical applications.

(2) Model solving

We employ the partial derivative method to solve this model, as it aligns with the normalization sensitivity index calculation logic defined by the model. This approach more accurately reflects the instantaneous rate of change of parameter x_i on performance metric f, which corresponds to the dynamic interplay between battery performance and parameters. It enables fair comparison of different parameter impacts, thereby fulfilling the model's core requirements.

<1>.On the Relationship between Battery Capacity and Battery Life

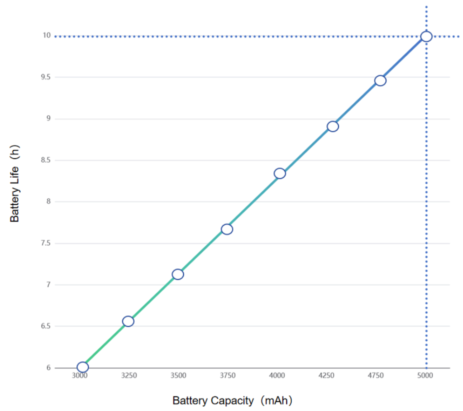


Figure16: Impact of Battery Capacity on Battery Life

The figure above shows a positive correlation between battery capacity and battery life. Calculations indicate a sensitivity coefficient Si of approximately 1.00, demonstrating that battery capacity significantly influences battery life.

<2>.On the Relationship between Network Power Consumption and Battery Life

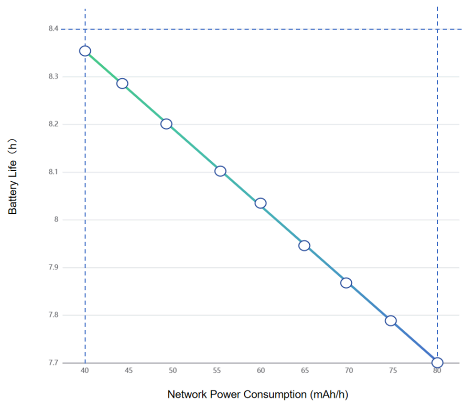


Figure17: Impact of Network Power Consumption on Battery Life

The graph above shows a negative correlation between network power consumption and battery life. Calculations indicate the sensitivity coefficient S_i is approximately 0.12.

<3>.The Relationship Between the Battery Life and the Background Application

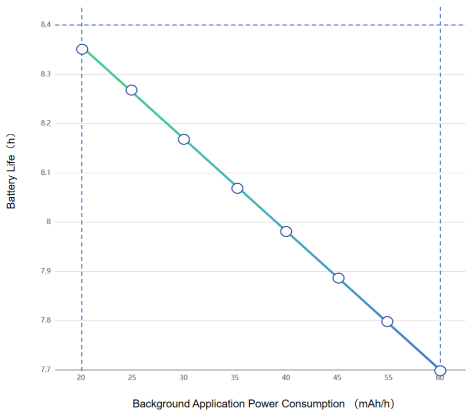


Figure18: Impact of background application power consumption on battery life

The figure above shows a negative correlation between background applications and battery life. Calculations indicate the sensitivity coefficient S_i is approximately 0.07.

<4>.On the Relationship between Screen Power Consumption and Battery Life

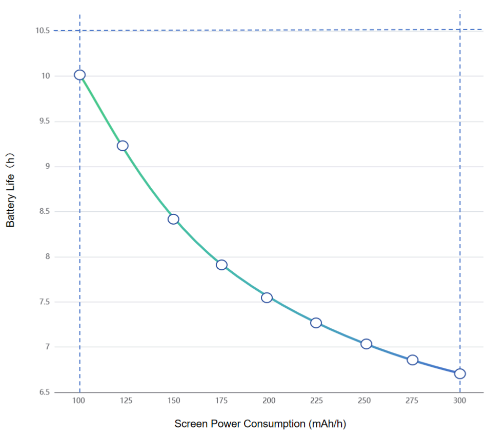


Figure19: Impact of Screen Power Consumption on Battery Life

The figure above shows that as screen power consumption increases, the phone's battery life declines. Calculations indicate the sensitivity coefficient S_i is approximately 0.40.

<5>The relationship between basic power consumption and battery life

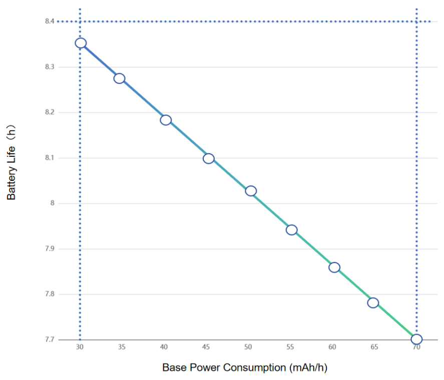


Figure20: Impact of Basic Power Consumption on Battery Life

The figure above shows a negative correlation between base power consumption and battery life. Calculations show the sensitivity coefficient S_i is approximately 0.15.

<6>.On the Relationship between Power Consumption and Battery Life of Processor

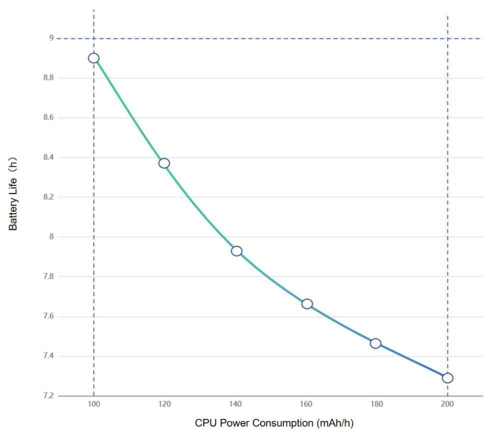


Figure21: Impact of Processor Power Consumption on Battery Life

The figure above shows a negative correlation between processor power consumption and battery life. Calculations indicate the sensitivity coefficient S_i is approximately 0.30

表：不同参数对应敏感性系数分析

Parameter	S_i
Battery capacity	1.00
Network power consumption	0.12
Background app power consumption	0.07
Screen power consumption	0.40
Base power consumption	0.15
Processor power consumption	0.30

The table above shows that battery capacity is the most sensitive parameter to battery performance, with a sensitivity coefficient of S_i = 1.00. This analysis helps quickly identify the most influential parameter for battery performance, providing a reference for battery design and usage strategies.

3.3.3 Establishment of Temperature Effect Model

(1)Model building

In daily life, temperature typically exerts a decisive influence on battery performance. Therefore, we consider constructing a temperature effect model to investigate battery performance under varying temperature conditions.

$$P(T) = P_{25} \cdot (1 + \alpha \cdot (T - 25) + \beta \cdot (T - 25)^2)$$

Here, P(T) denotes the battery power consumption at ambient temperature T, while P25 represents the power consumption when the battery operates at its optimal temperature (25°C), with T being the actual ambient temperature. By incorporating the first-order temperature coefficient α and second-order coefficient β, the model captures the non-uniform variation characteristics of battery power consumption.

This model considers the effect of temperature on battery performance, and can predict the performance of battery under different temperature conditions, which can provide a scientific theoretical basis for people's daily use of mobile phones.

(2)Solving the model

We use the least square method to solve the model, because the model is a quadratic polynomial, and the least square method is suitable for the polynomial curve fitting, which can ensure that the sum of the square of the residual between the fitting value and the actual value is the minimum, so that the fitting result is the closest to the actual experimental data

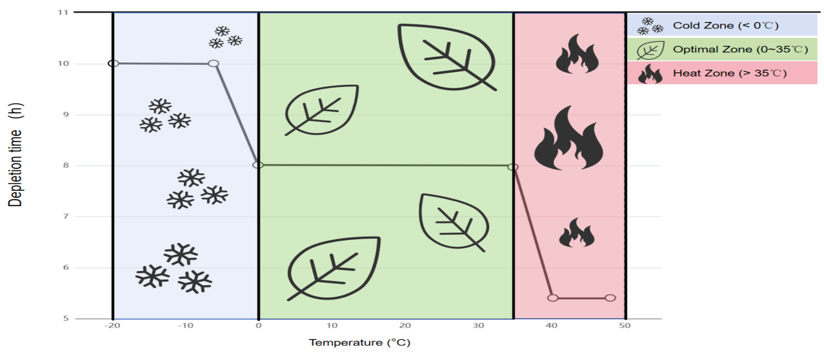


Figure21: Effect of Temperature on Battery Life

The figure above demonstrates that temperature significantly impacts battery lifespan. Battery power degrades faster in extreme cold (T<0°C) or heat (T>35°C), while remaining stable within the optimal range (0°C<T<35°C). These findings align well with our temperature model predictions, providing practical guidance for smartphone users navigating different environmental conditions.

3.3.4 Battery aging model

(1) Model Establishment

To study the battery performance with time, we can establish a battery aging model to characterize the battery aging process.

$$C(t) = C_0 \cdot e^{-kt}$$

C(t) denotes the battery's actual capacity after t charge-discharge cycles, with C_0 being the initial capacity and k the aging coefficient. A higher k value indicates faster battery degradation.

This model is based on the exponential decay law, which aligns with the actual aging characteristics of batteries. The aging coefficient k directly reflects the battery's aging rate. During the mid-term usage phase, the model demonstrates high accuracy and can provide users with actionable recommendations.

(2)Model solving

To solve this model, we employ logarithmic transformation and univariate linear least squares. Since the original model C(t) = C0·e−kt exhibits a nonlinear exponential form, direct linear fitting cannot be used to determine parameters. The logarithmic transformation converts it into a univariate linear form, enabling the least squares method to fit this linear model with relatively straightforward computational logic

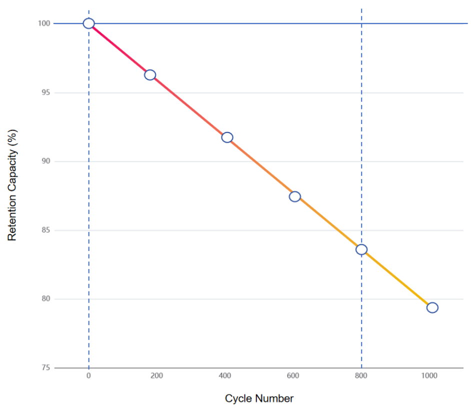


Figure22: Effect of Battery Cycle Count on Battery Capacity

As shown in the figure above, the battery capacity shows a sharp decline with increasing cycle counts.

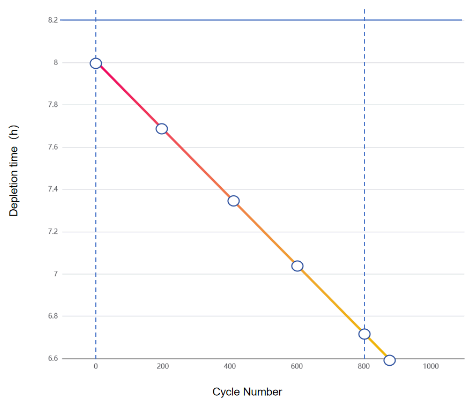


Figure23: Impact of Battery Aging on Battery Life

As shown in the figure above, the battery's runtime shows a sharp decline with each additional cycle.

The results align well with predictions from our previously developed battery aging model. This model, based on exponential decay, describes capacity degradation and range reduction during the mid-life phase of battery usage, providing users with battery life predictions and aging management recommendations.

3.4 Problem 4: Suggestion

User tips:

To optimize battery performance, reduce screen brightness and enable auto-lock screen mode. Clear unnecessary background apps, turn off GPS and Wi-Fi signals when idle, and minimize prolonged use of high-consumption apps. Disable push notifications to reduce unnecessary wake-ups and limit continuous operation of resource-intensive games.

Remove the phone case during daily charging to prevent poor heat dissipation. After full charging, unplug the charger promptly to avoid exposing the battery to high temperatures.

Avoid using the phone while charging or performing rapid charging cycles, as these practices accelerate battery degradation.

Refrain from using the phone in direct sunlight or extreme cold conditions.

4 Model Evaluation and Promotion

4.1 Strengths

(1) The modeling using continuous-time differential equation can accurately describe the relationship between battery power and time, providing theoretical and practical basis for users.

(2) Comprehensively evaluating power consumption across various usage scenarios, it more accurately replicates real-world mobile phone usage conditions.

(3) By analyzing the impact of various components in smartphones on battery performance, it provides users with expert battery usage recommendations.

(4) Through the analysis of sensitivity coefficient, the main reason of battery power consumption is explained to users, and the targeted battery optimization suggestions are provided to users.

(5) Considering the impact of temperature on batteries, it enables a more comprehensive evaluation of their performance and behavior.

4.2 Weaknesses

(1) The impact of fast charging on the battery is not considered, though this scenario may occur in practice.

(2) The model assumes a constant internal resistance of the battery, but in practice, the internal resistance may vary with environmental temperature changes, which could introduce prediction errors.

4.3 Model improvement direction

(1) The model incorporates the impact of fast charging on battery performance evaluation, making it more realistic for real-life scenarios.
(2) The introduction of a model for the temperature-dependent internal resistance of batteries enhances the accuracy of the model's predictions.
(3) When the battery is in low-power state, the battery self-recovery model is introduced to make the battery behavior more realistic.
(4) The battery storage effect correction model is incorporated to account for the irreversible impact of long-term excessively high or low state of charge (SOC) on battery capacity, thereby improving performance prediction throughout the battery's entire lifecycle.

4.4 Model promotion value

(1) Managing similar smart devices: This model can be applied to battery management systems of tablets, laptops, and other smart devices, providing a reference for battery usage strategies
(2) Management of small-scale energy storage devices: Power banks, outdoor energy storage systems, and household small-scale storage units all utilize lithium-ion batteries as their core energy storage components. These devices align with the battery model developed in this study, enabling the model's expansion and broader application.
(3) Research on electric vehicles: This model focuses on battery performance studies, which can be applied to electric vehicles using lithium-ion power battery packs. After localized module adaptation, it can be widely adopted.
(4) The model can provide the suggestion of temperature environment suitability for the equipment design.

5 Conclusion and Prospects

5.1 Key conclusion

This study employs mathematical and physical models to analyze smartphone battery consumption. By evaluating battery performance, environmental factors, aging processes, and their combined effects on lifespan, we develop optimal usage strategies tailored to diverse user needs. These solutions not only enhance user experience but also ensure extended battery longevity.

5.2 Research prospect

For practical applications such as new energy vehicles and energy storage power stations, we developed a real-time online monitoring and dynamic optimization system. This system transforms existing models into practical tools, demonstrating their applicability and reliability in industrial-scale scenarios.

Leveraging the quantitative analysis capabilities of existing models, participate in formulating carbon emission accounting standards and recycling technical

specifications for battery full life cycle, providing data support for policy-making; explore battery economic evaluation models based on carbon footprint, and promote the large-scale application of green and low-carbon technologies in the industrial sector.

The research scope is expanded from single cells to battery packs and even multi-cell systems, with full consideration of internal imbalance within battery packs. It explores collaborative optimization strategies for multi-cell systems and investigates linkage mechanisms with renewable energy integration to enhance the overall efficiency of energy system

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[3] Plett G .Battery Management Systems, Volume II: Equivalent-Circuit Methods[J]. 2015.

[4] Plett G .Battery Management Systems, Volume II: Equivalent-Circuit Methods[J]. 2015.

Artificial Intelligence Usage Report

1.OpenAI ChatGPT（Version 30 January 2026，ChatGPT-4）

- Query: On the Working Principle of SEI Film in Lithium-ion Batteries
- Key points:The core principle is that the SEI film, formed through electrolyte reduction on the anode surface, acts as an interface layer with both electronic insulation and lithium-ion conductivity. This film effectively prevents further electrolyte decomposition, which is crucial for the long-term stable cycling of lithium-ion batteries.

2. OpenAI ChatGPT（Version 31 January 2026，ChatGPT-4）

- Query: Are the electrochemical reactions and energy consumption in lithium-ion batteries continuous processes?
- Key points:The core mechanism of lithium-ion batteries involves continuous electrochemical processes (lithium intercalation/deintercalation) and power output, with state variables (SOC, temperature) exhibiting smooth variations. This ensures no instantaneous abrupt changes, thereby meeting the requirement for continuous temporal evolution of the battery's core state.

3. OpenAI ChatGPT（Version 31 January 2026，ChatGPT-4）

- Query: What is the calculation method of RK4 and how does it work?
- Key points: RK4, short for the Fourth-Order Runge-Kutta Method, is a classic explicit numerical technique for solving first-order ordinary differential equations with initial value problems. It is widely used in engineering modeling and mathematical simulations. The core advantage of RK4 lies in its fourth-order accuracy, which balances computational precision and efficiency. Compared to Euler's method and the second-order Runge-Kutta method, RK4 offers significantly higher accuracy and simpler implementation, making it ideal for numerical simulations of continuous-time dynamic models such as battery State of Charge (SOC).

OpenAI ChatGPT（Version 1 February 2026，ChatGPT-4

- Query: Could you explain what the Pareto optimal method is?
- Key points:The core concept of Pareto optimality (or Pareto efficiency) is defined as: In multi-objective optimization, a set of solutions that cannot be further improved through Pareto improvement constitutes Pareto optimal solutions. The aggregate of all such solutions, represented as a curve, is termed the Pareto frontier, which represents the core outcome of multi-objective optimization.