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# Sliding Mode observer for position and speed estimations in Brushless DC Motor (BLDCM)

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**Abstract**—Back electromotive force EMF is the only electric quantity able to provide instantaneous information on the mechanical variables of the electrical motor; that is true for the position and speed. In this paper, a new Sliding Mode Observer of the trapezoidal back EMF is applied to the BLDCM in order to ensure the control without speed and position sensors. A theoretical study is developed and the results obtained by simulation are presented to show the effectiveness of the method.

**Key-Words:** Permanent magnet, Brushless DC Motor, Sliding Mode Observer, sensorless control.

## I. INTRODUCTION

Today the demand for Permanent Magnet Brushless DC Motor (BLDCM) in the industrial and domestic applications is steadily rising. Their growing popularity is due to their high efficiency, silent operation, high power density, reliability and low maintenance. A BLDCM is one kind of synchronous motor, having permanent magnets on the rotor and trapezoidal shape back EMF.

The most popular way to control BLDCM is through voltage-source current-controlled inverters. The inverter must supply a rectangular current waveform whose magnitude,  $I_{MAX}$ , is proportional to the motor shaft torque. Three Hall Effect sensors are usually used as position sensors to perform current commutations; it is only required to know the position of commutation points, because the objective is to achieve rectangular current waveforms, with dead time periods of 60°. The sensors mentioned increase the cost and the size of the motor and reduce the reliability of the total system. Therefore, sensorless control of BLDCM has been receiving great interest in recent years. Many methods for obtaining rotor position and speed have been proposed in the literature. In most existing methods, the rotor position is detected every 60 electrical degrees, which is necessary to perform current commutations. These methods are based on: 1) using the back EMF of the motor [1], [2]; 2) detection of the conducting state of freewheeling diodes in the unexcited phase [3]; and 3) the stator third harmonic voltage components [4]. Since these methods cannot provide continual rotor position estimation,

they are not applicable for the sensorless drives in which high estimation accuracy of the speed rotor is required. In other methods, the estimation of the instantaneous rotor position is proposed. In [5], the instantaneous rotor position of the BLDCM is estimated indirectly from the estimating flux linkage using measured motor voltages and currents. The accuracy of the rotor position estimation depends significantly on the motor parameter variation and accuracy of measured voltages and currents. In [6], the extended Kalman filter (EKF) is used to estimate the instantaneous rotor position and speed of the BLDCM, but the speed estimation accuracy is decreased, particularly at low speeds. Despite of the drawbacks recalled in the previous section, a state observer based sensorless controller represent an excellent solution for a wide range of rated-speed and low-cost applications. The Extended Kalman Filter appeared to be the ultimate solution for sensorless control of Brushless DC motor. Unfortunately, this stochastic observer has some inherent disadvantages, such as the influence of noise characteristic, the computational burden and the absence of design and tuning criteria.

The goal of this paper is to present, a new Sliding Mode Observer of the trapezoidal back EMF in order to ensure the control of the BLDCM without speed and position sensors. This technique represents an attractive proposal because it is robust with respect to measurements noise and parametric uncertainties of the system. The trapezoidal back EMF which will be observed is the back EMF, induced between two phases, which does not depend neither on the stator voltage harmonics of order multiple of three nor on the noise of commutation introduced by the inverter (neutral point cannot connect). It is largely sufficient to provide the six positions of commutation and the speed of the rotor. However the continual estimation for the rotor position becomes not necessary.

The organization of this paper is given hereafter. In section 2, sensorless control strategy for BLDC Motor is presented.

In section 3, a formulation of the model of the BLDCM is given and a new Sliding mode observer of the trapezoidal FCEM is developed. In section 4, a simple zero crossing detectors (ZCD) of the back EMF is presented to obtain the position of commutation points. In section 5, a new approach to find directly the estimation rotor speed from the back EMF

is presented. Finally, in section 6, the simulation results are presented and explained.

## II. SENSORLESS CONTROL STRATEGY FOR BLDCM

The most popular way to control BLDCM is through voltage-source current-controlled inverters, the drive employs an inner current loop with an outer speed loop (figure1).

The inverter must supply a rectangular current waveform (fig.2) whose magnitude,  $I_{MAX}$ , is proportional to the machine shaft torque. The equivalent dc current is obtained through the sensing of two the three armature currents. From these currents, the absolute value is taken, and a dc component, which corresponds to the amplitude  $I_{MAX}$  of the original phase currents, is obtained. This dc component is compared with a reference coming from the output of the speed regulator 1, and the error signal is processed through a PI controller 2. The output of the PI controller2 is compared with a saw-teeth carrier signal, to generate the PWM for the power transistors. At the same time, the position sensor discriminates which couple of the six transistors of the inverter should receive this PWM signal, the several techniques of current control in such a system have been studied and described in the literature [7].

In this scheme, the position estimator is used to detect only six positions, which determine the switch commutations or commutations points (fig.2). From the outputs of the sliding mode observer the phase-to-phase back EMF is observed and using the zero crossing detectors (ZCD), the positions of commutation points are estimated. The speed rotor obtained by a new approach which based on the relation mathematical find between the magnitudes  $E_{max}$  of the phase-to-phase back EMF observed and speed rotor

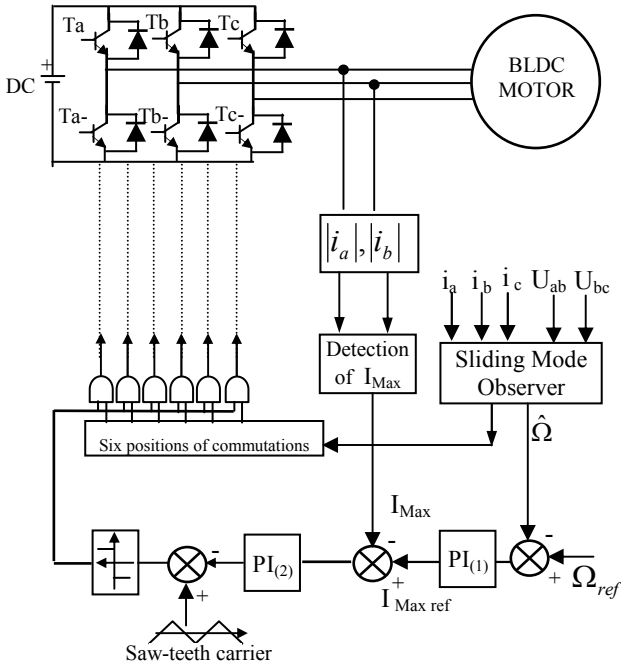


Fig.1: The proposed sensorless control scheme

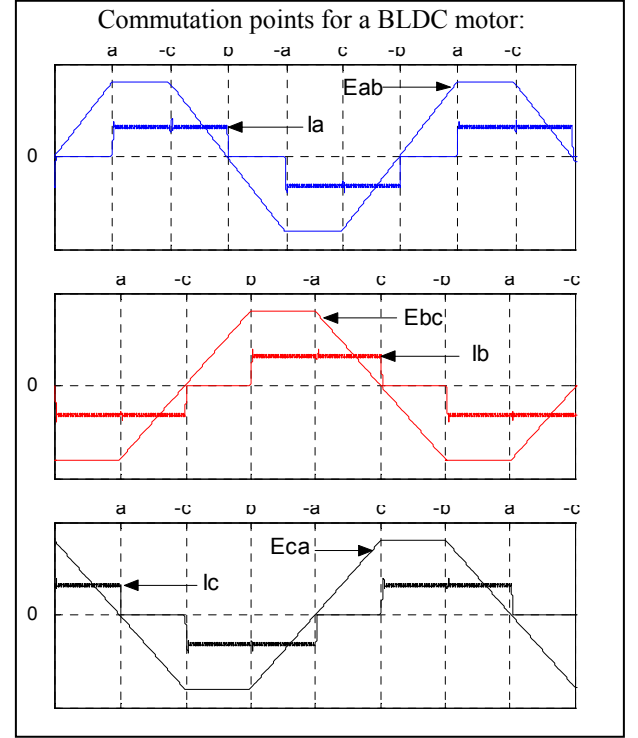


Fig.2: The currents phases ( $I_a$ ,  $I_b$  and  $I_c$ ) and the phase-to-phase back EMF ( $E_{ab}$ ,  $E_{bc}$  and  $E_{ca}$ )

## III. SLIDING MODE OBSERVER IN BRUSHLESS DC MOTOR

### A. Brushless DC motor model

In this kind of machine, this is only two of the three phases conducting at any time, as the stator winding neutral point of the machine is floating and not accessible, which makes it impossible to directly measure phase voltages. The BLDCM is modelled in the stationary reference frame  $abc$  using the subtractions currents ( $I_a - I_b$ ,  $I_b - I_c$ ,  $I_c - I_a$ ) which measured through tow phases, the phase-to-phase back EMF ( $E_{ab}$ ,  $E_{bc}$ ,  $E_{ca}$ ) (fig.2) and the phase-to-phase voltages ( $U_{ab}$ ,  $U_{bc}$ ,  $U_{ca}$ ). The following model has been derived:

$$\begin{cases} \frac{d(I_a - I_b)}{dt} = -\frac{R}{L}(I_a - I_b) - \frac{1}{L}E_{ab} + \frac{1}{L}U_{ab} \\ \frac{d(I_b - I_c)}{dt} = -\frac{R}{L}(I_b - I_c) - \frac{1}{L}E_{bc} + \frac{1}{L}U_{bc} \\ \frac{d(I_c - I_a)}{dt} = -\frac{R}{L}(I_c - I_a) - \frac{1}{L}E_{ca} + \frac{1}{L}U_{ca} \end{cases} \quad (1)$$

To reduce the order of the model (1), only the electric quantities of the two phases ( $a-b$ ) respectively ( $b-c$ ) are taken. The following traditional assumptions are made:

- The distribution of the phase-to-phase back EMF is trapezoidal, and its variation is very slow
- The motor is unsaturated.
- The armature reaction is negligible.

The model (2) is written:

$$\begin{cases} \frac{d(I_a - I_b)}{dt} = -\frac{R}{L}(I_a - I_b) - \frac{1}{L}E_{ab} + \frac{1}{L}U_{ab} \\ \frac{d(I_b - I_c)}{dt} = -\frac{R}{L}(I_b - I_c) - \frac{1}{L}E_{bc} + \frac{1}{L}U_{bc} \\ \frac{dE_{ab}}{dt} = 0 \\ \frac{dE_{bc}}{dt} = 0 \end{cases} \quad (2)$$

In order to deduce the third FCEM between the two phases ( $c-a$ ) and under the assumption that the system is balanced, the second model (3) can be easily described in the following form:

$$\begin{cases} E_{ab} + E_{bc} + E_{ca} = 0 \\ E_{ca} = -E_{ab} - E_{bc} \end{cases} \quad (3)$$

### B. Sliding Mode Observer

The observer presented in this paragraph is a very simple case of the sliding mode observer. It is a linear model disturbed by the phase-to-phase back EMF (external uncertainty) and the errors parametric (internal uncertainty) [8]. By using the measurements of the stator currents and the input phase-to-phase voltage, the objective is to observe the phase to phase back EMF (fig.3). Assuming the simplified model (2) the vector of state  $x = ((I_a - I_b), (I_b - I_c), E_{ab}, E_{bc})^T$ , and the vector input  $v = (U_{ab}, U_{bc})$ , the sliding mode observer can be easily described in the following form:

$$\begin{cases} \dot{\hat{x}}_1 = -\alpha_1 \hat{x}_1 - \alpha_2 \hat{x}_3 + \alpha_2 V_1 + K_1 I_s \\ \dot{\hat{x}}_2 = -\alpha_1 \hat{x}_2 - \alpha_2 \hat{x}_4 + \alpha_2 V_2 + K_2 I_s \\ \dot{\hat{x}}_3 = K_3 I_s \\ \dot{\hat{x}}_4 = K_4 I_s \end{cases} \quad (4)$$

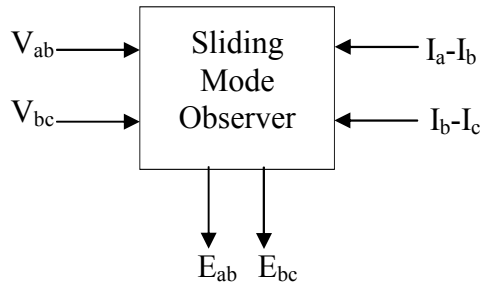


Fig.3: Sliding mode observer

Where  $\alpha_1 = \frac{R}{L}$ ,  $\alpha_2 = \frac{1}{L}$  and  $K_1 = (k_{11}, k_{12})$ ,  $K_2 = (k_{21}, k_{22})$ ,  $K_3 = (k_{31}, k_{32})$ ,  $K_4 = (k_{41}, k_{42})$  are the gains of sliding mode observer. The sliding surface  $S$  is given by:

$$S = \begin{bmatrix} x_1 - \hat{x}_1 \\ x_2 - \hat{x}_2 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (11) \quad I_s = \begin{bmatrix} \text{sign}(s_1) \\ \text{sign}(s_2) \end{bmatrix}$$

Let us define the tracking errors

$$e = \hat{x} - x \quad (5)$$

Its dynamic is governed by:

$$\begin{cases} \dot{e}_1 = -\alpha_2 e_3 - K_1 I_s \\ \dot{e}_2 = -\alpha_2 e_4 - K_2 I_s \\ \dot{e}_3 = -K_3 I_s \\ \dot{e}_4 = -K_4 I_s \end{cases} \quad (6)$$

In order to ensure the asymptotic convergence of the tracking errors  $e = [e_1 \ e_2 \ e_3 \ e_4]^T$  to zero, the following assumptions should be formulated:

**A1:** the system is uncoupled ( $X_1$  does not depend on  $X_2$ )

$$K_{12} = K_{21} = K_{32} = K_{41} = 0.$$

**A2:** The gain  $k_{11}$ ,  $k_{22}$  must be chosen so that

$$k_{11} > \alpha_2 |e_3|_{\max} \quad \text{and} \quad k_{22} > \alpha_2 |e_4|_{\max} \quad (7)$$

**A3:** The gain  $k_{31}$ ,  $k_{42}$  must be chosen so that

$$\frac{k_{31}}{k_{11}} < 0 \quad \text{and} \quad \frac{k_{42}}{k_{22}} < 0$$

**Proof:** Define the Lyapunov function candidate:

$$V(t) = \frac{1}{2} e_{1,2}(t) e_{1,2}(t)^T \quad \text{With } e_{1,2}(t) = [e_1(t) \ e_2(t)]^T$$

Its time derivative is calculated as:

$$\dot{V} = e_{1,2}^T \dot{e}_{1,2}$$

$$\dot{V} = [e_1 \ e_2] \left[ \begin{pmatrix} -\alpha_2 e_3 \\ -\alpha_2 e_4 \end{pmatrix} - \begin{pmatrix} k_{11} & 0 \\ 0 & k_{22} \end{pmatrix} \begin{pmatrix} \text{sign}(e_1) \\ \text{sign}(e_2) \end{pmatrix} \right] \quad (8)$$

Then:

$$\dot{V} = e_1 (-\alpha_2 e_3 - k_{11} \text{sign}(e_1)) + e_2 (-\alpha_2 e_4 - k_{22} \text{sign}(e_2)) \quad (9)$$

It is possible to find the conditions of  $K_{11}$  and  $K_{22}$  such as  $\dot{V} < 0$  in order to force the asymptotic convergence of the error  $e_{1,2}(t)$  to zero. According to the equation (9), these conditions are given by

- If the error  $e_1 > 0$  the gain switching is  $k_{11} > -\alpha_2 e_3$ ;

- If the error  $e_1 < 0$  the gain switching is  $k_{11} > \alpha_2 e_3$  ;
- If the error  $e_2 > 0$  the gain switching is  $k_{22} > -\alpha_2 e_4$  ;
- If the error  $e_2 < 0$  the gain switching is  $k_{22} > \alpha_2 e_4$  .

The vector of the tracking errors  $e(t)$  is bounded all time  $t$ , then this condition can be written in the following form:

$$k_{11} > \alpha_2 |e_3|_{\max} \quad \text{and} \quad k_{22} > \alpha_2 |e_4|_{\max} \quad (10)$$

Consequently  $e_{1,2}(t)$  is globally asymptotically stable. This system's behaviour once on the sliding surface is usually called *sliding mode*.

$$\dot{V} < 0 \quad \text{For } e \neq 0$$

$$\dot{V} = 0 \quad \text{For } e = 0$$

When the sliding mode occurs on the sliding surface (5), then  $e_{1,2}(t) = \dot{e}_{1,2}(t) = 0$ , and therefore the dynamic behaviour of the tracking problem can be written as:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\alpha_2 e_3 \\ -\alpha_2 e_4 \end{bmatrix} - \begin{bmatrix} k_{11} & 0 \\ 0 & k_{22} \end{bmatrix} I_s \quad (11)$$

$$\bar{I}_s = \begin{bmatrix} \frac{-\alpha_2 e_3}{k_{11}} \\ \frac{-\alpha_2 e_4}{k_{22}} \end{bmatrix} \quad (12)$$

The vector  $I_s \stackrel{\Delta}{=} \bar{I}_s$  represents the equivalent output error injection term necessary to maintain a sliding mode on  $S$ . Using the equivalent output (12), the reduced order sliding mode is governed by:

$$\begin{aligned} \dot{e}_3 &= \frac{k_{31} \alpha_2}{k_{11}} e_3 \\ \dot{e}_4 &= \frac{k_{42} \alpha_2}{k_{22}} e_4 \end{aligned} \quad (13)$$

Then, under assumption A3 the tracking error  $e(t)$  converge to zero exponentially.

#### IV. ROTOR POSITION ESTIMATION

From the phase-to-phase back EMF (fig.2), the detection of six positions of the rotor can be determined easily, by using a zero crossing detectors (ZCD) of this phase-to-phase back EMF. For example, during the switch commutation 'a' of the transistor 'Ta', one can deduce that the phase-to-phase back

EMF  $E_{ab}$  is always positive and that the phase-to-phase back  $E_{ca}$  is always negative, this is mean that it is possible to obtain a sufficient condition to extract the state switch commutation 'a' of the transistor 'Ta': it is thus enough to have  $E_{ab} > 0$  and  $E_{ca} < 0$ . For the other switch commutations the same logic of analysis is followed. A digital circuit (fig.4) can be now easily developed.

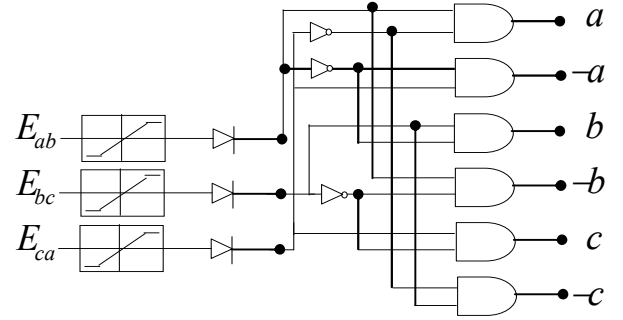


Fig.4: The zero crossing detector circuit

#### V. ROTOR SPEED ESTIMATION

For the application of the sensorless drives in which high estimation accuracy of rotor speed is required, the Extended Kalman Filter has been used to obtain the instantaneous position and speed. According to the drawbacks mentioned previously, the objective of this paper is to use only the observer sliding mode. Then a novel method is used, which based on the relation mathematical between the magnitude  $E_{\max(\text{phase-to-phase})}$  and rotor speed (14).

The rotor speed can be estimated using only the magnitude  $E_{\max(\text{phase-to-phase})}$  tracking by sliding mode observer.

$$\omega_r = \frac{E_{\max(\text{phase-to-phase})}}{2K_{EMF}} \quad (14)$$

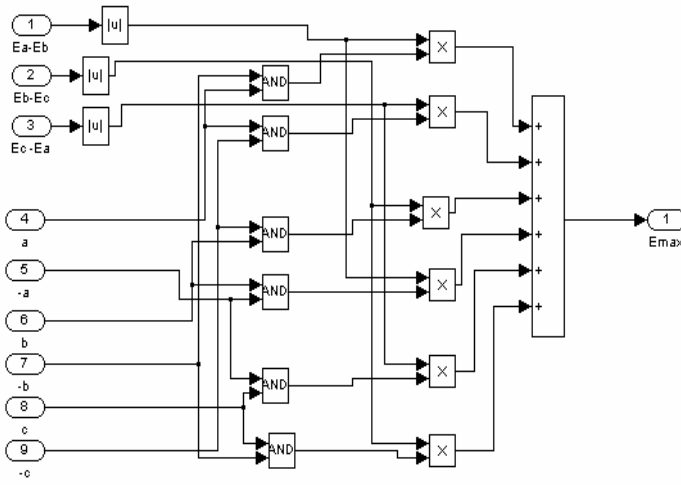
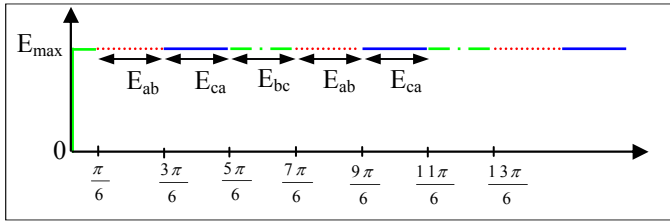
With:

$$E_{\max(\text{phase-to-neutral})} = K_{EMF} \omega_r$$

$$E_{\max(\text{phase-to-neutral})} = \frac{E_{\max(\text{phase-to-phase})}}{2}$$

Where  $K_{EMF}$  is the constant of the back EMF.

A logical circuit (fig.5) is developed using the phase-to-phase back EMF observed by observer sliding mode and the six points of commutations determined through the circuit digital of detection de switch commutations (fig.4), the  $E_{\max}$  amplitude is obtained (fig.6)

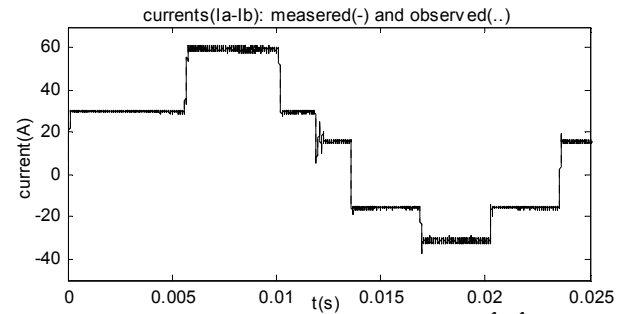
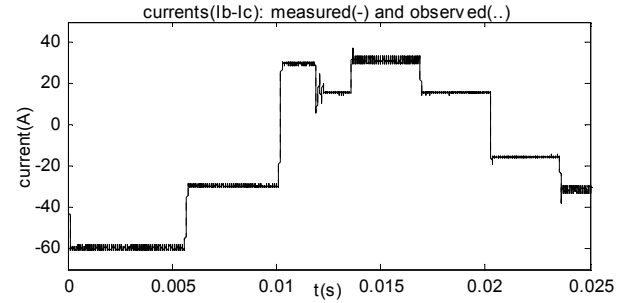
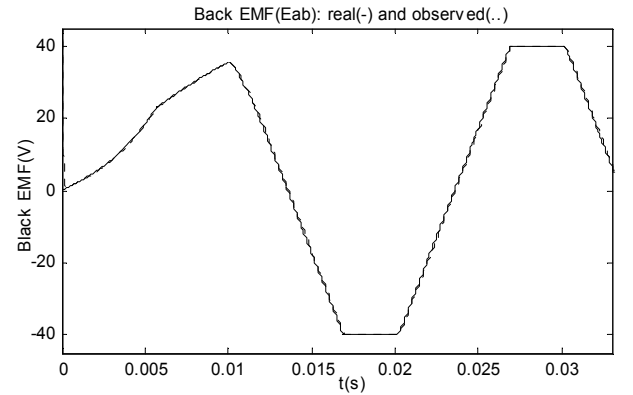
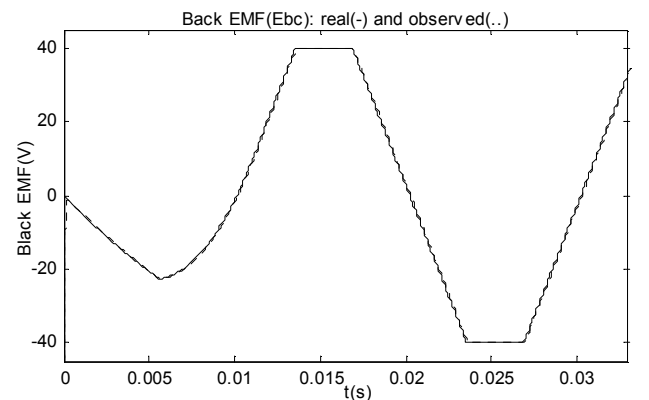
Fig. 5:  $E_{max}$  detection circuitFig. 6: The amplitude  $E_{max}$  of the phase-to-phase back EMF

## VI. APPLICATION AND SIMULATIONS

In order to study the performance of the proposed sliding mode observer for position and speed estimations in BLDCM drive, a detailed digital computer program using simulation package Matlab/simulink was developed. The parameters of the three-phase six-pole permanent magnet brushless DC motor are given by:

- permanent magnet brushless DC motor:  $V=144V$ ,  
 $I=120A$ ,  $R=12m\Omega$ ,  $L=150\mu H$ , and  
 $K_{EMF}=20V/krpm$

The fig.7-8 show the currents were applied as magnitude measurement to the sliding mode observer by using the subtraction of the two measured currents  $I_a$  and  $I_b$ , respectively ( $I_b$  and  $I_c$ ). This subtraction expresses the real current which circulates between two phases ( $a-b$ ) respectively ( $b-c$ ). In the response of the sliding mode observer, the current observed ( $\hat{I}_a-\hat{I}_b$ ) respectively ( $\hat{I}_b-\hat{I}_c$ ) follow the same measured current ( $I_a-I_b$ ) respectively ( $I_b-I_c$ ). The phase-to-phase back EMF follows closely the ideal trapezoidal phase-to-phase back EMF of the motor (fig.9 and fig.10).

Fig. 7: The measured  $I_a-I_b$  current and observed  $\hat{I}_a-\hat{I}_b$  currentFig. 8: The measured  $I_b-I_c$  current and observed  $\hat{I}_b-\hat{I}_c$  currentFig. 9: The real phase-to-phase back EMF  $E_{ab}$  and The observed phase-to-phase back EMF  $\hat{E}_{ab}$ Fig. 10: The real phase-to-phase back EMF  $E_{bc}$  and The observed phase-to-phase back EMF  $\hat{E}_{bc}$

The zero crossing detectors (ZCD) are given to the digital inputs (fig.11). These digital inputs forms a bus such as its digital value is used to change the firings sequence of transistors inverter and to detect the amplitude  $E_{max}$  of the trapezoidal back EMF observed which used to determine the speed estimation (fig.12). In this response the speed estimation  $\hat{\omega}_r$  follows closely the speed measurement  $\omega_r$  of the motor. It can be seen in (fig.13), the speed tracking error is reaching at zero point.

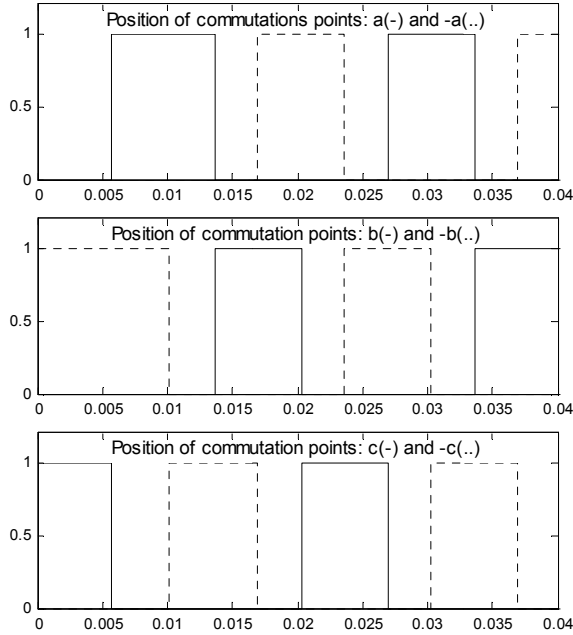


Fig.11: Estimated position of commutation points

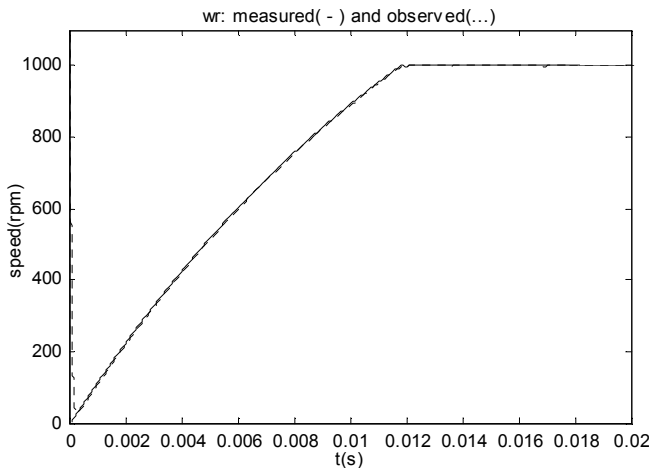


Fig.12: Actual speed  $\omega_r$  (measured) and estimated speed  $\hat{\omega}_r$  (observed).

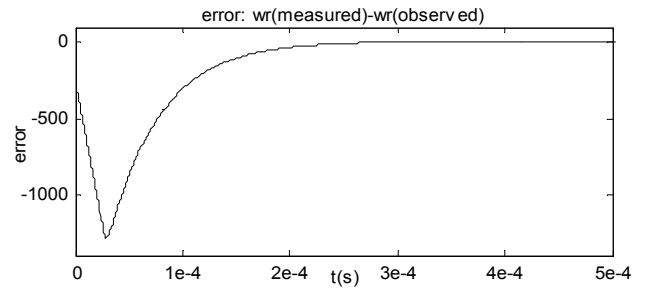


Fig.13: Speed tracking error:  $[\omega_r(\text{measured}) - \omega_r(\text{observed})]$

## VII. CONCLUSION

Using the presented method, which applied the sliding mode observer, the speed and rotor position of the BLDCM is estimated from the phase-to-phase back EMF. The results obtained by simulation show the effectiveness of the method. In our future work we plan to make an experimental verification of the proposal method.

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