International Conference on Power, Signals, Controls and Computation (EPSCICON), 8 – 10 January 2014

Robustness Evaluation of SMO in Sensorless Control of BLDC Motor under DTC Scheme

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***Abstract*-- Direct Torque Control (DTC) is a method to control the torque in variable frequency drives. This paper describes sensorless DTC of brushless dc (BLDC) motor drive operating in constant torque region under two phase conduction mode to get instantaneous torque control. Sliding Mode Observer (SMO) which is low sensitive to parameter variation is proposed to estimate the phase-to-phase trapezoidal back-EMF for the sensorless operation. This estimated back-EMF is used to deduce the rotor position and the angular velocity of the rotor. And instantaneous electromagnetic torque can be calculated by the product of back-EMF and current. The paper focuses the robustness evaluation of the Sliding Mode Observer to the variation of motor parameters and the effectiveness is also investigated with signum and saturation functions for DTC scheme.**

***Index Terms*-- Brushless dc motor (BLDCM); Direct torque control (DTC); non-sinusoidal back-EMF; Sliding mode observer(SMO); two-phase conduction**

I. INTRODUCTION

A BLDC motor is an inside-out DC commutator motor with the mechanical commutator replaced by an electronic switching converter. The demand of BLDC motor in domestic as well as industrial applications upturns due to their high efficiency, higher torque and power density, lower cost, simpler structure, better controllability and large torque to inertia ratio compared to brushless AC motors. The most popular way to control a BLDC motor is via voltage –source current-controlled inverters. The inverter must supply a rectangular current waveform whose magnitude is proportional to the motor's shaft torque. The back-EMF waveform of a BLDC motor is trapezoidal shape due to the concentrated winding.

In this paper BLDC with 120º conduction modes is proposed, that means only two-phase conduct at any instant of time. BLDC motor fed by two-phase conduction has higher power/weight and torque/current ratios [1]. Ideally a BLDC motor supplied with rectangular 120º elec. Phase currents produce a trapezoidal back-EMF waveform whose amplitude is constant over 120º elec. will result a ripple free torque. However, in a practical BLDC drive torque pulsation arise due to the deviation of back-EMF waveform from the ideal.

In DTC scheme, the torque command obtained from two-level hysteresis controller by comparing the estimated electro-magnetic torque with their reference value which is obtained from speed error. For the control, voltage vector is selected from a look-up table which depends on rotor flux vector position and the torque error to reduce switching frequency and torque ripple in commutation region [1].

Hall-effect sensors are usually used as position sensors to know the position of commutation points. Normally these sensors are mounted on the stator with 120º apart. These sensors increase the cost, size and weight of the motor and reduce the reliability of the total system. To overcome these problems, instead of using position sensors, the sensorless method has been developed to estimate the position and velocity of the rotor from the estimate of phase-to-phase back-EMF using sliding mode observer. The proposed observer is easy to design and has robustness against design parameters [2]-[7].

Sliding mode observer is a non-linear high gain observer has the ability to bring co-ordinates of the estimator error dynamics to zero in finite time. In this paper, robustness of sliding mode observer for the speed and position estimation is studied by varying the stator inductance and resistance.

The paper is organized as follows. In sect. II, the mathematical model of BLDC motor is described. In sect. III, the direct torque control of BLDC motor is explained. In sect. IV, design procedures of sliding mode observer is discussed. Sect. V deals with the estimation of electrical rotor speed and position. In sect. VI, the robustness of SMO is evaluated. The simulation results and its analysis are given in sect. VII.

II. MATHEMATICAL MODEL OF BLDC MOTOR

Three phase star connected BLDC motor can be described by the following equations,

$$\begin{aligned} v_{ab} &= R_s(i_a - i_b) + L_s\frac{d}{dt}(i_a - i_b) + e_a - e_b \\ v_{bc} &= R_s(i_b - i_c) + L_s\frac{d}{dt}(i_b - i_c) + e_b - e_c \\ v_{ca} &= R_s(i_c - i_a) + L_s\frac{d}{dt}(i_c - i_a) + e_c - e_a \end{aligned} \tag{1}$$

The equation of motion can be expressed as:

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$$T_e = T_L + B\omega_m + J\frac{d\omega_m}{dt} \quad (2)$$

Where v, i and e denote the phase-to-phase voltage, phase current and back-emf respectively in the three phase a, b and c. R_S and L_S denote the line-to-line resistance and inductance of stator winding. T_e is the electromagnetic generated torque, T_L is the load torque, B is the friction coefficient, J is the polar moment of inertia and ω_m is the angular velocity of rotor [3],[4].

III. Direct Torque Control of BLDC Motor

Transforming the state equation of BLDC motor in α-β stationary reference frame can be written as:

$$v_{s\alpha} = R_s i_{s\alpha} + L_s\frac{di_{s\alpha}}{dt} + e_\alpha$$
$$v_{s\beta} = R_s i_{s\beta} + L_s\frac{di_{s\beta}}{dt} + e_\beta \quad (3)$$

Where $v_{s\alpha}$, $v_{s\beta}$, $i_{s\alpha}$, $i_{s\beta}$, e_α, e_β are the stator voltage, stator current and back-emf respectively in the α-β stationary reference frame.

Electromagnetic torque for DTC can be expressed as:

$$T_e = \frac{3p}{4}\left[\frac{d\psi_{r\alpha}}{d\theta_e}i_{s\alpha} + \frac{d\psi_{r\beta}}{d\theta_e}i_{s\beta}\right] \quad (4)$$

Where $\psi_{r\alpha}$ and $\psi_{r\beta}$ are the α-β axis rotor flux vector components, p is the no. of poles and θ_e is the rotor electrical angle.

The differential forms of rotor flux components respect to θ_e can be derived from the ratio of the back-EMF to the electrical angular velocity ω_e. i.e.,

$$\frac{d\psi_{r\alpha}}{d\theta_e} = \frac{d\psi_{r\alpha}}{dt}\frac{dt}{d\theta_e} = \frac{1}{\omega_e}\frac{d\psi_{r\alpha}}{dt} = \frac{e_\alpha}{\omega_e}$$
$$\frac{d\psi_{r\beta}}{d\theta_e} = \frac{d\psi_{r\beta}}{dt}\frac{dt}{d\theta_e} = \frac{1}{\omega_e}\frac{d\psi_{r\beta}}{dt} = \frac{e_\beta}{\omega_e} \quad (5)$$

Where $\omega_e = \frac{d\theta_e}{dt}$

Then the electromagnetic torque can be written as,

$$T_e = \frac{3p}{4}\left[\frac{e_\alpha}{\omega_e}i_{s\alpha} + \frac{e_\beta}{\omega_e}i_{s\beta}\right] \quad (6)$$

Rotor angular velocity for the electromagnetic torque calculation can be obtained from the estimate of sliding mode observer.

For the direct torque control, a switching pattern of inverter is selected according to the output of hysteresis controller and the flux vector position.

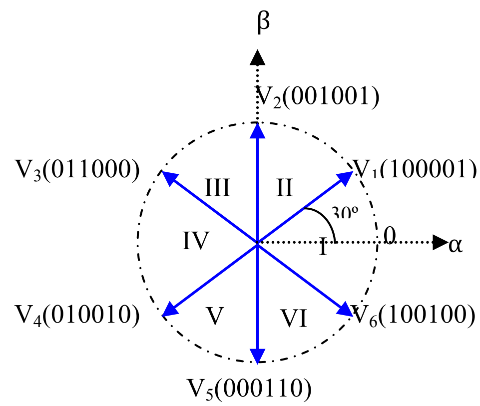

Fig. 1 Two-phase voltage space vectors and sectors

Fig. 1 shows the six voltage vectors and the sectors. The proper voltage vector, V_i for DTC scheme is selected from the switching table as shown in Table. I. τ denotes the output of hysteresis controller and the sector is denoted as θ_r. If the reference torque greater than the actual torque, within the hysteresis band limit, the output of the hysteresis controller is defined as τ=1 otherwise τ= -1, out of the six voltage vector, the proper voltage vector for inverter is selected based on the switching table.

Using rotor flux observer, the rotor flux vector components for (5) can be calculated,

$$\psi_{r\alpha} = -L_s i_{s\alpha} + \int\left(v_{s\alpha} - R_s i_{s\alpha}\right)dt$$
$$\psi_{r\beta} = -L_s i_{s\beta} + \int\left(v_{s\beta} - R_s i_{s\beta}\right)dt \quad (7)$$

To control electromagnetic torque in DTC scheme, the rotor flux vector position can be obtained as,

$$\theta_r = \tan^{-1}\left(\frac{\psi_{r\beta}}{\psi_{r\alpha}}\right) \quad (8)$$

TABLE I
Switching Table for DTC of BLDC Drive

τ	θ_r					
	I	II	III	IV	V	VI
1	V_2	V_3	V_4	V_5	V_6	V_1
-1	V_5	V_6	V_1	V_2	V_3	V_4

IV. Sliding Mode Observer

Sliding mode observer is used for the phase-to-phase back-EMF estimation accurately.

By rearranging (3) and neglecting the zero sequence components, the state equation for SMO can be written as,

$$\frac{di_{s\alpha}}{dt} = -\frac{R_s}{L_s}i_{s\alpha} - \frac{e_\alpha}{L_s} + \frac{v_{s\alpha}}{L_s}$$
$$\frac{di_{s\beta}}{dt} = -\frac{R_s}{L_s}i_{s\beta} - \frac{e_\beta}{L_s} + \frac{v_{s\beta}}{L_s} \quad (9)$$

In order to setup a back-EMF observer the phase-to-phase back-EMF components can be considered as disturbance with the following associated model,

$$\frac{de_\alpha}{dt} = 0 \text{ and } \frac{de_\beta}{dt} = 0$$

Sliding mode observer is proposed as:

$$\begin{aligned}
\frac{d\hat{i}_{s\alpha}}{dt} &= -\frac{R_s}{L_s}\hat{i}_{s\alpha} - \frac{\hat{e}_\alpha}{L_s} + \frac{v_{s\alpha}}{L_s} + K_{11}\operatorname{sgn}(\tilde{i}_{s\alpha}) \\
\frac{d\hat{i}_{s\beta}}{dt} &= -\frac{R_s}{L_s}\hat{i}_{s\beta} - \frac{\hat{e}_\beta}{L_s} + \frac{v_{s\beta}}{L_s} + K_{22}\operatorname{sgn}(\tilde{i}_{s\beta}) \\
\frac{d\hat{e}_\alpha}{dt} &= K_{31}\operatorname{sgn}(\tilde{i}_{s\alpha}) \\
\frac{d\hat{e}_\beta}{dt} &= K_{42}\operatorname{sgn}(\tilde{i}_{s\beta})
\end{aligned} \tag{10}$$

Where $\hat{i}_{s\alpha}, \hat{i}_{s\beta}, \hat{e}_\alpha, \hat{e}_\beta$ are the estimation of α-β axes stator currents and back-EMFs respectively.

The sliding surface S is defined as the error between the actual and estimated currents ($\tilde{i}_s = i_s - \hat{i}_s$), it is represented as

$$S = \left[S_{\alpha,} S_\beta\right]^T \tag{11}$$

Where, $S = \begin{pmatrix} S_\alpha \\ S_\beta \end{pmatrix} = \begin{pmatrix} i_{s\alpha} - \hat{i}_{s\alpha} \\ i_{s\beta} - \hat{i}_{s\beta} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

When the estimation error trajectories reach the sliding surface i.e., S=0, the observed currents will converge to actual.

To find the observer gain, consider the positive definite Lyapunov function as:

$$V(\tilde{i}_s) = \tfrac{1}{2}\left(\tilde{i}_{s\alpha}^{\,2} + \tilde{i}_{s\beta}^{\,2}\right) \tag{12}$$

Sliding mode occur when $\dot{V} = S^T \dot{S} \langle 0$. Once the sliding mode achieved, the system will remain on that sliding mode [5]-[7]. Derivative of $V(\tilde{i}_s)$ with respect to time concludes

$$\begin{aligned}
\frac{dV}{dt} &= -\frac{R_s}{L_s}\left(\tilde{i}_{s\alpha}^{\,2} + \tilde{i}_{s\beta}^{\,2}\right) - \frac{1}{L_s}\left(\tilde{e}_\alpha \tilde{i}_{s\alpha} + \tilde{e}_\beta \tilde{i}_{s\beta}\right) \\
&- K_{11}\left|\tilde{i}_{s\alpha}\right|^2 - K_{22}\left|\tilde{i}_{s\beta}\right|^2 < 0
\end{aligned} \tag{13}$$

In order to ensure the convergence of the observer, the gains should satisfy the following conditions,

(a) $K_{11} > \frac{1}{L_s}\left|e_\alpha\right|_{\max}$ and $K_{22} > \frac{1}{L_s}\left|e_\beta\right|_{\max}$

(b) $\frac{K_{31}}{K_{11}} < 0$ and $\frac{K_{42}}{K_{22}} < 0$

Here $K_{S1} = K_{11} = K_{22}$ and $K_{S2} = K_{31} = K_{42}$

The signum function is defined as:

$$sign(S) = \begin{cases} 1; S > 0 \\ 0; S = 0 \\ -1; S < 0 \end{cases} \tag{14}$$

To decrease chattering effect, the signum function in (14) is replaced with saturation function.
Saturation function is defined as:

$$sat(S) = \begin{cases} \dfrac{S}{\varepsilon}; |S| \le \varepsilon \\ sign(S); |S| > \varepsilon \end{cases} \tag{15}$$

Where ε is the sliding surface band.

In order to reduce the pure integrator influence in the back-EMF estimation, a PI controller is used in this observer [2].

V. ROTOR POSITION AND SPEED ESTIMATION

The detection of the six rotor positions (θ_r) for the proper commutation of a BLDC motor can be easily determined from the estimated back-EMFs in the α-β stationary reference frame [5]-[7]. The estimated rotor position as follows,

$$\hat{\theta}_r = \tan^{-1}\left(\frac{\hat{e}_\beta}{\hat{e}_\alpha}\right) \tag{16}$$

The back-EMFs signals contain the rotor speed information. In order to estimate the instantaneous rotor angular velocity, the following mathematical relation is considered.

$$\omega_r = \frac{E_{\max(phase\text{-}to\text{-}phase)}}{2K_e} \tag{17}$$

Where, $E_{\max(phase\text{-}to\text{-}phase)}$ is the amplitude of phase-to-phase back-EMFs and K_e is the back-EMF constant [4].

VI. ROBUSTNESS EVALUATION WITH PARAMETER VARIATION

Low sensitive to parameter variation is the well-known feature of sliding mode observer. This feature of SMO is evaluated by varying the motor parameters such as inductance and resistance.

- *Variation in R only,*

  The first set of simulations were carried out by keeping the value of inductance at L=0.6H for rated speed and changing the value of resistance, R. The original value of motor resistance is 70 ohms. The accurate back-EMF estimation is possible for a range of value of R, 64.6 – 95 ohms by using SMO with signum function and a range from 64 to 95 ohms with saturation function as control law.

- *Variation in L only,*

  The second set of simulations were carried out by keeping the value of resistance at R=70 ohms for rated speed and changing the value of inductance, L. The original value of motor inductance is 0.6H. For a range of value of L, 0.5 –0.7H, the proposed observer with signum and saturation functions estimate the back-EMF accurately.

- *Combined variation in R and L,*

The combined effect of motor parameters is evaluated by changing their values simultaneously. By changing motor inductance for a range 0.5 – 0.7H, the accurate estimation of back-EMF can be possible in the range of values of R, 67–75 ohms by using SMO with signum function and the range of R, 69.5 – 70.5 ohms with saturation function. By changing motor resistance for a range 64–95 ohms, the proposed observer with signum and saturation function can accurately estimate the back-EMF for a range of values of inductance in between 0.57–0.63H and 0.59–0.6H respectively.

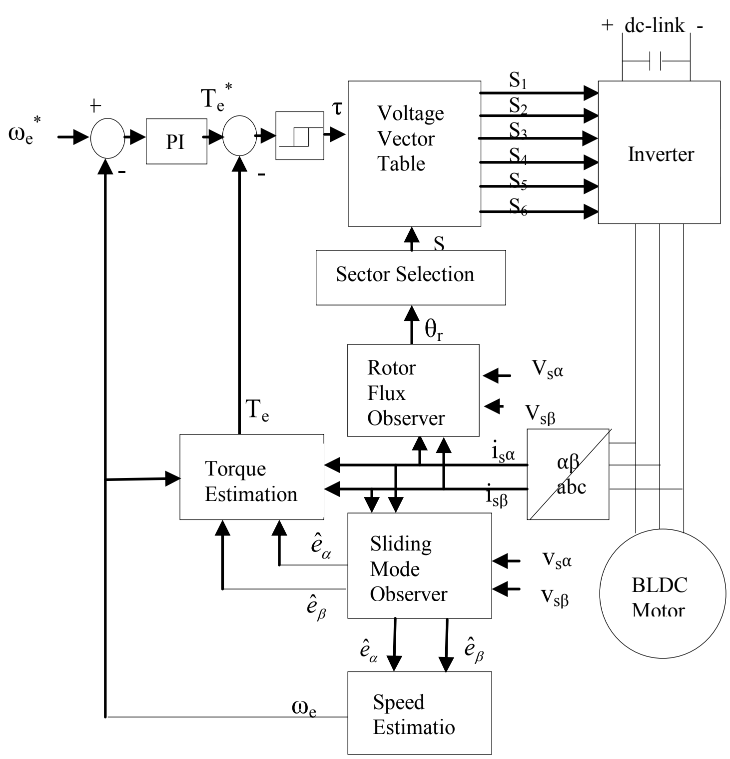

Fig 2.block diagram of direct torque control of BLDC drive

VII. SIMULATION RESULT

The overall block diagram of sliding mode observer based sensorless direct torque control of BLDC drive shown in fig. 2 was simulated by MATLAB/Simulink. Motor parameters used for the simulation are listed in Table. II and the sliding mode observer co-efficients are given in Table. III

In order to check the effectiveness of proposed observer against parameter variation, certain numbers of simulations were done. From the analysis of above simulations, it is found that the proposed observer can estimate the back-EMF accurately for the wide range of variation in motor parameters. And the sliding mode observer is less sensitive to parameter uncertainties.

TABLE II
PARAMETERS OF BLDC MOTOR

DC-link voltage(V)	300
No. of poles	16
Line-to-Line inductance(H)	0.6
Line-to-Line resistance(ohms)	70
Back-EMF constant(V/(rad/sec))	3.36
Polar moment of inertia(Kg.m^2)	0.012
Frictional co-efficient(N.m.sec)	0.001
Rated speed(rpm)	350

TABLE III
SLIDING MODE OBSERVER PARAMETERS

K_{S1}	900
K_{S2}	-260
K_P	30
K_I	1
Sliding surface band, ε	0.05

The actual and estimated α-β stationary reference frame phase-to-phase back-EMF, Eab using signum and saturation functions are shown in Fig.3 and Fig.4 respectively. Transforming the estimated $\hat{e}_\alpha$ and $\hat{e}_\beta$ in to *abc*-frame by modified inverse Clark's transformation [8], the phase-to-phase back-EMFs $\hat{e}_{ab}$ and $\hat{e}_{ca}$ is obtained. When sliding mode observer with signum function is implemented to estimate the back-EMF of BLDC motor the resulted graph has chattering due to the highly discontinuous nature of the signum function. In order to reduce the chattering, instead of signum function, saturation is given. Then it is observed that the chattering is reduced to great extent.

Fig.5 and Fig.6 shows the actual and estimated rotor position using SMO with signum and saturation functions respectively. In fig.5 the estimated rotor position shows the effect of chattering problem. When the saturation function used as a control signal in SMO, the effect of chattering considerably reduced which is shown in Fig.6. The above graphs show a phase delay from the actual rotor position because of the introduction of low pass filter used in the rotor position estimator. The estimated rotor position error is not more than 3 degrees.

Fig.7 and Fig.8 shows the actual and estimated rotor speed. When the reference speed of 350rpm was given, the estimated speed tracks the actual speed which is shown in Fig.7. Fig.8 shows the speed response of SMO with saturation function as control signal; the estimated speed reaches its reference value very quickly.

The estimated electromagnetic torque for DTC scheme with signum and saturation functions are shown in Fig.9 and Fig.10

respectively. By comparing these results it is observed that commutation torque ripple is reduced to great extent by the application of saturation function as the control signal.

The rotor flux vector position which is the output of rotor flux observer and the six sectors for DTC operation is shown in Fig.11and Fig.12 respectively.

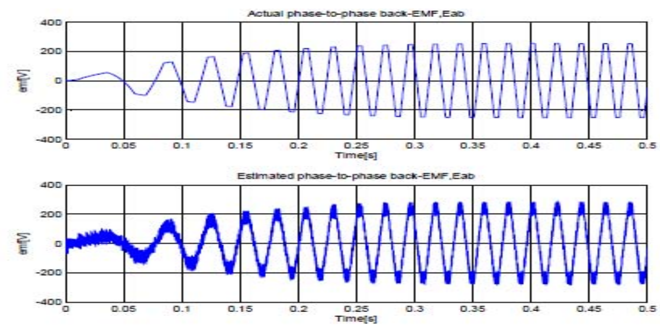

Fig.3 actual and estimated back-EMF using signum function

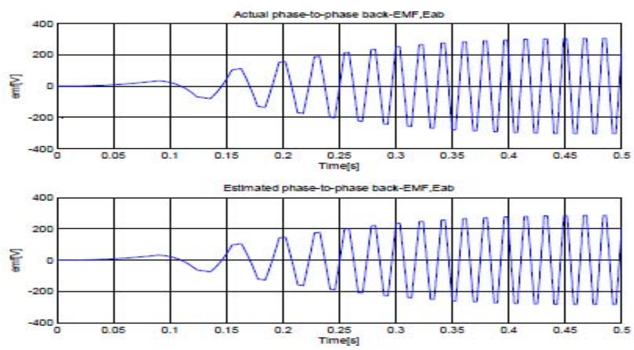

Fig.4 actual and estimated back-EMF using saturation function

actual estimated

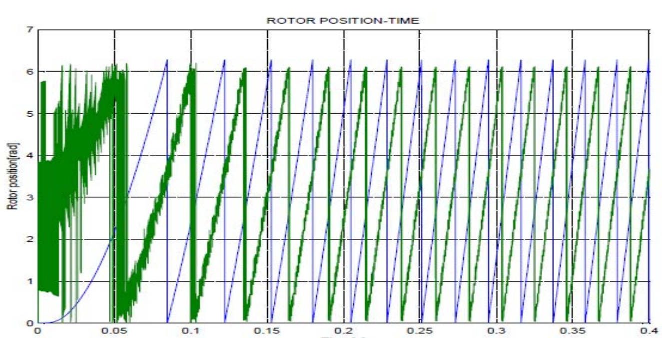

Fig.5 actual and estimated rotor position using signum function

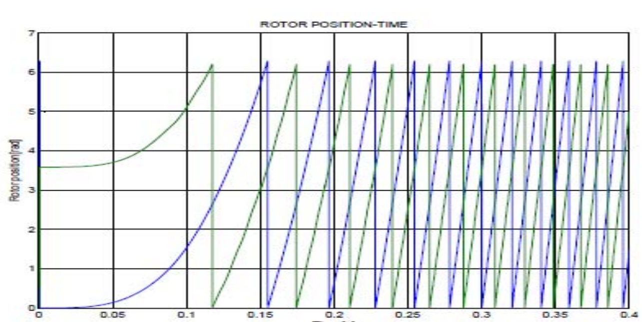

Fig.6 actual and estimated rotor position using saturation function

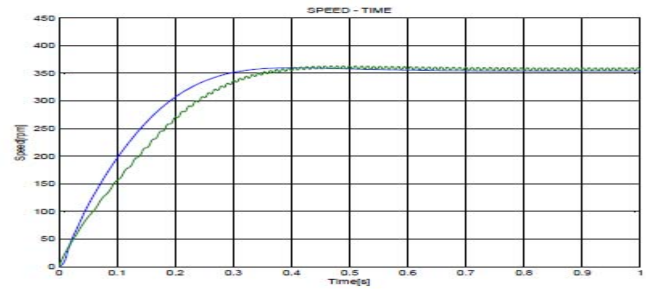

Fig.7 actual and estimated rotor speed using signum function

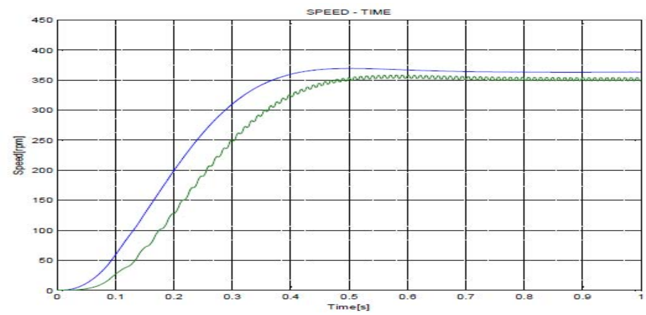

Fig. 8 actual and estimated rotor speed using saturation function

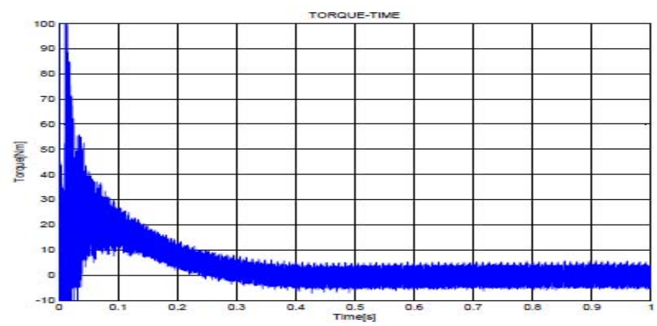

Fig.9 estimated torque using signum function

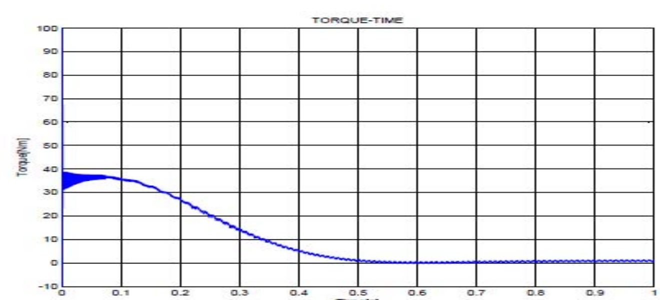

Fig.10 estimated torque using saturation function

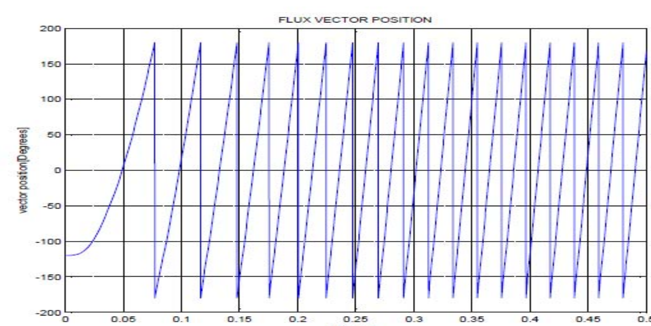

Fig.11 rotor flux vector position

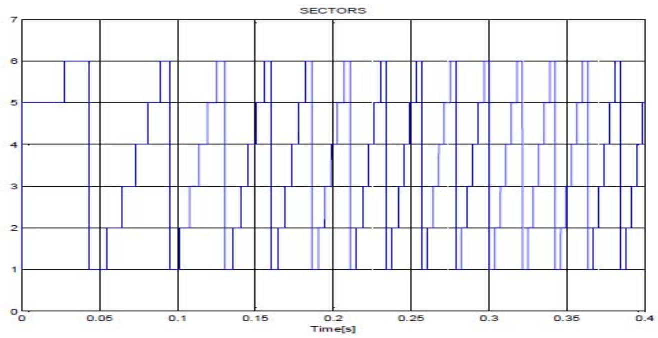

Fig.12 sector selection

From the analysis of these results, it is shown that the estimated outputs are accurate. However there is a problem of chattering effect in the estimated waveforms. By neglecting that effect of chattering, sliding mode observer is a very good sensorless method for the estimation of phase-to-phase back-EMFs, rotor position and angular velocity.

VIII. CONCLUSIONS

In this paper, an observer based sensorless direct torque control of brushless DC motor is developed. Using electromagnetic torque error and rotor flux vector position proper voltage vector for inverter switching was selected from look-up table to reduce the torque error within a predefined hysteresis band limit. The performance of SMO with signum and saturation functions for the electromagnetic torque estimation, rotor speed and position estimation and estimation of back-EMFs are verified under DTC scheme. The robustness of proposed observer against parameter uncertainties is validated by different values of motor parameters such as line-to-line inductance and resistance. The simulation results show the robustness is higher in SMO with signum function than the saturation function. However, SMO with saturation function has better speed response, reduced torque ripple and the less effect of chattering in the estimation of rotor position and back-EMF. It is a very good option for the estimation of back-EMFs, rotor position, and rotor speed respectively.

In sliding mode control, there is a problem with chattering effect due to the presence of switching imperfections, switching time delays and discontinuity in control. An analysis can be performed by replacing the discontinuous control functions such as signum and saturation functions with a continuous sigmoid function and compare the effect of chattering.

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