A Mathematics Student's Lament

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Introduction

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The HSC

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The SACE

3.1 Stage 1

3.1.1 Essential Mathematics

According to the SACE subject outline, Stage 1 Essential Mathematics covers the following topics.

1	Calculations, time, and ratio
2	Earning and spending
3	Geometry
4	Data in context
5	Measurement
6	Investing
7	Open topic

3.1.2 General Mathematics

According to the SACE subject outline, Stage 1 General Mathematics covers the following topics.

1	Investing and borrowing
2	Measurement
3	Statistical investigation
4	Applications of trigonometry
5	Linear and exponential functions and their graphs
6	Matrices and networks
7	Open topic

3.1.3 Mathematics

According to the SACE subject outline, Stage 1 Mathematics covers the following topics.

1	Functions and Graphs
2	Polynomials
3	Trigonometry
4	Counting and Statistics
5	Growth and Decay
6	Introduction to Differential Calculus
7	Arithmetic and geometric series and sequences
8	Geometry
9	Vectors in the plane
10	Further Trigonometry

3.2 Stage 2

3.2.1 Essential Mathematics

According to the SACE subject outline, Stage 2 Essential Mathematics covers the following topics.

1	Scales, plans, and models
2	Measurement
3	Business applications
4	Statistics
5	Investments and loans
6	Open topic

3.2.2 General Mathematics

According to the SACE subject outline, Stage 2 General Mathematics covers the following topics.

1	Modelling with linear relationships
2	Modelling with matrices
3	Statistical models
4	Financial models
5	Discrete models
6	Open topic

3.2.3 Mathematical Methods

According to the SACE subject outline, Stage 2 Mathematical Methods covers the following topics.

1	Further differentiation and applications
2	Discrete random variables
3	Integral calculus
4	Logarithmic functions
5	Continuous random variables
6	Sampling and confidence intervals

3.2.4 Specialist Mathematics

According to the SACE subject outline, Stage 2 Mathematical Methods covers the following topics.

1	Mathematical induction
2	Complex numbers
3	Functions and sketching graphs
4	Vectors in three dimensions
5	Integration techniques and applications
6	Rates of change and differential equations

Chapter 4

The QCE

Chapter 5

What's Missing?

Chapter 6

An Alternative

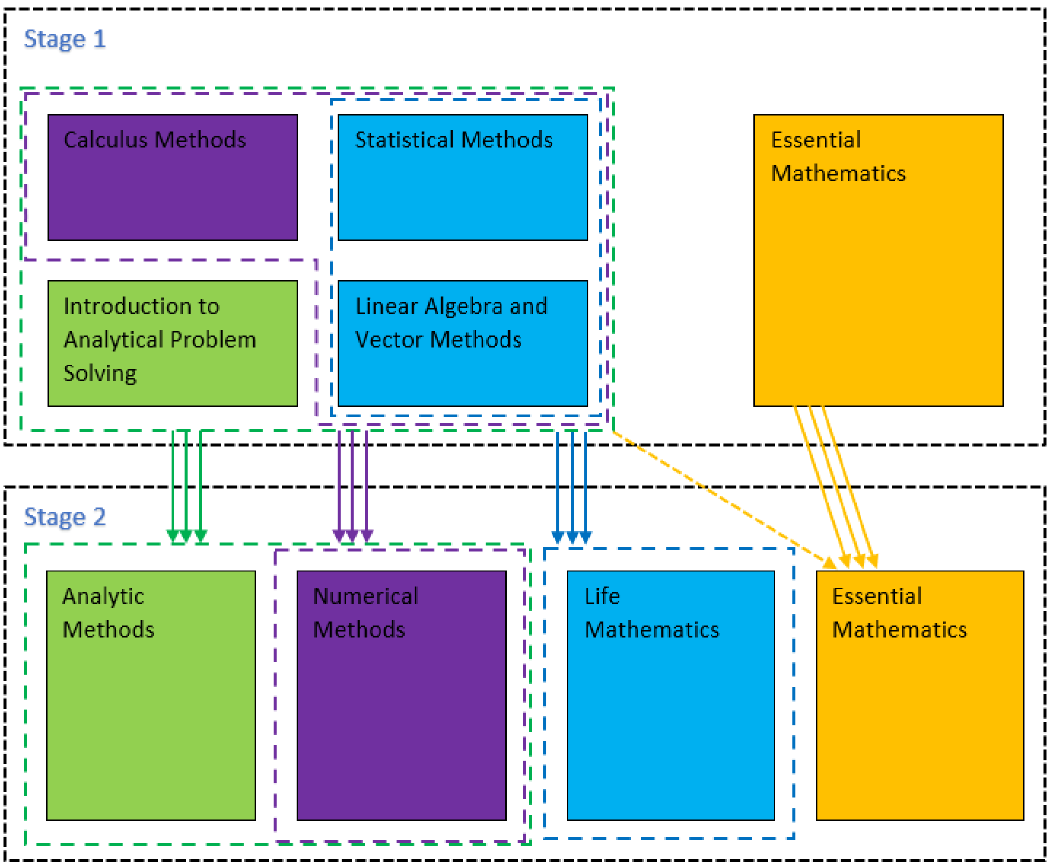

Figure 6.1: Subject structure

6.1 Stage 1 - Year 11

S1.1	The notion of probability and nPr/nCr Calculations
S1.2	Measures and Centre and Spread
S2	Continuous random variables
S3	Sampling, confidence intervals and hypothesis testing
L1	Geometric Trigonometry
L2	Vectors in the plane
L3	Matrices, networks and linear systems
C1.1	Functions and Graphs
C1.2	Polynomials
C2.1	Trigonometric functions
C2.2	Growth and Decay
C3	Introduction to Differential Calculus
A1	Geometry, Mathematical Problem Solving and Direct Proofs
A2	Sets, Elementary Set Operations and Relations
A3	Sequences, Series and Inductive Proofs
AX	The map of mathematics

Table 6.1: Main Cluster

6.1.1 Essential Mathematics

6.1.2 Statistical Methods

6.1.3 Calculus Methods

6.1.4 Linear Algebra Methods

6.1.5 Introduction to analytic problem solving

6.2 Stage 2 - Year 12

6.2.1 Essential Mathematics

6.2.2 Life Mathematics

6.2.3 Analytical Methods

Analytical methods should serves to develop the analytical skills necessary for the mathematical, engineering and physical sciences and an appreciation of proof, logic and the fundamental structures of mathematics.

Proposed topics are:

1.1	Logic and Proofs
1.2	Introduction to algebra and real analysis
2	Functions and graphs
3	Polynomials and Complex numbers
4	Analytic Integration
5.1	Analytic solutions to differential equations
5.2	Vectors and Vector Calculus in three dimensions

Introduction to mathematics

Logic and Proofs (Including $\forall$, $\exists$, Negation, Direct, Contrapositive, Contradiction and Induction.)

Introduction to algebra and real analysis ($\varepsilon - N$ and $\varepsilon - \delta$ limit definitions, groups, permutation groups, cyclic groups.)

Functions and Graphs

(with links to analysis)

Polynomials and Complex Numbers

(Lead from solutions to Polynomials to Complex numbers to realizing roots of unity as a cyclic group)

Analytic integration

(Parts, Substitution and inverse trigonometric functions)

Differential equations, vectors and vector calculus

Analytic solutions to differential equations (Separable DEs, harmonic oscillators and logistic growth)

Vectors and Vector Calculus in three dimensions (Volume integrals, parametric curves, vector fields and partial differentiation)

6.2.4 Numerical Methods

Skills in numerical modelling and computation are incredibly important to all sciences with the authours having expressed how important such skills are in preparing for university studies in multiple disciplines. Numerical Methods aims to address this shortcoming by developing the topics learned in stag 1 with emphasis computing. Students will be left with the appreciation of computer driven calculation necessary for further study in engineering, the sciences and economics.

The language Julia has been recommended by the authours due to it having syntax familiar to users of both MATLAB and Python but with the capabilities of MATLAB and R (commonly used in university) inbuilt requiring no imported LA or Statistical packages to achieve the aims of the course. This eases the barrier of entry to students whilst also being free (including when pared with an IDE like VS Code).

Proposed topics are:

1.1	Introduction to computational approaches and the julia language
1.2	Revision of common differential functions
2	Further differentiation and applications
3	Integral calculus
4	Discretization of calculus models
5	Computational linear algebra
6.1	Statistics and computation
6.2	Computational problem solving

Introduction and revision

As commonly done students should first revise logarithmic, exponential and trigonometric functions for later use in calculus. Subsequently the Julia language should be introduced and the motivation behind the choice. Students should then have a practical understanding variables, simple types, digital boolean logic including `If-Then-Else` statements and loops such as `for`, `while` and recursion. Students should understand the difference between scripting and repl based interaction.

Further differentiation and applications

Students should be able to differentiate trigonometric, exponential and logarithmic functions. They should also understand the chain product and quotient rules.

Integral calculus

Understand antidifferentiation, the fundamental theorem of calculus and integrating trigonometric and exponential functions.

Discretization of calculus models

Cover the disscretization of calculus problems and implement in Julia discrete differentiation, the midpoint rule, trapezoid rule and simpson's rule. Introduce differential equations and how to simulate their behaviour such as harmonic oscillators. Understand the precision of their algorithms.

Computational linear algebra

Be able to perform elementary row operations to reduced row echelon form and solve consistent systems of linear equations. Understand geometrical interpretation of inconsistent systems in $\mathbb{R}^2$ and $\mathbb{R}^3$. Write both row reduction Cramer's rule algorithms so find solutions to consistent systems. Find the shortest distance between planes and line problems in n dimensions. Be able to simulate markov chains and compute steady states as well as matrix transformations.

Further computation

Statistics and computation Import .csv files and navigate datasets using julia. Perform visualization and statistical analysis on large datasets applying previously learned statistics.

Computational problem solving Study intersectional areas such as Monte-carlo methods, financial applications, and reason on algorithmic complexity (linear, quadratic and exponential time). Select topics from cellular automata and introduce simple game strategies in theory such as greedy and minimax. Present a map of numerical and computational fields.