$$-u''(x) - u(x) = \sin x \qquad u: [0,2] \to \mathbb{R}$$

$u(0) = 0$ (warunek Dirichleta)

$u'(2) - u(2) = 0$ (warunek Robina) $\Rightarrow u'(2) = u(2)$

$$\int_0^2 \left(-u''(x) - u(x)\right) \cdot v(x)\,dx = \int_0^2 (\sin x) \cdot v(x)\,dx$$

$$-\int_0^2 u''(x)v(x)\,dx - \int_0^2 u(x)v(x)\,dx = \int_0^2 (\sin x)v(x)\,dx$$

$$-\left[u'(x)v(x)\right]_0^2 + \int_0^2 u'(x)v'(x)\,dx - \int_0^2 u(x)v(x)\,dx = \int_0^2 (\sin x)v(x)\,dx$$

$$-u'(2)v(2) + u'(0)v(0) + \int_0^2 u'(x)v'(x)\,dx - \int_0^2 u(x)v(x)\,dx = \int_0^2 (\sin x)v(x)\,dx$$

$u'(2) = u(2)$ (WR) $\qquad v(0) = 0$ (WD)

$$\underbrace{-u(2)v(2) + \int_0^2 u'(x)v'(x) - \int_0^2 u(x)v(x)\,dx}_{B(u,v)} = \underbrace{\int_0^2 (\sin x)v(x)\,dx}_{L(v)}$$

$$e_i(x) = \begin{cases} \dfrac{x - x_{i-1}}{x_i - x_{i-1}}, & x \in (x_{i-1}, x_i) \\ \dfrac{x_{i+1} - x}{x_{i+1} - x_i}, & x \in (x_i, x_{i+1}) \\ 0, & \text{w p. p.} \end{cases} \qquad e_i'(x) = \begin{cases} \dfrac{1}{x_i - x_{i-1}} \\ \dfrac{1}{x_i - x_{i+1}} \\ 0 \end{cases}$$

(te same przedziały)

Po zastosowaniu uproszczenia

$$B\left(\sum_{i=0}^{N} u_i e_i, v_j\right) = L(v_j) \quad \text{dla } j = 0 \ldots N$$

$$\sum_{i=0}^{N} u_i \cdot B(e_i, e_j) = L(e_j) \quad \text{dla } e_j = v_j$$

Z czego wynika z układ równań:

$$\begin{bmatrix} B(e_0, e_0) & \cdots & B(e_N, e_0) \\ \vdots & \ddots & \vdots \\ B(e_0, e_N) & \cdots & B(e_N, e_N) \end{bmatrix} \begin{bmatrix} u_0 \\ \vdots \\ u_N \end{bmatrix} = \begin{bmatrix} L(e_0) \\ \vdots \\ L(e_N) \end{bmatrix}$$

A po uwzględnieniu warunku Dirichleta:

$$\begin{bmatrix} B(e_1, e_1) & \cdots & B(e_N, e_1) \\ \vdots & \ddots & \vdots \\ B(e_1, e_N) & \cdots & B(e_N, e_N) \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} = \begin{bmatrix} L(e_1) \\ \vdots \\ L(e_N) \end{bmatrix}$$

Po którego rozwiązaniu mamy:

$$u(x) = \sum_{i=1}^{N} u_i \cdot e_i(x)$$