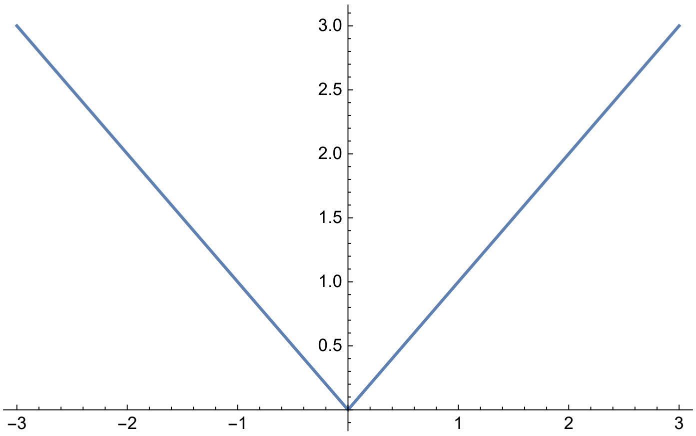
```
(*Лагранж, неопределен коэфф. Фурье, МНК*)

(*интерполяция функции*)
Plot[Abs[x], {x, -2, 2}]
```



```
pxk = {x1, x2, x3, x4};(*для теста *)
Lk = Table[(*создаем все часные многочлены Лагранжа*)
  ϕk = 1;
  For[jj = 1, jj ≤ Length[pxk], jj++,
   If[k == jj, ϕk, ϕk = ϕk (x - pxk[[jj]])/(pxk[[k]] - pxk[[jj]])] (*ϕj[x] = ∏_{k=1,k≠j}^{n} (x-xk)/(xj-xk) *)
  ]; ϕk
  , {k, 1, Length[pxk]}]
```

$$\left\{\frac{(x-x2)(x-x3)(x-x4)}{(x1-x2)(x1-x3)(x1-x4)},\ \frac{(x-x1)(x-x3)(x-x4)}{(-x1+x2)(x2-x3)(x2-x4)},\ \frac{(x-x1)(x-x2)(x-x4)}{(-x1+x3)(-x2+x3)(x3-x4)},\ \frac{(x-x1)(x-x2)(x-x3)}{(-x1+x4)(-x2+x4)(-x3+x4)}\right\}$$

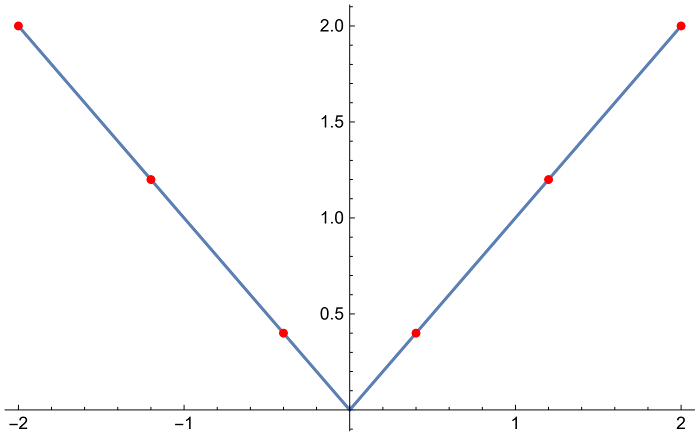
```
(*================================*)
```

In[1]:=
```
(*Вычисления*)
nn = 5; (*число отрезков, точек nn+1*)
pxk = Table[xk, {xk, -2, 2, 4/nn}]; (*точки где p[xk]=f[xk]=pfk *)
pfk = Table[Abs[pxk[[k]]], {k, 1, Length[pxk]}];
(*собираем точки для графика*)
tbpts = Table[{pxk[[k]], Abs[pxk[[k]]]}, {k, 1, Length[pxk]}]
```

Out[2]=
$$\left\{\{-2, 2\}, \left\{-\frac{6}{5}, \frac{6}{5}\right\}, \left\{-\frac{2}{5}, \frac{2}{5}\right\}, \left\{\frac{2}{5}, \frac{2}{5}\right\}, \left\{\frac{6}{5}, \frac{6}{5}\right\}, \{2, 2\}\right\}$$

In[3]:=
```
Show[
 Plot[Abs[x], {x, -2, 2}],
 ListPlot[tbpts, PlotStyle → Red]
]
```

Out[3]=



In[4]:=
```
Lk = Table[(*создаем все часные многочлены Лагранжа*)
  (*pxk={x1,x2,x3,x4} для теста *)
  ϕk = 1;
  For[jj = 1, jj ≤ Length[pxk], jj++,
   If[k == jj, ϕk, ϕk = ϕk (x - pxk[[jj]])/(pxk[[k]] - pxk[[jj]])] (*ϕj[x] = ∏_{k=1,k≠j}^n (x-xk)/(xj-xk)*)
  ]; ϕk
  , {k, 1, Length[pxk]}]
```

Out[4]=

$$\left\{\frac{625\,(2-x)\left(-\frac{6}{5}+x\right)\left(-\frac{2}{5}+x\right)\left(\frac{2}{5}+x\right)\left(\frac{6}{5}+x\right)}{24\,576},\ \frac{3125\,(-2+x)\left(-\frac{6}{5}+x\right)\left(-\frac{2}{5}+x\right)\left(\frac{2}{5}+x\right)(2+x)}{24\,576},\right.$$

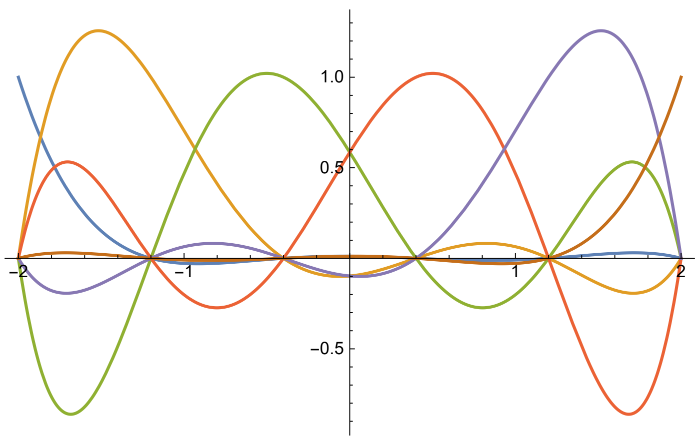
$$-\frac{3125\,(-2+x)\left(-\frac{6}{5}+x\right)\left(-\frac{2}{5}+x\right)\left(\frac{6}{5}+x\right)(2+x)}{12\,288},$$

$$\frac{3125\,(-2+x)\left(-\frac{6}{5}+x\right)\left(\frac{2}{5}+x\right)\left(\frac{6}{5}+x\right)(2+x)}{12\,288},$$

$$\left.-\frac{3125\,(-2+x)\left(-\frac{2}{5}+x\right)\left(\frac{2}{5}+x\right)\left(\frac{6}{5}+x\right)(2+x)}{24\,576},\ \frac{625\left(-\frac{6}{5}+x\right)\left(-\frac{2}{5}+x\right)\left(\frac{2}{5}+x\right)\left(\frac{6}{5}+x\right)(2+x)}{24\,576}\right\}$$

In[5]:=
```
Plot[Lk, {x, -2, 2}]
```

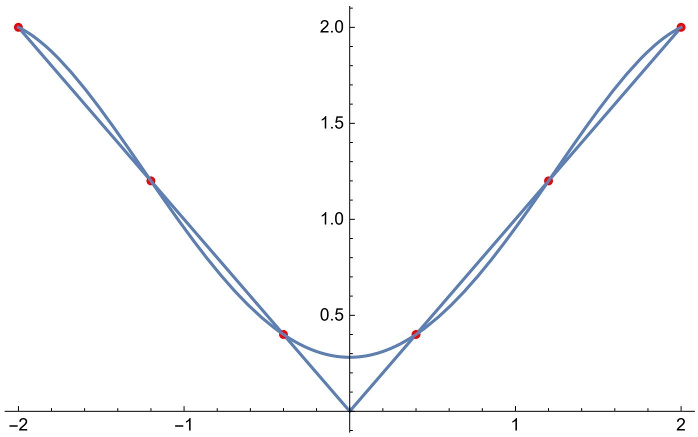
Out[5]=



In[6]:=
```
PolyN = Tr[Table[Lk[[k]] pfk[[k]], {k, 1, Length[pxk]}]];

Show[
 Plot[Abs[x], {x, -2, 2}],
 ListPlot[tbpts, PlotStyle → Red],
 Plot[PolyN, {x, -2, 2}]
]
```

Out[7]=




```
(*многочлен Лагранжа через метод неопределенных коэффициентов*)

pfk = {x1, x2, x3, x4};

ϕϕL = a x^3 + b x^2 + c x + d;
equ = {
    a x1^3 + b x1^2 + c x1 + d == 1,
    a x2^3 + b x2^2 + c x2 + d == 0,
    a x3^3 + b x3^2 + c x3 + d == 0,
    a x4^3 + b x4^2 + c x4 + d == 0
   };
Simplify[a x^3 + b x^2 + c x + d /. Solve[equ, {a, b, c, d}]]
```

$$\left\{\frac{(x-x2)\,(x-x3)\,(x-x4)}{(x1-x2)\,(x1-x3)\,(x1-x4)}\right\}$$

```
(*Находим интерполирующий многочлен из решения СЛАУ *)

equL = Table[ϕϕL /. x → pxk[[k]], {k, 1, Length[pxk]}]
Simplify[ϕϕL /. Solve[equL == pfk, {a, b, c, d}]]
```

$$\left\{-8\,a+4\,b-2\,c+d,\ -\frac{8\,a}{27}+\frac{4\,b}{9}-\frac{2\,c}{3}+d,\ \frac{8\,a}{27}+\frac{4\,b}{9}+\frac{2\,c}{3}+d,\ 8\,a+4\,b+2\,c+d\right\}$$

$$\left\{\frac{1}{8}\left(4+3\,x^2\right)\right\}$$

```
Simplify[PolyN]
```

$$\frac{1}{8}\left(4+3\,x^2\right)$$

```
(*==================================*)

(*Фурье*)
```

```
nn = 4;
```

$$\left\{\text{Table}\left[\int_0^{2\pi}\text{Cos}[k\,s]\,\text{Cos}[m\,s]\,ds,\ \{k,0,nn\},\ \{m,0,nn\}\right]\ //\ \text{MatrixForm},\right.$$
$$\text{Table}\left[\int_0^{2\pi}\text{Cos}[k\,s]\,\text{Sin}[m\,s]\,ds,\ \{k,0,nn\},\ \{m,0,nn\}\right]\ //\ \text{MatrixForm},$$
$$\left.\text{Table}\left[\int_0^{2\pi}\text{Sin}[k\,s]\,\text{Sin}[m\,s]\,ds,\ \{k,0,nn\},\ \{m,0,nn\}\right]\ //\ \text{MatrixForm}\right\}$$

$$\left\{\begin{pmatrix}2\pi&0&0&0&0\\0&\pi&0&0&0\\0&0&\pi&0&0\\0&0&0&\pi&0\\0&0&0&0&\pi\end{pmatrix},\begin{pmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix},\begin{pmatrix}0&0&0&0&0\\0&\pi&0&0&0\\0&0&\pi&0&0\\0&0&0&\pi&0\\0&0&0&0&\pi\end{pmatrix}\right\}$$

$$\left\{\text{Table}\left[\int_0^{4}1.\,\text{Cos}[k\,s]\,\text{Cos}[m\,s]\,ds,\ \{k,0,nn\},\ \{m,0,nn\}\right]\ //\ \text{MatrixForm},\right.$$
$$\text{Table}\left[\int_0^{4}1.\,\text{Cos}[k\,s]\,\text{Sin}[m\,s]\,ds,\ \{k,0,nn\},\ \{m,0,nn\}\right]\ //\ \text{MatrixForm},$$
$$\left.\text{Table}\left[\int_0^{4}1.\,\text{Sin}[k\,s]\,\text{Sin}[m\,s]\,ds,\ \{k,0,nn\},\ \{m,0,nn\}\right]\ //\ \text{MatrixForm}\right\}$$

$$\left\{\begin{pmatrix}4.&-0.756802&0.494679&-0.178858&-0.0719758\\-0.756802&2.24734&-0.46783&0.211352&0.00186571\\0.494679&-0.46783&1.96401&-0.287107&0.171875\\-0.178858&0.211352&-0.287107&1.92454&-0.359051\\-0.0719758&0.00186571&0.171875&-0.359051&2.03446\end{pmatrix},\right.$$
$$\begin{pmatrix}0.&1.65364&0.57275&0.0520487&0.489415\\0.&0.286375&0.852846&0.531082&0.0852161\\0.&-0.800797&0.244707&0.886014&0.33436\\0.&-0.0416676&-0.76763&0.0479851&0.967008\\0.&0.0331675&-0.23839&-0.686636&0.010361\end{pmatrix},$$
$$\left.\begin{pmatrix}0.&0.&0.&0.&0.\\0.&1.75266&-0.288972&0.283327&-0.180723\\0.&-0.288972&2.03599&-0.469696&0.322804\\0.&0.283327&-0.469696&2.07546&-0.397752\\0.&-0.180723&0.322804&-0.397752&1.96554\end{pmatrix}\right\}$$

$$f[x] = a_0 + \sum_{k=1}^{nn} a_k\,\text{Cos}[k\,x] + \sum_{k=1}^{nn} b_k\,\text{Sin}[k\,x]\ ,\quad |\ \text{применим, например},\ \int_0^{2\pi}(\_)\ \text{Cos}[3\,x]\,dx$$

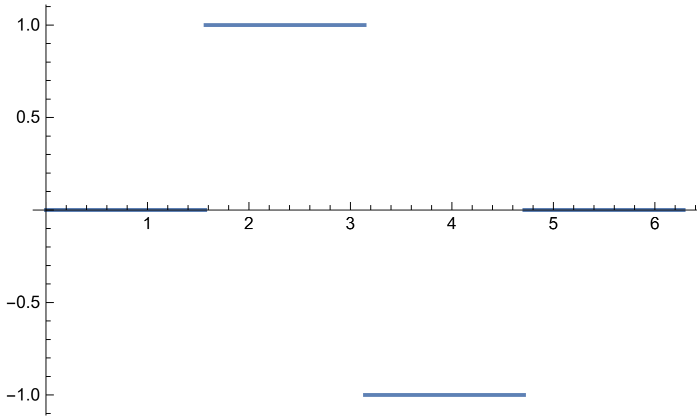
$$\int_0^{2\pi} f[x]\,\text{Cos}[3\,x]\,dx = \int_0^{2\pi}\left(a_0 + \sum_{k=1}^{nn} a_k\,\text{Cos}[k\,x] + \sum_{k=1}^{nn} b_k\,\text{Sin}[k\,x]\right)\text{Cos}[3\,x]\,dx\ =\ \pi\,a_3$$

```
Piecewise[{{Piecewise[{{1, x < 1}}, 2], x > 0}}, 3]
```

$$\text{ff}[xx\_] := \text{Piecewise}\left[\left\{\left\{0,\ xx < \frac{\pi}{2}\right\},\ \left\{1,\ \frac{\pi}{2} \le xx < \pi\right\},\ \left\{-1,\ \pi \le xx < \frac{3\pi}{2}\right\},\ \left\{0,\ xx > \frac{3\pi}{2}\right\}\right\}\right]$$

```
(*график функции*)
ff[x]
Plot[ ff[x]
 , {x, 0, 2 π}, PlotStyle → Thick]
```

$$\begin{cases} 0 & x < \frac{\pi}{2} \\ 1 & \frac{\pi}{2} \le x < \pi \\ -1 & \pi \le x < \frac{3\pi}{2} \\ 0 & \text{True} \end{cases}$$



$\int_0^{2\pi} ff[x]\, dx$ (*означает, что a_0=0*)

0

$cck = Table\left[\int_0^{2\pi} ff[s]\, Cos[k\, s]\, ds, \{k, 1, nn\}\right]$

$ssk = Table\left[\int_0^{2\pi} ff[s]\, Sin[k\, s]\, ds, \{k, 1, nn\}\right]$

{0, 0, 0, 0}

$\left\{2, -2, \frac{2}{3}, 0\right\}$

$pF[x\_] := \sum_{k=1}^{nn} Ck[[k]]\, Cos[k\, x] + \sum_{k=1}^{nn} Sk[[k]]\, Sin[k\, x];$

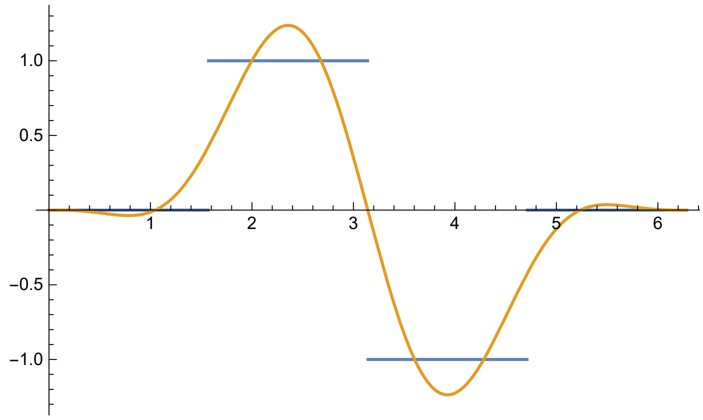
```
pF[x] /. {Ck → {a1, a2, a3, a4}, Sk → {b1, b2, b3, b4}}
```

a1 Cos[x] + a2 Cos[2 x] + a3 Cos[3 x] +
 a4 Cos[4 x] + b1 Sin[x] + b2 Sin[2 x] + b3 Sin[3 x] + b4 Sin[4 x]

```
pF2 = pF[x] /. {Ck → cck, Sk → ssk}
```

$2\, Sin[x] - 2\, Sin[2\, x] + \frac{2}{3} Sin[3\, x]$

$\texttt{Plot}\left[\left\{\texttt{ff[x]}, \frac{1}{\pi}\texttt{pF2}\right\}, \{\texttt{x}, 0, 2\pi\}\right]$



```
nnn = 40;
```

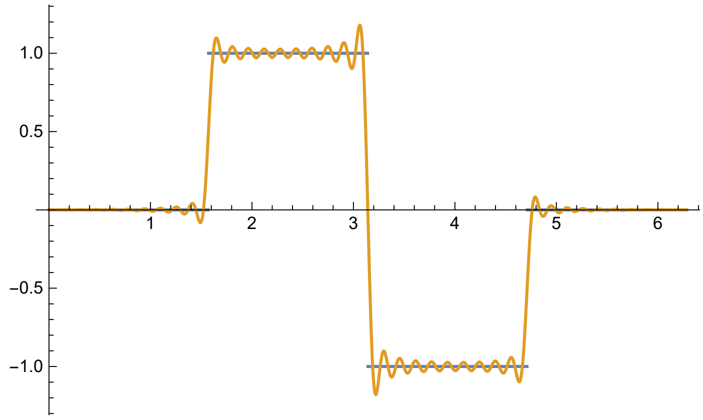
$\texttt{ccck} = \texttt{Table}\left[\int_0^{2\pi} \texttt{ff[s] Cos[k s]}\, ds, \{k, 1, \texttt{nnn}\}\right];$

$\texttt{sssk} = \texttt{Table}\left[\int_0^{2\pi} \texttt{ff[s] Sin[k s]}\, ds, \{k, 1, \texttt{nnn}\}\right];$

$\texttt{pF3} = \sum_{k=1}^{\texttt{nnn}} \texttt{ccck}[[k]]\, \texttt{Cos}[k\, x] + \sum_{k=1}^{\texttt{nnn}} \texttt{sssk}[[k]]\, \texttt{Sin}[k\, x]$

$$2\,\mathrm{Sin}[x] - 2\,\mathrm{Sin}[2x] + \frac{2}{3}\mathrm{Sin}[3x] + \frac{2}{5}\mathrm{Sin}[5x] - \frac{2}{3}\mathrm{Sin}[6x] + \frac{2}{7}\mathrm{Sin}[7x] + \frac{2}{9}\mathrm{Sin}[9x] - \frac{2}{5}\mathrm{Sin}[10x] + \frac{2}{11}\mathrm{Sin}[11x] + \frac{2}{13}\mathrm{Sin}[13x] - \frac{2}{7}\mathrm{Sin}[14x] + \frac{2}{15}\mathrm{Sin}[15x] + \frac{2}{17}\mathrm{Sin}[17x] - \frac{2}{9}\mathrm{Sin}[18x] + \frac{2}{19}\mathrm{Sin}[19x] + \frac{2}{21}\mathrm{Sin}[21x] - \frac{2}{11}\mathrm{Sin}[22x] + \frac{2}{23}\mathrm{Sin}[23x] + \frac{2}{25}\mathrm{Sin}[25x] - \frac{2}{13}\mathrm{Sin}[26x] + \frac{2}{27}\mathrm{Sin}[27x] + \frac{2}{29}\mathrm{Sin}[29x] - \frac{2}{15}\mathrm{Sin}[30x] + \frac{2}{31}\mathrm{Sin}[31x] + \frac{2}{33}\mathrm{Sin}[33x] - \frac{2}{17}\mathrm{Sin}[34x] + \frac{2}{35}\mathrm{Sin}[35x] + \frac{2}{37}\mathrm{Sin}[37x] - \frac{2}{19}\mathrm{Sin}[38x] + \frac{2}{39}\mathrm{Sin}[39x]$$

$\texttt{Plot}\left[\left\{\texttt{ff[x]}, \frac{1}{\pi}\texttt{pF3}\right\}, \{\texttt{x}, 0, 2\pi\}\right]$



```
(*как близко Фурье к первичной функции*)
```

$$|f - a_k \phi_k|^2_{L_2} = (f - a_k \phi_k, f - a_k \phi_k) = (f, f) - 2\left(f, \sum_k a_k \phi_k\right) + \sum_k a_k{}^2 \pi$$

```
(*Ряд Тейлора для f[x,y] *)
nn = 4;
TaylorSer = Normal[Series[f[x, y], {x, 0, nn}, {y, 0, nn}]]
```

$$f[0,0] + y f^{(0,1)}[0,0] + \frac{1}{2} y^2 f^{(0,2)}[0,0] + \frac{1}{6} y^3 f^{(0,3)}[0,0] + \frac{1}{24} y^4 f^{(0,4)}[0,0] + x\left(f^{(1,0)}[0,0] + y f^{(1,1)}[0,0] + \frac{1}{2} y^2 f^{(1,2)}[0,0] + \frac{1}{6} y^3 f^{(1,3)}[0,0] + \frac{1}{24} y^4 f^{(1,4)}[0,0]\right) + x^2 \left(\frac{1}{2} f^{(2,0)}[0,0] + \frac{1}{2} y f^{(2,1)}[0,0] + \frac{1}{4} y^2 f^{(2,2)}[0,0] + \frac{1}{12} y^3 f^{(2,3)}[0,0] + \frac{1}{48} y^4 f^{(2,4)}[0,0]\right) + x^3 \left(\frac{1}{6} f^{(3,0)}[0,0] + \frac{1}{6} y f^{(3,1)}[0,0] + \frac{1}{12} y^2 f^{(3,2)}[0,0] + \frac{1}{36} y^3 f^{(3,3)}[0,0] + \frac{1}{144} y^4 f^{(3,4)}[0,0]\right) + x^4 \left(\frac{1}{24} f^{(4,0)}[0,0] + \frac{1}{24} y f^{(4,1)}[0,0] + \frac{1}{48} y^2 f^{(4,2)}[0,0] + \frac{1}{144} y^3 f^{(4,3)}[0,0] + \frac{1}{576} y^4 f^{(4,4)}[0,0]\right)$$

$$\text{Simplify}\left[\text{TaylorSer} - \sum_{i=0}^{nn}\left(\sum_{j=0}^{nn} f^{(i,j)}[0,0] \frac{x^i y^j}{i!\, j!}\right)\right]$$

0

```
(*{i,j}→{k,j}, k=i+j*)
```

$$\text{TaylorP} = \sum_{k=0}^{4}\left(\sum_{j=0}^{k} f^{(j,k-j)}[0,0] \frac{x^j y^{k-j}}{j!\,(k-j)!}\right)$$

$$f[0,0] + y f^{(0,1)}[0,0] + \frac{1}{2} y^2 f^{(0,2)}[0,0] + \frac{1}{6} y^3 f^{(0,3)}[0,0] + \frac{1}{24} y^4 f^{(0,4)}[0,0] + x f^{(1,0)}[0,0] + x y f^{(1,1)}[0,0] + \frac{1}{2} x y^2 f^{(1,2)}[0,0] + \frac{1}{6} x y^3 f^{(1,3)}[0,0] + \frac{1}{2} x^2 f^{(2,0)}[0,0] + \frac{1}{2} x^2 y f^{(2,1)}[0,0] + \frac{1}{4} x^2 y^2 f^{(2,2)}[0,0] + \frac{1}{6} x^3 f^{(3,0)}[0,0] + \frac{1}{6} x^3 y f^{(3,1)}[0,0] + \frac{1}{24} x^4 f^{(4,0)}[0,0]$$

```
Collect[TaylorSer - TaylorP, {x, y}]
```

$$\frac{1}{24} x y^4 f^{(1,4)}[0,0] + x^2 \left(\frac{1}{12} y^3 f^{(2,3)}[0,0] + \frac{1}{48} y^4 f^{(2,4)}[0,0]\right) + x^3 \left(\frac{1}{12} y^2 f^{(3,2)}[0,0] + \frac{1}{36} y^3 f^{(3,3)}[0,0] + \frac{1}{144} y^4 f^{(3,4)}[0,0]\right) + x^4 \left(\frac{1}{24} y f^{(4,1)}[0,0] + \frac{1}{48} y^2 f^{(4,2)}[0,0] + \frac{1}{144} y^3 f^{(4,3)}[0,0] + \frac{1}{576} y^4 f^{(4,4)}[0,0]\right)$$

```
(*интерполяция сплайнами*)

(*пусть две точки x1,x2 но производная тоже учитывается*)
```

$$equ = \{ a\,x1^3 + b\,x1^2 + c\,x1 + d == f1,\; a\,x2^3 + b\,x2^2 + c\,x2 + d == f2,\; 3\,a\,x1^2 + 2\,b\,x1 + c == df1,\; 3\,a\,x2^2 + 2\,b\,x2 + c == df2 \};$$

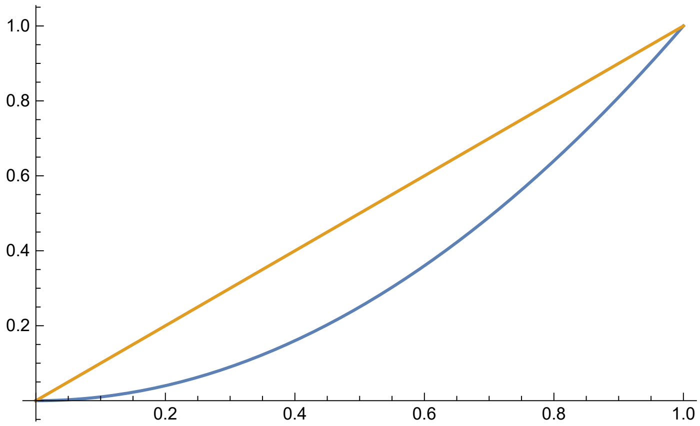
$$\text{Simplify}\left[a\,x^3 + b\,x^2 + c\,x + d\;/.\;\text{Solve}[equ, \{a, b, c, d\}]\right]$$

$$\left\{\left((x - x1)\,\left(f2\,(x - x1)\,(2\,x + x1 - 3\,x2) + \left(df2\,(x - x1) + df1\,(x - x2)\right)\,(x - x2)\,(x1 - x2)\right) - f1\,(x - x2)^2\,(2\,x - 3\,x1 + x2)\right) \Big/ (x1 - x2)^3\right\}$$

```
(*размытие Гаусса*)
(*антиразмытие Гаусса*)
```

(*апроксимация $f[x]=x^2$ при помощи линейной функции *)
```
(*Лагранж. берем две крайние точки*)
```

$$\text{Plot}\left[\left\{x^2, 1\,x + 0\,(1 - x)\right\}, \{x, 0, 1\}\right]$$




```
(*хотим, чтобы площадь между графиками =0 ,
но получается 1-параметрическое семейство прямых*)
```

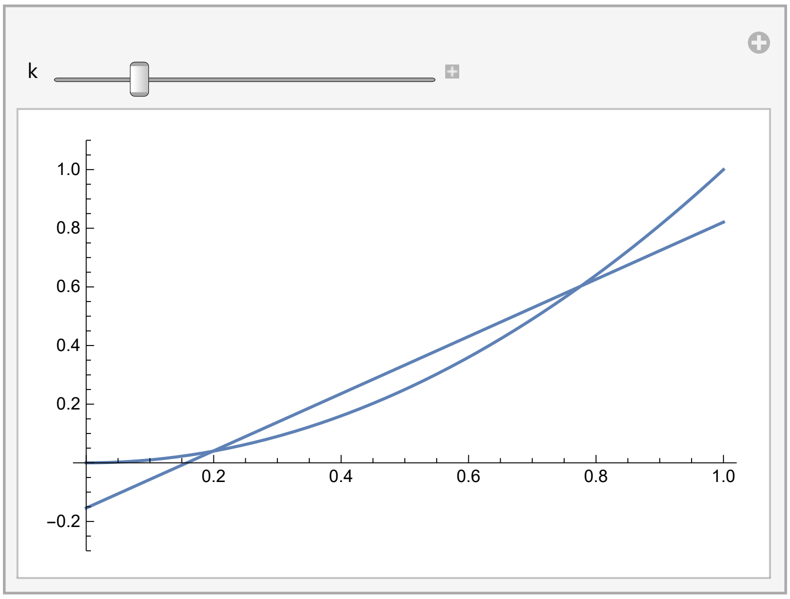
$$\int_0^1 \left(x^2 - k\,x - b\right) dx$$

$$\text{Solve}\left[\frac{1}{3} - b - \frac{k}{2} == 0, \{b\}\right][[1]]$$

$$\frac{1}{3} - b - \frac{k}{2}$$

$$\left\{b \to \frac{1}{6}\,(2 - 3\,k)\right\}$$

```
Manipulate[Show[
  Plot[x^2, {x, 0, 1}, PlotRange → {-0.3, 1.1}],
  Plot[k x + 1/6 (2 - 3 k), {x, 0, 1}]
 ], {k, 0, 5}]
```

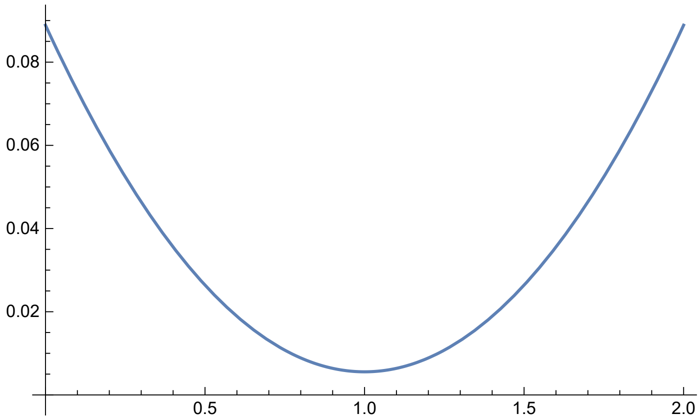



```
(*хотим чтобы *)
```

$$\int_0^1 \left(x^2 - k\,x - \frac{1}{6}(2 - 3\,k)\right)^2 dx$$

$$\frac{4}{45} - \frac{k}{6} + \frac{k^2}{12}$$

```
Plot[4/45 - k/6 + k^2/12, {k, 0, 2}]
```



```
Solve[D[4/45 - k/6 + k^2/12, k] == 0, k]
1/6 (2 - 3 k) /. k -> 1
```

$\{\{k \to 1\}\}$

$-\frac{1}{6}$

заменяем на

$(x^2 - k\,x - b)^2 \to \min,\ x \in [0, 1]$

если было бы много точек x_j, то было бы $\sum_j (x_j{}^2 - k\,x_j - b)^2 \to \min$

т.к. точек ∞, то напишем интеграл, получим функцию двух переменных fkb[k, b] $\to$ min

т.е. минимизация функции 2 – х переменных

```
fkb = Integrate[(x^2 - k x - b)^2, {x, 0, 1}]
```

$\frac{1}{5} - \frac{2\,b}{3} + b^2 - \frac{k}{2} + b\,k + \frac{k^2}{3}$

```
Solve[{D[fkb, k], D[fkb, b]} == 0, {k, b}]
```

$\{\{k \to 1,\ b \to -\frac{1}{6}\}\}$

```
Manipulate[Show[
  Plot[x^2, {x, 0, 1}, PlotRange -> {-0.3, 1.1}],
  Plot[k x + 1/6 (2 - 3 k), {x, 0, 1}],
  Plot[x - 1/6, {x, 0, 1}, PlotStyle -> Red]


 ], {k, 0, 5}]
```