Real–Hidden (RH) Ring O(1) Bit-Primitive Math and Proofs

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July 8, 2025

1. RH Ring Definition

An *RH element* is a pair of complex numbers

$$x = (a, b), \quad a, b \in \mathbb{C}.$$

We define the following operations on RH elements:

$$\text{ADD}: \quad (a, b) + (c, d) = (\, a + c,\ b + d\,), \tag{1}$$

$$\text{MUL}: \quad (a, b) * (c, d) = \big(ac + bd,\ ad + bc\big), \tag{2}$$

$$\text{TRACE}: \quad \text{trace}(a, b) = a. \tag{3}$$

2. Enumeration Operator

To enumerate the n-th binary operation in constant time, set

$$P_n = (n, 0), \quad X = (x, 0),$$

and define

$$O_n(x) \;=\; \text{trace}\Big(U * P_n * X\Big),$$

where U is a fixed RH element encoding the master operator, and $*$ denotes the RH MUL.

3. Theorem (Constant-Time Complexity)

Theorem 1. *The computation of $O_n(x)$ requires a constant number of RH primitives (ADD, MUL, TRACE) and thus runs in $O(1)$ time.*

Proof. Each RH element (a, b) is stored as exactly two complex numbers. The costs are:

- ADD: two complex additions $\longrightarrow O(1)$.
- MUL: four complex multiplications and two complex additions $\longrightarrow O(1)$.
- TRACE: one move/copy of the "even" component $\longrightarrow O(1)$.

Computing

$$O_n(x) = \text{trace}\big(U * P_n\big) * X = \text{TRACE}\big((U * P_n) * X\big)$$

requires exactly

$$2 \times \mathsf{MUL} \;+\; 1 \times \mathsf{TRACE} \;=\; \text{constant work, no loops.}$$

Since there are no size-dependent iterations or recursive steps, the total time is bounded by a fixed constant. □