Optimal Margin Requirements

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Introductory Notes

What follows below is research carried out by Vest Labs with the primary intent of it being discussed on the labs' forum. This particular session can be found here: https://vest.discourse.group/t/optimal-margin-requirements/26. *"The most that can be expected from any model is that it can supply a useful approximation to reality: All models are wrong; some models are useful."* - George Box

Motivation

We aim to build an exchange that can facilitate trading contracts on illiquid underlyings. To manage risk, we need to set proper margin requirements. Typical industry approaches involve simulating scenarios and setting margins according to worst-case losses (e.g. CME's SPAN). On the other hand, the advent of crypto exchanges open opportunities for more prudent, near real-time risk management. We now explore the existing literature on optimal margin requirements focused on accounts with a single risky asset and will leave extensions to multiple assets to the future.

Risk measures

To quantify risk, we will choose a risk measure, $\rho : \mathcal{G} \mapsto \mathbb{R}$, where $\mathcal{G}$ is the set of all functions $X : \Omega \mapsto \mathbb{R}$ that represents the future returns of a portfolio. In particular, we are interested in convex risk measures (i.e. coherent risk measures that relax positive homogeneity) because they satisfy monotonicity, translation invariance, and convexity [3]. Note however that because crypto assets are highly correlated and have fat-tailed return distributions, it may be more desirable to require subadditivity conditional on two assets being negatively correlated.

In addition to their theoretical soundness, we desire a risk measure that is practical. This entails two more requirements: *elicitability* and *robustness*. A risk measure ρ is *elicitable* if there exists a scoring function whose minimizer is an optimal forecast for ρ (see [18]). Elicitability is important for backtesting, which allows us to compare the performance of different forecasting methods.

We consider a risk measure ρ to be *robust* if it is continuous with respect to the Wasserstein distance (see [9]). Robustness is important because a robust risk measure is insensitive to a small perturbation8-09kll in the probability measure, which implies greater tolerance t8; o model misspecification. Indeed, all risk measures require the specification of a model $\mathbb{P}$, a distribution over future returns of an account (unless we directly forecast risk measure, or rather statistic, from historical samples). Approaches include fat-tailed parametric distribution fitting (Laplace or generalized Pareto distribution) [7], maximum likelihood estimation [1][11] or GARCH-based volatility forecasting. We leave model specification as a topic for the future.

Existing risk measures include:

1. (*Value-at-Risk*) $VaR_\alpha(X) = q_\alpha^+(X) = -\inf\{x \mid \mathbb{P}[X \leq x] > \alpha\}$ where $0 < \alpha < 1$, which is robust and elicitable but not coherent.

2. (*Expected Shortfall*) $ES_\alpha(X) = -\frac{1}{\alpha}\int_0^\alpha VaR_p(X)dp$ where $0 < \alpha < 1$, which is coherent but not elicitable (although jointly elicitable with VaR).

3. (*Spectral Risk Measure*) $SR_\phi(X) = -\int_0^1 q_p(X)\phi(p)dp$ for some nonincreasing function $\phi(p)$ such that $\phi(p) \geq 0$ and $\int_0^1 \phi(p)dp = 1$ [8]. Spectral risk measures are coherent, but not robust or elicitable. ES is a special cases of spectral risk measure.

4. (*Median Shortfall*) $MS_\alpha(X) = -median[X \mid X < -VaR_\alpha(X)]$ (equivalently $VaR_{\alpha/2}(X)$ if the distribution for X is continuous) is the median tail value-at-risk. MS is robust and elicitable but not coherent because it does not satisfy subadditivity but instead satisfies comonotonic subadditivity [11].

5. (*Liquidity-adjusted Risk Measure*) Liquidity-adjusted risk measure takes into account liquidity risk that can be observed in thin markets or during crisis, in particular limited access to financing (e.g. restrictions on short-selling) and price impact of trades [17]. One example is LaVaR (liquidity-adjusted VaR), where it is defined as the sum of VaR and half the bid-ask spread, representing additional cost for exiting the position [2].

Instead of the above, we choose to explore expectiles

$$e_\tau(X) = -\arg\min_{x\in\mathbb{R}} \mathbb{E}[\tau(X-x)_+^2 + (1-\tau)(x-X)_+^2]$$

for some $0 < \tau < 1$ and $(x)_+ = \max(0, x)$ [9]. It follows from the first order condition $\tau\mathbb{E}[(X - e_\tau(X))_+] = (1-\tau)\mathbb{E}[(e_\tau(X) - X)_+]$ that $e_\tau(X)$ is uniquely defined for τ given X.

We can then define *expectile value-at-risk* $EVaR_\tau(X) = -e_\tau(X)$. We choose to explore $EVaR_\tau$ because it is the only risk measure to be coherent, robust, and elicitable for $\tau \leq \frac{1}{2}$ [9]. We can characterize $EVaR_\tau$ by its acceptance set $\mathcal{A}_{EVaR_\tau} = \{X \mid \frac{\mathbb{E}[(X-e_\tau(X))_+]}{\mathbb{E}[(e_\tau(X)-X)_+]} \geq \frac{1-\tau}{\tau}\}$ [20]. Note that $\Omega(X,\tau) = \frac{\mathbb{E}[(X-e_\tau(X))_+]}{\mathbb{E}[(e_\tau(X)-X)_+]}$ is a performance measure referred to as the omega ratio [19], the ratio of expected gains to expected losses. Hence, $\mathcal{A}_{EVaR_\tau}$ defines a set of accounts with sufficiently large omega ratios. τ can then be intuitively thought of as the weight given to expected gains relative to expected losses.

Designing an optimization problem

We can simply set margin requirements equal to some risk measure. But clearly, this could lead to high margins that disincentivize trading (and certainly would not reflect the risk appetite of crypto-traders). Instead, optimal margin requirements involve balancing two factors: *prudentiality* and *opportunity cost* [14]. Prudentiality can be captured by a risk measure, while opportunity cost is more abstract. Opportunity cost could be defined as the expected overcharge $\mathbb{E}[(M-L)_+]$ where M is margin and L is loss, or as some risk-free rate multiplied by the required margin, sometimes known as funding cost. It follows that higher margin products demand higher expected returns in an equilibrium [10].

Formulations of setting optimal margin requirements start with Brennan (1986) [5] who advocates for efficient contract design that is self-enforcing by minimizing margin subject to the constraint that the expected profitability of reneging the contract is zero. Shanker (2014) [16] minimizes margin subject to the constraints that the initial margin is above a fair value and that the probability of cumulative losses exceeding the margin during some grace period is acceptable. Berlinger et al. (2019) [4] presents an optimization framework that takes overall funding liquidity as an exogenous parameter to characterize a trader's ability-to-pay versus willingness-to-pay (i.e. the difference between margin requirement and account value). Interestingly, Capponi and Cheng (2018) [6] considers an equilibrium where the exchange chooses trading fees and margin requirements that maximize its expected profits, and the traders chooses direction and size to maximize ex ante expected profits.

For our formulation, we will assume that we do not extend grace periods for margin calls. We want to balance between prudentiality and opportunity cost without relying on exogenous parameters that we need to calibrate, so we will simply minimize the sum of expected margin shortfall and expected margin overcharge. The only constraint we will impose is that we want to remain solvent under this margin requirement. Hence, we formulate our optimization problem with expectile value-at-risk as the following:

$$\begin{aligned} \underset{0\le\tau\le\frac{1}{2}}{\text{minimize}} \quad & \sum_{i=1}^{n} \mathbb{E}[(EVaR_\tau(X_i)+X_i)_+] + \mathbb{E}[(-X_i - EVaR_\tau(X_i))_+] \\ \text{subject to} \quad & EVaR_\tau(X_i) \ge -x_i^t \end{aligned}$$

where n is the number of traders, and x_i^t denotes the unrealized profit and loss of the account for trader i and current time t. In words, the first term in the summation of our objective function is expected margin overcharge $\mathbb{E}[(M-L)_+]$ and the second term is expected margin shortfall $\mathbb{E}[(L-M)_+]$. The constraints require the new margin requirement for each trader to be no less than their current loss.

Discussion

It remains to show if there exists an efficient solution for our optimization problem. One issue could be that the optimal margin requirement $EVaR_{\tau^*}$ is too lenient compared to $\tau \approx 0.00145$, which results in similar values to the industry standards VaR_α with $\alpha = 0.01$ and $ES_{\alpha'}$ with $\alpha' = 0.025$ under normal distributions [20]. In addition, it remains to find an optimal forecast model and time horizon to evaluate $EVaR_\tau$. The elicitability of $EVaR_\tau$ will be helpful in employing a variety of backtests to continuously measure and potentially improve the accuracy of our model.

Additionally, in contrast to traditional daily settlement of futures, crypto exchanges typically mark-to-market in near real-time. Hence, the procyclical effects of increasing margin during periods of increased market risk (volatility) and decreased liquidity will be magnified. To combat this, Lam et al. (2010) [13] introduces "margin bands" calibrated to achieve desired coverage probability and expected frequency of margin change. Moreover, since the margin requirement covered in this paper refers to maintenance (variation) margin, we will also need to provide extensions in the future on proper initial margin requirements (that are sufficiently far from the maintenance margin) and partial liquidation mechanisms. In light of the recent Mango Markets exploit, we will also need to introduce additional measures to combat spot market manipulation such as requiring additional margin for accounts holding large position sizes.

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