Vest Labs' Dynamic Virtual Automated Market Maker

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October 2022

Abstract

In this explainer, we explore a new dynamic virtual automated market maker (vAMM) model which updates the model with every transaction such that the mark price determined by the curve will always equal the index price, without any cost or reliance on funding rate. Some notable DEXes such as Curve [3] or Drift [4], as well as a study by Krishnamachari et al (2021) [5] explore dynamic extensions of constant function market makers (CFMMs). Our model uses a combination of constant sum market makers (CSMMs) and linear splining to meet the constraints above and achieve lower slippage compared to the existing constant product market maker (CPMM) models.

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1 Introduction

In short, we aim to build an exchange that enables traders to speculate on any asset–an extension of their holistic beliefs of the world. To do so, we initially decided to improve upon the capabilities of vAMMs, pioneered by those before us. Previous vAMMs faced issues with high slippage, unexpected liquidations, large spreads, and nontrivial financial costs. We developed our vAMM to address these issues. In this paper, we introduce a new type of vAMM designed to efficiently market-make long-tail assets.

Beforehand, we would like to note that Vest Labs has now moved away from the vAMM for our exchange. Our prior research found that vAMMs work well for liquid markets where the oracle price is reliable and the objective is to minimize slippage. However, we have since made the decision to focus on minimizing solvency risk while simultaneously minimizing slippage, thus moving away from vAMMs. This new approach enables fully permissionless trading constrained to the necessary considerations for solvency risk. Our prior research before this decision is outlined below.

2 Glossary

- $f : \mathbb{R} \to \mathbb{R}$ is a state function which determines current state in a dual asset market. In token A/B swap market, $(x, f(x))$ represents x amount of token A and $f(x)$ amount of token B. In perpetual futures market, which we explore in this paper, $(x, f(x))$ represents x long-short imbalance and $f(x)$ net notional value.

- p_{oracle}: Oracle price
- $p_{mark}(x, f(x))$: Mark price, which is determined by the current state $(x, f(x))$. For notational convenience, we will denote as p_{mark}.
- $l, s \in \mathbb{R}_{\geq 0}$: Notional size of open long, short position
- $\mathcal{F}$: Family of convex state functions that define the vAMM
  e.g.) CPMM $\mathcal{F} = \{f : x \mapsto \frac{k}{x} | k \in \mathbb{R}_+\}$
- $\mathcal{A}$: Update rule, which returns a new vAMM model $f' \in \mathcal{F}$ given some inputs such as previous model $f \in \mathcal{F}$, long-short imbalance x and p_{oracle}
- Spread: $|p_{mark} - p_{oracle}|$
- Slippage: $|\frac{f(x+x')-f(x)}{x'} - p_{oracle}|/p_{oracle}$ where x' is the size of new trade
- Update cost: $C(f_0, f) \in \mathbb{R}$. $C(f_0, f) > 0$ implies that the protocol experiences a loss by updating the function from f_0 to f

Lemma 1. Update cost can be expressed as $C(f_0, f) = f_0(x_{l=s}) - f(x_{l=s})$, where f_0 is the initial function, f is the new function, and $x_{l=s}$ is the equilibrium market state (i.e. no long short imbalance).

Proof. By definition, $x = x_{l=s} + l - s$ holds. Assume that the new function satisfies $f_0(x_{l=s} + l - s) = f(x_{l=s} + l - s)$, i.e. update maintains the current market state. We will follow the derivation in [2].

Given any vAMM f_0, l and s:

1. First, we close all long positions. The market state before closing is $(x_{l=s} + l - s, f_0(x_{l=s} + l - s))$ and after closing is $(x_{l=s} - s, f_0(x_{l=s} - s))$. The capital gain of all long positions is $P_l = f_0(x_{l=s} + l - s) - f_0(x_{l=s} - s) - K_l$, where K_l is the opening notional value all the long positions.

2. Second, we close all short positions. The market state before closing is $(x_{l=s} - s, f_0(x_{l=s} - s))$ and after closing is $(x_{l=s}, f_0(x_{l=s}))$. The capital gain of all short positions is $P_s = K_s - (f_0(x_{l=s}) - f_0(x_{l=s} - s))$, where K_s is the opening notional value of all the short positions.

3. Therefore, the total capital gain of all long and short positions is $\delta(f_0, l, s) = P_s + P_l = K_s - f_0(x_{l=s}) + f_0(x_{l=s} + l - s) - K_l$.

Thus, the cost of updating from the initial vAMM $y = f_0(x)$ to $y = f(x)$ can be derived as:

$$\begin{aligned}
C(f_0, f) &= \delta(f, l, s) - \delta(f_0, l, s) \\
&= K_s - f(x_{l=s}) + f(x_{l=s} + l - s) - K_l \\
&\quad - \{K_s - f_0(x_{l=s}) + f_0(x_{l=s} + l - s) - K_l\} \\
&= \{f_0(x_{l=s}) - f(x_{l=s})\} - \{f_0(x_{l=s} + l - s) - f(x_{l=s} + l - s\}] \\
&= f_0(x_{l=s}) - f(x_{l=s}) \qquad (\because f_0(x_{l=s} + l - s) = f(x_{l=s} + l - s))
\end{aligned}$$

□

3 Problem Formalization

We can formalize the update problem as follows: find a family of convex functions $\mathcal{F}$ and an update rule $\mathcal{A} : (\mathbb{R} \to \mathbb{R}) \times \mathbb{R} \times \mathbb{R}_+ \to (\mathbb{R} \to \mathbb{R})$ such that given a state function $f \in \mathcal{F}$, market state $x_{curr} \in \mathbb{R}$, and $p_{oracle} \in \mathbb{R}_+$, the output function $\mathcal{A}(f, x_{curr}, p_{oracle}) = f^* \in \mathcal{F}$ satisfies the following:

1. Current market state is maintained, i.e. $f^*(x_{curr}) = f(x_{curr})$
2. Spread is 0 after updating, i.e. $p_{mark}(x_{curr}, f^*(x_{curr})) = p_{oracle}$
3. Update cost is minimized, i.e. $f^* \in \operatorname{argmin}_{f' \in \mathcal{F}} C(f, f')$

4 vAMM

Our vAMM uses a combination of CSMM and a strictly convex function defined by linear splining to simultaneously achieve low slippage as well as lossless convergence of the mark price to the index price of the underlying asset. Compared to most existing AMMs where $x \in \mathbb{R}_{\geq 0}$ represents the amount of quote assets, our vAMM uses long-short imbalance $l - s \in \mathbb{R}$ for the x-axis, thereby allowing the functions to be defined on a real line, i.e. allowing any trade size under any market state (compared to the existing protocols, where the size of a new long position is capped at x_{curr}). Note that this leads to $x_{l=s} = 0$ for our model.

For any market, our vAMM is initialized to $f_0(x) = p_{oracle}x$ with $x = 0$. Before executing each trade, the vAMM checks to see if updating to CSMM induces a loss. Here, the target CSMM is given by the update rule

$$\mathcal{A}_{CS}(f, x_{curr}, p_{oracle}) := x \mapsto p_{oracle}x - p_{oracle}x_{curr} + f(x_{curr})$$

Derivation of $\mathcal{A}_{CS}(f, x_{curr}, p_{oracle})$

We want to find a function of the form $g(x) = ax + k$ such that the current market state is maintained and spread is 0. Current market state is maintained iff $g(x_{curr}) = ax_{curr} + k = f(x_{curr})$. Spread is 0 iff $p_{mark}(g, x_{curr}) = g'(x_{curr}) = a = p_{oracle}$. Combining the two, $k = f(x_{curr}) - p_{oracle}x_{curr}$, and $\mathcal{A}_{CS}(f, x_{curr}, p_{oracle}) = g(x) = p_{oracle}x - p_{oracle}x_{curr} + f(x_{curr})$

Proposition 1. Updating to CSMM induces a cost (i.e. $C(f_0, f) > 0$) when $p_{oracle} > \frac{f(x_{curr})}{x_{curr}}$ in a long-heavy market ($x_{curr} > 0$) and $p_{oracle} < \frac{f(x_{curr})}{x_{curr}}$ in a short-heavy market.

Proof. From Lemma 1,

$$\begin{aligned} C(f_0, f) &= f_0(x_{l=s}) - f(x_{l=s}) \\ &= f_0(x_{l=s}) - (p_{oracle}x_{l=s} - p_{oralce}x_{curr} + f(x_{curr})) \\ &= 0 - (p_{oracle} \cdot 0 - p_{oracle}x_{curr} + f(x_{curr}) \\ &= p_{oracle}x_{curr} - f(x_{curr}) \end{aligned}$$

Then, $C(f_0, f)$ is positive iff $p_{oracle}x_{curr} - f(x_{curr}) > 0$. This implies that the protocol experiences a loss iff $p_{oracle} > \frac{f(x_{curr})}{x_{curr}}$ for $x_{curr} > 0$ and $p_{oracle} < \frac{f(x_{curr})}{x_{curr}}$ for $x_{curr} < 0$. $\square$

Thus, we define our updating rule $\mathcal{A}(f, x_{curr}, p_{oracle})$ based on two cases:

Case 1: $(x_{curr} = 0) \lor (p_{oracle} < \frac{f(x_{curr})}{x_{curr}} \land x_{curr} > 0) \lor (p_{oracle} > \frac{f(x_{curr})}{x_{curr}} \land x_{curr} < 0)$

In this case, we update the curve to CSMM. In other words, we define

$$\mathcal{A}_1(f, x_{curr}, p_{oracle}) = \mathcal{A}_{CS}(f, x_{curr}, p_{oracle})$$

Proposition 2. $\mathcal{A}_1(f, x_{curr}, p_{oracle})$ defined above maintains current market state, achieves zero spread with zero update cost.

Proof. By definition, $\mathcal{A}_{CS}(f, x_{curr}, p_{oracle})$ maintains current market state and achieves 0 spread. By proposition 1., updating to CSMM in case 1 doesn't induce any cost. $\square$

Proposition 3. Any function defined by $\mathcal{A}_1(f, x_{curr}, p_{oracle})$ achieves zero slippage.

Proof. Note that slippage is defined as $|\frac{f(x+x')-f(x)}{x'} - p_{oracle}|/p_{oracle}$ where $x' \in \mathbb{R}$ is the new trade size. Here,

$$\begin{aligned}
&|\frac{\mathcal{A}_{CS}(f, x_{curr}, p_{oracle})(x+x') - \mathcal{A}_{CS}(f, x_{curr}, p_{oracle})(x)}{x'} - p_{oracle}|/p_{oracle} \\
=&|\frac{p_{oracle}(x+x') - p_{oracle}x}{x'} - p_{oracle}|/p_{oracle} \\
=&0
\end{aligned}$$

$\square$

Case 2: $(p_{oracle} > \frac{f_0(x_{curr})}{x_{curr}} \wedge x_{curr} > 0) \vee (p_{oracle} < \frac{f_0(x_{curr})}{x_{curr}} \wedge x_{curr} < 0)$

In this case, updating to CSMM will induce a loss (proposition 1). Here, we use a linear spline. In particular, we define

$$\mathcal{A}_2(f, x_{curr}, p_{oracle})(x) = \begin{cases} \mathcal{A}_{CS}(f, x_{curr}, p_{oracle})(x) & (0 \le x_{curr} \le x \text{ or } x \le x_{curr} \le 0) \\ \mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})(x) & (0 \le x \le x_{curr} \text{ or } x_{curr} \le x \le 0) \end{cases}$$

where

$$\begin{aligned}
\mathcal{A}_{BZ}(f, x_{curr}, p_{oracle}) &:= x \mapsto t^2(x)y_1 + 2t(x)(1-t(x))y_2 + (1-t(x))^2 f(x_{curr}) \\
t(x) &= \begin{cases} \frac{x-x_1}{2(x_2-x_1)} & (x_{curr} = -x_1 + 2x_2) \\ \frac{x_{curr}-x_2 - \sqrt{-x_1x_{curr}+x_2^2+(x_1-2x_2+x_{curr})x}}{x_1-2x_2+x_{curr}} & (x_{curr} > 0) \\ \frac{x_{curr}-x_2 + \sqrt{-x_1x_{curr}+x_2^2+(x_1-2x_2+x_{curr})x}}{x_1-2x_2+x_{curr}} & (x_{curr} \le 0) \end{cases} \\
(x_1, y_1) &= (0, 0) \\
(x_2, y_2) &= \begin{cases} (x_{curr} - \frac{f(x_{curr})}{p_{oracle}}, 0) & (x_{curr} > 0) \\ (0, -p_{oracle}x_{curr} + f(x_{curr})) & (x_{curr} \le 0) \end{cases}
\end{aligned}$$

Note that the market state (x, y) under $\mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})$ can also be expressed as a function of t $(0 \le t \le 1)$, where

$$\begin{cases} x(t) = t^2(x_1) + 2t(1-t)x_2 + (1-t)^2 x_{curr} \\ y(t) = t^2(y_1) + 2t(1-t)y_2 + (1-t)^2 f(x_{curr}) \end{cases} \quad \cdots \quad (*)$$

$t = 0$ corresponds to the current state $(x_{curr}, f(x_{curr}))$, and $t = 1$ corresponds to the equilibrium state $(0, 0)$.

Derivation of $\mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})$

The parameterized expression $(*)$ is a formula for quadratic Bézier curve that "smooths out" piecewise linear segments (line segment from $(0, 0)$ to (x_2, y_2) and from (x_2, y_2) to $(x_{curr}, f(x_{curr}))$). Given the parameterized expression for x, we can solve for t in the quadratic equation $(x_1 - 2x_2 + x_{curr})t^2 - 2(-x_2 + x_{curr})t + x_{curr} - x = 0$. Here, when $x_1 - 2x_2 + x_{curr} = 0, t = \frac{x_{curr}-x}{2(x_{curr}-x_2)}$. Otherwise,

$$\begin{aligned}
t &= \frac{x_{curr} - x_2 \pm \sqrt{(x_{curr}-x_2)^2 - (x_{curr}-x)(x_1 - 2x_2 + x_{curr})}}{x_1 - 2x_2 + x_{curr}} \\
&= \frac{x_{curr} - x_2 \pm \sqrt{-x_1x_{curr} + x_2^2 + (x_1 - 2x_2 + x_{curr})x}}{x_1 - 2x_2 + x_{curr}}
\end{aligned}$$

When $x_{curr} > 0$, $\mathcal{A}_{BZ}$ is defined on $x \in [0, x_{curr}]$. If $x_1 - 2x_2 + x_{curr} > 0$, or $x_{curr} > 2x_2$, the term

$$\begin{aligned}\sqrt{-x_1 x_{curr} + x_2^2 + (x_1 - 2x_2 + x_{curr})x} &\leq \sqrt{-x_1 x_{curr} + x_2^2 + (x_1 - 2x_2 + x_{curr})x_{curr}} \\ &= \sqrt{x_2^2 - 2x_2 x_{curr} + x_{curr}^2} \\ &= x_{curr} - x_2\end{aligned}$$

Thus, $t = \frac{x_{curr} - x_2 - \sqrt{-x_1 x_{curr} + x_2^2 + (x_1 - 2x_2 + x_{curr})x}}{x_1 - 2x_2 + x_{curr}}$ in order for $t \in [0,1]$. By similar reasoning, this should also hold when $x_{curr} < 2x_2$. Similarly, when $x_{curr} \leq 0, t(x) = \frac{x_{curr} - x_2 + \sqrt{-x_1 x_{curr} + x_2^2 + (x_1 - 2x_2 + x_{curr})x}}{x_1 - 2x_2 + x_{curr}}$.

We can plug in the derived t into the expression for $y(t)$, resulting in the formula
$y = \mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})(x) = t^2(x)y_1 + 2t(x)(1 - t(x))y_2 + (1 - t(x))^2 f(x_{curr})$
Expressing y in terms of x allows us to easily calculate the notional value given a position size.

Proposition 4. Given any f, x_{curr}, and p_{oracle}, $\mathcal{A}_2(f, x_{curr}, p_{oracle})$ maintains current market state, achieves zero spread with zero update cost.

Proof. Plugging in $x = x_{curr}$, by definition of $t(f, x_{curr})$ and $\mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})$, $t(x_{curr}) = 0$ and $\mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})(x_{curr}) = f(x_{curr})$ hold.
Now, since $p_{mark} = \frac{d\mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})}{dx}|_{x=x_{curr}}$,

$$\begin{aligned}p_{mark} &= \frac{d\mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})}{dx}|_{x=x_{curr}} \\ &= \frac{d\mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})/dt}{dx/dt}|_{t=0} \\ &= \frac{2ty_1 + (2-4t)y_2 - 2(1-t)f(x_{curr})}{2tx_1 + (2-4t)x_2 - 2(1-t)x_{curr}}|_{t=0} \\ &= \frac{y_2 - f(x_{curr})}{x_2 - x_{curr}} \\ &= p_{oracle}\end{aligned}$$

Finally,

$$\begin{aligned}C(f_0, \mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})) &= \mathcal{A}_0(x_{l=s}) - f_{BZ}(f, x_{curr}, p_{oracle})(x_{l=s}) \\ &= f_0(0) - \mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})(0)\end{aligned}$$

Here, $f_0(0) = 0$, $t(0) = 1$ and $\mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})(0) = y_1 = 0$. Thus, $C(\mathcal{A}_{BZ}(f, x_{curr}, p_{oracle})) = 0$ □

5 Discussion

In this explainer, we explored a new dynamic vAMM that allows us to update the model with no cost such that the mark price determined by the new curve will always be aligned with the index price provided by the oracle. However, the current design has some limitations. Specifically, since the curve only takes in long-short imbalance $l - s$ as the input, the output doesn't take into account the total liquidity $l + s$. For example, opening a long position in a market with 10 long and 10 short positions will face as much slippage as it would for opening the same position with 1000 long and 1000 short positions. This may be counter-intuitive, as the slippage is expected to decrease with an increase in liquidity.

One way to combat this is to change the curvature based on liquidity. Note that in our update rule $\mathcal{A}_{BZ}$, the second control point (x_2, y_2) was fixed to be x-intercept / y-intercept in a long / short heavy market. We can instead make it a function of total liquidity $l+s$ such that $x_2^* = x_3$ when $l+s=0$ and $\lim_{l+s\to\infty} x_2^* = x_2$. For example, we can use $x_2^* = sx_2 + (1-s)x_3$ where $s = \frac{1-e^{-(l+s)}}{1+e^{-(l+s)}}$ as a possible way of choosing the second control point.

Existing studies on automated market makers have also investigated the addition of liquidity function or profit function ($f, g : \mathbb{R}_+ \to \mathbb{R}$ such that $f(0) = g(0) = 0$) to incorporate liquidity into the pricing mechanism, such as constant-utility profit-charging market maker by Othman and Sandholm (2012) [6] or perspective market maker by Abernethy et al (2014) [1]. It may be interesting to explore appropriate liquidity functions that fits our mechanism.

6 Disclaimer

This paper is intended soley for general informational purposes. It does not constitute investment advice or a recommendation or solicitation to buy or sell any investment and should not be used in the evaluation of the merits of making any investment decision. It should not be relied upon for accounting, legal or tax advice or investment recommendations. This paper reflects current opinions of the authors. The opinions reflected herein are subject to change without being updated.

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