```
> restart; libname := libname, "/home/stephen/research" :
> with(cl) :
> ps1 := {alpha[1]=0.1, beta[1]=0.8, alpha[2]=0.2, beta[2]=0.7, alpha[3]=0.3, beta[3]
      =1.2}
```

$$ps1 := \{\alpha_1 = 0.1, \alpha_2 = 0.2, \alpha_3 = 0.3, \beta_1 = 0.8, \beta_2 = 0.7, \beta_3 = 1.2\} \quad (1)$$

```
> assume(op(flist(j→0 < alpha[j] < beta[j], 1..10)))
```

$$> \text{nu} := P \to \frac{sum(\text{alpha}[j]\cdot\exp(-\text{beta}[j]\cdot t), j=1..P)}{sum\left(\frac{\text{alpha}[j]}{\text{beta}[j]}, j=1..P\right)}$$

$$\nu := P \mapsto \frac{\sum_{j=1}^{P} \alpha_j \mathrm{e}^{-\beta_j t}}{\sum_{j=1}^{P} \frac{\alpha_j}{\beta_j}} \quad (2)$$

```
>
>
```

$$> F := P \to \frac{\sum_{j=1}^{P} \frac{\alpha_j \left(1 - \mathrm{e}^{-\beta_j t}\right)}{\beta_j}}{\sum_{j=1}^{P} \frac{\alpha_j}{\beta_j}}$$

$$F := P \mapsto \frac{\sum_{j=1}^{P} \frac{\alpha_j \left(1 - \mathrm{e}^{-\beta_j t}\right)}{\beta_j}}{\sum_{j=1}^{P} \frac{\alpha_j}{\beta_j}} \quad (3)$$

```
> simp(F(2))
```

$$-\frac{\alpha_2 \beta_1 \mathrm{e}^{-\beta_2 t} + \alpha_1 \beta_2 \mathrm{e}^{-\beta_1 t} - \alpha_1 \beta_2 - \alpha_2 \beta_1}{\alpha_1 \beta_2 + \alpha_2 \beta_1} \quad (4)$$

```
> simp(int(nu(2), t=0..s))
```

$$-\frac{\alpha_1 \mathrm{e}^{-\beta_1 s} \beta_2 + \alpha_2 \mathrm{e}^{-\beta_2 s} \beta_1 - \alpha_1 \beta_2 - \alpha_2 \beta_1}{\alpha_1 \beta_2 + \alpha_2 \beta_1} \quad (5)$$

```
> eval((4), ps1)
```

$$-0.6956521739\, \mathrm{e}^{-0.7\, t} - 0.3043478261\, \mathrm{e}^{-0.8\, t} + 1 \quad (6)$$

```
> eval((5), ps1)
```

$$-0.3043478261\, \mathrm{e}^{-0.8\, s} - 0.6956521739\, \mathrm{e}^{-0.7\, s} + 1 \quad (7)$$

> $int(\exp(-t), t = 0..s)$

$$1 - \mathrm{e}^{-s} \tag{8}$$

>

> $H := unapply\left(simp\left(\frac{\mathrm{nu}(P)}{1 - F(P)}\right), P\right)$

$$H := P \mapsto \frac{\sum_{j=1}^{P} \alpha_j \, \mathrm{e}^{-\beta_j t}}{\left(\sum_{j=1}^{P} \frac{\alpha_j}{\beta_j}\right) + \left(\sum_{j=1}^{P} \frac{\alpha_j \left(-1 + \mathrm{e}^{-\beta_j t}\right)}{\beta_j}\right)} \tag{9}$$

> $eval(F(3), ps1)$

$$1 - 0.1891891892\,\mathrm{e}^{-0.8\,t} - 0.4324324325\,\mathrm{e}^{-0.7\,t} - 0.3783783785\,\mathrm{e}^{-1.2\,t} \tag{10}$$

> $eval(H(3), ps1)$

$$\frac{0.1\,\mathrm{e}^{-0.8\,t} + 0.2\,\mathrm{e}^{-0.7\,t} + 0.3\,\mathrm{e}^{-1.2\,t}}{0.1250000000\,\mathrm{e}^{-0.8\,t} + 0.2857142857\,\mathrm{e}^{-0.7\,t} + 0.2500000000\,\mathrm{e}^{-1.2\,t}} \tag{11}$$

> $eval(\mathbf{(11)}, t = 0)$

$$0.9081081081 \tag{12}$$

> $limit(\mathbf{(11)}, t = -inf)$

$$1.200000000 \tag{13}$$

> $limit(\mathbf{(11)}, t = inf)$

$$0.7000000000 \tag{14}$$

> $eval(\mathbf{(11)}, t = 0)$

$$0.9081081081 \tag{15}$$

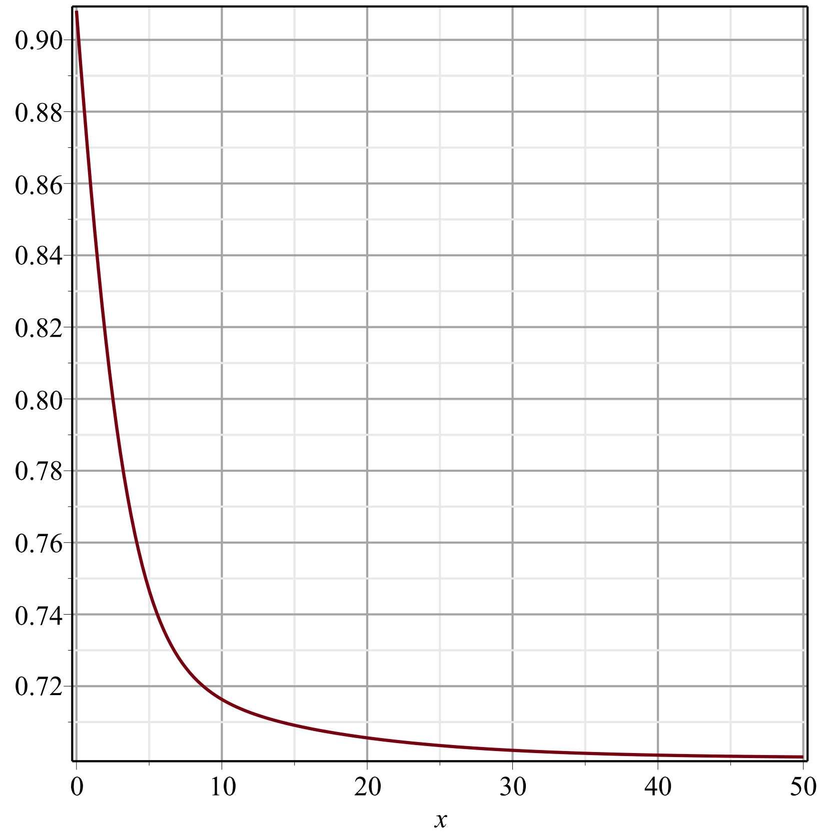
> $eval(\mathbf{(11)}, t = 20) \cdot 0.01$

$$0.007056084848 \tag{16}$$

>

>

> $pgl(\mathbf{(11)}, 0..50)$




> $-\ln(1 - int(\exp(-t), t=0..s))$ assuming $s > 0$

$$s \tag{17}$$

>

> $limit(F(1), t=inf)$ assuming $0 < \text{alpha}[1] < \text{beta}[1]$

$$1 \tag{18}$$

> $simp(F(2))$

$$-\frac{\alpha_2 \beta_1 e^{-\beta_2 t} + \alpha_1 \beta_2 e^{-\beta_1 t} - \alpha_1 \beta_2 - \alpha_2 \beta_1}{\alpha_1 \beta_2 + \alpha_2 \beta_1} \tag{19}$$

> $eval(\text{nu}(2), ps1)$

$$0.2434782609\, e^{-0.8\, t} + 0.4869565218\, e^{-0.7\, t} \tag{20}$$

>

> $simp(\text{nu}(3))$

$$\frac{\left(\alpha_1 e^{-\beta_1 t} + \alpha_2 e^{-\beta_2 t} + \alpha_3 e^{-\beta_3 t}\right) \beta_1 \beta_2 \beta_3}{\alpha_1 \beta_2 \beta_3 + \alpha_2 \beta_1 \beta_3 + \alpha_3 \beta_1 \beta_2} \tag{21}$$

> $inthaz := unapply(-\ln(1 - F(P)), P)$

$$inthaz := P \mapsto -\ln\left(1 - \frac{\sum_{j=1}^{P} \frac{\alpha_j \left(1 - e^{-\beta_j t}\right)}{\beta_j}}{\sum_{j=1}^{P} \frac{\alpha_j}{\beta_j}}\right) \quad \textbf{(22)}$$

> $invinthaz := P \rightarrow solve(inthaz(P) = y, t)$

$$invinthaz := P \mapsto solve(inthaz(P) = y, t) \quad \textbf{(23)}$$

> $invF := P \rightarrow solve(F(P) = y, t)$

$$invF := P \mapsto solve(F(P) = y, t) \quad \textbf{(24)}$$

> $hmm := to1pf('evalf'(eval(invinthaz(2), ps1)))$

$$hmm := y \mapsto evalf\left(-1.428571429\, RootOf\left(0.16\, e^{-\_Z + y} + 0.07\, e^{1.142857143\, \_Z + y} - 0.23\right)\right) \quad \textbf{(25)}$$

> $hmmF := to1pf('evalf'(eval(invF(2), ps1)));$
> # this solution needs to be restricted to the real-valued one

$$hmmF := y \mapsto evalf\left(-1.428571429\, RootOf\left(0.07\, e^{1.142857143\, \_Z} + 0.16\, e^{-\_Z} + 0.23\, y - 0.23\right)\right) \quad \textbf{(26)}$$

> $invinthaz(3)$

$$-\frac{1}{\beta_3}\left(RootOf\left(\alpha_2 \beta_1 \beta_3\, e^{\frac{\beta_2 \_Z}{\beta_3} + y} + \alpha_1 \beta_2 \beta_3\, e^{\frac{\beta_1 \_Z}{\beta_3} + y} + \alpha_3 \beta_1 \beta_2\, e^{\_Z + y} - \alpha_1 \beta_2 \beta_3 - \alpha_2 \beta_1 \beta_3 - \alpha_3 \beta_1 \beta_2\right)\right) \quad \textbf{(27)}$$

> $antiIntegratedHazard := P \rightarrow RootOf(sum((\exp(y - \mathrm{beta}[k] \cdot z) - 1) \cdot product(piecewise(j = k, \mathrm{alpha}[j], \mathrm{beta}[j]), j = 1..P), k = 1..P), z)$

$$antiIntegratedHazard := P \mapsto RootOf\left(\sum_{k=1}^{P} \left(e^{-\beta_k z + y} - 1\right) \left(\prod_{j=1}^{P} \begin{cases} \alpha_j & j = k \\ \beta_j & otherwise \end{cases}\right), z\right) \quad \textbf{(28)}$$

> $antiIntegratedHazardDerivative := unapply(eval(diff(op(1, antiIntegratedHazard(P)), \_Z), \_Z = z), P)$

$$antiIntegratedHazardDerivative := P \mapsto \sum_{k=1}^{P} \left(-\beta_k\, e^{-z \beta_k + y} \left(\prod_{j=1}^{P} \begin{cases} \alpha_j & j = k \\ \beta_j & otherwise \end{cases}\right)\right) \quad \textbf{(29)}$$

> $antiIntegratedHazardNewton := unapply\left(\_Z - \frac{op(1, antiIntegratedHazard(P))}{antiIntegratedHazardDerivative(P)}, \_Z\right)$

$$antiIntegratedHazardNewton := \_Z \mapsto \_Z - \frac{\sum_{k=1}^{P} \left(e^{-\_Z \beta_k + y} - 1\right) \left(\prod_{j=1}^{P} \begin{cases} \alpha_j & j = k \\ \beta_j & otherwise \end{cases}\right)}{\sum_{k=1}^{P} \left(-\beta_k\, e^{-z \beta_k + y} \left(\prod_{j=1}^{P} \begin{cases} \alpha_j & j = k \\ \beta_j & otherwise \end{cases}\right)\right)} \quad \textbf{(30)}$$

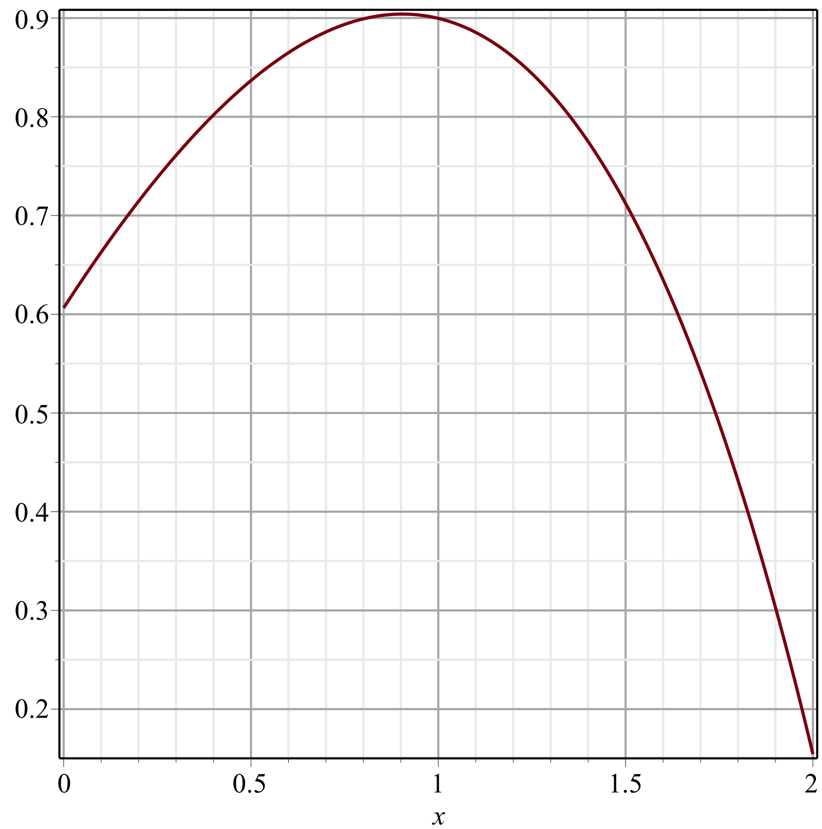
> $eval(eval(antiIntegratedHazardNewton(t), P=3), \{op(ps1), P=3\})$

$$t - \frac{0.084\,\mathrm{e}^{-0.8\,t+y} - 0.444 + 0.192\,\mathrm{e}^{-0.7\,t+y} + 0.168\,\mathrm{e}^{-1.2\,t+y}}{-0.0672\,\mathrm{e}^{y-0.8\,z} - 0.1344\,\mathrm{e}^{y-0.7\,z} - 0.2016\,\mathrm{e}^{y-1.2\,z}} \quad (31)$$

> $thefunc := to1pf(eval(eval(eval(eval(antiIntegratedHazardNewton(z), \{P=3\}), ps1), \{y=0.8\}), z=t))$

$$thefunc := t \mapsto t - \frac{0.084\,\mathrm{e}^{0.8-0.8\,t} - 0.444 + 0.192\,\mathrm{e}^{0.8-0.7\,t} + 0.168\,\mathrm{e}^{0.8-1.2\,t}}{-0.0672\,\mathrm{e}^{0.8-0.8\,t} - 0.1344\,\mathrm{e}^{0.8-0.7\,t} - 0.2016\,\mathrm{e}^{0.8-1.2\,t}} \quad (32)$$

> $pgl(thefunc, 0..2)$




> $fsolve(thefunc(t) = t, t = 0..2)$

$$0.9040293934 \quad (33)$$

> $eval(inthaz(3), ps1)$

$$-\ln\left(0.1891891892\,\mathrm{e}^{-0.8\,t} + 0.4324324325\,\mathrm{e}^{-0.7\,t} + 0.3783783785\,\mathrm{e}^{-1.2\,t}\right) \quad (34)$$

> $eval(\mathbf{(34)}, t = \mathbf{(33)})$

$$0.7999999998 \quad (35)$$

> $iterate(thefunc, 0.3, 10)$

[0.3, 0.7609487863, 0.8949929389, 0.9039920489, 0.9040293929, 0.9040293937, 0.9040293934, 0.9040293931, 0.9040293928, 0.9040293936, 0.9040293933] (36)

└>
[>