jz5880_hw2 _q5to9

Question5

1.12.2 sections b, e

(b)

$$\begin{array}{l} p \to (q \land r) \\ \neg q \\ \hline \therefore \neg p \end{array}$$

1.Hypothesis: ¬q

2.Addition: ¬q ∨ ¬r

3.De Morgan's law 2: ¬(q ∧ r)

4.Hypothesis: p → (q ∧ r)

5.Modus tollens 4, 5: ¬p

(e)

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \underline{\neg q} \\ \therefore r \end{array}$$

1.Hypothesis: ¬q

2.Hypothesis: p ∨ q

2.Disjunctive syllogism 1, 2: p

3.Hypothesis: ¬p ∨ r

4.Disjunctive syllogism 2, 3: r

1.12.3 section c

(c) One of the rules of inference is Disjunctive syllogism :

$$\begin{array}{l} p \vee q \\ \underline{\neg p} \\ \therefore q \end{array}$$

Prove that Disjunctive syllogism is valid using the laws of propositional logic and any of the other rules of inference besides Disjunctive syllogism. (Hint: you will need one of the conditional identities from the laws of propositional logic).

1.Hypothesis: p ∨ q

2.conditional identify 1: ¬p → q

3.Hypothesis: $\neg p$

4.Modus ponens 2, 3: q

1.12.5

section c

(c) I will buy a new car and a new house only if I get a job.
I am not going to get a job.

$\therefore$ I will not buy a new car.

Given:

p: will get a job

q: will buy a new car

r: will buy a new house

symbolic terms:

$(q \land r) \rightarrow p$

$\neg p$

$\therefore \neg q$

1.Hypothesis: $\neg p$

2.Hypothesis: $(q \land r) \rightarrow p$

3.Modus tollens 1, 2: $\neg(q \land r)$

4.De Morgan's law 3: $\neg q \lor \neg r$

The argument is invalid, the conclusion indicates that either I will not buy a new car, or I will not buy a new house, or both. It does not directly states I will not buy a car.

section d

> I will buy a new car and a new house only if I get a job.
> I am not going to get a job.
> I will buy a new house.
> ______________________________
> ∴ I will not buy a new car.

Given:

p: will get a job

q: will buy a new car

r: will buy a new house

symbolic terms:

(q ∧ r) -> p

¬p

r

____

∴¬q

1.Hypothesis: ¬p

2.Hypothesis: (q ∧ r) -> p

3.Modus tollens 1, 2: ¬(q ∧ r)

4.De Morgan's law 3: ¬q ∨ ¬r

5.Hypothesis: r

6.Disjunctive syllogism 4,5: ¬q

The argument is valid!

1.13.3

(b)

$$\begin{array}{l} \exists x(P(x) \vee Q(x)) \\ \exists x \neg Q(x) \\ \hline \therefore \exists x P(x) \end{array}$$

define P and Q as follows over the domain {a, b}:

P(a) = false, P(b) = false

Q(a) = true, Q(b) = false

In this counterexample, there is no x in the domain {a,b} for P(x) is true.

Therefore, the argument is invalid.

1.13.5

(d)

Every student who missed class got a detention.
Penelope is a student in the class.
Penelope did not miss class.

Penelope did not get a detention.

Defined:

- M(x): Student x missed class.
- D(x): Student x got a detention.

symbolic terms:

$\forall x(M(x) \rightarrow D(x))$

Penelope, a student in the class

$\neg M(\text{Penelope})$

$\therefore \neg D(\text{Penelope})$

Truth table:

M(Penelope)	D(Penelope)
F	T

When Penelope did not miss class, but get a detention. both hypothesis both evaluate to true. However, ¬D(x) evaluates to false. Therefore, this statement is invalid

(e)

Every student who missed class or got a detention did not get an A.
Penelope is a student in the class.
Penelope got an A.

Penelope did not get a detention.

Defined:

- M(x): Student x missed class.
- D(x): Student x got a detention.
- A(x): Student x got an A.

symbolic terms:

$\forall x((M(x) \lor D(x)) \to \neg A(x))$

Penelope, a student in the class

A(Penelope)

$\therefore \neg D(\text{Penelope})$

1.Hypothesis: Penelope, a student in the class

2.Hypothesis: $\forall x((M(x) \lor D(x)) \to \neg A(x))$

3.Universal Instantiation 1, 2: $M(\text{Penelope}) \lor D(\text{Penelope}) \to \neg A(x)$

4.Hypothesis: A(Penelope)

5.Modus Tollens 3, 4: $\neg(M(\text{Penelope}) \lor D(\text{Penelope}))$

6.De Morgans Law 5: $\neg M(\text{Penelope}) \land \neg D(\text{Penelope})$

7.Simplification 6: $\neg D(\text{Penelope})$

The Statement is vaild!!

Question 6:

Solve Exercise 2.4.1, section d; Exercise 2.4.3, section b, from the Discrete Math zyBook:

> (d) The product of two odd integers is an odd integer.

proof:

suppose x = 2m + 1, where m is an integer

suppose y = 2n + 1, where n is an integer

xy = (2m+1)(2n+1) = 4mn + 2m + 2n + 1

since 4mn, 2m, and 2n are even number because they're are divisible by 2, any even numbers puls one will make it an odd number. Therefore, "The product of two odd integers is an odd integer" is valid.

(b) If x is a real number and $x \leq 3$, then $12 - 7x + x^2 \geq 0$.

Given:

Statement 1: x ≤ 3

Statement 2: 12 - 7x + x^2 ≥ 0

$$12 - 7x + x^2 = (x - 2/7)^2 - 1/4$$

plug in x = 3, we get $(-1/2)^2 - 1/4 = 0$.

Therefore, the statement is valid.

Question 7

Solve Exercise 2.5.1, section d; Exercise 2.5.4, sections a, b; Exercise 2.5.5, section c, from the Discrete Math zyBook:

2.5.1, section d

(d) For every integer n, if $n^2 - 2n + 7$ is even, then n is odd.

suppose n = 2x, where x is an integer

plug into the equation we get:

$(2x)^2 - 2 * 2x + 7$

= $4x^2 - 4x + 7$

=$4x^2 - 4x + 6 + 1$

=$2(2x^2 - 2x + 3) + 1$

The terms $2(2x^2 - 2x + 3)$ will always be an even number, puls one make it an odd number. Therefore, the statement is true.

2.5.4, sections a, b

(a) For every pair of real numbers x and y, if $x^3 + xy^2 \leq x^2y + y^3$, then $x \leq y$.

(b) For every pair of real numbers x and y, if $x + y > 20$, then $x > 10$ or $y > 10$.

a).

Rearrange the equation we get: $x^3 + xy^2 - x^2y - y^3 \leq 0$

$x^2(x-y) + y^2(x-y) \leq 0$

$(x^2 + y^2)(x-y) \leq 0$

$(x^2 + y^2)$ is always bigger than 0, x - y must be less or equal to 0 in order to hold the statement true.

Therefore, (x - y) ≤ 0 which is x ≤ y.

b).

supposed x ≤ 10 and y ≤ 10

the maximum possible value for x + y = 10 + 10 =20, it is not possible to have x+ y ≥ 20.

Therefore, x ≤ 10 and y ≤ 10 is **false**.

Therefore, if x + y > 20, then either x > 10 or y > 10 or both.

2.5.5, section c

(c) For every non-zero real number x, if x is irrational, then $\frac{1}{x}$ is also irrational.

Assume *x* is irrational

if 1/x is rational, it can be expressed as p/q for some integers p and q(q not equal to 0)

Since 1/x not equal to 0, then p not equal to 0.

$1/x = p/q$

$x = q/p$

Since x is equal to a ratio of two non-zero integers , it follows that x must be a rational number.

Question 8

Solve Exercise 2.6.6, sections c, d

(c) The average of three real numbers is greater than or equal to at least one of the numbers.

(d) There is no smallest integer.

c).

proof

supposed we have three number a, b, and c

Average = $(a+b+c)/3$

assume $(a+b+c)/3 < a$, $(a+b+c)/3 < b$, $(a+b+c)/3 < c$

adding three inequalities we get $(a + b + c) < (a + b + c)$

There is a contradiction exists. therefore, the statement “The average of three real numbers is greater than or equal to at least one of the numbers” is true

d).

proof

supposed we have the smallest integer n.

n-1 is the smaller integer we can find with respect to n.

There is a contradiction exists. therefore, the statement “There is no smallest integer” is true

Question 9

Solve Exercise 2.7.2, section b

(b) If integers x and y have the same parity, then $x + y$ is even.
The parity of a number tells whether the number is odd or even. If x and y have the same parity, they are either both even or both odd.

x and y have the same parity

1). x and y are both even, supposed x = 2m, y = 2n, where m and n are integers.

x + y = 2m + 2n

x + y = 2(m+n)

2(m + n) is even number where x and y are integers.

2). x and y are both odd, supposed x = 2m + 1, y = 2n + 1, where m and n are integers.

x + y = 2m + 1 + 2n + 1

x + y = 2m + 2n + 2

x + y = 2(m + n + 1)

2(m + n + 1) is even number where x and y are integers.

In both cases, the sum of x + y is even. Therefore, the statement is true