

DEPARTMENT OF ELECTRONIC AND TELECOMMUNICATION  
UNIVERSITY OF MORATUWA

EN 3150: PATTERN RECOGNITION

This is offered as a "EN 3150: Pattern Recognition" module's partial completion.



**Assignment 04 : Kernel methods**

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## Abstract

*This report deals with the explanation of the solutions for the given questions in the assignment 02 of the EN3150 module. The solutions are explained in a way that it is easy to understand and follow. The solutions are explained with the help of the code snippets and the results. We are mainly focussed on using the kernel methods and utilizing it for Support vector machine algorithm in the case of non seperable data.*

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## 1 Kernel Methods

### 1.1 Question 01

$$\phi(x) = (1, \sqrt{2}x, x^2) \quad (1)$$

for one dimensional data,

$$\phi(z) = (1, \sqrt{2}z, z^2) \quad (2)$$

$$\phi(z) * \phi(x) = (1 + 2xz + x^2z^2) \quad (3)$$

$$K(x, z) = (1 + 2xz + x^2z^2) \quad (4)$$

$$K(x, z) = (1 + xz)^2 \quad (5)$$

### 1.2 Question 02

$$K(x, z) = (1 + xz)^2 \quad (6)$$

for 2 dimensional input data,

$$K(x, z) = (1 + x_1z_1 + x_2z_2)^2 \quad (7)$$

### 1.3 Question 03

Finding the mapping function for the given kernel function,

$$K(x, z) = (1 + x_1z_1 + x_2z_2)^2 \quad (8)$$

$$K(x, z) = (1 + x_1z_1 + x_2z_2)(1 + x_1z_1 + x_2z_2) \quad (9)$$

$$K(x, z) = (1 + (x_1z_1)^2 + (x_2z_2)^2 + 2x_1z_1 + 2x_2z_2 + 2x_1z_1x_2z_2) \quad (10)$$

$$\phi(x_1, x_2) = (1, \sqrt{2}x_1, \sqrt{2}x_2, x_1^2, x_2^2, \sqrt{2}x_1x_2) \quad (11)$$

### 1.4 Question 04

$$G = \begin{bmatrix} k(x_1, x_1) & k(x_1, x_2) & k(x_1, x_3) & k(x_1, x_4) \\ k(x_2, x_1) & k(x_2, x_2) & k(x_2, x_3) & k(x_2, x_4) \\ k(x_3, x_1) & k(x_3, x_2) & k(x_3, x_3) & k(x_3, x_4) \\ k(x_4, x_1) & k(x_4, x_2) & k(x_4, x_3) & k(x_4, x_4) \end{bmatrix} \quad (12)$$

$$G = \begin{bmatrix} 27^2 & 24^2 & 15^2 & 71^2 \\ 24^2 & 26^2 & 21^2 & 79^2 \\ 15^2 & 21^2 & 21^2 & 65^2 \\ 71^2 & 79^2 & 65^2 & 245^2 \end{bmatrix} \quad (13)$$

$$G = \begin{bmatrix} 729 & 576 & 225 & 5041 \\ 576 & 676 & 441 & 6241 \\ 225 & 441 & 441 & 4225 \\ 5041 & 6241 & 4225 & 60025 \end{bmatrix} \quad (14)$$

List of eigen values,

$$\lambda_1 = 61397.1 \quad (15)$$

$$\lambda_2 = 382.206 \quad (16)$$

$$\lambda_3 = 81.5348 \quad (17)$$

$$\lambda_4 = 10.116 \quad (18)$$

All eigen values are **non-negative** as well as the matrix is a **symmetric matrix**. Therefore the matrix is **positive semi definite matrix**. And also this will be a valid kernel.

## 2 Implementation of Kernel and Results

### 2.1 Data Generation and Visualization

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 from sklearn.datasets import make_circles
4 # Generate data with make_circles
5 np.random.seed(5)
6 X, y = make_circles(n_samples=500,
7                     factor=0.3, noise=0.1)
8 # Plot data
9
10 plt.scatter(X[:, 0], X[:, 1], c=y, cmap=
11             plt.cm.Paired)
12 plt.show()
```

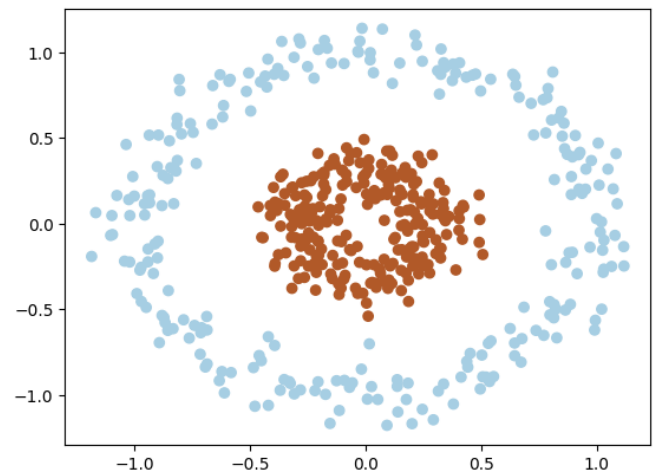


Figure 1: Data Generation

## 2.2 Using mapping functions

### Function 01

$$\phi : x = (x_1, x_2) \rightarrow \phi(x) = (x_1, x_2, x_1^2 + x_2^2) \quad (19)$$

The following code was used to implement the above mentioned function.

```
1 def mappingKernel (x1,x2) :
2     return x1,x2,x1**2 + x2**2
3
4
5 mappedx_1 ,mappedy_1 ,mappedz_1 =
6     mappingKernel (X[:, 0], X[:, 1])
7
8 # visualize in 3D
9
10 from mpl_toolkits.mplot3d import Axes3D
11 fig = plt.figure ()
12 ax = fig.add_subplot (111 , projection ='3d
13                        ')
14 ax.scatter (mappedx_1 ,mappedy_1 ,mappedz_1
15            , c=y, cmap=plt.cm.Paired)
16 plt.show()
```

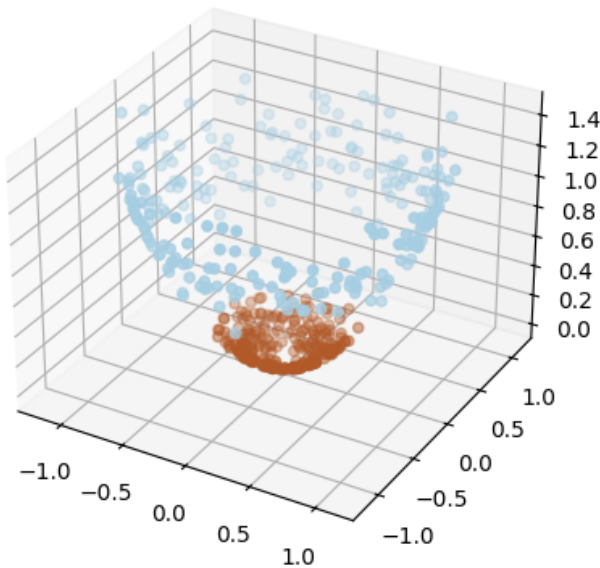


Figure 2: Data Generation

We have created a data set with a factor of 0.3 and we have used the above mentioned function to map the data into a higher dimensional space.

From this Visualization we can see that the data is linearly separable. We have mapped the data into a higher dimensional space and we can use this data for our linear support vector machine algorithm.

If we try to increase the factor from 0.3 to 0.5,

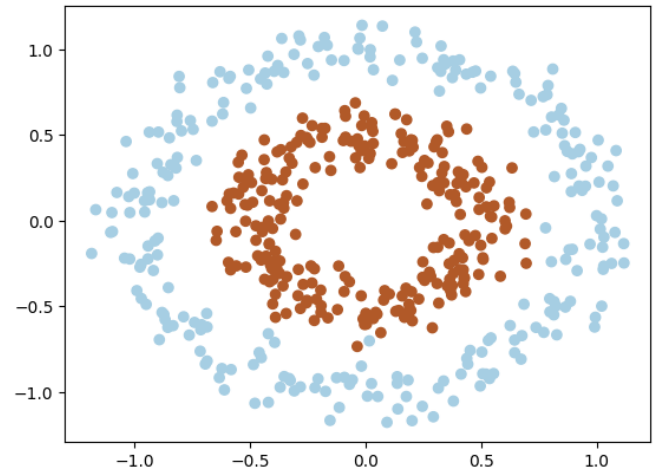


Figure 3: Data Generation

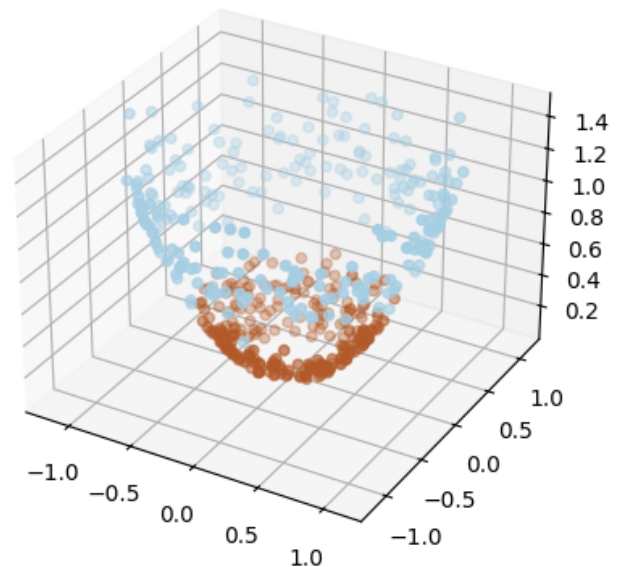


Figure 4: Data Generation

Normally factor determines the radius of the inner circle with respect to the radius of the outer circle. In this scenario where we increased the factor from 0.3 to 0.5 we can clearly see that the data tends to mix up, even in the higher dimensional space, the data from the two classes tends to mix up.

### Function 02

Now for the same data (factor = 0.3) if we use the following mapping function,

$$\phi : x = (x_1, x_2) \rightarrow \phi(x) = (x_1^2, x_2^2, x_1x_2) \quad (20)$$

And the following code was used to implement the above mentioned function.

```
1 def mappingKernel (x1,x2) :
2     return x1**2,x2**2,x1*x2
3
4
5 mappedx_2 ,mappedy_2 ,mappedz_2 =
6     mappingKernel (X[:, 0], X[:, 1])
7
8 # visualize in 3D
9
10 from mpl_toolkits.mplot3d import Axes3D
11 fig = plt.figure ()
12 ax = fig.add_subplot (111 , projection ='3d
13                        ')
14 ax.scatter (mappedx_2 ,mappedy_2 ,mappedz_2
15             , c=y, cmap=plt.cm.Paired)
16 plt.show()
```

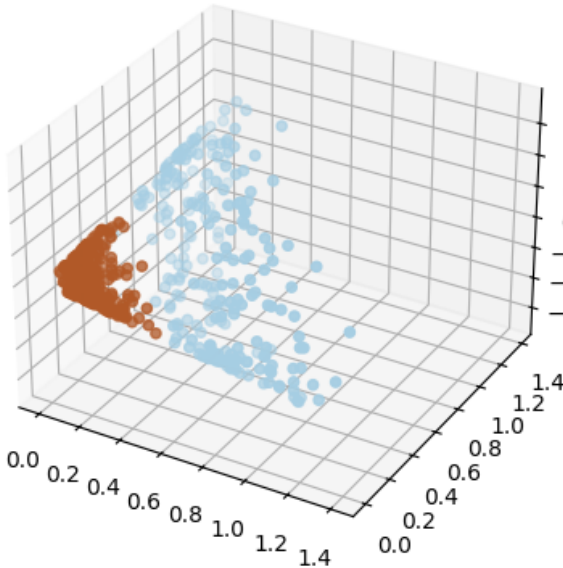


Figure 5: Data Generation

Comparing to the above mapping this mapping seems to be somewhat better as we can clearly see two clusters of data in the higher dimensional space which will be easy to separate using a linear boundary surface. What happens here is as we are squaring the  $x_1$  and  $x_2$  features because of the non-linear mapping and the feature expansion we can get a clear separable data which can be distinguished by a linear decision boundary.

## 2.3 Running Linear SVC

### For data without mapping

```
1 from sklearn.model_selection import
2     train_test_split
3 import sklearn.svm as svm
4 from sklearn.metrics import accuracy_score,
5     precision_score, recall_score, f1_score
6
7 # Split the data into training and testing
8 sets
9 X_train, X_test, y_train, y_test =
10     train_test_split(X, y, test_size=0.2)
11
12 # Create an instance of the LinearSVC class
13 and fit it to the training data
14 svm_classifier = svm.SVC(kernel='linear')
15 svm_classifier.fit(X_train, y_train)
16
17 # Make predictions on the testing data
18 y_pred = svm_classifier.predict(X_test)
19
20 # Evaluate the performance of the model
21 using metrics such as accuracy,
22 precision, recall, and F1 score
23 print("Accuracy:", accuracy_score(y_test,
24 y_pred))
25 print("F1 score:", f1_score(y_test, y_pred))
26
27 plt.scatter (X[:, 0], X[:, 1], c=y, cmap=
28             plt.cm.Paired)
29
30 xx = np.linspace(X.min(), X.max(), 50)
31 yy = (svm_classifier.intercept_ -
32       svm_classifier.coef_[0][0] * xx)
33 plt.plot(xx, yy, alpha=0.3, color='Black')
```

and following were the results,

- Accuracy:
- Precision:
- Recall:
- F1 Score:

### For data with first mapping

```
1 from sklearn.svm import SVC
2 from mpl_toolkits.mplot3d import Axes3D
3
4 # Create a linear SVM
5 svm_classifier = SVC(kernel='linear')
6
7 # Use the mapped 3-dimensional data
8 mapped_data = np.column_stack((mappedx_1,
9 mappedy_1, mappedz_1))
10
11 # Split the test and training data sets.
```

```

12 X_train, X_test, y_train, y_test =
    train_test_split(mapped_data, y,
        test_size=0.2)
13
14 # Fit the SVM model
15 svm_classifier.fit(X_train, y_train)
16
17 # Make predictions using the model
18
19 y_pred = svm_classifier.predict(X_test)
20
21 # Visualize the decision boundary
22 fig = plt.figure()
23 ax = fig.add_subplot(111, projection='3d')
24
25 # Plot the points
26 ax.scatter(mappedx_1[y == 0], mappedy_1[y
    == 0], mappedz_1[y == 0], color='r',
    marker='o', label='Class 0')
27 ax.scatter(mappedx_1[y == 1], mappedy_1[y
    == 1], mappedz_1[y == 1], color='b',
    marker='^', label='Class 1')
28
29 # Plot the decision boundary
30 xx, yy = np.meshgrid(np.linspace(mappedx_1.
    min(), mappedx_1.max(), 50),
    np.linspace(mappedy_1.
    min(), mappedy_1.
    max(), 50))
31
32 zz = (-svm_classifier.intercept_[0] -
    svm_classifier.coef_[0][0] * xx -
    svm_classifier.coef_[0][1] * yy) /
    svm_classifier.coef_[0][2]
33 ax.plot_surface(xx, yy, zz, alpha=0.3,
    color='gray')
34
35 # Set labels
36 ax.set_xlabel('X')
37 ax.set_ylabel('Y')
38 ax.set_zlabel('Z')
39 ax.legend()
40
41 # Show the plot
42 plt.show()
43
44
45 # Evaluate the performance of the model
    using metrics such as accuracy,
    precision, recall, and F1 score
46 print("Accuracy:", accuracy_score(y_test,
    y_pred))
47 print("Precision:", precision_score(y_test,
    y_pred))
48 print("Recall:", recall_score(y_test,
    y_pred))
49 print("F1 score:", f1_score(y_test, y_pred)
    )

```

and following were the results,

- Accuracy: 1.0
- Precision: 1.0
- Recall: 1.0
- F1 Score: 1.0

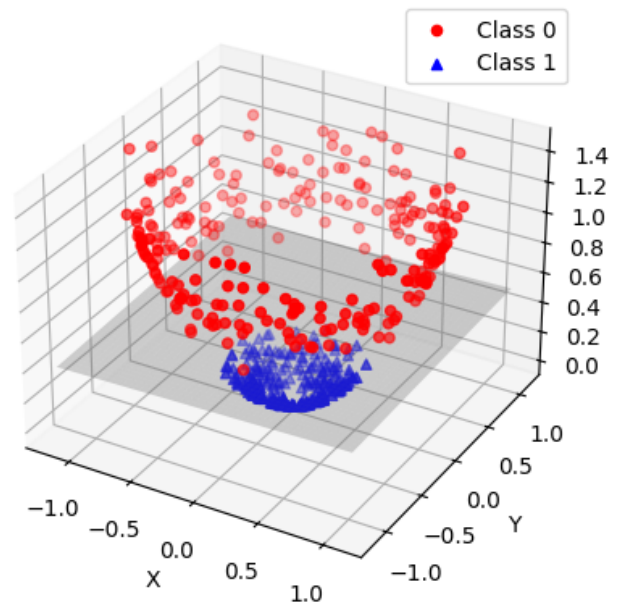


Figure 6: Data Generation

*For data with second mapping*

```

1 from sklearn.svm import SVC
2 from mpl_toolkits.mplot3d import Axes3D
3
4 # Create a linear SVM
5 svm_classifier = SVC(kernel='linear')
6
7 # Use the mapped 3-dimensional data
8 mapped_data = np.column_stack((mappedx_1,
    mappedy_1, mappedz_1))
9
10 # Split the test and training data sets.
11
12 X_train, X_test, y_train, y_test =
    train_test_split(mapped_data, y,
        test_size=0.2)
13
14 # Fit the SVM model
15 svm_classifier.fit(X_train, y_train)
16
17 # Make predictions using the model
18
19 y_pred = svm_classifier.predict(X_test)
20
21 # Visualize the decision boundary
22 fig = plt.figure()
23 ax = fig.add_subplot(111, projection='3d')
24
25 # Plot the points
26 ax.scatter(mappedx_1[y == 0], mappedy_1[y
    == 0], mappedz_1[y == 0], color='r',
    marker='o', label='Class 0')
27 ax.scatter(mappedx_1[y == 1], mappedy_1[y
    == 1], mappedz_1[y == 1], color='b',
    marker='^', label='Class 1')
28
29 # Plot the decision boundary
30 xx, yy = np.meshgrid(np.linspace(mappedx_1.
    min(), mappedx_1.max(), 50),

```

```

31         np.linspace(mappedy_1.
                    min(), mappedy_1.
                    max(), 50))
32 zz = (-svm_classifier.intercept_[0] -
        svm_classifier.coef_[0][0] * xx -
        svm_classifier.coef_[0][1] * yy) /
        svm_classifier.coef_[0][2]
33 ax.plot_surface(xx, yy, zz, alpha=0.3,
        color='gray')
34
35 # Set labels
36 ax.set_xlabel('X')
37 ax.set_ylabel('Y')
38 ax.set_zlabel('Z')
39 ax.legend()
40
41 # Show the plot
42 plt.show()
43
44
45 # Evaluate the performance of the model
    using metrics such as accuracy,
    precision, recall, and F1 score
46 print("Accuracy:", accuracy_score(y_test,
    y_pred))
47 print("Precision:", precision_score(y_test,
    y_pred))
48 print("Recall:", recall_score(y_test,
    y_pred))
49 print("F1 score:", f1_score(y_test, y_pred)
    )

```

and following were the results,

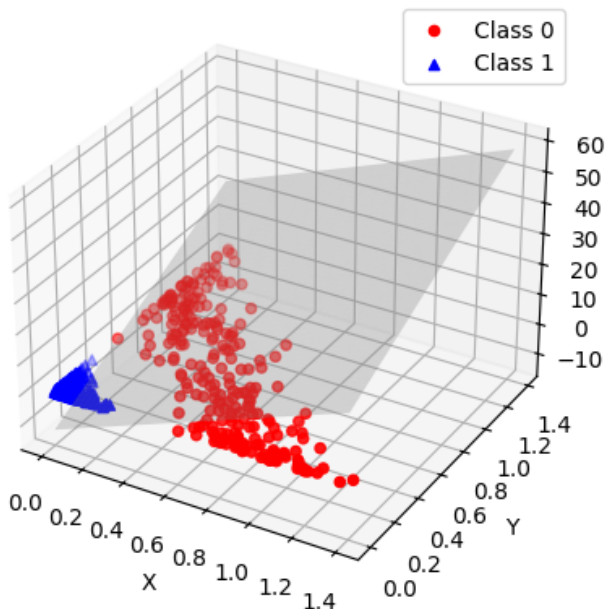


Figure 7: Data Generation

- Accuracy: 1.0
- Precision: 1.0
- Recall: 1.0
- F1 Score: 1.5

### 3 References

- [Sci-kit Learn Documentation on Logistic regression](#)
- [Pipelining](#)
- [Grid Search](#)

### 4 Github Repository

Following is the link to my Github repository for this assignment.

[Github/EN3150\\_Assignment\\_04](#)