Ejercicio 5 PRIMITIVAS

a) $\int \cos^3(x)\,dx$

$$\int \cos^3(x)\,dx = \int (1-\text{sen}^2(x))\cdot\cos(x)\,dx \quad \left\{ \begin{array}{l} t = \text{sen}(x) \\ dt = \cos(x)dx \end{array} \right.$$

$$\int (1-t^2)\,dt = t - \frac{t^3}{3} \Rightarrow \boxed{\text{sen}(x) - \frac{\text{sen}^3(x)}{3} + C}$$

b) $\int \text{sen}^5(x)\,dx$

$$\int \text{sen}^5(x)dx = \int \text{sen}^4(x)\cdot\text{sen}(x)\,dx = \int (1-\cos^2 x)^2\cdot\text{sen}(x)\,dx \quad \left\{ \begin{array}{l} t = \cos(x) \\ dt = -\text{sen}(x)dx \end{array} \right.$$

$$\int -(1-t^2)^2\,dt = \int -(1+t^4-2t^2)\,dt = -t - \frac{t^5}{5} + \frac{2t^3}{3} \Rightarrow$$

$$\boxed{-\cos(x) - \frac{\cos^5(x)}{5} + \frac{2\cos^3(x)}{3} + C}$$

c) $\int \text{sen}^2(x)\cdot\cos^3(x)\,dx$

$$\int \text{sen}^2(x)\cos^3(x)dx = \int \text{sen}^2(x)\cdot(1-\text{sen}^2(x))\cdot\cos(x)\,dx \quad \left\{ \begin{array}{l} t = \text{sen}(x) \\ dt = \cos(x)dx \end{array} \right.$$

$$\int u^2(1-u^2)\,dt = \int (t^2 - t^4)\,dt = \frac{t^3}{3} - \frac{t^5}{5} = \frac{\text{sen}^3(x)}{3} - \frac{\text{sen}^5(x)}{5}$$

d) $\int sen^2(x) \cdot cos^2(x)\, dx$

Primero: $\left\{ \right.$ Sabemos que $cos(2x) = cos^2 x - sen^2(x)$ y que $cos^2(x) + sen^2(x) = 1$

$$2 sen^2(x) + cos(2x) = \overbrace{cos^2(x) - \cancel{sen^2(x)} + \cancel{sen^2(x)} + sen^2(x)}^{1} \Rightarrow$$

$$2sen^2(x) = 1 - cos(2x) \Rightarrow sen^2(x) = \frac{1 - cos(2x)}{2}$$

∴ (con un razonamiento parecido llegamos a que $cos^2(x) = \frac{1 + cos(2x)}{2}$

$$\int \left(\frac{1 - cos(2x)}{2}\right)\left(\frac{1 + cos(2x)}{2}\right) dx = \frac{1}{4}\int (1 - cos(2x))(1 + cos(2x))\, dx = \frac{1}{4}\int (1 - cos^2(2x))\, dx =$$

$$\frac{1}{4} \cdot \int sen^2(2x)\, dx = \frac{1}{4} \cdot \int \frac{1 - cos(4x)}{2}\, dx = \int \frac{1}{8}\, dx - \int \frac{cos(4x)}{8}\, dx =$$

$$\boxed{\frac{1}{8} x - \frac{sen(4x)}{4} \cdot \frac{1}{8} + C}$$

e) $\int cos^6(3x)\, dx \quad \left\{ \begin{array}{l} u = 3x \\ du = 3\, dx \Rightarrow dx = \frac{du}{3} \end{array} \right.$

$$\frac{1}{3}\int cos^6(u)\, du \Rightarrow \frac{1}{3}\int \left(\frac{1 + cos 2u}{2}\right)^3 du = \frac{1}{3}\int \left[\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 \left(\frac{cos 2u}{2}\right) + 3\left(\frac{1}{2}\right)\left(\frac{cos 2u}{2}\right)^2 + \right.$$

$$\left. + \left(\frac{1}{2} cos 2u\right)^3 \right) du = \frac{1}{3}\int \left(\frac{1}{8} + \frac{3}{8} cos 2u + \frac{3}{8} cos^2 2u + \frac{1}{8} cos^3 2u\right) du =$$

$$\frac{1}{24}\int du + \frac{1}{8}\int cos 2u\, du + \frac{1}{8}\int cos^2 2u\, du + \frac{1}{24}\int cos^3 2u\, du =$$

$$\frac{1}{24} u + \frac{1}{8} \cdot \frac{sen 2u}{2} + \frac{1}{8}\int cos^2 2u\, du + \frac{1}{24}\int cos^3 2u\, du$$

$$\rightarrow \int \cos^2 2u \, du \quad \begin{cases} t = 2u \\ dt = 2du \end{cases} \Rightarrow$$

$$\frac{1}{2} \cdot \int \cos^2 t \, dt = \frac{1}{2} \cdot \int \frac{1+\cos 2t}{2} dt = \int \frac{1}{4} dt + \frac{1}{2} \int \frac{2\cos 2t}{4} dt =$$

$$\frac{1}{4} t + \frac{\operatorname{sen}(2t)}{8} = \frac{1}{2} u + \frac{\operatorname{sen}(4u)}{8} + C$$

$$\int \cos^3 2u \, du = \begin{cases} t = 2u \\ \frac{dt}{2} = du \end{cases} \Rightarrow \frac{1}{2} \int \cos^3(t) \, dt \quad \text{(Resuelta en el ap. a))}$$

$$\frac{1}{2} \cdot \int \cos^3(t) dt = \frac{1}{2} \left(\operatorname{sen}(t) - \frac{\operatorname{sen}^3(t)}{3} \right) = \frac{\operatorname{sen}(2u)}{2} - \frac{\operatorname{sen}^3(2u)}{6} + C$$

$$\int \cos^6 3x \, dx = \boxed{\frac{x}{8} + \frac{\operatorname{sen} 6x}{16} + \frac{3x}{16} + \frac{\operatorname{sen}(12x)}{64} + \frac{\operatorname{sen}(6x)}{48} - \frac{\operatorname{sen}^3(6x)}{144} + C}$$

f) $\displaystyle\int \frac{\cos^5(x)}{\operatorname{sen}^3(x)} dx$

$$\int \frac{\cos^4(x) \cdot \cos(x)}{\operatorname{sen}^3(x)} dx = \int \frac{(1-\operatorname{sen}^2 x)^2 \cdot \cos(x)}{\operatorname{sen}^3(x)} dx \quad \begin{cases} t = \operatorname{sen}(t) \\ dt = \cos t \end{cases}$$

$$= \int \frac{(1-t^2)^2}{t^3} dt = \int \frac{t^4 - 2t^2 + 1}{t^3} dt = \int t \, dt - 2 \int \frac{1}{t} dt + \int \frac{1}{t^3} dt =$$

$$\frac{t^2}{2} - 2\ln|t| - \frac{1}{t^2} + C = \boxed{\frac{\operatorname{sen}^2(t)}{2} - 2\ln(|\operatorname{sen}(x)|) - \frac{1}{\operatorname{sen}^2(x)} + C}$$