→ /·. 1. Calcula los 10 nodos de Chebyshev x0, x1, . . . , x9 en el intervalo [0, 91/11]. ·/;

```
linsolve([91/11=a+b, 0=−a+b],[a,b]);
```

(%o4) $$\left[a=\frac{91}{22},b=\frac{91}{22}\right]$$

(%i8)
```
a:91/22;
```

(a) $$\frac{91}{22}$$

(%i9)
```
b:91/22;
```

(b) $$\frac{91}{22}$$

(%i78)
```
nodos_cheb:makelist(cos(%pi·(2·i+1)/(2·10)),i,0,9);
```

(nodos_cheb) $$\left[\cos\left(\frac{\pi}{20}\right),\cos\left(\frac{3\,\pi}{20}\right),\frac{1}{\sqrt{2}},\cos\left(\frac{7\,\pi}{20}\right),\cos\left(\frac{9\,\pi}{20}\right),\cos\left(\frac{11\,\pi}{20}\right),\cos\left(\frac{13\,\pi}{20}\right),-\frac{1}{\sqrt{2}},\cos\left(\frac{17\,\pi}{20}\right),\cos\left(\frac{19\,\pi}{20}\right)\right]$$

(%i79)
```
funcion(x):= a·x + b;
```

(%o79) $$\text{funcion}(x):=a\,x+b$$

→ /·Me he equivocado al aplicar la función afin para calcular los nodos correspondientes ·/

(%i93) `nodos:makelist(float(funcion(nodos_cheb)),i,0,9);`

(nodos)
$$\left[\begin{pmatrix} 12.24517292334283 & 5.124051976958775 \\ 11.01365061934811 & 5.027370160552005 \\ 8.941272252551446 & 4.843470417550185 \\ 6.656502726960694 & 4.590354136103183 \\ 4.783433469030046 & 4.292798101403868 \\ 3.710604720607508 & 3.979929171323406 \\ 3.465565278416705 & 3.68237313662409 \\ 3.744507934560977 & 3.429256855177089 \\ 4.080372200105839 & 3.245357112175269 \\ 4.074296651146691 & 3.148675295768499 \end{pmatrix},\right.$$
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→ /·2. Determina la forma de Newton del polinomio de interpolaci´on del problema:
p □ P9 : j = 0, 1, . . . , 9 ⇒ p(xj) = log (1 + √xj) + M
y calcula p(3).·/

(%i20)
```
p(x):=log(1+sqrt(x))+8;
```
(%o20) $p(x):=\log(1+\sqrt{x})+8$

(%i24)
```
y:makelist(p(nodos[i+1]),i,0,9);
```
(y) [9.352036703919852,9.329255536052356,9.282808226234117,
9.210771751578863,9.1098176768277,8.97446173758483,
8.795500408470724,8.55642462936176,8.22368771533288,
8.22368771533288]

(%i81)
```
A:genmatrix(float(lambda([i,j], 0)),10,10);
```
(A)

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

(%i82)
```
for i:1 thru 10 do(
   A[i][1]:float(y[i])
);
```
(%o82) *done*

(%i83)
```
for i : 2 thru 10 do(
   for j: i thru 10 do(
      A[j][i]:(A[j][i−1]−A[j−1][i−1]) / (nodos[j]−nodos[j−i+1])
   )
);
```
(%o83) *done*

(%i84)
```
diferencias_divididas:makelist(0,i,1,10);
```
(diferencias_divididas) [0,0,0,0,0,0,0,0,0,0]

```
(%i85) for i:1 thru 10 do (
           diferencias_divididas[i]:A[i][i]);
(%o85) done

(%i86) funcion_omega(x,j):=product(x−nodos[i],i,1,j);
```

(%o86) funcion_omega$(x, j) :=$ $\prod_{i=1}^{j} (x - nodos_i)$

```
(%i87) for i:2 thru 10 do(
           omega[i]:funcion_omega(x,i));
(%o87) done

(%i88) Newton(x):=sum(diferencias_divididas[i]·funcion_omega(x,i−1),i,1,10);
```

(%o88) Newton$(x) :=$ $\sum_{i=1}^{10} diferencias\_divididas_i \; \text{funcion\_omega}(x, i-1)$

(%i90) expand(Newton(x));

(%o90)

$$\begin{pmatrix} -1.453198727201292\,10^{-8}\,x^9+6.535730205190279\,10^{-7}\,x^8-1.109228603023934\,10^{-5}\,x^7+8.20430782 \\ -2.532404469088604\,10^{-8}\,x^9+1.127201852463143\,10^{-6}\,x^8-1.926187600275744\,10^{-5}\,x^7+1.50283796 \\ -7.971834434979665\,10^{-8}\,x^9+3.478887996118258\,10^{-6}\,x^8-6.006431175425851\,10^{-5}\,x^7+5.078129 \\ -4.919939105226894\,10^{-7}\,x^9+2.089287772099296\,10^{-5}\,x^8-3.638045968737653\,10^{-4}\,x^7+0.0033 \\ -6.946899853015684\,10^{-6}\,x^9+2.857307164134449\,10^{-4}\,x^8-0.004983965704676399\,x^7+0.04 \\ -3.005229565261909\,10^{-4}\,x^9+0.01195950225774534\,x^8-0.2070456994702476\,x^7+2.0 \\ -0.07335883197862628\,x^9+2.833236803952235\,x^8-48.31055677886295\,x^7+477.2 \\ -499.561460790414\,x^9+18858.56012926017\,x^8-316097.8114735905\,x^7+3087620.531605273 \\ -1.611361293256593\,10^{11}\,x^9+6.008214251760914\,10^{12}\,x^8-9.95657087144911\,10^{13}\,x^7+9.6246594324592 \\ -1.611361293256593\,10^{11}\,x^9+6.008214251760914\,10^{12}\,x^8-9.95657087144911\,10^{13}\,x^7+9.6246594324592 \end{pmatrix}$$

(%i91) Newton(3);

(%o91)

$$\begin{pmatrix} 8.984462588943755 & 9.907410847744528 \\ 8.979602426460092 & 9.907410847744528 \\ 8.968360896949795 & 9.907410847744528 \\ 8.94664337410972 & 9.907410847744528 \\ 8.904606104919118 & 9.907410847744528 \\ 8.817963433752182 & 9.907410847744528 \\ 8.451062340068901 & 9.907410847744528 \\ 1512.140774278936 & 9.907410847744528 \\ 5.330764655471618\,10^{11} & 9.907410847744528 \\ 5.330764655471618\,10^{11} & 9.907410847744528 \end{pmatrix}$$

➔ /·3Halla la recta que mejor aproxima, en el sentido de los m´ınimos cuadrados, los datos (xj , lo M), j = 0, 1, . . . , 9 y dibuja simult´aneamente los puntos y la recta.·/

(%i68) u:[1,1,1,1,1,1,1,1,1,1];

(u) [1,1,1,1,1,1,1,1,1,1]

(%i69) a:transpose(matrix(nodos, u));

(a)
$$\begin{pmatrix} 8.209886614732316 & 1 \\ 7.718559624744724 & 1 \\ 6.795166930976141 & 1 \\ 5.551083320119813 & 1 \\ 4.136363636363637 & 1 \\ 2.721643952607462 & 1 \\ 1.477560341751133 & 1 \\ 0.5541676479825495 & 1 \\ 0.06284065799495764 & 1 \\ 0.06284065799495764 & 1 \end{pmatrix}$$

(%i70) recta : invert(transpose(a).a).transpose(a).y;

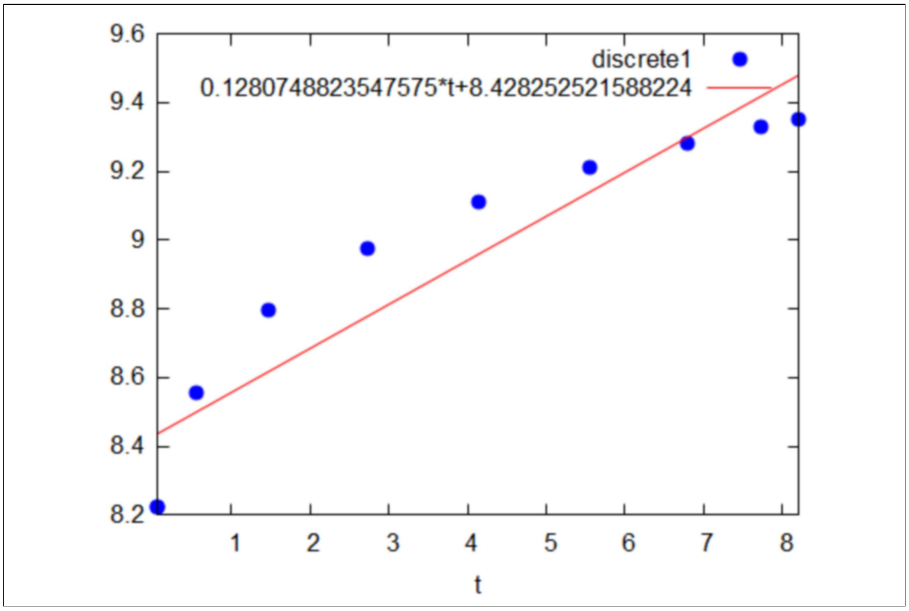
(%o70)
$$\begin{pmatrix} 0.1280748823547575 \\ 8.428252521588224 \end{pmatrix}$$

(%i73) lista:[[nodos[1],y[1]],[nodos[2],y[2]],[nodos[3],y[3]],[nodos[4],y[4]],[nodos[5],y[5]],[nodos[6],y[6]],[

(lista) [[8.209886614732316,9.352036703919852],[7.718559624744724,9.329255536052356],[6.795166930976141,9.282808226234117],[5.551083320119813,9.210771751578863],[4.136363636363637,9.1098176768277],[2.721643952607462,8.97446173758483],[1.477560341751133,8.795500408470724],[0.5541676479825495,8.55642462936176],[0.06284065799495764,8.22368771533288],[0.06284065799495764,8.22368771533288]]

(%i77) wxplot2d([[discrete, lista], recta[1][1]·t+recta[2][1]],[t, nodos[10],nodos[1]], [style, points, lines]);

(%t77)



(%o77)