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① Tomando $B_u = \{1, x, x^2\}$

obtengamos primero $M(g, B_u)$.

$$g(1,1) = \int_0^1 1 \cdot dx = [x]_0^1 = 1$$

$$g(1,x) = \int_0^1 x\, dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

$$g(1,x^2) = \int_0^1 x^2\, dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

$$g(x,x) = \int_0^1 x^2\, dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

$$g(x^2,x^2) = \int_0^1 x^4\, dx = \left[\frac{x^5}{5}\right]_0^1 = \frac{1}{5}$$

$$g(x,x^2) = \int_0^1 x^3 dx = \frac{1}{4}$$

$$M(g, B_u) = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix}$$

a) $U = L(\{x - \frac{1}{2}, x - 3\})$

Hacemos gram-Schmidt

sigue

La Base ortonormal: $\{u_1\ u_2\}$

$$u_1 = x - \frac{1}{2} = \left(-\frac{1}{2}, 1, 0\right)$$

$$u_2 = (-3,1,0) - \frac{g\left(\left(-\frac{1}{2},1,0\right),(-3,1,0)\right)}{g\left(\left(-\frac{1}{2},1,0\right)\left(-\frac{1}{2},1,0\right)\right)}\left(-\frac{1}{2},1,0\right)$$

$$g\left(\left(-\frac{1}{2},1,0\right)(-3,1,0)\right) = \left(-\frac{1}{2},1,0\right)\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix}(-3\ 1\ 0)^t = \frac{1}{12}$$

$$g\left(\left(-\frac{1}{2},1,0\right)\left(-\frac{1}{2},1,0\right)\right) = \left(-\frac{1}{2},1,0\right)\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix}\left(-\frac{1}{2},1,0\right)^t = \frac{1}{12}$$

$$u_2 = (-3,1,0) - \frac{\frac{1}{12}}{\frac{1}{12}}\left(-\frac{1}{2},1,0\right) = (-3,1,0) - \left(\frac{1}{2},1,0\right) =$$

$$= (-3{,}5,\ 0,0)$$

$$B = \left\{\left(-\frac{1}{2},1,0\right)(-3'5,0,0)\right\}$$

$$g\left((3'5,0,0)(-3'5,0,0)\right) = \frac{49}{4}$$

$$B_{ortonormal} = \left\{\frac{\left(-\frac{1}{2},1,0\right)}{\sqrt{\frac{1}{12}}},\ \frac{(-3'5,0,0)}{\frac{7}{2}}\right\} = \left\{\sqrt{12}\left(-\frac{1}{2},1,0\right),\ \frac{2(3'5,0,0)}{7}\right.$$

Luego dicha base es la ortonormal del espacio métrico $(U, g_{|U})$. Calculemos $U^\perp$ pues $\mathbb{R}_2[x] = U \oplus U^\perp$ al ser ~~es~~ una métrica euclídea.

b)

$U^{\perp}$: Viene dada por:

$$\sqrt{12}\left(-\frac{1}{2},1,0\right)\begin{pmatrix}1 & \frac{1}{2} & \frac{1}{3}\\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4}\\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5}\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\left(0,\frac{\sqrt{3}}{6},\frac{\sqrt{3}}{6}\right)\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \quad \cdot\ \frac{\sqrt{3}}{6}y+\frac{\sqrt{3}}{6}z=0$$

$$\frac{2}{7}\left(3'5,0,0\right)\begin{pmatrix}1 & \frac{1}{2} & \frac{1}{3}\\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4}\\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5}\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\left(-1,-\frac{1}{2},-\frac{1}{3}\right)\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \quad \cdot\ -x-\frac{y}{2}-\frac{z}{3}=0$$

$$\begin{cases}\frac{\sqrt{3}}{6}y+\frac{\sqrt{3}}{6}z=0\\ -x-\frac{y}{2}-\frac{z}{3}=0\end{cases} \qquad \begin{cases}\frac{\sqrt{3}}{6}y=-\frac{\sqrt{3}}{6}z \quad y=-z\\ -x-\frac{y}{2}=\frac{z}{3} \quad -x=\frac{z}{3}-\frac{z}{2}=\end{cases}$$

$$=\frac{z}{2}-\frac{z}{3}=\frac{z}{6}$$

Luego $U^{\perp}=\left\{\left(\frac{1}{6},-1,1\right)\right\}$

$$g\left(\left(\frac{1}{6},-1,1\right)\left(\frac{1}{6},-1,1\right)\right)=\frac{1}{180}$$

Luego la base de $(\mathbb{R}^2[X],g)$ es:

$$B=\left\{\sqrt{12}\left(-\frac{1}{2},1,0\right),\ \frac{2(3'5,0,0)}{7},\ \frac{\left(\frac{1}{6},-1,1\right)}{\sqrt{\frac{1}{180}}}\right\}$$

(Siendo B_0 la base de $\mathbb{R}_2[x]$ calculada antes)

b) $$M(\sigma, B_0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

c) $$M(r, B_0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \frac{\sqrt{2}}{2} & \sqrt{2}/2 \end{pmatrix}$$

d) Para clasificarla multiplicamos las dos matrices ~~prebi~~ previas.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \frac{\sqrt{2}}{2} & \sqrt{2}/2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix} = \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{2}{4} = -1$$

Calculamos el ángulo: $\theta = \arccos\left(\frac{1}{2}(Tr(A)+1)\right)$

$Tr(A) = 1$ $\qquad \theta = \arccos\left(\frac{1}{2} + \cancel{1}\right) = \arccos(1) =$

$=$

~~Luego se trata de una simetría especular respecto de V_1~~

Simetría especular respecto V_1