Biyección que lo lleve a su forma canónica?

$C = \{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+2xy+2x-2y-2z+1=0\}$

Tenemos $M_R(C) = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 0 \\ -1 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$ y $N_R(C) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = A$

Calculamos los subespacios propios del núcleo lineal:

$$\det(A-\alpha I_3) = \begin{vmatrix} 1-\alpha & 1 & 0 \\ 1 & 1-\alpha & 0 \\ 0 & 0 & -\alpha \end{vmatrix} = -\alpha\left((1-\alpha)^2-1\right) = 2\alpha^2-\alpha^3 = \alpha^2(2-\alpha)$$

$\alpha = 0$

$\alpha = 0$

$\alpha = 2$

Entonces

$$V_0 = \left\{ u : \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \right\} = \begin{matrix} x+y=0 \\ x=-y \end{matrix} = L\{(1,-1,0),(0,0,1)\} = L\left\{\tfrac{1}{\sqrt{2}}(1,-1,0),(0,0,1)\right\}$$

Es base ortonormal

$$V_2 \equiv \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \quad \begin{matrix} -x+y=0 \\ -2z=0 \end{matrix} \Rightarrow L\{(1,1,0)\} \Rightarrow L\left\{\tfrac{1}{\sqrt{2}}(1,1,0)\right\}$$

Dividimos por el valor propio $\Rightarrow V_2 = L\left\{\tfrac{1}{2}(1,1,0)\right\}$

Entonces en $R_1 = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, B = \left\{ \tfrac{1}{2}(1,1,0), \tfrac{1}{\sqrt{2}}(1,-1,0), (0,0,1) \right\}\right.$

$$M(I_d, R_1, R_0) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/\sqrt{2} & 0 \\ 0 & 1/2 & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$M_{R_1}(C) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/\sqrt{2} & 0 \\ 0 & 1/2 & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^t \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 0 \\ -1 & 1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/\sqrt{2} & 0 \\ 0 & 1/2 & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & \sqrt{2} & -1 \\ 0 & 1 & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

Entonces si $(a,b,c)_{R_1}$ tenemos

$$(1 \;\; y_1 \; y_2 \; y_3) \cdot \begin{pmatrix} 1 & \sqrt{2} & -1 & 0 \\ \sqrt{2} & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix} = 0 \Rightarrow -2y_3 + 2\sqrt{2}\,y_2 + y_1^2 + 1 = 0$$

$$\boxed{-2y_3 + 2\sqrt{2}y_2 + y_1^2 + 1 = 0}$$

$$\left.\begin{array}{l} z_1 = y_1 \\ z_2 = y_2 \\ z_3 = \sqrt{2}y_2 - y_3 + \frac{1}{2} \end{array}\right\} \quad z_1^2 + 2z_3 = 0 \Rightarrow \text{cilindro parabólico}$$

Es el cambio
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\frac{1}{2} & 0 & \sqrt{2} & -1 \end{pmatrix} = M(I, R_1, R)$$

$$M_R(C) = M(I, R_1, R)^t \cdot M_{R_1}(C) \cdot M(I, R_1, R)$$
$$= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$M(I, R_2, R_0) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/\sqrt{2} & 0 \\ 0 & 1/2 & -1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\frac{1}{2} & 0 & \sqrt{2} & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/\sqrt{2} & 0 \\ 0 & 1/2 & \frac{-1}{\sqrt{2}} & 0 \\ 1/2 & 0 & \sqrt{2} & -1 \end{pmatrix}$$

y entonces $f(x,y,z) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/\sqrt{2} & 0 \\ 0 & 1/2 & \frac{-1}{\sqrt{2}} & 0 \\ 1/2 & 0 & \sqrt{2} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix}$ es una aplicación afín biyectiva que lleve C a su forma canónica