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(2) Resolver:

$$x_{n+2} + x_{n+1} + x_n = 0 \qquad \begin{cases} x_0 = 1 \\ x_1 = 0 \end{cases}$$

Solución: $p(x) = \lambda^2 + \lambda + 1 \qquad p(\lambda) = 0 \iff \lambda^2 + \lambda + 1 = 0.$

$$\lambda = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{1 \cdot 2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3} \cdot i}{2}$$

$$\lambda_1 = \frac{-1}{2} + \frac{\sqrt{3}}{2} \cdot i \qquad \lambda_2 = \frac{-1}{2} - \frac{\sqrt{3}}{2} \cdot i$$

Raíces complejas, una conjugado de la otra.

$$\rho = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$$\theta = \operatorname{arctg}\left(\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right) = \operatorname{arctg}(-\sqrt{3}) = -60^\circ = 300^\circ$$

Luego la solución:

$$x_n = \rho^n \cdot (A \cdot \cos(n \cdot \theta) + B \cdot \operatorname{sen}(n \cdot \theta)) =$$

$$= 1^n \cdot (A \cdot \cos(-n60^\circ) + B \cdot \operatorname{sen}(-n \cdot 60^\circ)) =$$

$$= A \cdot \cos(-n \cdot 60^\circ) + B \cdot \operatorname{sen}(-n \cdot 60^\circ)$$

Tomando los valores iniciales x_0 y x_1:

$$\begin{cases} 1 = A \cdot \cos(0) + B \cdot \operatorname{sen}(0) \Rightarrow \boxed{1 = A} \\ 0 = A \cdot \cos(-60^\circ) + B \cdot \operatorname{sen}(-60^\circ) \Rightarrow \end{cases}$$

$$\Rightarrow 0 = \frac{A}{2} + B \cdot \frac{\sqrt{3}}{2} \Rightarrow B = \frac{2}{\sqrt{3}} \cdot \left(-\frac{1}{2}\right) = \boxed{\frac{-1}{\sqrt{3}}}$$

$$x_n = 1 \cdot \cos(-n \cdot 60^\circ) - \frac{1}{\sqrt{3}} \cdot \operatorname{sen}(-n \cdot 60)$$