⑥ sucesión $x_n = \frac{1}{2}(4n + 1 + (-1)^n) \ \forall n \geq 0$

$x_0 = \frac{1}{2} \cdot (4 \cdot 0 + 1 + (-1)^0) = 1$

$x_1 = \frac{1}{2} \cdot (4 \cdot 1 + 1 + (-1)^1) = 2$

$\vdots$

¿ $x_n = 4 + x_{n-2} \ \forall n \geq 2$?

~~Sabemos~~ $x_2 = \frac{1}{2} \cdot (4 \cdot 2 + 1 + (-1)^2) = \frac{10}{2} = 5 = x_0 + 4 \Rightarrow$

$\Rightarrow$ Se cumple para el primer término

Suponemos que se cumple para x_n.

Comprobamos que se cumple para x_{n+1}:

$x_{n+1} = \frac{1}{2} \cdot (4n + 4 + 1 + (-1)^{n+1}) =$

$= \frac{1}{2}(4n + 5 + (-1)^{n+1}) = \frac{1}{2} \cdot (4n + 8 - 3 + (-1)^{n+1}) =$

$= \frac{1}{2} \cdot 8 + \frac{1}{2} \cdot (4n - 3 + (-1)^{n+1}) = 4 + \underbrace{\frac{1}{2}(4n - 3 + (-1)^{n+1})}_{* \, x_{n-1}}$

$* \ x_{n-1} = \frac{1}{2} \cdot (4(n-1) + 1 + (-1)^{n-1}) = \frac{1}{2} \cdot (4n - 3 + (-1)^{n-1})$

Resulta que $(-1)^{n+1}$ y $(-1)^{n-1}$ son iguales.

Luego por inducción hemos demostrado que

$x_n = 4 + x_{n-2}$

⑦

1) $X_n = 4n+1$
$X_{n-1} = 4(n-1)+1$ } Restamos ⇒ $X_n - X_{n-1} = 4n+1 - 4(n-1) - 1 =$

$= +4$

$X_n = X_{n-1} + 4$
$X_{n-1} = X_{n-2} + 4$ } Restamos ⇒ $X_n - X_{n-1} = X_{n-1} - X_{n-2}$ ⇒

⇒ $X_n = 2X_{n-1} - X_{n-2}$

2) $y_n = 2^n + n = 2^n + n \cdot 1^n$ ⇒

⇒ Polinomio característico: $(x-2)\cdot(x-1)^2 =$

$= (x-2)(x^2-2x+1) = x^3 - 2x^2 + x - 2x^2 + 4x - 2 =$

$= x^3 - 4x^2 + 5x - 2$ ⇒

⇒ $x^3 - 4x^2 + 5x - 2 = 0$ ⇒ $x^3 = +4x^2 - 5x + 2$ ⇒

⇒ $x_n = 4x_{n-1} - 5x_{n-2} + 2 \cdot x_{n-3}$

3) $z_n = 2^n + (n+1)3^n$ ⇒ Polinomio característico:

$(x-2)\cdot(x-3)^2 = (x^2 - 6x + 9)(x-2) = x^3 - 2x^2 - 6x^2 + 12x + 9x - 18 =$

$= x^3 - 8x^2 + 21x - 18$ ⇒

⇒ $z_n = 8z_{n-1} - 21z_{n-2} + 18z_{n-3}$

(8)

a) $F_{n+2} > 2F_n \quad \forall n \geq 2$

$$F_{n+2} = F_{n+1} + F_n = F_n + F_{n-1} + F_n = 2F_n + F_{n-1} > 2F_n$$

b) $\sum_{i=0}^{n} (F_i)^2 = F_n \cdot F_{n+1} \ \forall n \geq 0$

Para $n=0$ $\quad \sum_{i=0}^{0} (F_0)^2 = F_0 \cdot F_1 = 0$ ✓

Suponemos cierto para n.

Comprobamos para $n+1$:

$$\sum_{i=0}^{n+1} (F_i)^2 = F_{n+1} \cdot F_{n+2} \quad \forall n \geq 0$$

$$\sum_{i=0}^{n+1} (F_i)^2 = \sum_{i=0}^{n} (F_i)^2 + (F_{n+1})^2 = F_n \cdot F_{n+1} + (F_{n+1})^2 =$$

$$= F_{n+1} (F_n + F_{n+1}) = F_{n+1} \cdot F_{n+2}$$ ✓

c) 5 divide a $F_{5n} \quad \forall n \geq 0$

Para $n=0$ $\quad F_{5 \cdot 0} = F_0 = 0$

Suponemos cierto para n.

Comprobamos para $n+1$:

$$F_{5n+5} = F_{5n+4} + F_{5n+3} = F_{5n+3} + F_{5n+2} + F_{5n+2} + F_{5n+1} =$$

$$= F_{5n+2} + F_{5n+1} + F_{5n+2} + F_{5n+2} + F_{5n+1} =$$

$$= 5(F_{n+1}) + 3(F_{5n}) = 5F_{n+1} + 3 \cdot 5 \cdot c =$$

$$= 5(F_{n+1} + 3 \cdot c) \Rightarrow$$ ✓

d) $F_{n-1} \cdot F_{n+1} = (F_n)^2 + (-1)^n \quad \forall n \geq 1$

$F_0 \cdot F_2 = 0 = 1^2 - 1$ ✓ Para $n=1$

supavemos para n y comprobamos para $n+1$:

$F_{n-1} \cdot F_{n+1} = (F_n)^2 + (-1)^n \Rightarrow F_{n-1} \cdot F_{n+1} - (-1)^n = (F_n)^2 =$

$= F_{n-1} \cdot F_{n+1} + (-1)^{n+1}$

$F_n \cdot F_{n+2} = F_n \cdot (F_n + F_{n+1}) = (F_n)^2 + F_n \cdot F_{n+1} = F_{n-1} F_{n+1} + (-1)^{n+1} + F_n \cdot F_{n+1} =$

$= F_{n+1}(F_{n-1} + F_n) + (-1)^{n+1} = (F_{n+1})^2 + (-1)^{n+1}$ ✓

e) $mcd(F_n, F_{n+1}) = 1 \quad \forall n \geq 0$

Para $n=0$ $\quad mcd(F_0, F_1) = 1$ ✓

Supavemos para n. Comprobamos para $n+1$

$mcd(F_{n+1}, F_{n+2}) = mcd(F_{n+1}, F_{n+1} + F_n) = mcd(F_{n+1}, F_n) = 1$ ✓

(a)

1) $x_0 = 1 \quad x_1 = 1 \quad x_n = 2x_{n-1} - x_{n-2} \quad \forall n \geq 2$

Ec. característica $\quad x^2 - 2x + 1 = 0 \Rightarrow (x+1)^2 = 0$

Sol. g: $\quad An + B$

~~Sol~~ p. $\quad x = 1 \Rightarrow (An+B) \cdot 1^n = An + B.$

~~B~~ Sol par. $\left.\begin{cases} B = 1 \\ A + B = 1 \Rightarrow A = 0 \end{cases}\right\} a_n = 1$

2) $x_0 = 1 \quad x_1 = 2 \quad x_n = 5x_{n-1} - 6x_{n-2} \quad \forall n \geq 2.$

Ec. cara. $\quad x^2 - 5x + 6 = 0 \Rightarrow (x-3)(x-2) = 0$

Sol g: $\cancel{(An+B)^n}$ $A \cdot 3^n + B \cdot 2^n$

sol p. $\begin{cases} A + B = 1 & A = 1 - B \\ 3A + 2B = 2 & 3 - 3B + 2B = 2 \quad -B = -1 \Rightarrow B = 1 \quad A = 0 \end{cases}$

$a_n = 2^n \cdot B = 2^n$

3) $x_0 = 1 \quad x_1 = 1 \quad x_n = 3x_{n-1} + 4x_{n-2} \quad \forall n \geq 2$

$x^2 - 3x - 4 = 0 \Rightarrow (x-4)(x+1) = 0$

Sol g: $\quad A \cdot (4)^n + B \cdot (-1)^n$

Sol p: $\begin{cases} A + B = 1 & A = 1 - B \\ 4A - B = 1 & 4 - 4B - B = 1 \quad B = \frac{3}{5} \quad A = \frac{2}{5} \end{cases}$

$a_n = \frac{2}{5} \cdot 4^n + \frac{3}{5}(-1)^n$

4) $X_0=1 \quad X_1=2 \quad X_n = -X_{n-1}+6X_{n-2} \quad \forall \ n\geq 2$

Ec. cara. $\quad x^2+x-6=0 \qquad (x-2)(x+3)=0$

Sol g. $\quad A\cdot(2)^n + B\cdot(-3)^n$

Sol p. $\begin{cases} A+B=1 & A=1-B \\ 2A-3B=2 & 2-2B-3B=2 \qquad B=0 \Rightarrow A=1 \end{cases}$

$a_n = 2^n$

5) $x_0=0 \quad x_1=1 \quad x_n = 2x_{n-1} - 2x_{n-2} \quad \forall \ n\geq 2.$

Ec. c. $\quad x^2-2x+2=0 \qquad (x-(1+i))(x-(1-i))=0$

Sol g. $\quad A\cdot(1+i)^n + B(1-i)^n$

$\begin{cases} A+B=0 \qquad A=-B \\ A(1+i)+B(1-i)=1 \Rightarrow -B(1+i)+B(1-i)=1 \Rightarrow \end{cases}$

$\Rightarrow -B(1+i-1+i)=1 \qquad -B(2i)=1 \qquad B=-\frac{1}{2i}$

$A=\frac{1}{2i}$

$a_n = \frac{1}{2i}(1+i)^n - \frac{1}{2i}(1-i)^n$

6) $X_0=5 \quad X_1=12 \quad X_n = 6X_{n-1} - 9X_{n-2} \quad \forall \ n\geq 2$

Ec. car. $\quad x^2-6x+9=0 \qquad (x-3)^2=0$

Sol g. $\quad (An+B)3^n$

Sol p. $\begin{cases} B=5 \\ (A+B)\cdot 3=12 \qquad 3A+15=12 \qquad A=-1 \end{cases}$

$a_n=(-n+5)3^n$

7) $x_0=1 \quad x_1=1 \quad x_2=2 \quad x_n = 5x_{n-1} - 8x_{n-2} + 4x_{n-3} \ \forall n \geq 3$.

Ec. c. $\quad x^3 - 5x^2 + 8x - 4 = 0 \qquad (x-1)(x-2)^2 = 0$

Sol. g. $\quad (An+B)\cdot 2^n + C\cdot 1^n$

Sol p. $\begin{cases} B + C = 1 \\ 2A + 2B + C = 1 \\ 8A + 4B + C = 2 \end{cases} \quad B = 1 - C \quad \begin{cases} 2A + 2 - 2C + C = 1 \\ 8A + 4 - 4C + C = 2 \end{cases} \quad \begin{cases} 2A - C = -1 \\ 8A - 3C = -2 \end{cases}$

$\begin{cases} 2A + 1 = C \Rightarrow \quad 8A - 3(2A+1) = -2 \quad 8A - 6A - 3 = -2 \\ \qquad 2A = 1 \quad A = \frac{1}{2} \end{cases}$

$C = 2 \qquad B = -1$

$a_n = \left(\frac{n}{2} - 1\right)2^n + 2\cdot 1^n$

8) $x_0=1 \quad x_1=1 \quad x_2=2 \quad x_n = x_{n-1} + x_{n-2} - x_{n-3} \quad \forall n \geq 3$

Ec. ca. $\quad x^3 - x^2 - x + 1 = 0 \qquad (x-1)^2(x+2) = 0$

Sol. g. $\quad (An+B)(1)^n + C\cdot(-2)^n$

Sol p. $\begin{cases} B + C = 1 \\ A + B - 2C = 1 \\ 2A + B + 4C = 2 \end{cases} \quad B = 1 - C \quad \begin{cases} A + 1 - C - 2C = 1 \\ 2A + 1 - C + 4C = 2 \end{cases} \quad \begin{cases} A - 3C = 0 \\ 2A + 3C = 1 \end{cases}$

$\{ \ 3A = 1 \quad A = \frac{1}{3} \quad C = \frac{1}{9} \quad B = \frac{8}{9}$

$a_n = \left(\frac{n}{3} + \frac{8}{9}\right) + \frac{1}{9}\cdot(-2)^n$

9)

$x_0=0 \quad x_1=1 \quad x_2=2 \quad x_n = x_{n-1}+2x_{n-2}-x_{n-3} \; \forall n \geq 3.$

Ec. carac. $\quad x^3-x^2-2x+1=0 \quad (x+2)(x+1)(x$

$\boxed{\text{NO SE HACE}}$

10)

$x_0=1 \quad x_1=1 \quad x_2=3 \quad x_n = 4x_{n-1}-5x_{n-2}+2x_{n-3} \; \forall n \geq 3$

Ec.c: $x^3-4x^2+5x-2=0 \quad (x-1)^2(x-2)$

Sol g: $(An+B)(1)^n + C(2)^n = An+B+C2^n$

Sol p: $\begin{cases} B+C=1 \\ A+B+2C=1 \\ 2A+B+4C=3 \end{cases}$ $\quad B=1-C \quad \begin{cases} A+1-C+2C=1 \\ 2A+1-C+4C=3 \end{cases} \quad \begin{cases} A+C=0 & A=-C \\ 2A+3C=2 \end{cases}$

$-2C+3C=2 \Rightarrow$

$C=2$

$A=-2 \quad B=-1$

$a_n = (-2n-1)+2^{n+1}$

11)

$x_0=1 \quad x_1=3 \quad x_2=7 \quad x_n=3x_{n-1}-3x_{n-2}+x_{n-3} \; \forall n \geq 3$

Ec. carac.

$x^3-3x^2+3x-1=0 \Rightarrow (x-1)^3=0$

Sol. g: $(An^2+Bn+C)1^n = An^2+Bn+C$

Sol. p: $\begin{cases} C=1 \\ A+B+C=3 \\ 4A+2B+C=7 \end{cases}$ $\quad \begin{cases} A+B=2 \\ 4A+2B=6 \end{cases} \quad A=2-B$

$8-4B+2B=6$

$-2B=-2$

$B=1$

$A=1$

$C=1$

$a_n = n^2+n+1$

12) $x_0=0 \quad x_n = 2x_{n-1}+1 \quad \forall n \geq 1$

$x_n - 2x_{n-1} = 1$ $\qquad 1 = b^n \cdot p(n) \Rightarrow b=1 \quad p(n)=1$

$gr(p)=0$

P. caracte. $\quad (x-2)(x-1).$

Sol. g. $\quad A\cdot 2^n + B$

Extendemos las condiciones.

$x_0=0 \quad x_1=1 \quad x_2=3$

Sol p.: $\begin{cases} A+B=0 \\ 2A+B=1 \end{cases}$ $\quad A=-B \quad -B=1 \Rightarrow B=-1 \quad A=1$

Extendemos las condiciones para una sucesión arbitraria.

$x_0=a \quad x_1=2a+1$

Sol g. $\begin{cases} a = A+B \\ 2a+1 = 2A+B \end{cases}$ $\begin{cases} 2a+1 = A+a \\ A = a+1 \\ B=-1 \end{cases}$ $\begin{cases} S_n = (a+1)2^n - 1 \end{cases}$

13) $x_0=1 \quad x_n = x_{n-1}+n \quad \forall n \geq 1$

$x_n - x_{n-1} = n$ $\qquad b^n \cdot p(n) = n \Rightarrow b=1 \quad p(n)=n \quad gr(p)=1$

Po.ca. $\quad (x-1)(x-1)^2 = (x-1)^3$

Sol g. $\quad An^2+Bn+C$

Extendemos ~~condiciones~~ iniciales.

$x_0=1 \quad x_1=2 \quad x_2=4$

Sol p. $\begin{cases} C=1 \\ A+B+C=2 \\ 4A+2B+C=4 \end{cases}$ $\begin{cases} A+B=1 \\ 4A+2B=3 \end{cases}$ $\quad A=1-B \quad 4-2B=3 \quad -2B=-1 \quad B=\frac{1}{2}$

$A=\frac{1}{2}$

Sol par $\quad a_n = \frac{1}{2}n^2 + \frac{1}{2}n + 1$

Sol g. de la no homogénea.

$x_0 = a \qquad x_1 = a+1 \qquad x_2 = a+1+2 = a+3.$

$$\begin{cases} a = C \\ A+B+C = a+1 \\ 4A+2B+C = a+3 \end{cases} \quad \begin{cases} A+B = 1 \\ 4A+2B = 3 \end{cases} \quad \begin{array}{|l} A = \frac{1}{2} \\ B = \frac{1}{2} \end{array} \qquad S_n = \frac{n(n+1)}{2} + a$$

14) $x_0 = 1 \quad x_n = 2x_{n-1} + n \quad \forall\, n \geq 1$

$x_n - 2x_{n-1} = n \qquad b^n \cdot p(n) = n \Rightarrow b^n = 1 \quad p(n) = n \quad gr(p) = 1$

Pol. car: $(x-2)(x-1)^2$

Sol g: $A \cdot 2^n + (Bn+C) \cdot 1^n = A \cdot 2^n + Bn + C$

Extendemos condiciones iniciales:

$x_0 = 1 \qquad x_1 = 3 \qquad x_2 = 8$

$$\begin{cases} A + C = 1 \\ 2A + B + C = 3 \\ 4A + 2B + C = 8 \end{cases} \quad A = 1 - C \quad \begin{cases} \cancel{2A+B} \; \cancel{2+B} \; B - C = 1 \quad \cancel{B = 1 - C} \\ \cancel{4A} \; \cancel{2B - 3C = 4} \quad \cancel{2 - 2C - 5C = 4} \end{cases}$$

$$\begin{cases} 4A + 2B + 2C = 6 \\ 4A + 2B + C = 8 \end{cases} \qquad \cancel{3C} \quad C = -2 \qquad A = 3 \qquad B = -1$$

$a_n = 3 \cdot 2^n - n - 2$

Sol. g. no homogénea

$x_0 = a \qquad x_1 = 2a+1 \qquad x_2 = 4a + 2 + 2 = 4a + 4 = 4(a+1)$

$$\begin{cases} A + C = a \\ 2A + B + C = 2a+1 \\ 4A + 2B + C = 4(a+1) \end{cases} \quad \begin{cases} 4A + 2B + 2C = 4a + 2 \\ 4A + 2B + C = 4(a+1) \end{cases} \qquad \begin{array}{l} C = -2 \\ A = a + 2 \\ B = -1 \end{array}$$

$S_n = (a+2) \cdot 2^n - n - 2$

15) $x_0 = 0 \quad x_n - 2x_{n-1} = 3^n \quad \forall n \geq 1$

$3^n = b^n \; p(n) \Rightarrow b = 3 \quad p(n) = 1 \quad gr(p) = 0$

Pol car. $(x-2)(x-3)$

Sol g. $A \cdot 2^n + B \cdot 3^n$

$x_0 = 0 \quad x_1 = 3$

$$\begin{cases} A + B = 0 & A = -B & A = -3 \\ 2A + 3B = 3 & -2B + 3B = 3 & B = 3 \end{cases}$$

$a_n = -3 \cdot 2^n + 3^{n+1}$

Sol. g. no homo.

$x_0 = a \quad x_1 = 2a + 3$

$$\begin{cases} A + B = a & A = a - B & \\ 2A + 3B = 2a + 3 & 2a - 2B + 3B = 2a + 3 & B = 3 \quad A = a - 3. \end{cases}$$

$S_n = (a-3) \cdot 2^n + 3 \cdot 3^n$

16) $x_0 = 0 \quad x_n - 2x_{n-1} = (n+1) \cdot 3^n \quad \forall n \geq 1$

$(n+1) \cdot 3^n = b^n \cdot p(n) \Rightarrow b = 3 \quad p(n) = n+1 \quad gr(p) = 1$

Po. carac: $(x-2)(x-3)^2$

Sol g. $A \cdot 2^n + (Bn + C) \cdot 3^n \qquad x_0 = 0 \; x_1 = 6 \; x_2 = 39$

$$\begin{cases} A + C = 0 \\ 2A + 3B + 3C = 6 \\ 4A + 28B + 9C = 39 \end{cases} \quad \begin{cases} 12A + 18B + 18C = 36 \\ 4A + 18B + 9C = 39 \end{cases} \quad \begin{cases} 8A + 9C = -3 \\ A + C = 0 \end{cases}$$

$C = -3 \quad A = 3$

$B = 3$

$a_n = 3 \cdot 2^n + (3n - 3) \cdot 3^n.$

Sol g:

$x_0 = a \quad x_1 = 2a+6 \quad x_2 = 4a+39$

$$\begin{cases} a = A + C \\ 2a+6 = 2A + 3B + 3C \\ 4a+39 = 4A + 20B + 9C \end{cases} \quad \begin{cases} 8A + 9C = 8a - 3 \\ 8A + 8C = 8a \\ C = -3 \end{cases} \quad \begin{matrix} A = a+3 \\ B = 3 \\ C = -3 \end{matrix}$$

$S_n = (a+3)\,2^n + (3n-3)\cdot 3^n$

17) $x_0 = \frac{1}{2} \quad x_1 = 3 \quad x_n = 2x_{n-1} + x_{n-2} + 3 \quad \forall n \geq 2$

$x_n - 2x_{n-1} - x_{n-2} = 3 \qquad 3 = b^n p(n) \quad b = 1 \quad p(n) = 3 \quad gr(p) = 0$

$\cancel{x^2 - 2x - 1}$ p. carac $(x-1-\sqrt{2})(x-1+\sqrt{2})(x-1) = 0$

Sol g: $A(1+\sqrt{2})^n + B(1-\sqrt{2})^n + C$

$x_0 = \frac{1}{2} \quad x_1 = 3 \quad x_2 = \frac{19}{2}$

$$\begin{cases} A + B + C = \frac{1}{2} \\ A(1+\sqrt{2}) + B(1-\sqrt{2}) + C = 3 \\ A(1+\sqrt{2})^2 + B(1-\sqrt{2})^2 + C = \frac{19}{2} \end{cases} \quad \begin{cases} A + \sqrt{2}A + B - \sqrt{2}B + C = 3 \\ \sqrt{2}(A-B) = \frac{5}{2} \\ A + 2\sqrt{2} + 2A + B - 2\sqrt{2}B + 2B + C = \frac{19}{2} \end{cases}$$

... $A = 1 + \frac{5}{8}\sqrt{2} \quad B = 1 - \frac{5}{8}\sqrt{2} \quad C = \frac{-3}{2}$

$a_n = (1 + \frac{5}{8}\sqrt{2})(1+\sqrt{2})^n + (1 - \frac{8}{5}\sqrt{2})(1-\sqrt{2})^n - \frac{3}{2}$

Sol g: $x_0 = a \quad x_1 = b \quad x_2 = 2b + a + 3.$

$$\begin{cases} A + B + C = a \\ A(1+\sqrt{2}) + B(1+\sqrt{2}) + C = b \\ A(1+\sqrt{2})^2 + B(1-\sqrt{2})^2 + C = 2b + a + 3 \end{cases}$$

19) $x_0 = 0 \qquad x_n - 2x_{n-1} = n + 2^n \qquad \forall n \geq 1$

$n = b^n p(n) \Rightarrow b = 1 \quad p(n) = n \quad gr(p) = 1$

$2^n = c^n \cdot p(n) \Rightarrow c = 2 \quad p = 1 \quad gr(p) = 0$

P. cara. $\quad (x-2)^2 (x-1)^2$.

Sol g: $\quad An + B + (Cn + D) \cdot 2^n$

$x_0 = 0 \quad x_1 = 3 \quad x_2 = 12 \quad x_3 = 35$

$$\left\{\begin{array}{l} B + D = 0 \\ A + B + 2C + 2D = 1 \\ 2A + B + 8C + 4D = 12 \\ 3A + B + 24C + 8D = 35 \end{array}\right. \qquad \begin{array}{ll} A = -1 & C = 1 \\ B = -2 & D = 2 \end{array}$$

$$a_n = -n - 2 + (n+2)\,2^n = (2^n - 2)(n+2).$$

Sol g: $\quad x_0 = a \quad x_1 = 2a + 3 \quad x_2 = 4a + 12 \quad x_3 = 8a + 35$

$$\left\{\begin{array}{l} B + D = a \\ A + B + 2C + 2D = 2a + 3 \\ 2A + B + 8C + 4D = 4a + 12 \\ 3A + B + 24C + 8D = 8a + 35 \end{array}\right. \qquad S_n = -n - 2 + (n + 2 + a) \cdot 2^n$$

20) $x_0 = 0 \quad x_1 = 1 \quad x_n = 3x_{n-1} - 2x_{n-2} + 2^n + 2n.$

$$x_n - 3x_{n-1} + 2x_{n-2} = 2^n + 2\cdot n$$

$$2^n = b^n \cdot p(n) \Rightarrow \quad b = 2 \quad p(n) = 1 \quad gr(p) = 0$$

$$2n = b^n \cdot p(n) \Rightarrow \quad b = 1 \quad p(n) = 2n \quad gr(p) = 1$$

Pol. ca. $\quad (x-2)^2 (x-1)^3 = 0$

Sol g. $\quad An^2 + Bn + C + (Dn + E)\cdot 2^n$

$x_0 = 0 \quad x_1 = 1 \quad x_2 = 11 \quad x_3 = 45 \quad x_4 = 137$

$$\begin{cases} C + E = 0 \\ A + B + C + 2D + 2E = 1 \\ 4A + 2B + C + 8D + 4E = 11 \\ 9A + 3B + C + 24D + 8E = 45 \\ 16A + 4B + C + 64D + 16E = 137 \end{cases} \Rightarrow A = -1 \quad B = -5 \quad C = -3 \quad D = 2 \quad E = 3.$$

$$a_n = -n^2 - 5n - 3 + (2n+3)\cdot 2^n$$

$x_0 = a \quad x_1 = b \quad x_2 = 3b - 2a + 8 \quad x_3 = 7b - 6a + 38$

$x_4 = 15b - 12a + 122$

$$\begin{cases} C + E = a \\ A + B + C + 2D + 2E = b \\ 4A + 2B + C + 8D + 4E = 3b - 2a + 8 \\ 9A + 3B + C + 24D + 8E = 7b - 6a + 38 \\ 16A + 4B + C + 64D + 16E = 15b - 12a + 122 \end{cases} \Rightarrow$$

$$\Rightarrow \quad S_n = (a-1) n^2 + (3a-5)n + 8a - b - 2 + ((a+2)\cdot n - 7a + b + 2)\cdot 2^n$$