(2.15)

- Formas canónicas disyuntivas y conjuntivas:

x	y	z	f_{63}	f_{82}	f_{104}	f_{116}	f_{128}	f_{143}	f_{172}	f_{188}	f_{217}	f_{231}
0	0	0	0	0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1	0	0	0	1	1
0	1	0	1	0	1	1	1	0	1	1	0	1
0	1	1	1	1	0	1	1	0	0	1	1	0
1	0	0	1	0	1	0	1	1	1	1	1	0
1	0	1	1	0	0	1	1	1	1	1	0	1
1	1	0	1	1	0	0	1	1	0	0	0	1
1	1	1	1	0	0	0	0	1	0	0	1	1

DISYUNTIVAS

$$f_{63}(x,y,z) = x^*y^*z^* + x^*y\cdot z + x\cdot y^*z^* + x\cdot y\cdot z^* + x\cdot y\cdot z$$

$$f_{82}(x,y,z) = x^*y^*\cdot z + x^*\cdot y\cdot z^* + x\cdot y^*\cdot z^*$$

$$f_{104}(x,y,z) = x^*y^*\cdot z + x^*\cdot y\cdot z^* + x\cdot y^*\cdot z^*$$

$$f_{116}(x,y,z) = x^*y\cdot z + x^*\cdot y^*\cdot z + x^*\cdot y\cdot z^* + x\cdot y^*\cdot z^*$$

$$f_{128}(x,y,z) = x^*y^*\cdot z^* + x^*y^*\cdot z + x^*\cdot y\cdot z^* + x^*\cdot y\cdot z + x\cdot y^*\cdot z^* + x\cdot y^*\cdot z + x\cdot y\cdot z^* + x\cdot y\cdot z$$

$$f_{143}(x,y,z) = x^*y^*\cdot z + x^*\cdot y^*\cdot z^* + x^*\cdot y\cdot z + x\cdot y^*\cdot z^* + x\cdot y^*\cdot z + x\cdot y\cdot z^* + x\cdot y\cdot z$$

$$f_{172}(x,y,z) = x^*y\cdot z + x^*\cdot y^*\cdot z + x^*\cdot y\cdot z^* + x\cdot y^*\cdot z^*$$

$$f_{188}(x,y,z) = x^*y\cdot z^* + x^*\cdot y^*\cdot z + x^*\cdot y\cdot z^* + x\cdot y^*\cdot z^* + x\cdot y\cdot z^*$$

$$f_{217}(x,y,z) = x^*y^*\cdot z^* + x^*\cdot y\cdot z + x\cdot y^*\cdot z + x\cdot y\cdot z^* + x\cdot y\cdot z$$

$$f_{231}(x,y,z) = x^*y^*\cdot z + x^*\cdot y\cdot z^* + x^*\cdot y\cdot z + x\cdot y^*\cdot z^* + x\cdot y^*\cdot z + x\cdot y\cdot z$$

• CONJUNTIVAS

$f_{63}(x,y,z) = (x+y+z)\cdot(x+y+\bar{z})$

$f_{82}(x,y,z) = (x+y+z)\cdot(x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+\bar{z})$

$f_{104}(x,y,z) = (x+y+z)(x+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z})$

$f_{116}(x,y,z) = (x+y+z)(\bar{x}+y+z)(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z})$

$f_{126}(x,y,z) = (x+y+z)(\bar{x}+\bar{y}+\bar{z})$

$f_{143}(x,y,z) = (x+y+\bar{z})(x+\bar{y}+z)(x+\bar{y}+\bar{z})$

$f_{172}(x,y,z) = (x+y+\bar{z})(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z})$

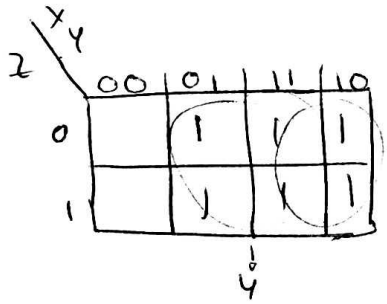
$f_{188}(x,y,z) = (x+y+\bar{z})(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z})$

$f_{217}(x,y,z) = (x+\bar{y}+z)(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+z)$

$f_{231}(x,y,z) = (x+\bar{y}+\bar{z})(\bar{x}+y+z)$

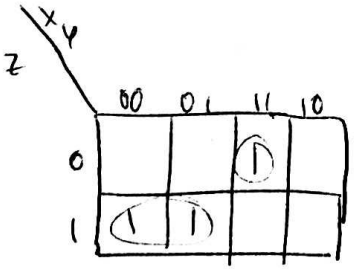
• IMPLICANTES PRIMOS [KARNAUGH]

f_{63}:



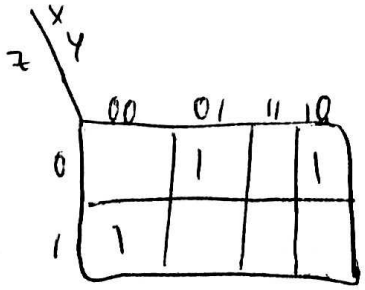
$\Rightarrow f_{63}(x,y,z) = x + y$

f_{82}:



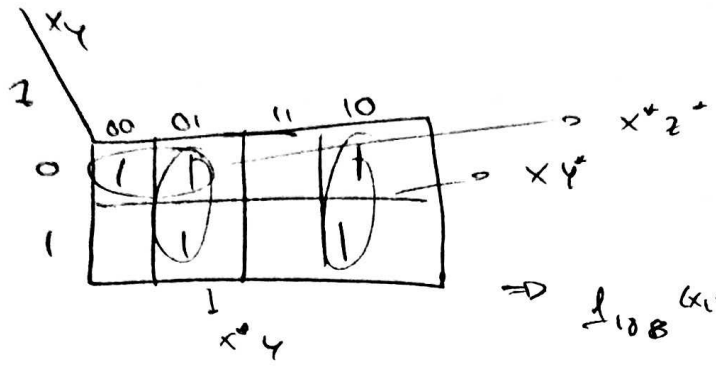
$\Rightarrow f_{82}(x,y,z) = x\cdot y\cdot\bar{z} + \bar{x}\cdot z$

f_{104}:



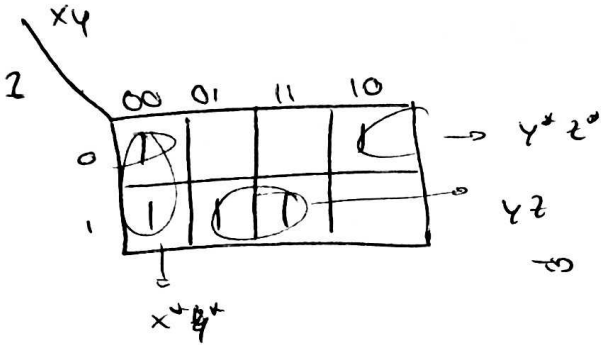
$\Rightarrow f_{104}(x,y,z) = \bar{x}\cdot y\cdot\bar{z} + x\cdot\bar{y}\cdot\bar{z} + \bar{x}\cdot\bar{y}\cdot z$

f_{188} :



$\Rightarrow f_{188}(x,y,z) = x^*z^* + xy^* + x^*y.$

f_{217} :



$\Rightarrow f_{217}(x,y,z) = x^*y^* + yz + y^*z^*$

[QUINE]

f_{116} :

~ 101	-01
~ 011	01-
~ 010	0-1
~ 001	

$\Rightarrow f_{116}(x,y,z) = y^*z + x^*y + x^*z.$

f_{126} :

110	-10
101	1-0
011	-01
001	10-
010	0-1
100	01-

$\Rightarrow f_{126}(x,y,z) = yz^* + xz^* + y^*z + xy^* + x^*z + x^*y.$

f_{143} :

~ 111	11-	1--
~ 110	1-1	1--
~ 101	1-0	
100	10-	
000	-00	

$\Rightarrow f_{143}(x,y,z) = x + y^*z^*$

[CONSENSO]

$$f_{172}(x,y,z) = x'y'z' + x'yz' + xy'z' + xy'z =$$

$$= x'z' + xy'$$

[FCC]

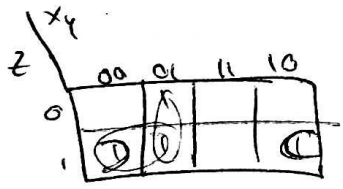
$$f_{231}(x,y,z) = (x + y' + z')(x' + y + z) =$$

$$= xy + xz + x'y' + y'z + x'z' + yz'$$

- Formas no simplificables.

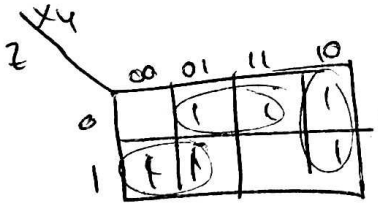
  Hallaremos aquellas a las que no se le ha aplicado KARNAUGH:

f_{116}:



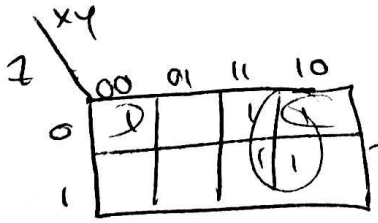
$\Rightarrow f_{116}(x,y,z) = x'y + y'z$

f_{126}:



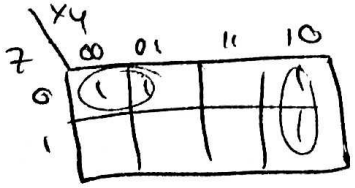
$\Rightarrow f_{126}(x,y,z) = x'z + yz' + xy'$

f_{143}:



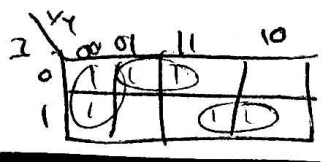
$\Rightarrow f_{143}(x,y,z) = x + y'z'$

f_{172}:



$\Rightarrow f_{172}(x,y,z) = x'z' + xy'$

f_{231}



$\Rightarrow f_{231}(x,y,z) = x'y' + yz' + xz.$

Aplicaremos a los que se le aplicó QUINE, PETRICK:

f_{116}:

		m_1	m_2	m_3	m_5	
A	$y'z$	X			X	✓
B	$x'y$		X	X		✓
C	$x'z$	X				✓

$\Rightarrow$

~~A·B·B·C~~ = A·B·C

$\Rightarrow f_{116}(x,y,z) = x'y + y'z$.

f_{126}

		m_1	m_2	m_3	m_4	m_5	m_6	
A	yz'		X				X	
B	xz'				X		X	
C	$y'z$	X				X		
D	xy'	\			X	X		
E	$x'z$	X ·		X				
F	$x'y$		X	X				

$\Rightarrow$

$\Rightarrow f_{126}(x,y,z) = x'z + yz' + xy'$

f_{143}:

		m_0	m_4	m_5	m_6	m_7	
A	X		X	X	X	X	
B	$y'z'$	X	X				

$\Rightarrow$

$\Rightarrow f_{143}(x,y,z) = x + y'z'$

2.16

13244 = 00 11 00 11 10 11 11 00

43944 = 10 10 10 11 10 10 10 00

62640 = 11 11 01 00 10 11 00 00

- Formas canónicas disyuntivas y conjuntivas.

DISYUNTIVAS

$f_{13244}(x,y,z,t) = m_2 + m_3 + m_6 + m_7 + m_8 + m_{10} + m_{11} + m_{12} + m_{13} =$

$= x^* y^* z t^* + x^* y^* z t + x^* y z t^* + x^* y z t + x \bar{y} z t^* + x y^* z t^* + x y^* z t +$

$+ x y z^* t^* + x y z^* t$

$f_{43944} = m_0 + m_2 + m_4 + m_6 + m_7 + m_8 + m_{10} + m_{12} =$

$= x^* y^* z^* t^* + x^* y z t^* + x y^* z^* t^* + x^* y z t^* + x^* y z t + x y^* z^* t^* + x y^* z t^* + x y z^* t^*$

$f_{62640} = m_0 + m_1 + m_2 + m_3 + m_5 + m_8 + m_{10} + m_{11} =$

$= x^* y^* z^* t^* + x^* y^* z^* t + x^* y^* z t^* + x^* y^* z t + x^* y z^* t + x y^* z^* t^* +$

$+ x y^* z t^* + x y^* z t$

CONJUNTIVAS

$f_{13244}(x,y,z,t) = M_0 \cdot M_1 \cdot M_4 \cdot M_5 \cdot M_9 \cdot M_{14} \cdot M_{15} =$

$= (x + y + z + t)(x + y + z + \bar{t})(x + \bar{y} + z + t)(x + \bar{y} + z + \bar{t})$

$(\bar{x} + y + z + \bar{t})(\bar{x} + \bar{y} + \bar{z} + t)(\bar{x} + \bar{y} + \bar{z} + \bar{t})$

$f_{43944}(x,y,z,t) = M_1 M_3 M_4 M_8 M_{11} M_{13} M_{14} M_{15} =$

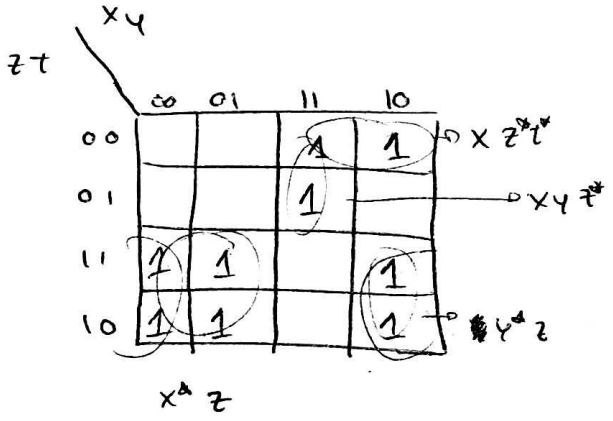
$= (x+y+z+\bar{t})(x+y+\bar{z}+\bar{t})(x+\bar{y}+z+\bar{t})(\bar{x}+y+z+\bar{t})$
$(\bar{x}+y+\bar{z}+\bar{t})(\bar{x}+\bar{y}+z+\bar{t})(\bar{x}+\bar{y}+\bar{z}+\bar{t})(\bar{x}+\bar{y}+\bar{z}+\bar{t})$

$f_{62640}(x,y,z,t) = M_4 \cdot M_6 M_7 M_8 M_{12} M_{13} M_{14} M_{15} =$

$= (x+\bar{y}+z+\bar{t})(x+\bar{y}+\bar{z}+t)(x+\bar{y}+\bar{z}+\bar{t})(\bar{x}+y+z+\bar{t})$
$(\bar{x}+\bar{y}+z+\bar{t})(\bar{x}+\bar{y}+z+t)(\bar{x}+\bar{y}+\bar{z}+t)(\bar{x}+\bar{y}+\bar{z}+\bar{t})$

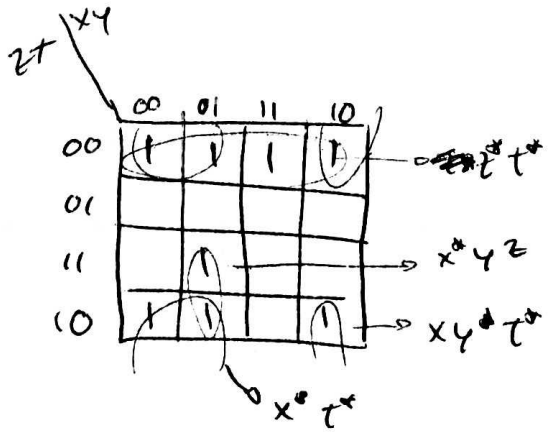
- Implicantes primos:

f_{13244}:



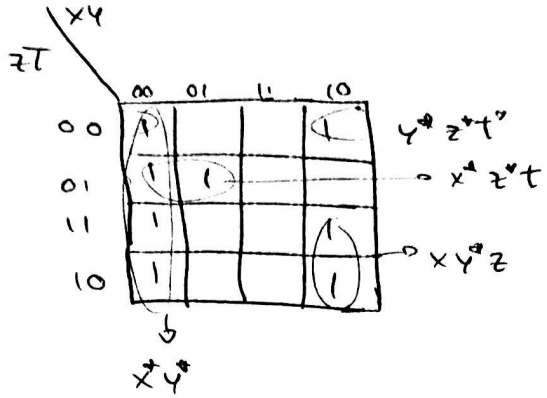
$f_{13244}(x,y,z,t) = \bar{x}z + x\bar{z}\bar{t} + xy\bar{z} + \bar{y}\bar{z}$.

f_{43944}:



$f_{43944}(x,y,z,t) = \bar{x}\bar{t} + z\bar{t} + \bar{x}yz + xy\bar{z}\bar{t}$

1_{62640}:



$$1_{62640}(x,y,z,t) = x^*y^* + xy^*z + x^*z^*t + y^*z^*t^*$$

- Formas canónicas disyuntivas reducidas.

Calculadas en el apartado anterior.

- Formas no simplificables. → Se han hallado por Karnaugh.

2.17

$$f(x,y,z) = (x \vee y) \wedge z = (x+y)\cdot z$$

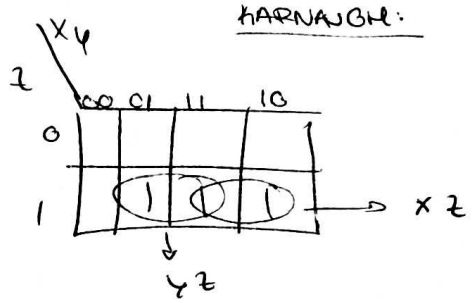
x y z	f
0 0 0	0
0 0 1	0
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

FORMA CANÓNICA:

$f(x,y,z) = m_3 + m_5 + m_7 =$

$= x^*yz + xy^*z + xyz.$ → forma canónica

KARNAUGH:



$\Rightarrow f(x,y,z) = xz + yz.$

↓

forma reducida

PETRICK:

		m_3	m_5	m_7	
A	xz		X	X	✓
B	yz	X		X	✓

$\Rightarrow f(x,y,z) = xz + yz$ ~~(NO SIMPLIFICABLE)~~

↓

forma no simplificable.

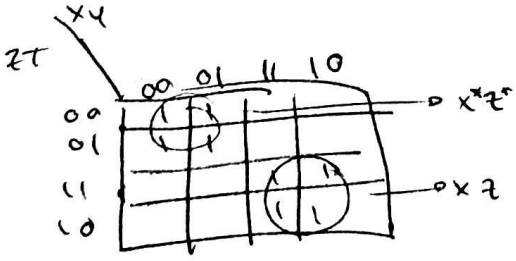
2.18

1) $f(x,y,z,t) = xyzt + xy^*zt + xy^*z^*t + xy^*z^*t^* + x^*yzt + x^*yz^*t + x^*y^*zt + x^*y^*z^*t =$

$= xzt + xzt^* + x^*yz^* + x^*yz^* = xz + x^*z^*$

2) $f(x,y,z,t) = m_{15} + m_{11} + m_{14} + m_{10} + m_0 + m_4 + m_1 + m_5 =$

$= m_0 + m_1 + m_4 + m_5 + m_{10} + m_{11} + m_{14} + m_{15}$



		m_0	m_1	m_4	m_5	m_{10}	m_{11}	m_{14}	m_{15}	
A	x^*z^*	X	X	X	X					✓
B	xz					X	X	X	X	✓

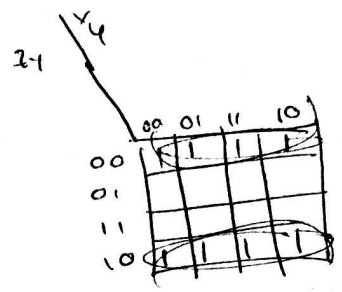
$\Rightarrow f(x,y,z,t) = xz + x^*z^*$

(2.19)

$f_{(2)}$ → múltiplos de 2.
$f_{(3)}$ → múltiplos de 3.
$f_{(4)}$ → múltiplos de 4.

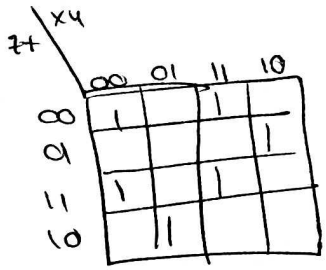
x	y	z	t	$f_{(2)}$	$f_{(3)}$	$f_{(4)}$
0	0	0	0	1	1	1
0	0	0	1	0	0	0
0	0	1	0	1	0	0
0	0	1	1	0	1	0
0	1	0	0	1	0	1
0	1	0	1	0	0	0
0	1	1	0	1	1	0
0	1	1	1	0	0	0
1	0	0	0	1	0	1
1	0	0	1	0	1	0
1	0	1	0	1	0	0
1	0	1	1	0	0	0
1	1	0	0	1	1	1
1	1	0	1	0	0	0
1	1	1	0	1	0	0
1	1	1	1	0	1	0

$f_{(2)}$



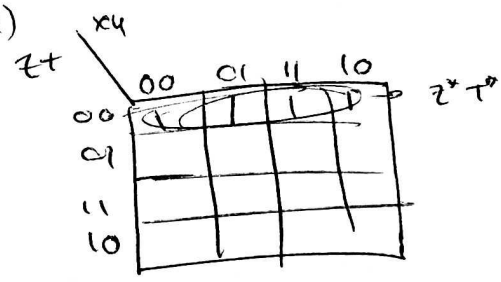
$f_{(2)}(x,y,z,t) =$

$f_{(3)}$



$f_{(3)}(x,y,z,t) = m_0 + m_3 + m_6 + m_9 + m_{12} + m_{15}$

$f_{(4)}$



$f_{(4)}(x,y,z,t) = z^* t^*$

(2.20)

Si = 1 No = 0. {Si, No, Si, Si} = {1, 0, 1, 1}

Para aprobar solo puede diferir una cifra con 1011:

$f(x,y,z,t) = xy^*z\bar{t} + x^*y^*z\bar{t} + xy^*z^*t + xy^*z\bar{t} + x^*yz\bar{t} =$

$= x^*y^*z\bar{t} + xy^*t + xy^*z + xz\bar{t} \Rightarrow$

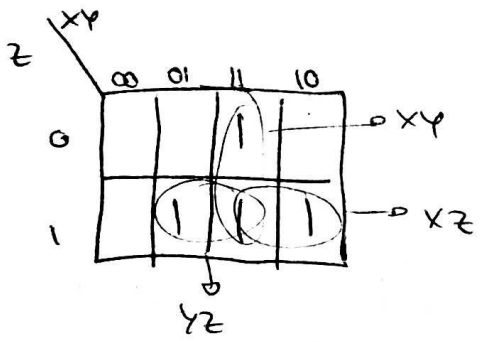
$\Rightarrow$ Si $f(x,y,z,t) = 1$ aprobado si $f(x,y,z,t) = 0$ suspenso.

(2.21)

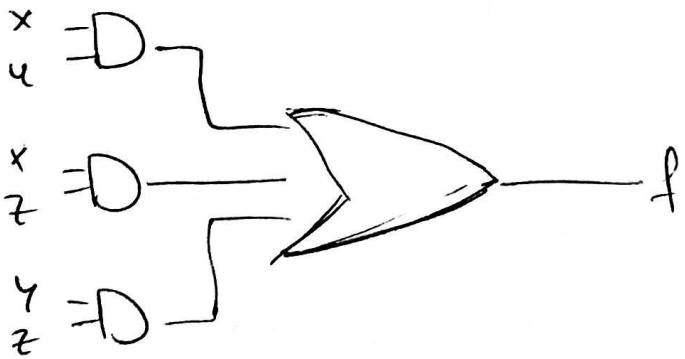
Si = 1 No = 0

Mayoría: 2 ó más votos.

$f(x,y,z) = xyz + x^*yz + xy^*z + xyz^*$



$\Rightarrow f(x,y,z) = xy + xz + yz.$



2.22

$$f(x,y,z) = (x \uparrow y) \downarrow z = \overline{\overline{(x \cdot y)} + z} = \left((x \cdot y)^* + z\right)^* =$$

$$= (x^* + y^* + z)^* = xyz^*$$

2.23

$$f(x,y,z) = (x^* y + z)^* + x \cdot z^* =$$

$$= (x^* y)^* \cdot z^* + x \cdot z^* = (x + y^*) \cdot z^* + x \cdot z^* = x \cdot z^* + y^* \cdot z^* =$$

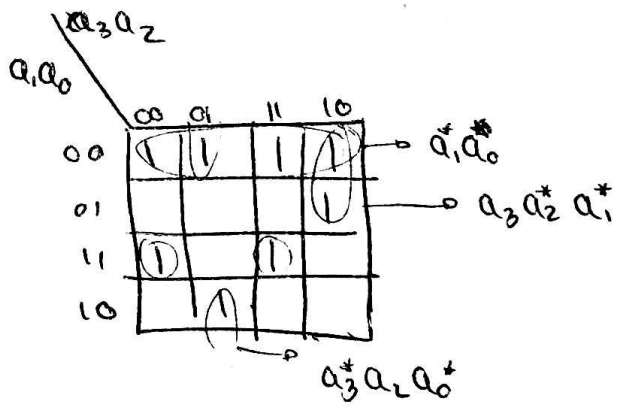
$$= xy \cdot z^* + xy^* z^* + x^* y^* z^* = m_0 + m_4 + m_6$$

$$f(x,y,z) = M_1 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_7 =$$

$$= (x + y + z^*)(x + y^* + z)(x + y^* + z^*)(x^* + y + z^*)(x^* + y^* + z^*)$$

2.24

	a_3	a_2	a_1	a_0	f
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	1



$$f(a_3a_2a_1a_0) = a_3a_2^*a_1^* + a_1^*a_0^* + a_3^*a_2a_1a_0 +$$

$$+ a_3a_2a_1a_0 + a_3^*a_2a_0^*$$

2.25

$f(x,y,z,t) = x'y'z't' + x'y'z't + x'yz't + xy'z't + xyz't' + xyz't + xyzt' + xyzt = m_0 + m_1 + m_4 + m_7 + m_{11} + m_{14} + m_{15}$

[QUINE]

x	$xyzt$	$-yzt$
x	$x'yzt$	xzt
x	$xy'zt$	$xyz-$
-	$xyz't$	
x	$xy'z't'$	
x	$xyz't'$	
x	$xyzt'$	

$\Rightarrow f(x,y,z,t) = yzt + xzt + xyz + xy'z't' + xyz't'$

[PETRICK]

		m_0	m_1	m_4	m_7	m_{11}	m_{14}	m_{15}	O.
A	xyz						X	X	✓
B	yzt				X			X	✓
C	xzt					X		X	✓
D	$xy'z'$	X	X						✓
E	$xy'z't'$	X		X					✓

$\Rightarrow$

$\Rightarrow$ f no se puede reducir más, f es irredundante.