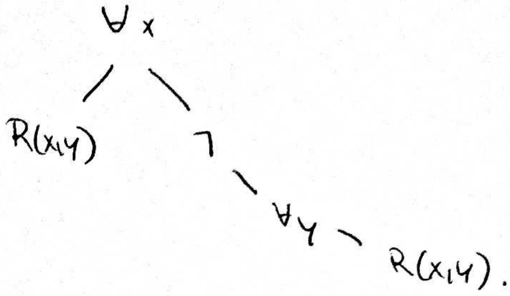
5.1

1.- $\forall x\,(R(x,y) \land \neg \forall y R(x,y)) = \alpha$

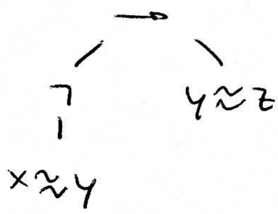


$Sub(\alpha) = \{ R(x,y) \land \neg \forall y R(x,y),\ R(x,y),\ \neg \forall y R(x,y),\ \forall y R(x,y) \}$

x: ligada

y: una ocurrencia libre, una ocurrencia ligada.

2.- $x \not\approx y \to y \approx z = \alpha$.



$Sub(\alpha) = \{ x \not\approx y,\ x \approx y,\ y \approx z \}$

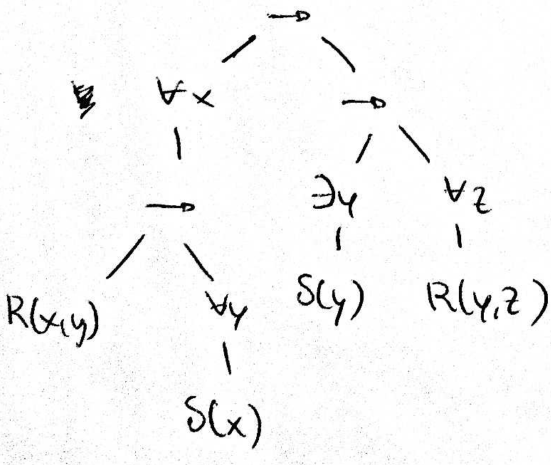
x : libre

y : 2 ocurrencias libres

z : libre

Todas variables libres.

3.- $\forall x\,(R(x,y) \to \forall y\, S(x)) \to (\exists y\, S(y) \to \forall z R(y,z)) = \alpha$



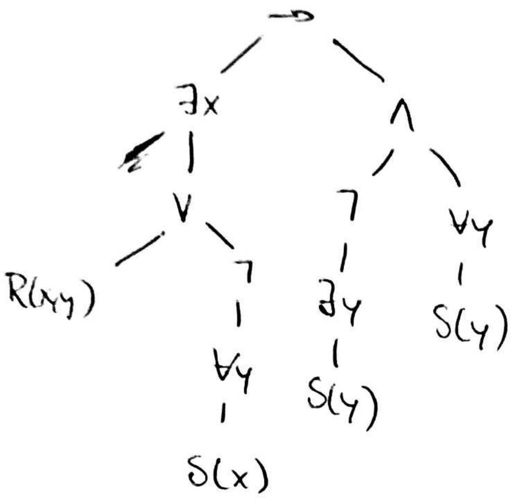
$Sub(\alpha) = \{ \forall x (R(x,y) \to \forall y\, S(x),\ \exists y (S(y) \to \forall z R(y,z)),\ R(x,y) \to \forall y\, S(x),\ R(x,y),\ \forall y S(x),\ S(y),\ \exists y (S(y)),\ \forall z\, R(x,z),\ S(y),\ R(x,z) \}$

x: ligada (2 ocurrencias)

y: 3 ligadas, 2 libres

z: 2 ligadas.

4.- $\exists x\ (R(x,y) \vee \neg \forall y\, S(x)) \rightarrow (\neg \exists y\, S(y) \wedge \forall y\, S(y)) = \alpha$

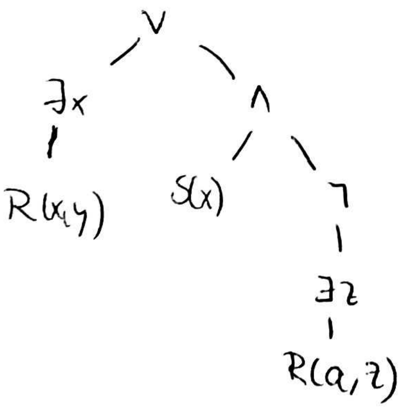


$Sub(\alpha) = \{ \exists x\, (R(x,y) \vee \neg \forall y\, S(x)),$
$\neg \exists y\, S(y) \wedge \forall y\, S(y),$
$R(x,y) \vee \neg \forall y\, S(x),$
$R(x,y),\ \neg \forall y\, S(x),$
$\forall y\, S(x),\ S(x),\ \neg \exists y\, S(y),$
$\forall y\, S(y),\ \exists y\, S(y),\ S(y) \}$

x: 3 ligadas

y: 1 libre, 5 ligadas.

5.- $\exists x\ R(x,y) \vee [S(x) \wedge \neg \exists z\, R(a,z)] = \alpha$



$Sub(\alpha) = \{ \exists x\ R(x,y),\ S(x) \wedge \neg \exists z\, R(a,z),$
$R(x,y),\ S(x),\ \neg \exists z\, R(a,z),$
$\exists z\, R(a,z),\ R(a,z) \}$

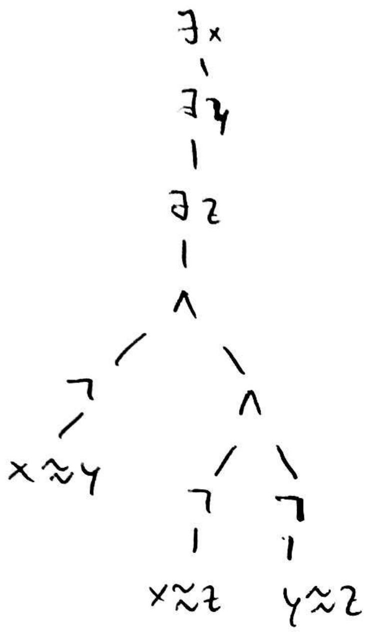
x: 2 ligadas 1 libre

y: libre

z: 2 ligadas

a: libre.

6.- $\exists x \exists y \exists z \, (x \not\approx y \wedge x \not\approx z \wedge y \not\approx z) = \alpha$



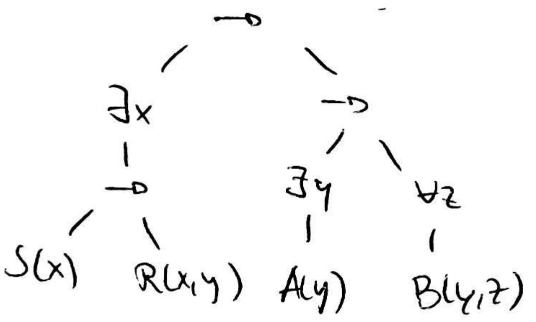
$Sub(\alpha) = \{ \exists y \exists z (x \not\approx y \wedge x \not\approx z \wedge y \not\approx z),$
$\exists z \, (x \not\approx y \wedge x \not\approx z \wedge y \not\approx z),$
$x \not\approx y \wedge x \not\approx z, \; y \not\approx z,$
$x \not\approx y, \; x \not\approx z, \; y \not\approx z,$
$x \approx y, \; x \approx z, \; y \approx z \}$

x: ligada

y: ligada

z: ligada

7.- $\exists x \, (S(x) \rightarrow R(x,y)) \rightarrow (\exists y A(y) \rightarrow \forall z B(y,z)) = \alpha$



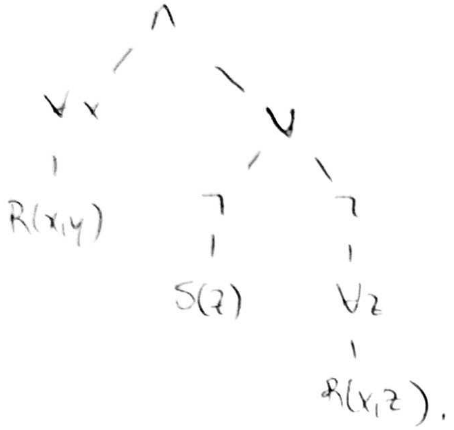
$Sub(\alpha) = \{ \exists x \, (S(x) \rightarrow R(x,y)),$
$\exists y A(y) \rightarrow \forall z B(y,z),$
$S(x) \rightarrow R(x,y), \; S(x), \; R(x,y),$
$\exists y A(y), \; A(y), \; \forall z \; B(y,z), \; B(y,z) \}$

x: 3 ligadas

y: 2 libres, 2 ligadas

z: 2 ligadas

8.- $\forall x\, R(x,y) \wedge (\neg S(z) \vee \neg \forall z\, R(x,z)) = \alpha$



$Sub(\alpha) = \{ \forall x\, R(x,y),\ \neg S(z) \vee \neg \forall z\, R(x,z),$
$\neg S(z),\ S(z),\ \neg \forall z\, R(x,z),$
$\forall z\, R(x,z),\ R(x,z),\ R(x,y) \}$

x: 2 ligadas

y: 1 libre

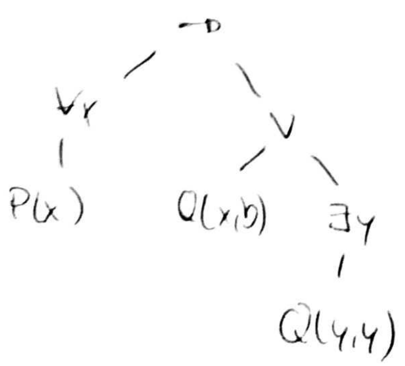
z: 1 libre, 2 ligadas.

9.- $\forall x \forall y \forall z\, (x \approx y \vee x \approx z \vee y \approx z) = \alpha$

$Sub(\alpha) = \{ \forall y \forall z\, (x \approx y \vee x \approx z \vee y \approx z),$
$\forall z\, (x \approx y \vee x \approx z \vee y \approx z),$
$(x \approx y \vee x \approx z \vee y \approx z),$
$x \approx y,\ x \approx z,\ y \approx z \}.$

x, y, z siempre ligadas.

10.- $\forall x\, P(x) \rightarrow Q(x,b) \vee \exists y\, Q(y,y) = \alpha$



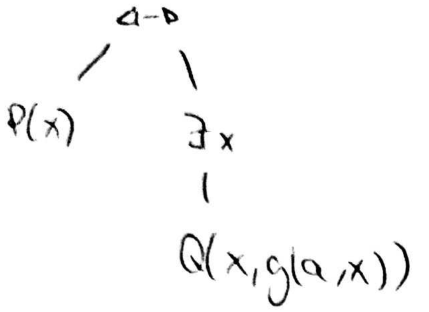
$Sub(\alpha) = \{ \forall x\, P(x),\ Q(x,b) \vee \exists y\, Q(y,y),$
$P(x),\ Q(x,b),\ \exists y\, Q(y,y),$
$Q(y,y) \}.$

x: 2 ligadas, 1 libre.

y: 3 ligadas.

b: libre.

11.- $P(x) \leftrightarrow \exists x\, Q(x, g(a,x)) = \alpha$

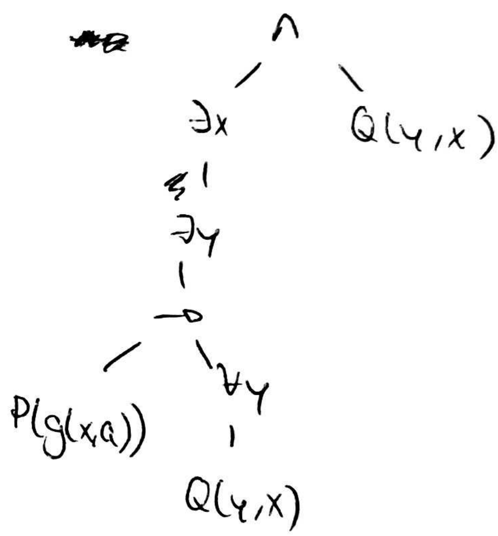


$Sub(\alpha) = \{P(x), \exists x\, Q(x, g(a,x)), Q(x, g(a,x))\}$.

x: 1 libre, 3 ligadas.

a: libre.

12.- $\exists x \exists y\, (P(g(x,a)) \rightarrow \forall y\, Q(y,x)) \wedge Q(y,x) = \alpha$



$Sub(\alpha) = \{\exists x \exists y\, (P(g(x,a)) \rightarrow \forall y\, Q(y,x)), Q(y,x),$

$\exists y (P(g(x,a)) \rightarrow \forall y\, Q(y,x)),$

$P(g(x,a)) \rightarrow \forall y\, Q(y,x), P(g(x,a)), \forall y\, Q(y,x)\}$

x: 3 ligadas, 1 libre

y: 3 ligadas, 1 libre.

5.2

1. $P(b,c) \land M(b) \land M(c)$

2. $\exists x\,(Hr(b,x) \land P(x,a)) \land M(b) \land H(a)$.

3. $\exists x\,(P(a,x) \land P(x,b)) \land M(b) \land H(a)$

4. $\exists x\,(P(x,b) \land P(a,x)) \land M(b) \land H(a)$

5. $\forall x\, \exists y\,(P(y,x) \land H(y))$.

6. $\forall x\; \exists y\, \exists z\,(P(y,x) \land P(z,x) \land y \neq z)$

7. $\neg \exists x\, P(x,x)$

8. $\exists x\, \neg \exists y\,(Hr(x,y))$.

9. $\forall x\,(A(x,b) \to A(x,c)) \land M(b) \land M(c)$.

10. $\exists x\, \exists y\; P(x,y) \land \exists x\, \forall y\, \neg P(x,y)$.

11. $\exists x\, \exists y\,(Hr(x,y) \leftrightarrow \forall z\,(P(z,x) \approx P(z,y)))$.

12. $\exists x\,(Hr(b,x) \land P(a,x) \land H(x)) \land H(a) \land M(b)$.

13. $\forall x\, \forall y\, \forall z\,((P(x,y) \land A(y,z)) \to A(x,z))$

14. $\forall x\, \forall y\,(P(x,y) \to A(x,y))$.

15. $\forall x\, \forall y\,(Hr(x,y) \to \neg P(x,y))$.

16. $\forall x\, \forall y\,(Hr(x,y) \to \neg P(x,y))$.

17. $\exists x\,(P(b,x) \land P(x,c) \land M(x)) \land M(b) \land M(c)$

18. $\exists x\, \exists y\,(P(c,x) \land P(x,y) \land P(y,a)) \land M(c) \land H(a)$.

19. $\forall x \exists y \exists z (P(y,z) \land P(z,x))$

20. $\forall x \exists y \exists z \exists K(P(K,z) \land P(z,y) \land P(y,x))$.

21. $\exists x (A(x,b) \land \neg A(x,c)) \land M(b) \land M(c)$

22. $\exists x \exists y (Hr(x,b) \land Hr(y,b) \land x \not\approx y) \land M(b)$

23. $\exists x \exists y (Hr(b,\cancel{x}) \land Hr(b,x) \land x \not\approx y \land \forall z ((z \not\approx y \land z \not\approx x) \to \neg Hr(\cancel{z,b})$

$\neg Hr(b,z))$

Segunda parte.

1. $b \approx m(c)$
2. $\exists x (Hr(b,x) \land (x \approx m(a) \lor x \approx p(a)) \land M(b) \land H(a)$
3. $a \approx p(p(b)) \lor a \approx p(m(b))$
4. $\exists x (a \approx p(x) \land (x \approx p(b) \lor x \approx m(b)))$
5. $\forall x \exists y (y \approx p(x))$
6. $\forall x \exists y \exists z (y \approx p(x) \land z \approx m(x))$
7. $\forall x (x \not\approx p(x) \land x \not\approx m(x))$

10. $\exists x \exists y (x \approx p(y)) \land \exists x \forall y (x \not\approx p(y))$.

11. $Hr(x,y) \leftrightarrow (p(x) \approx p(y) \land m(x) \approx m(y))$.

12. $\exists x (Hr(b,x) \land a \approx p(x))$.

13. $\forall x \forall y (A(x,y) \to A(p(x),y))$.

14. $\forall x A(p(x),x)$

15. $\forall x \forall y (Hr(x,y) \to (x \not\approx p(y) \land x \not\approx m(y)))$.

16. $\forall x \exists y (y \hat{\approx} m(x) \wedge \forall z (z \hat{\not\approx} y \rightarrow z \hat{\not\approx} m(x)))$

17. $b \hat{\approx} m(m(c))$.

18. $c \hat{\approx} m(m(m(a))) \vee c \hat{\approx} m(p(m(a))) \vee c \hat{\approx} m(m(p(a))) \vee$

$\vee\ c \hat{\approx} m(p(p(a)))$.

19. $\forall x\, \cancel{\exists}\, \exists y\, (y \hat{\approx} p(p(x)) \vee y \hat{\approx} p(m(x)))$.

20 $\forall x\, \exists y\, (y \hat{\approx} p(p(p(x))) \vee y \hat{\approx} p(m(m(x))) \vee$

$\vee\ y \hat{\approx} p(p(m(x))) \vee y \hat{\approx} p(m(p(x)))$.

5.4

- 1. $\{\forall x\ P(x)\} \models P(a)$

$I = (\mathcal{E}, v)$ $\quad I^v(\forall x P(x)) = 1 \Rightarrow I^v(P(x)) = 1 \quad \forall x$ luego

$I^v(P(a)) = 1. \Rightarrow$ Cierto

- 2.- $\exists x P(x) \not\models P(a)$.

$D = \{0, 1\}$ $\quad a = 0$ $\quad P(x) =$ "$x = 1$" $\quad P = \{1\}$

x	$P\{x\}$	$\exists P\{x\}$	$P(a)$
0	0	1	0
1	1	1	0

$\Rightarrow$ falso

- 3. $\emptyset \models \exists x P(x) \to P(a)$

T. de deducción : $\{\exists x P(x)\} \models P(a)$ que por ② es falsa

- 4. $\emptyset \models \forall x P(x) \to P(a)$

T. de deducción

$\{\forall x P(x)\} \models P(a) \Rightarrow$ cierta

- 5 $\emptyset \models \forall x (P(x) \to P(a))$

$D = \{0,1\}$ $P(x) = $ "$x=1$" $a=0$ $P = \{1\}$

x	$P(a)$	$P(x)$	$P(x) \to P(a)$	$\forall x(P(x) \to P(a))$
0	0	0	1	0
1	0	1	0	0

$\Rightarrow$ falsa

- 6. $\emptyset \models P(a) \to \exists x P(x)$

T. de deducción: $P(a) \models \exists x P(x)$

$(P(a)) = 1 \Rightarrow$ Si $x = a$ $I(P(x)) = 1 \Rightarrow I(\exists x P(x)) = 1.$

Cierta.

- 7. $\{\exists x \forall y R(x,y)\} \models \forall x \exists y R(x,y)$

$D = \{\mathbb{Z}_2\}$ $R = \{(0,0), (0,1)\}$

x	y	$R(x,y)$	$\forall y$	$\exists x$	$\exists y$	$\forall x$
0	0	1	1	1	1	0
0	1	1	1	1	1	0
1	0	0	0	1	0	0
1	1	0	0	1	0	0

$\Rightarrow$ falsa.