Example Parsing $LR(0)$

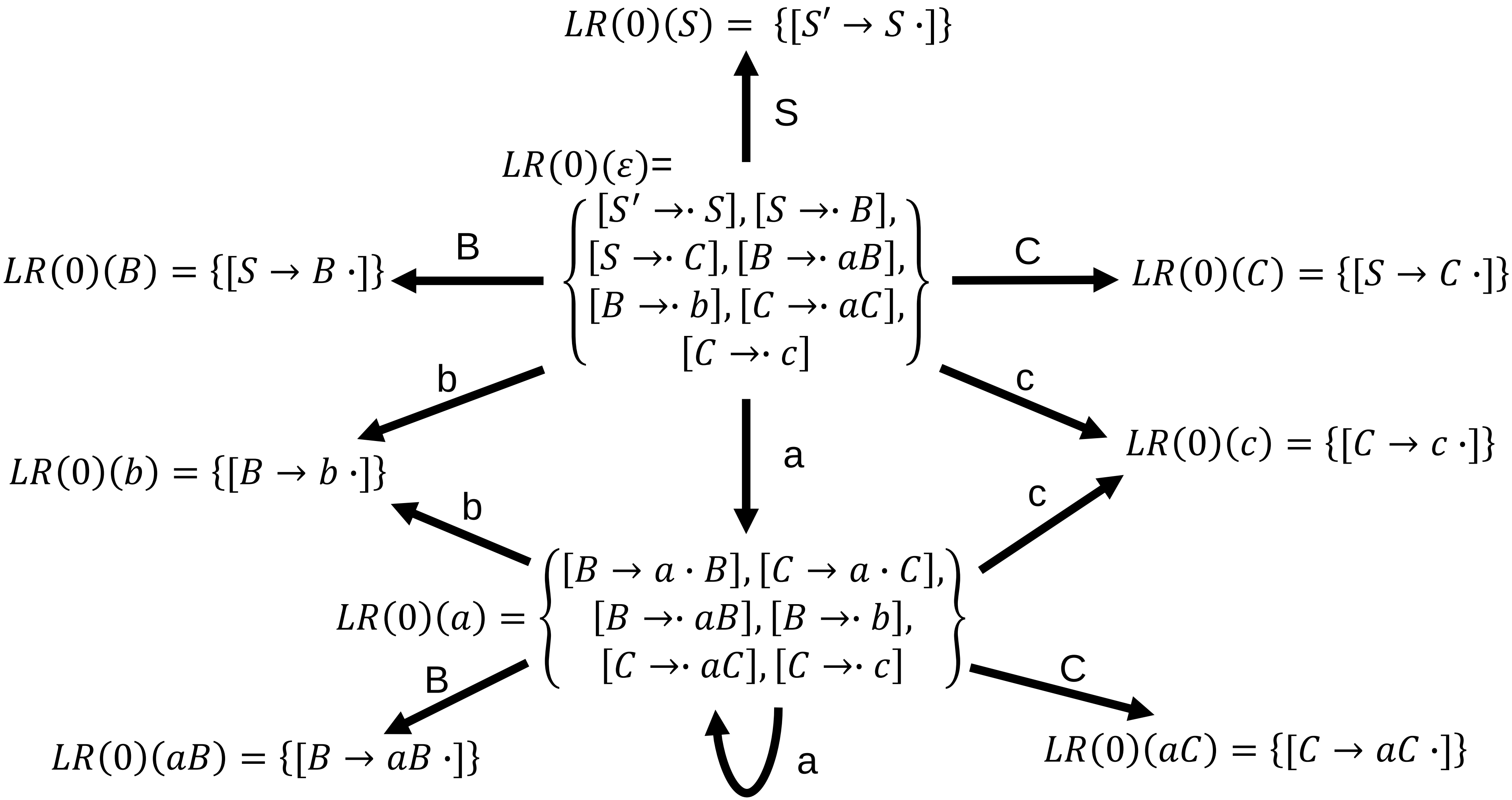
Recap: Deterministic Parsing Automaton $LR(0)$

- We define the deterministic parsing automaton of $LR(0)$ grammar $G =< \Sigma, N, P, S >$ with rule 0: $S' \to S$
  - Input alphabet Σ
  - Output alphabet U = the rule numbers 0,1,2,3,...
  - States $\Sigma^* \times \Gamma^* \times U^*$
  - Pushdown alphabet $\Gamma = LR(0)(G)$!! item sets !!
  - Initial state (w, I_0, ε) for $w \in \Sigma^*$ where $I_0 = LR(0)(\varepsilon)$
  - Final state final state $(\varepsilon, \varepsilon, u)$ where $u \in U^*$
  - Action depends on the item set I in the state $(x, \alpha I, z)$:
    - **Shift** $(aw, \alpha I, z) \to (w, \alpha IJ, z)$ if $[A \to \alpha_1 \cdot a\alpha_2] \in I$ and $I \underset{goto}{\overset{a}{\to}} J$
    - **Reduce** $(aw, \alpha I I_1 \dots I_n, z) \to (w, \alpha IJ, zi)$ with rule $i \neq 0$ $A \to Y_1 \dots Y_n$ if $[A \to Y_1 \dots Y_n \cdot] \in I_n$ and $I \underset{goto}{\overset{A}{\to}} J$
    - **Accepting** state $(\varepsilon, I_0 I, z) \to (\varepsilon, \varepsilon, z0)$ if $[S' \to S \cdot] \in I$
    - **Error** in state $(w, \alpha I, z)$ if $I = \emptyset$

Example, step 1: Construct the goto-automaton

Example: $S' \to S$ (0) $\quad S \to B|C$ (1,2)

$B \to aB|b$ (3,4) $\quad C \to aC|c$ (5,6)



Example, step 2: goto-automaton as table

Example: $S' \rightarrow S$ (0) $S \rightarrow B|C$ (1,2)

$B \rightarrow aB|b$ (3,4) $C \rightarrow aC|c$ (5,6)

- For the parsing, it is more convenient to write the transitions of the goto automaton as a table (all empty cells are "illegal" and go to I_9)

Item set	S	B	C	a	b	c
$I_0 = LR(0)(\varepsilon)$	I_1	I_2	I_3	I_4	I_5	I_6
$I_1 = LR(0)(S)$						
$I_2 = LR(0)(B)$						
$I_3 = LR(0)(C)$						
$I_4 = LR(0)(a)$		I_7	I_8	I_4	I_5	I_6
$I_5 = LR(0)(b)$						
$I_6 = LR(0)(c)$						
$I_7 = LR(0)(aB)$						
$I_8 = LR(0)(aC)$						
$I_9 = \emptyset$						

$$I_0 = LR(0)(\varepsilon) = \left\{ \begin{matrix} [S' \rightarrow \cdot\, S], [S \rightarrow \cdot\, B], \\ [S \rightarrow \cdot\, C], [B \rightarrow \cdot\, aB], \\ [B \rightarrow \cdot\, b], [C \rightarrow \cdot\, aC], \\ [C \rightarrow \cdot\, c] \end{matrix} \right\}$$

$$I_1 = LR(0)(S) = \{[S' \rightarrow S\, \cdot]\}$$

$$I_2 = LR(0)(B) = \{[S \rightarrow B\, \cdot]\}$$

$$I_3 = LR(0)(C) = \{[S \rightarrow C\, \cdot]\}$$

$$I_4 = LR(0)(a) = \left\{ \begin{matrix} [B \rightarrow a \cdot B], [C \rightarrow a \cdot C], \\ [B \rightarrow \cdot\, aB], [B \rightarrow \cdot\, b], \\ [C \rightarrow \cdot\, aC], [C \rightarrow \cdot\, c] \end{matrix} \right\}$$

$$I_5 = LR(0)(b) = \{[B \rightarrow b\, \cdot]\}$$

$$I_6 = LR(0)(c) = \{[C \rightarrow c\, \cdot]\}$$

$$I_7 = LR(0)(aB) = \{[B \rightarrow aB\, \cdot]\}$$

$$I_8 = LR(0)(aC) = \{[C \rightarrow aC\, \cdot]\}$$

$$I_9 = \emptyset$$

Example, step 3: Determine actions for parser automaton

Example: $S' \to S$ (0) $\quad S \to B|C$ (1,2)

$B \to aB|b$ (3,4) $\quad C \to aC|c$ (5,6)

- For every item set I, we determine the action for the automaton to take when seeing I in a state $(w, \gamma I, z)$

Item set	action
$I_0 = LR(0)(\varepsilon)$	shift
$I_1 = LR(0)(S)$	accept
$I_2 = LR(0)(B)$	reduce 1
$I_3 = LR(0)(C)$	reduce 2
$I_4 = LR(0)(a)$	shift
$I_5 = LR(0)(b)$	reduce 4
$I_6 = LR(0)(c)$	reduce 6
$I_7 = LR(0)(aB)$	reduce 3
$I_8 = LR(0)(aC)$	reduce 5
$I_9 = \emptyset$	error

$$I_0 = LR(0)(\varepsilon) = \left\{ \begin{matrix} [S' \to \cdot\, S], [S \to \cdot\, B], \\ [S \to \cdot\, C], [B \to \cdot\, aB], \\ [B \to \cdot\, b], [C \to \cdot\, aC], \\ [C \to \cdot\, c] \end{matrix} \right\}$$

$$I_1 = LR(0)(S) = \{[S' \to S \cdot]\}$$

$$I_2 = LR(0)(B) = \{[S \to B \cdot]\}$$

$$I_3 = LR(0)(C) = \{[S \to C \cdot]\}$$

$$I_4 = LR(0)(a) = \left\{ \begin{matrix} [B \to a \cdot B], [C \to a \cdot C], \\ [B \to \cdot\, aB], [B \to \cdot\, b], \\ [C \to \cdot\, aC], [C \to \cdot\, c] \end{matrix} \right\}$$

$$I_5 = LR(0)(b) = \{[B \to b \cdot]\}$$

$$I_6 = LR(0)(c) = \{[C \to c \cdot]\}$$

$$I_7 = LR(0)(aB) = \{[B \to aB \cdot]\}$$

$$I_8 = LR(0)(aC) = \{[C \to aC \cdot]\}$$

$$I_9 = \emptyset$$

Run the parser automaton, part 1

Example: $S' \to S$ (0) $S \to B|C$ (1,2)
$B \to aB|b$ (3,4) $C \to aC|c$ (5,6)

- Input aac
- Parsing:
  - The initial state is $(aac,\ I_0,\ \varepsilon)$ with $I_0 = LR(0)(\varepsilon)$
  - What do we have to do next? Let's look at the table:

Item set	action
I_0	shift

  - We have to shift, taking the first "a" from the input
  - What will happen next? Consult the table:

Item set	S	B	C	a	b	c
I_0	I_1	I_2	I_3	I_4	I_5	I_6

  - We go now to I_4. The new state is $(ac,\ I_0 I_4,\ \varepsilon)$
  - Note that I_0 is not removed. We are not yet done with it!

Run the parser automaton, part 2

Example: $S' \to S$ (0) $S \to B|C$ (1,2)

$B \to aB|b$ (3,4) $C \to aC|c$ (5,6)

- Parsing:
  - The current state is $(ac,\ I_0 I_4,\ \varepsilon)$
  - What do we have to do next? Let's look at the table:

Item set	action
I_4	shift

Item set	S	B	C	a	b	c
I_4		I_7	I_8	I_4	I_5	I_6

  - The new state is $(c,\ I_0 I_4 I_4,\ \varepsilon)$

Run the parser automaton, part 3

Example: $S' \rightarrow S$ (0) $S \rightarrow B|C$ (1,2)

$B \rightarrow aB|b$ (3,4) $C \rightarrow aC|c$ (5,6)

- Parsing:
  - The current state is $(c,\ I_0 I_4 I_4,\ \varepsilon)$
  - What do we have to do next? Let's look at the table:

Item set	action
I_4	shift

Item set	S	B	C	a	b	c
I_4		I_7	I_8	I_4	I_5	I_6

  - The new state is $(\varepsilon,\ I_0 I_4 I_4 I_6,\ \varepsilon)$

Run the parser automaton, part 4

Example: $S' \to S$ (0) $S \to B|C$ (1,2)

$B \to aB|b$ (3,4) $C \to aC|c$ (5,6)

- Parsing:
  - The current state is $(\varepsilon,\ I_0 I_4 I_4 I_6, \qquad \varepsilon)$
  - What do we have to do next? Let's look at the table:

Item set	action
I_6	reduce 6

  - That means we are ready to reduce to C using rule 6: $C \to c$
  - The right-hand side of $C \to c$ contains one symbol, so we remove one element from the stack:

$$(\varepsilon,\ I_0 I_4 I_4, \qquad 6)$$

And we consult the table to see where $I_4 \xrightarrow{C}$ leads us:

Item set	S	B	C	a	b	c
I_4		I_7	I_8	I_4	I_5	I_6

  - The new state is $(\varepsilon,\ I_0 I_4 I_4 I_8, \qquad 6)$

Run the parser automaton, part 5

Example: $S' \rightarrow S$ (0) $S \rightarrow B|C$ (1,2)

$B \rightarrow aB|b$ (3,4) $C \rightarrow aC|c$ (5,6)

- Parsing:
  - The current state is $(\varepsilon,\ I_0 I_4 I_4 I_8,\qquad 6)$
  - What do we have to do next? Let's look at the table:

Item set	action
I_8	reduce 5

  - That means we are ready to reduce to C using rule 5: $C \rightarrow aC$
  - The right-hand side of $C \rightarrow aC$ contains two symbols, so we remove two elements from the stack:

$$(\varepsilon,\ I_0 I_4,\qquad 65)$$

And we consult the table to see where $I_4 \xrightarrow{C}$ leads us:

Item set	S	B	C	a	b	c
I_4		I_7	I_8	I_4	I_5	I_6

  - The new state is $(\varepsilon,\ I_0 I_4 I_8,\qquad 65)$

Run the parser automaton, part 6

Example: $S' \to S$ (0) $S \to B|C$ (1,2)

$B \to aB|b$ (3,4) $C \to aC|c$ (5,6)

- Parsing:
  - The current state is $(\varepsilon,\ I_0 I_4 I_8, \qquad 65)$
  - What do we have to do next? Let's look at the table:

Item set	action
I_8	reduce 5

  - That means we are ready to reduce to C using rule 5: $C \to aC$
  - The right-hand side of $C \to aC$ contains two symbols, so we remove two elements from the stack:

$$(\varepsilon,\ I_0, \qquad 655)$$

And we consult the table to see where $I_0 \xrightarrow{C}$ leads us:

Item set	S	B	C	a	b	c
I_0	I_1	I_2	I_3	I_4	I_5	I_6

  - The new state is $(\varepsilon,\ I_0 I_3, \qquad 655)$

Run the parser automaton, part 7

Example: $S' \to S$ (0) $S \to B|C$ (1,2)

$B \to aB|b$ (3,4) $C \to aC|c$ (5,6)

- Parsing:
  - The current state is $(\varepsilon,\ I_0 I_3,\qquad 655)$
  - What do we have to do next? Let's look at the table:

Item set	action
I_3	reduce 2

  - That means we are ready to reduce to S using rule 2: $S \to C$
  - The right-hand side of $S \to C$ contains one symbol, so we remove one element from the stack:

$$(\varepsilon,\ I_0,\qquad 6552)$$

And we consult the table to see where $I_0 \xrightarrow{S}$ leads us:

Item set	S	B	C	a	b	c
I_0	I_1	I_2	I_3	I_4	I_5	I_6

  - The new state is $(\varepsilon,\ I_0 I_1,\qquad 6552)$

Run the parser automaton, part 8

Example: $S' \to S$ (0) $S \to B|C$ (1,2)

$B \to aB|b$ (3,4) $C \to aC|c$ (5,6)

- Parsing:
  - The current state is $(\varepsilon,\ I_0 I_1,\qquad 6552)$
  - What do we have to do next? Let's look at the table:

Item set	action
I_1	accept

  - We are done. The last state is $(\varepsilon, \varepsilon, 65520)$

Everything in one picture

Example: $S' \to S$ (0) $S \to B|C$ (1,2)

$B \to aB|b$ (3,4) $C \to aC|c$ (5,6)

- Input aac
- Parsing:

$(aac,\ I_0,\ \varepsilon)$

$(ac,\ I_0 I_4,\ \varepsilon)$

$(c,\ I_0 I_4 I_4,\ \varepsilon)$

$(\varepsilon,\ I_0 I_4 I_4 I_6,\ \varepsilon)$

$(\varepsilon,\ I_0 I_4 I_4 I_8,\ 6)$

$(\varepsilon,\ I_0 I_4 I_8,\ 65)$

$(\varepsilon,\ I_0 I_3,\ 655)$

$(\varepsilon,\ I_0 I_1,\ 6552)$

$(\varepsilon,\ \varepsilon,\ 65520)$

Item set	action	S	B	C	a	b	c
$I_0 = LR(0)(\varepsilon)$	shift	I_1	I_2	I_3	I_4	I_5	I_6
$I_1 = LR(0)(S)$	accept						
$I_2 = LR(0)(B)$	red 1						
$I_3 = LR(0)(C)$	red 2						
$I_4 = LR(0)(a)$	shift		I_7	I_8	I_4	I_5	I_6
$I_5 = LR(0)(b)$	red 4						
$I_6 = LR(0)(c)$	red 6						
$I_7 = LR(0)(aB)$	red 3						
$I_8 = LR(0)(aC)$	red 5						
$I_9 = \emptyset$	error						