

Guión presentación Game Theory:

Slide 08:

Explicación del ejemplo.

- X = are the prizes {Barça, R. Madrid, Tie}
- For bet A: $\Delta(X) = (\text{Barça} : 0.50, \text{R. Madrid} : 0.10, \text{Tie} : 0.40)$
- For bet B: $\Delta(X) = (\text{Barça} : 0.20, \text{R. Madrid} : 0.70, \text{Tie} : 0.10)$

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Given a lottery $f : \Omega \rightarrow \Delta(X)$,

- $f(\cdot|t) \in \Delta(X)$ = prob. distribution for the prizes if state = t
- $f(x|t)$ = prob. of receiving the prize x if state = t

$[x]$ denote a lottery that will always give the prize x

It is also remarkable this notation as we will use it during the rest of the presentation.

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Thanks Trini. Now I will start with the main part of the chapter, which is the concerning to the Utility Maximization theorem. But before we have to do 3 more definitions. This definitions involve some of the previous concepts that my workmates have showed to you, so I will remind you these concepts before start.

So Santi introduced the decision theory, remember that a player have to be rational and intelligent; and some of the notation we will use, as delta of X . Just after Trini and me introduced the idea of lottery which is one of the most important definition in the Chapter as the following definitions use this concept, and finally Trini gave us the axiomatic for establishing preferences between lotterys.

Do you have any question concerning to these concepts?

So, now I will explain you the first concept which is utility function.

I will try to give you a intuitive idea of all these definitions with an example that is not that one. So It is a function that assigns a value to each prize in each state of the world. We may confuse it with the concept of Lottery but is not the same as in the codomain you can take any real value.

Let see the example. We have X and we have Ω Everyone will agree that our preferences for this problem would be having a high-paying job with a good economy, as we would get more money. So we should give the highest value to this pair, and probably the lowest to the lower-paying job and bad economy pair, right?

Okei now I will explain you another example that will be used during the rest of the presentation.

We have two possible outcomes, Car A with no roof, and Car B with roof. And two possible states in the world, SUNNY DAY and RAYNY DAY.

I will also define a lottery.

$$\begin{aligned} f(A, \text{SUNNY DAY}) &= 0.2 \\ f(A, \text{RAINY DAY}) &= 0.4 \end{aligned}$$

$$\begin{aligned} f(B, \text{SUNNY DAY}) &= 0.8 \\ f(B, \text{RAINY DAY}) &= 0.6 \end{aligned}$$

Is this a lottery? Yes, as it is defined from ω to $\Delta(\omega)$, and the probabilities distributions sum a total of 1 for each state.

So given this problem, we will build a utility function. This utility function might assign to each pair of the plain $X \times \Omega$ a real value, but taking in count our preferences

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Okei lets continue with the next definition. Dont hesitate to ask me if you have any doubt.

Now lets talk about conditional probability function. Just a question before, did anyone of you study probability in any course? Okei I will ask you a simple question after, dont panic hjahahaha.

So a conditional probability....

And Now i want to ask you which would be the $p(A | A)$. Okei so know lets build the conditonal probability function for the example.

$$P(\text{rainy day} / R, S) = 0.8$$

$$P(\text{sunny day} / R, S) = 0.2$$

$$P(S / S) = 1$$

$$P(R / R) = 1$$

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Okei, so now lets define what is the expected utility value. The name says it all, but i will try to give you a intuitive idea of what each factor means.

So the expected utility value of a lottery f

Lets understand carefully this equation.

The first part is just notation so lets focus just in the right part of the equation. The first sum illustrate the conditional probability function p for each element on the event S , which is multiplied by another sum. This sum is the multiplication of the utility function, remember that it helps to characterize our preferences with a real value, and the lottery, both defined for the state t . Very important.

All right? Any doubt?

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Okei so, now gathering all these concepts we will be able to describe how an individual plays and to states that a player's behaviour is driven by two main elements, which was described in the previous equation. A subjective conditional probability function. And a Utility function transcribing the preferences of the players for the different prizes once the uncertainty is revealed.

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So Finally we reach the main concept of this part, the utility maximization theorem.
This theorem holds the following:

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So, this theorem allow us to get some functions for which the axioms are satisfied which is key in order to analyze the behaviour of the player and of course the way he or she will play.

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So with this theorem we have been able to model or to represent the behaviour or mentality of a player. But is this the unique possible model? Is there any other representation ? I mean there are another different functions that allows us to represent the same behaviour?

The answer is that it is unique, up to some linear scaling.

As you can see in the slide, if we have an utility function and a conditional probaibility function satisfying the bayes theorem exists another conditional probaibility and utility function that represent the same preferences as u and p , if and only if there exists a constant A and a function B which satisfy the following equation.

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Finally I will talk about the bayesian probability system. We introduced the idea of the bayesian theorem just in the previous theorem.

So we have to define a bayesian probability system, which is just a conditional probability function which satisfied the Bayes Formula.

If a player takes decision using a Bayesian probability system as a reference has an interesting interpretation regarding his planning of events. In particular, even if he known that an event S should not happen according to the fundamental theorem, he still needs to define $p(\cdot|S)$.

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The behaviour of a player is axiomatized as we said before. The theorem explained before allows us to ensure the existance of a Bayesian probability and a utility function that characterize preferences between lotteries (and therefore between decisions)

Any way as every axiomatic representation, as used in mathematics, has its own limits. There would be situations that couldnt be represented with these axioms.

In conclusion of this part, this preference's characterization allows us to predict what people will do, and to use it as a guide for what we should do, or what a general decision-maker should do in order of their preferences.