LINMA 2345
Game Theory

Introduction
2023

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Sup. Prof. **M. Philippe**,
T.A. **B. Pinon**,
T.A. **A. Rubbens**

Outline

- Who are we, what do we have in common?
- About: Game Theory
- Ice-Breaker game!
- Material & rules
- Course structure

Who are we, what do we share ?

UCLouvain

OXFORD

Prof. **Raphaël Jungers**

Chair of Applied Math department,
Cyber Physical systems lab

- Cyber-Physical systems research
  - Computer science && dynamics (&& algebra && fun stuff)
  - Multi-agent systems, multi-objectives
- **Introduced the game-theory course to INMA.**
  - On a fun topic
  - **Students contribute to the course through material**

Who are we, what do we share ?

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Ir. **Brieuc Pinon**

Ph.D. Student in Applied Math

UCLouvain

Ir. **Anne Rubbens**

Ph.D. Student in Applied Math

I’m glad to be helped by **Anne** & **Brieuc** :-)

- Brieuc has seen several iterations of the course,
  Anne has seen last year.
- Both are really up to speed both technically & pedagogically :-)
- **They bring balance to the force.**

I’ll mainly be interacting with them, and they with you :-)

Who are we, what do we share ?

- Applied Math Researcher in Academia then,
- Data-Scientist & Developer, then
- ML-Engineer, Mentor, sometimes doing clever things

RecSys, Customer Segmentation,
Customer Lifetime value,
Predictive maintenance,
Visualization,
...
MLOps platforms,
Lots of automation,
Infra as Code
...
Also, here today :-)

Dr. **Matthew Philippe**

I save time through code and sometimes being clever (Senior DS, Junior-ish MLops, researcher, ...)

and substitute professor for LINMA2345

Who are we, what do we share ?

What about you?
Why here?
What are your expectations?

Outline

- Who are we?
- What/who is game theory about?
- Ice-Breaker game!
- Material & rules
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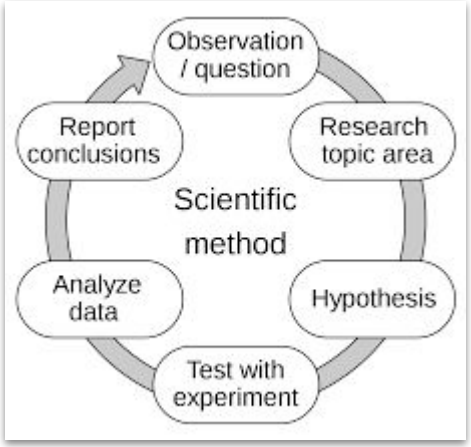
Our objectives:

Observe multi-agent situations as a 3rd-person.
(Actors, Actions, Motives, Interactions)
(Players, Strategies, Utilities, Games)



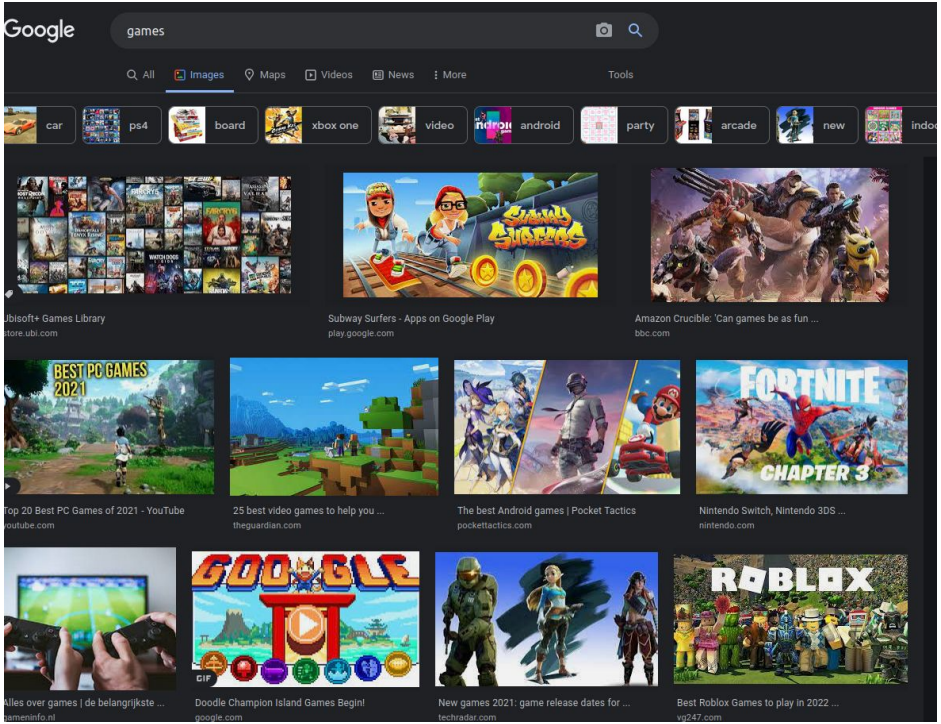
Know what to test,
(bet vs outcome)

Make use of relevant tools in this setting.
(Equilibrium, Models)

The aim is for you to tool up :-)

What is game theory about?

Games



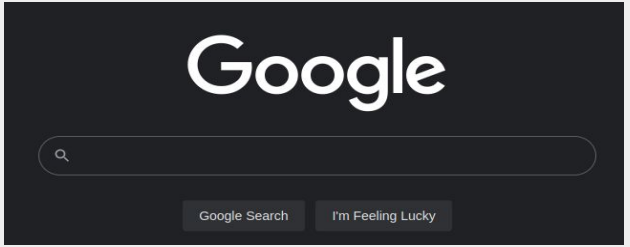
What is game theory about?

Order attack? Retreat? How?

Follow group? Flee? Attack? St
ground?

What to buy?
What to sell?
What will the market do?

What to pick?
What to sell?



What should I type in to learn about game theory?

What are valid results for the tool to show?

Narrowing down the scope

Game theory is the study of mathematical models of strategic interactions among rational agents.

- Agents: **formal mathematical definition**,
  (~ humans as models)
- Games: **formal mathematical definition**,
  (~ real life strategic situations as models)

The models, may be inadequate for specific cases (accuracy, or considering benefits they provide).
Yet, great tools to reflect on many situations

Outline

- Who are we?
- What/who is game theory about?
- Let's play!
  - Perudo
  - Menti
- Material & rules
- Course structure

I’d like to share some recent experience I had involving this game.

Context:

- 4 well trained engineers
- Last round of perudo (2 dices, 2 players)
- Two schools of thought:
  - The game is solved
  - Bluff matters, blehbleh

Debunk a statement of opponent.

Players look at dice privately.

P1: *“X dices of value ‘a’”, with ‘a’ in “2 to 6.”*

P2: FALSE or *Y > X dices of value ‘a’.*

(Game goes on until one player says FALSE).

Winner is the one who was right when FALSE happens.

Who wants to play?

We will play 4 games

We will play 4 games

- Pick number between 0 and 10.
- Answer on web-browser
  - On www.menti.com (I'll give you a code for each game).

- If not able to (no phone is ok), try to guess lower, higher and average number picked.

- Do not overthink, but be prepared to answer a "why" question.
- Results will be aggregated later.

- www.menti.com
- 3236 6815

Game 1

"Your score is the **average of all numbers picked**."

Alice picks 5, Bob picks 7
Their scores are 6 each.

- www.menti.com
- 3269 9290

Game 2

"Your score is the **2 X average of all numbers picked - your number."**

Alice picks 10, Bob picks 5.

Alice gets 15 - 10 = 5
Bob gets 15 - 5 = 10

- www.menti.com
- 7505 1571

Game 3

"**If** you picked **the smallest number**, your score **that number divided by the number of players having picked it.** Else, 0**.**

Alice picks 10, Bob picks 5, Charles 2 and Danielle 2.

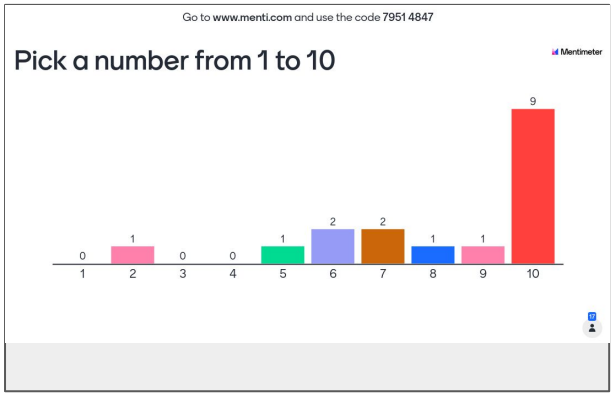
Alice gets 0.
Bob gets 0.
Charles and Danielle get 1.

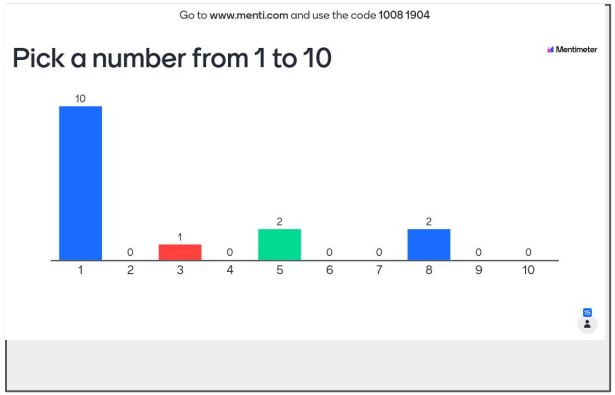
Game 4

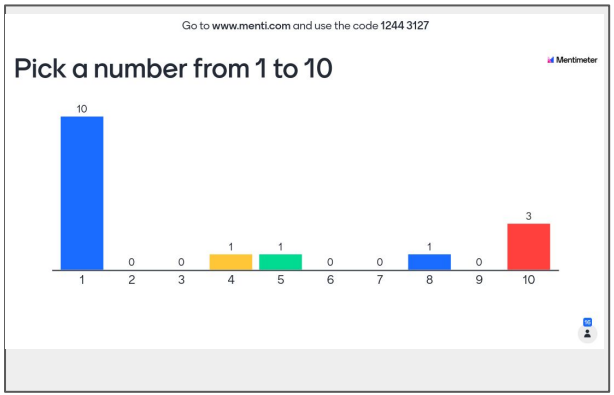
“**If** you picked **the smallest number**, your score **that number divided by the number of players having picked it.** Else, 0**.**

You have 10 seconds to answer.
Everyone can see your move.
If you don’t play, you get 0.

Results!







Skipped :-)

Live reactions/comments:

Someone: I don’t understand why people wouldn’t pick 10 at the first game
Answer: I didn’t understand properly
Teacher: Yup, we’re human. Humans have lot’s of things going on, their goal isn’t always to solve the problem we present them. For instance, it’s known that we often substitute a problem for an easiest one, or a version of it that’s easier for us to handle.

Outline

- Who are we?
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- Ice-Breaker game!
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Purpose of material

- You don’t need us to learn about game theory - great material online, different perspectives, etc....
  - Let’s be pragmatic :-)
- Want to be able to solve problems?
  - **Exercises sessions.** (prompts, questions, to solve).
- Want to be introduced to a concept? Want to structure knowledge?
  - **Course notes** and **lectures** are way to go.
  - **Recaps** are very good for structuration.
- Want to understand, motivate concepts?
  - Use **examples** (available in notes, lectures, and exercises (solved))

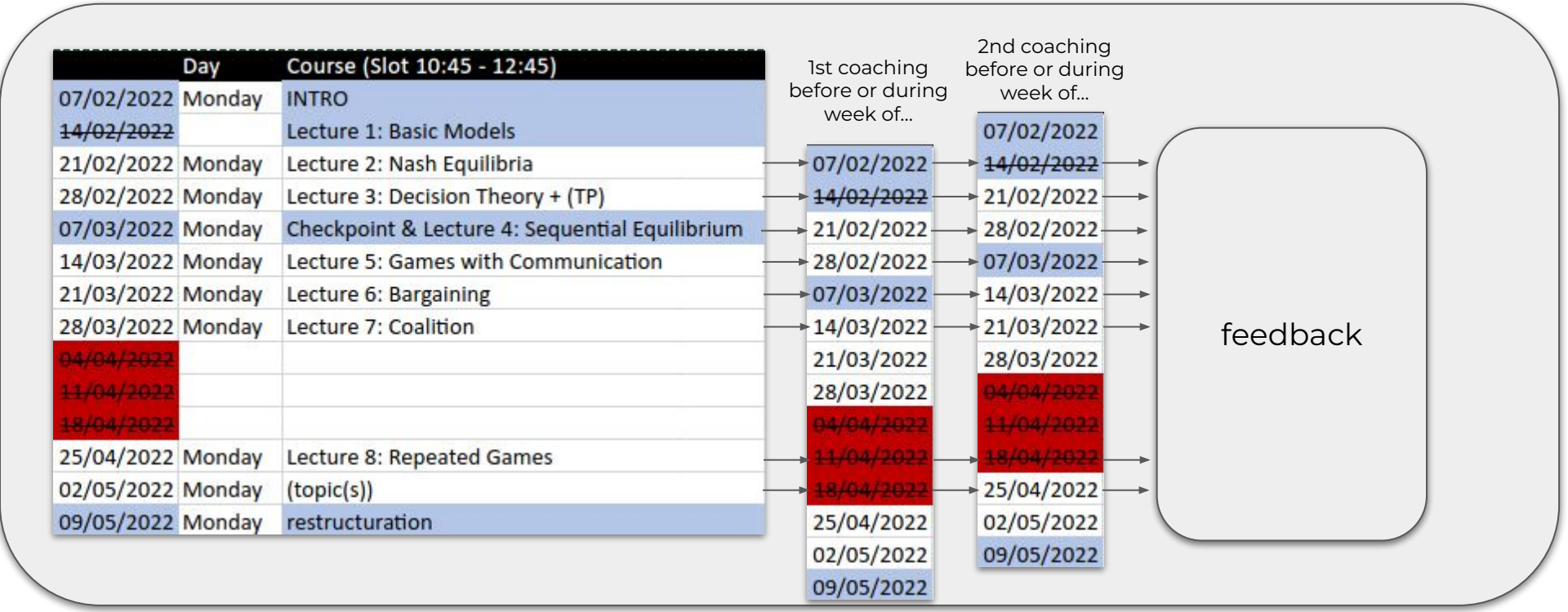
Classes

	A	B	C	D	E
1	Week no	Week of	Day	Course (Slot 10:45 - 12:45)	Practice (Slot 14:00 - 16:00)
2	1	2/6/2023	Monday	INTRO	
3	2	2/13/2023		Lecture 1: Basic Models	APE 2 - Models Brieuc
4	3	2/20/2023	Monday	Lecture 2: Nash Equilibria Anne	APE 3 - Domination Anne
5	4	2/27/2023	Monday	Lecture 3: Decision Theory + (TP) Brieuc	APE 4 - Nash Brieuc
6	5	3/6/2023	Monday	Checkpoint & Lecture 4: Sequential Equil	APE 5 - Nash 2 Anne
7	6	3/13/2023	Monday	Lecture 5: Games with Communication A	APE 7 - Sequential Brieuc
8	7	3/20/2023	Monday	Lecture 6: Bargaining Brieuc	APE 8 - Correlated_Equilibria Anne
9	8	3/27/2023	Monday	Lecture 7: Coalition Anne	APE 9 - Bargaining Brieuc
10		4/3/2023			
11		4/10/2023			
12	9	4/17/2023	Monday	Lecture 8: Repeated Games Anne	APE 10 - Coalitions Anne
13	10	4/24/2023	Monday	Lecture 9: Special topic (Auctions/...) Brie	APE 11 - Repeated_Games Anne
14	11	5/1/2023	Monday	/	
15	12	5/8/2023	Monday	APE 1 - Intro Brieuc	APE 12 - Auctions Brieuc
16	13	5/15/2023	Monday	restructuration	APE X - Exams Brieuc

I'll be handling those in blue :-), you do the rest

What's the seminar

- Typically, a 1h session during which you present a course.
  - We may adapt if relevant (e.g., cool activity that takes some time, smaller groups, ...)
- Two prior coachings, during which:
  - We help you in making links in the material, finding ways to present it, Q&A, ...
  - You help us understand what you need to move forward.
    - You must prepare (read the material, understand it).
- One feedback session afterwards
  - **NOT AN EXAM, it's me talking to you.**
  - The why's behind the good, the bad, and the ugly
- We'll try to make it as remote friendly as possible
  - Please give us feedback
  - We won't force you to record your seminar
  - Slides should be shared on teams ASAP.



1st coaching:

- check that you understand the material
- prior work required
- advices, Q&A

2nd coaching:

- check course structure
- check for issues
- advices, Q&A

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Evaluations

Seminars

We will **evaluate** you on the following grounds:

- **structure** that's made to help the audience.
- **main concepts** that adequately put forward, allowing the audience to focus on what matters.
- **take home messages** are clearly underlined.
- **keep the audience engaged**.
- **material well mastered** by presenters.
- **timing** are respected (usually, the course should last between 50 minutes and 1h, with possible adaptations if prior agreement).
- **discipline and seriousness in their preparations** (be honest with us, be as prepared as possible, be proactive).

~ 40%

Exam

- **Limited theoretical content** (Multiple choice questions), about basic understanding and vocab.
- Exercises as in **Practice Sessions**
- Manipulate concepts to **solve a new problem**

~ 60%

About repartition:
we take into account several things for the final repartition:

- quality of seminars,
- difficulty of exams,
- grade variance ...

So far, anyone who did well in the seminar passed the exam, and therefore the course).

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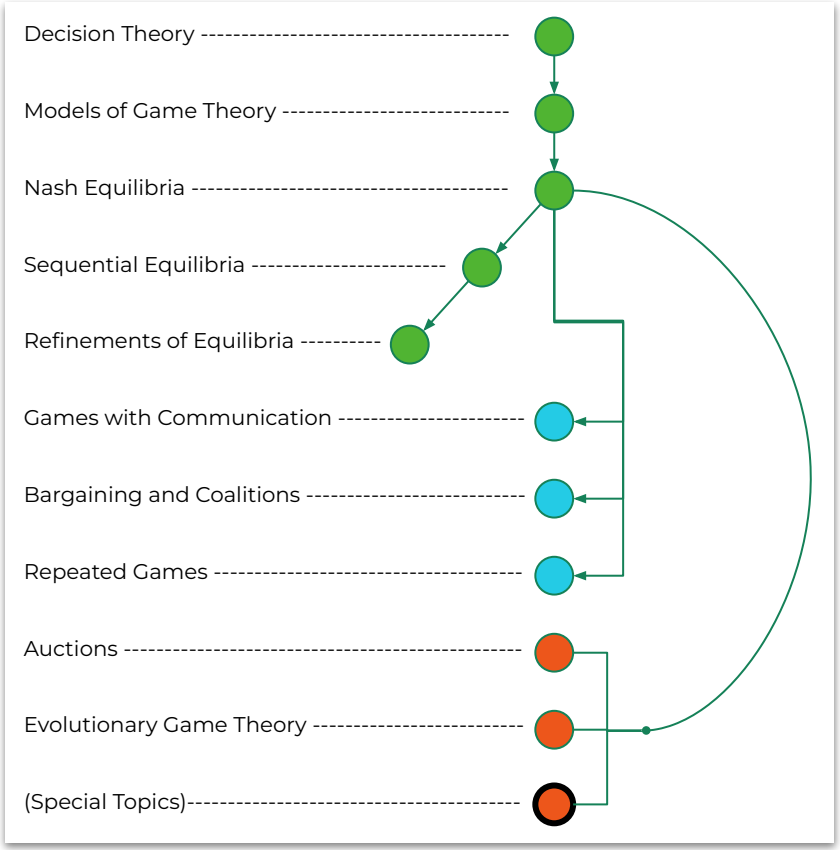
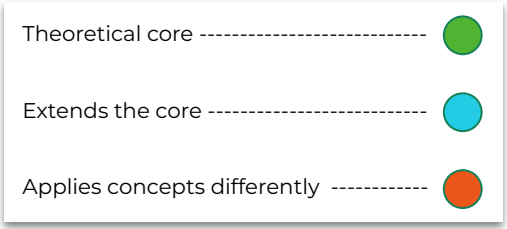
Course structure



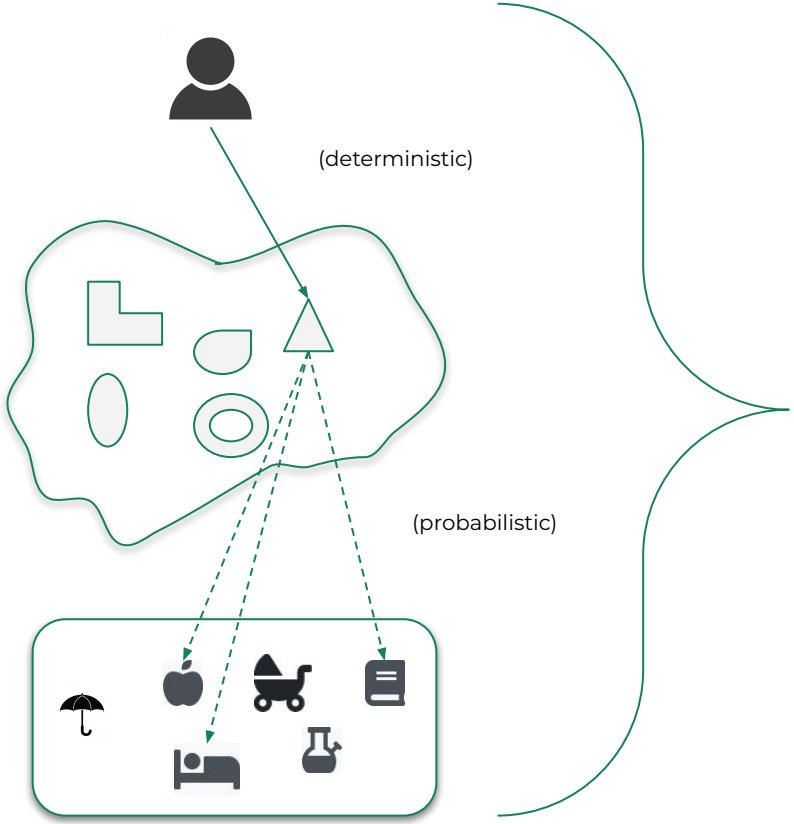


Decision theory

Agent

Decision

Outcome(s)



Axiomatic theory:
Intelligence
Rationality

How will an intelligent and rational agent decide?

Basic models of Game Theory

Several Agents

Decisions

Outcome(s)

How can we model those situations?

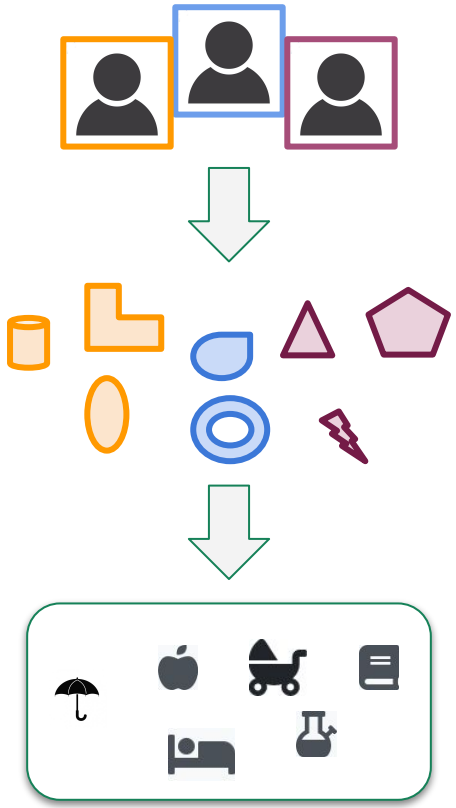
What kind of insights can we get from model?

Nash Equilibria

Several Agents

Decisions

Outcome(s)



Understanding how agents will handle the game

Solution for “normal forms”

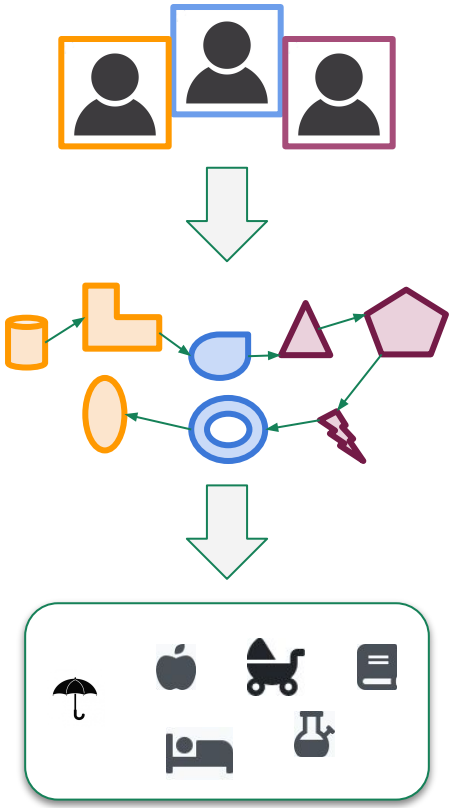
(all agents play once, together)

Sequential Equilibria

Several Agents

Decisions

Outcome(s)

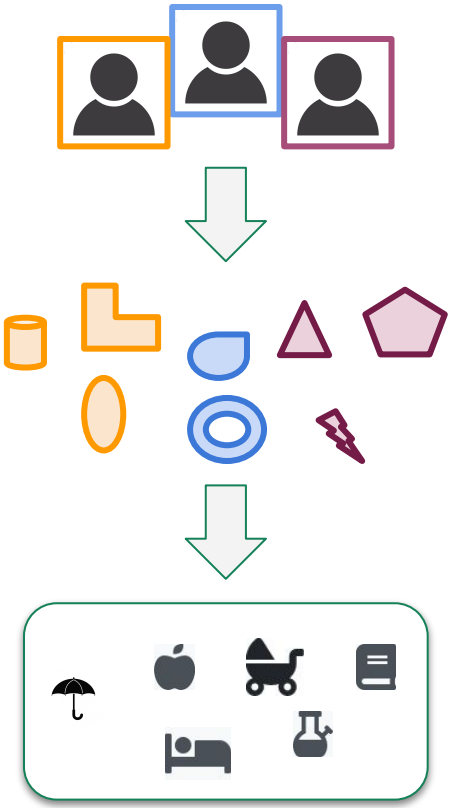


Solution for “extended form”.

Allow to model sequences of decisions.

Characterisation of solutions to a game

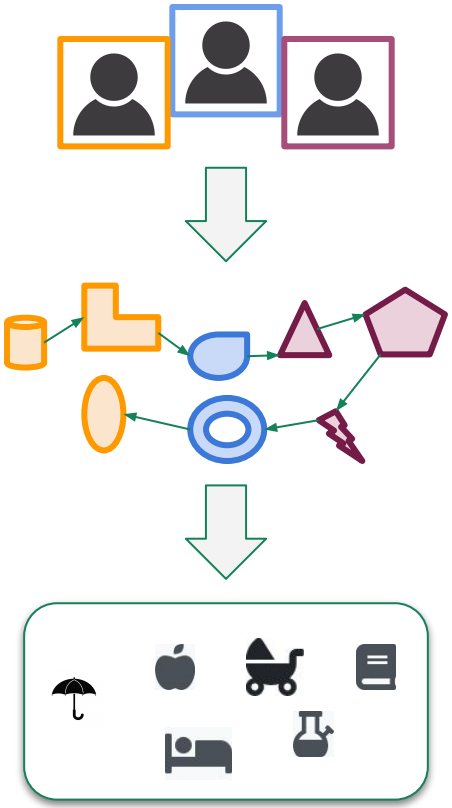
Refinement of equilibria



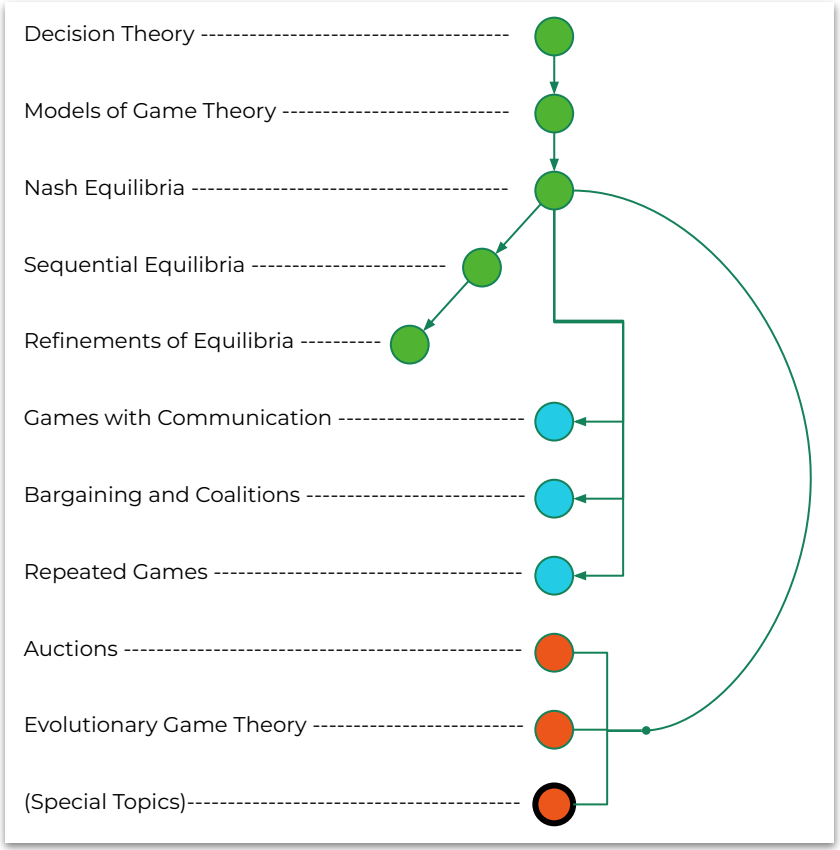
Seq. eq. ⇒ Nash eq.
Nash eq. can be “wrong”

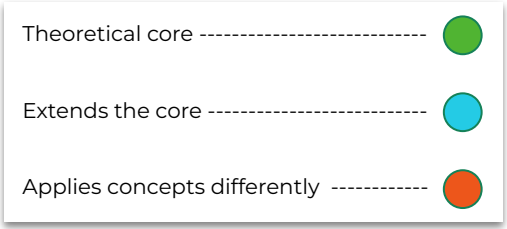
Seq. is more complex.

Solutions to “fix” Nash eq?



Course structure

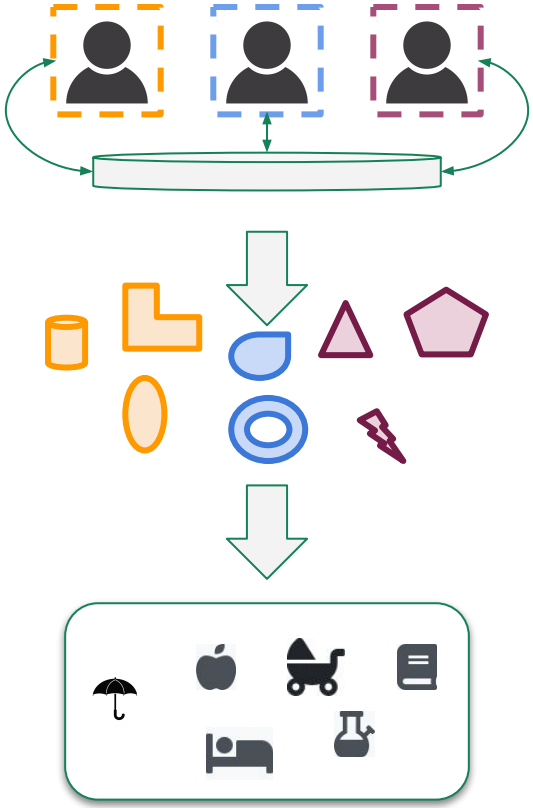




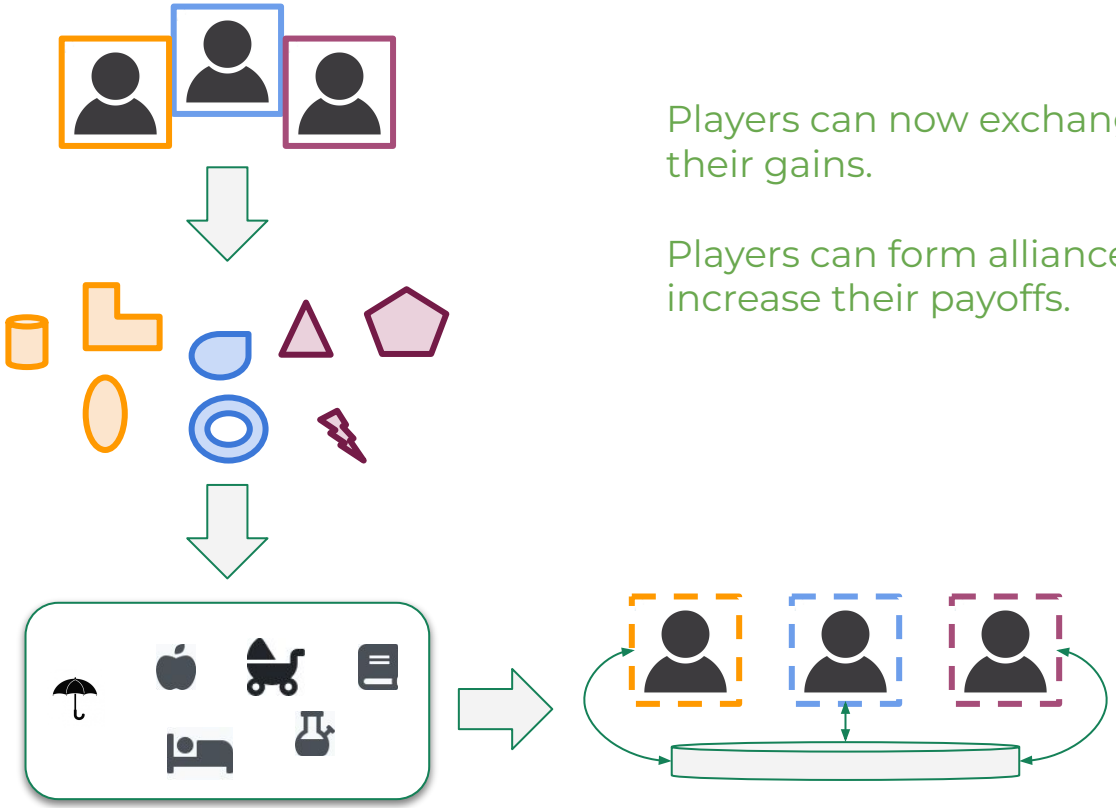
Games with communication

Adding the possibility for the agents to communicate

Leads to negotiations, mediation, ...



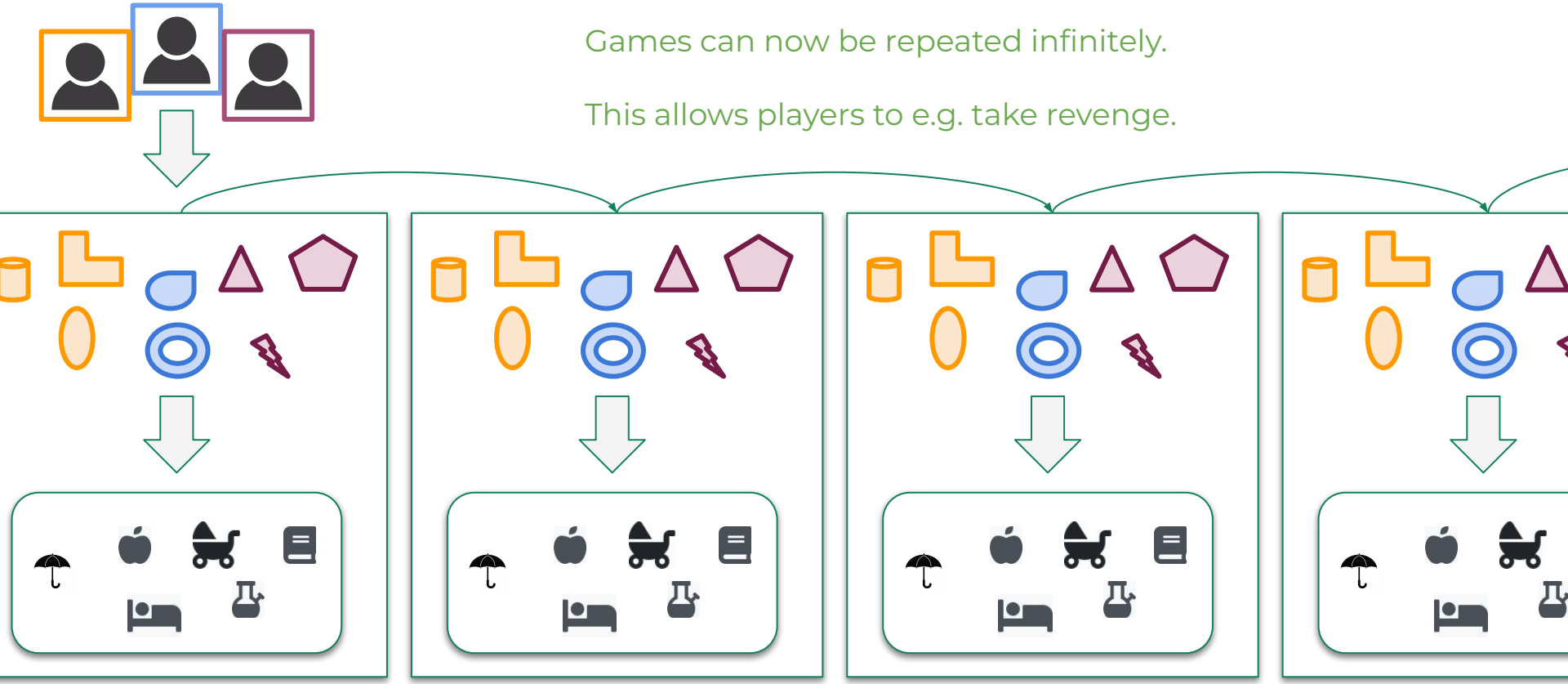
Bargaining and coalitions



Players can now exchange their gains.

Players can form alliance to increase their payoffs.

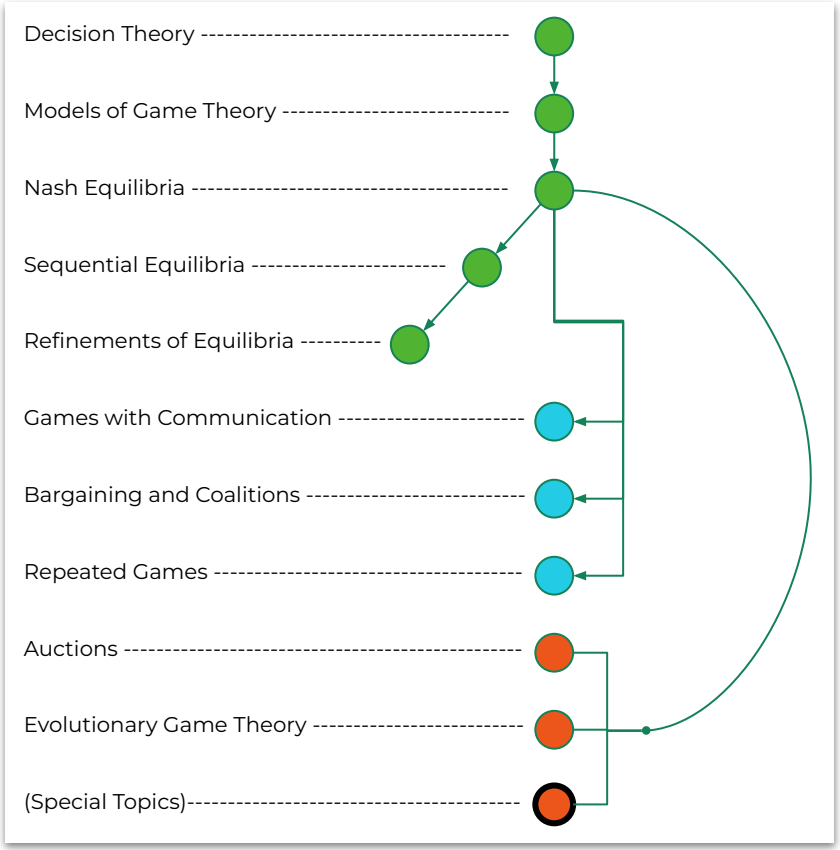
Repeated games

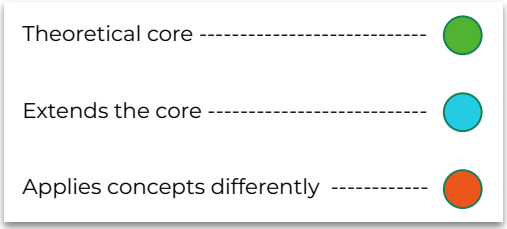


Games can now be repeated infinitely.

This allows players to e.g. take revenge.

Course structure





The last topics are not on the exam.

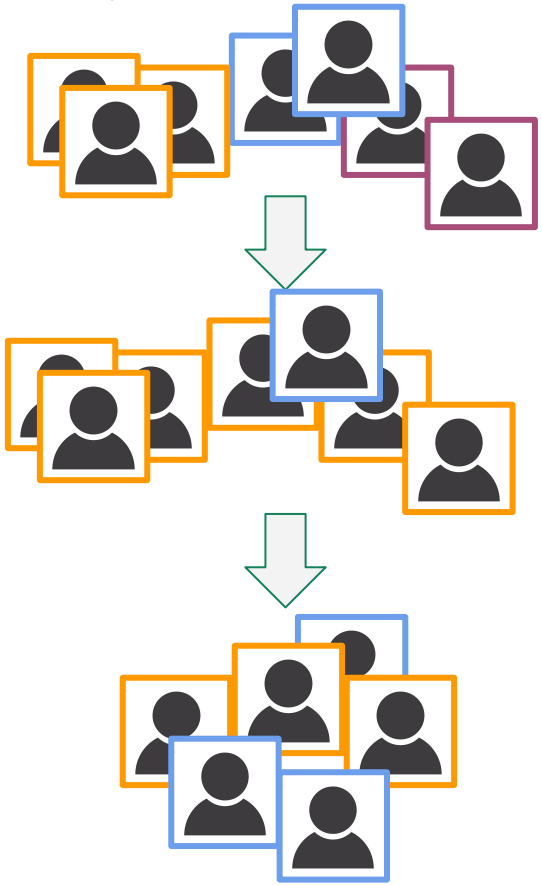
Possible to pic a special topic instead of auctions/evolutionary GT.

Auctions

How do agents bid?

When do they bid true?

Evolutionary game theory



Dynamics of how species
Interact, compete, and win
the evolutionary game

Special topics

- Related with Game Theory
    - Could be application (direct, or partial, or coincidental) e.g.
        - Control & Game Theory (there are books on it),
        - Generative adversarial networks,
        - AI safety,
        - ...
    - Could be a dig into special subtopic, e.g.
        - Computational aspects & algorithms,
        - Bounded rationality,
        - ...
- Different rules will apply
        - Not on exam (obviously),
        - Groups of 3,
        - We expect a document on top of the presentation, which we could use in a future iteration of the course.

We have 1 slots available for this!

Outline

- Who are we?
- What/who is game theory about?
- Ice-Breaker game!
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See you soon!