## LSINF2275 - Data mining and decision making

# Markov Decision Processes

### Project guidelines

*Professor*: Marco Saerens

*Teaching Assistants*: Sylvain Courtain and Pierre Leleux

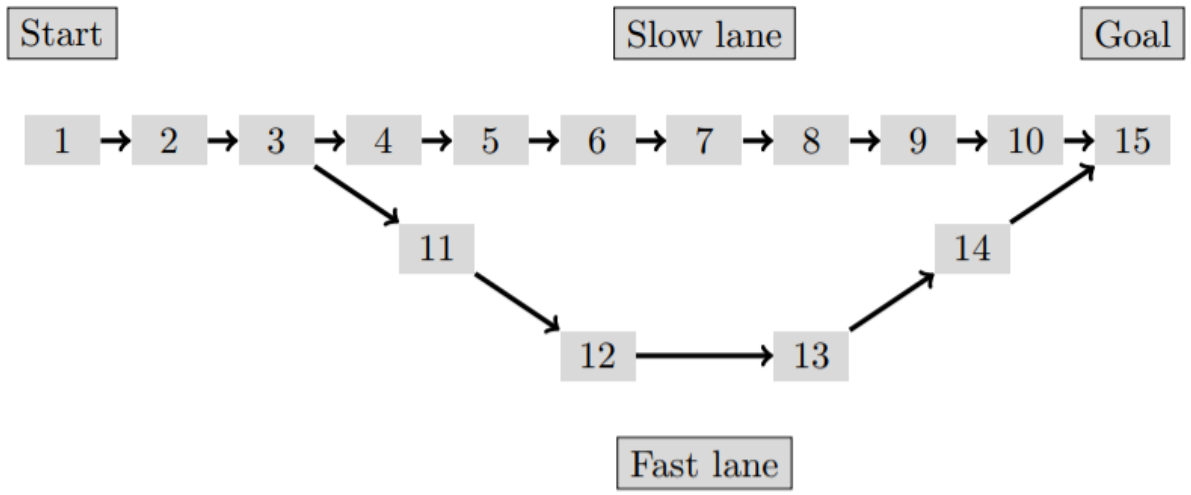
## 1 Objective

The objective of this project is to put into practice some of the techniques introduced in the data mining and decision making lectures. This is done through the study of a practical case which requires the use of a scientific programming language related to the statistical processing of data. This work aims at applying algorithms (mainly the value iteration) solving "Markov decision processes" in the framework of a Snakes and Ladders game.

## 2 Problem statement

The realization of the project will be carried out in groups of maximum 3 students (2 students only if 3 is not possible). You are asked to solve the following problem.

Let us assume that you are playing a Snakes and Ladders game which is made of 15 squares. Square number 1 is the initial square (start) and square 15 is the winning square (goal). If you reach square 15, you win. The board can contain traps that slow down your progression if triggered. A schematic representation of this game is shown below:




At any time during the game, to move forward, you can choose between three dices:

- The "security" dice which can only take two possible values: 0 or 1. It allows you to move forward by 0 or 1 square, with a probability of 1/2. With that dice, you are invincible,

which means that you ignore the presence of traps when playing with the security dice – you are immune to traps and bonuses (if any).

- The "normal" dice, which allows the player to move by 0, 1 or 2 squares with a probability of 1/3. If you land on a trapped square using this dice, you have 50 % chance of triggering the trap.
- The "risky" dice which allows the player to move by 0, 1, 2 or 3 squares with a probability of 1/4. However, when playing with the risky dice, you automatically trigger any trap you land on (100 % trigger probability).

At square 3, there is a junction with two possible paths. If the player just passes through square 3 without stopping, he continues to square 4 or beyond (5, 6, etc), as if the other path does not exist. Conversely, if the player stops on square 3, he will have, on the following turn, an equal probability of taking the slow or the fast lane. For instance, for the security dice, if the player is on square 3 and the result is 0, the player stays on square 3 with probability 1. If the result is 1, he reaches square 4 or 11, each with probability 1/2. Both paths ultimately lead to the final, goal, square.

You should also define "trap" squares on the game, that can be triggered when stopping exactly on that square. Recall that the player is only exposed to traps when drawing the normal or the risky dice. There are different types of trap, each one has its own effect:

- Type 1 – Restart: Immediately teleports the player back to the first square.
- Type 2 – Penalty: Immediately teleports the player 3 squares backwards.
- Type 3 – Prison: The player must wait one turn before playing again.
- Type 4 – Gamble: Randomly teleports the player anywhere on the board[1], with equal, uniform, probability.

Note that the first and final squares cannot be trapped.

We ask you to compute the optimal strategy, i.e., for each square, which dice should be used to reach the goal square in a minimal number of turns, on average (the cost represents the number of turns to reach the goal square). You should determine this solution in two different scenarios:

- You should exactly stop on the arrival, goal, square to win. The game board is designed as a circle, which means that, if you overstep the last square, you have to restart from the 1st square.
- You win as soon as you land on or overstep the final square (i.e. if you are on square 15 or further).

[1]This means that the trap may teleport the player on any of the 15 squares, including the final square.

## 3 Implementation

You are asked to implement, in *Python* 3, the following function

`markovDecision(layout,circle)`

This function launches the Markov Decision Process algorithm to determine the optimal strategy regarding the choice of the dice in the Snakes and Ladders game, using the "value iteration" method.

**Inputs**:

- `layout`: a vector of type `numpy.ndarray` that represents the layout of the game, containing 15 values representing the 15 squares of the Snakes and Ladders game:

  `layout[i]` = 0 if it is an ordinary square
  = 1 if it is a "restart" trap (go back to square 1)
  = 2 if it is a "penalty" trap (go back 3 steps)
  = 3 if it is a "prison" trap (skip next turn)
  = 4 if it is a "gamble" trap (random teleportation)

- `circle`: a boolean variable (type `bool`), indicating if the player must land exactly on the final, goal, square 15 to win (`circle = True`) or still wins by overstepping the final square (`circle = False`).

**Output**: Your function `markovDecision` is expected to return a type `list` containing the two vectors `[Expec,Dice]`:

- `Expec`: a vector of type `numpy.ndarray` containing the expected cost (= number of turns) associated to the 14 squares of the game, excluding the goal square. The vector starts at index 0 (corresponding to square 1) and ends at index 13 (square 14).

- `Dice`: a vector of type `numpy.ndarray` containing the choice of the dice to use for each of the 14 squares of the game (1 for "security" dice, 2 for "normal" dice and 3 for "risky"), excluding the goal square. Again, the vector starts at index 0 (square 1) and ends at index 13 (square 14).

In addition, it is also interesting to compare this strategy with other (sub-optimal) strategies, e.g. the use of dice 1, dice 2 or dice 3 only, a mixed random strategy, or a purely random choice, etc. Launch/simulate an important number of games and compare empirically the performance of each of these strategies with the optimal strategy (or policy) obtained by value iteration. You should then report (i) the *theoretical expected cost*, obtained by value iteration, and compare it with the *empirical average cost* when playing (simulating) a large number of games with the optimal strategy, and (ii) compare the empirical average cost obtained by the optimal strategy with the other (sub-optimal) strategies mentioned before (dice 1 only, etc). You could test this with a few different configurations of traps, as well as for the different scenarios (circle or not), depending on your time.

Finally, if you have some time left, and thus *as a bonus*, feel free to investigate the Q-learning value iteration on the same problem. What are your conclusions, in comparison with the use of the standard value iteration?

# 4 Report

Please do not forget to mention your affiliation on the cover page, together with your name (SINF, INFO, MAP, STAT, BIR, DATS, etc) and group number. You are asked to write a report (PDF) in English of maximum 6 pages (everything included). This report must have a professional look & feel, like a scientific or a technical report. Therefore, your report can integrate plot (only if pertinent), but no screenshot of equations and/or screenshot of code outputs. Do not forget to provide references for your sources of information[2] and be consistent in your notations.

Your report must at least contain:

- a short introduction;
- a brief theoretical explanation of the method used to determine the optimal strategy regarding the choice of the dice;
- a short description of your implementation[3];
- for various trap layouts (trap squares at relevant places), comparisons between theoretical and empirical results for the optimal strategy as well as between the optimal and suboptimal strategies. Analyze and discuss your results.
- a conclusion.

Your project will be evaluated on the following aspects:

- The quality of your report and code;
- The amount of experimental work (including potential additional experiments);
- The correctness and efficiency of your function `markovDecision`.

**Important:** Beware that the correctness and efficiency of your function will be graded by an automated script that will execute your function several times using different board layouts to check if your function always returns the correct policy and expected costs. It is therefore important that you strictly comply with the syntax of the function (name, inputs and outputs). Moreover, your code must be a .py, not a Jupyter notebook.

This report must be uploaded before April 02, 2023, 23:55, together with the code (all files zipped together) on Moodle in the section "Assignments". Do not forget to comment your code. This first project accounts for 5 points in the final grade of the course. Your project can be handed behind schedule, but your group will get a penalty of $-0.5$ for each day late. For instance, a project worth 4/5, but handed on April 03 (1 day late), will be graded 3.5/5. Submissions will no longer be accepted after April 07, 23:55.

[2] A good reference available online is the book of Sutton and Barto (2018) "Reinforcement learning: an introduction", second edition.

[3] Do not go into details about your code itself. If you still wish to provide details about the code and the functions of your implementation, do not hesitate to integrate a ReadMe file in your zip, but your report should not focus on the code.

## 5 Practicalities

Consider the following practical details concerning the implementation:

1. Traps can be triggered when drawing a 0 from the dice. For instance, if you use the normal or the risky dice and get a 0, while standing on a trapped square, you (may) trigger the trap of the square you are standing on.

2. There is no "cascade triggering" of traps: if you trigger a trap that teleports you (type 2 or 4 traps), you ignore the potential presence of a trap/bonus on the square you got teleported to. Note that, however, if you play with the normal or risky dice, you may still trigger the trap next turn by drawing a 0 (see previous point).

3. Penalty traps (type 2) situated too close to the beginning (on square 2 or 3) that cannot make the player move 3 squares backwards, simply teleport the player on the first square like a restart trap (type 1) would. This rule also applies in the case of **circular** boards (`circle = True`).

**Good work !!**

---