In[35]:=
```
Remove["Global`*"]
```

Numerical solution to a partial differential equation, namely the transverse vibrational motion of a stretched string, fixed at both ends, as a function of time.
Transverse vibrations of a stretch string follows from the wave equation we where u(x,t) is the shape of the string for a longitudinal position x at time t.

Consider a string with shape u(x,0) = (L/2 - x)^2 (L/2 + x)^2 where the string extends from x = -L/2 to x = L/2 and is fixed at the endpoints so u(+-L/2,t) = 0
Find u(x,t), L = l

In[36]:=
```
v = 2;
l = 5;
```

In[38]:=
```
we = (1 / v^2) * (D[u[x, t], {t, 2}]) == D[u[x, t], {x, 2}];
```

In[39]:=
```
sol =
 NDSolve[{we, Derivative[0, 1][u][x, 0] == 0, u[x, 0] == ((l / 2 - x) ^2 ) * ((l / 2 + x) ^2),
   u[-l / 2, t] == 0, u[l / 2, t] == 0}, u[x, t], {x, -l / 2, l / 2}, {t, 0, l / v}]
```

NDSolve: Warning: scaled local spatial error estimate of 644.9482606398883` at t = 2.5` in the direction of independent variable x is much greater than the prescribed error tolerance. Grid spacing with 25 points may be too large to achieve the desired accuracy or precision. A singularity may have formed or a smaller grid spacing can be specified using the MaxStepSize or MinPoints method options.

Out[39]=
```
{{u[x, t] → InterpolatingFunction[ Domain: {{-2.5, 2.5}, {0., 2.5}} Output: scalar ][x, t]}}
```

In[40]:=
```
U = u[x, t] /. sol
```

Out[40]=
```
{InterpolatingFunction[ Domain: {{-2.5, 2.5}, {0., 2.5}} Output: scalar ][x, t]}
```

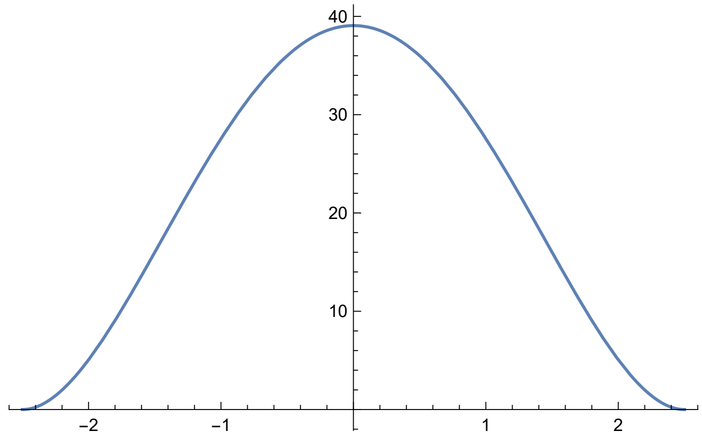
In[41]:=
```
U0 = u[x, t] /. sol /. t → 0
```

Out[41]=
```
{InterpolatingFunction[ Domain: {{-2.5, 2.5}, {0., 2.5}} Output: scalar ][x, 0]}
```

In[43]:=

In[44]:=
```
Plot[U0, {x, -l / 2, l / 2}]
```

Out[44]=



In[45]:=

In[46]:=
```
Animate[Plot[u[x, t] /. sol /. t → ti, {x, -l / 2, l / 2}, PlotRange → {-40, 40}],
 {ti, 0, l / v}, AnimationRunning → True]
```

Out[46]=