In[47]:=

```
Remove["Global`*"]
```

Principle axes and moments of inertia for a “rigid body” that consists of eight equal masses m at the corners of a rectangular block in three dimensional space. The corners are at (x,y,z) = (0, 2a, +-b), (-a sqrt 3, a, +-b), (a sqrt3, -a, +-b), (0, -2a, +-b).

First we define a variable that is a list of the vectors for each of the eight corners, then we pick values for a & b and graph the corners in 3D.

In[48]:=

```
c = {{0, 2 a, b}, {-a Sqrt[3], a, b}, {a Sqrt[3], -a, b}, {0, -2 a, b},
  {0, 2 a, -b}, {-a Sqrt[3], a, -b}, {a Sqrt[3], -a, -b}, {0, -2 a, -b}}
a = 1
b = 1
xval = 1
```

Out[48]= $\{\{0, 2a, b\}, \{-\sqrt{3}\, a, a, b\}, \{\sqrt{3}\, a, -a, b\}, \{0, -2a, b\}, \{0, 2a, -b\}, \{-\sqrt{3}\, a, a, -b\}, \{\sqrt{3}\, a, -a, -b\}, \{0, -2a, -b\}\}$

Out[49]= 1

Out[50]= 1

Out[51]= 1

In[52]:=

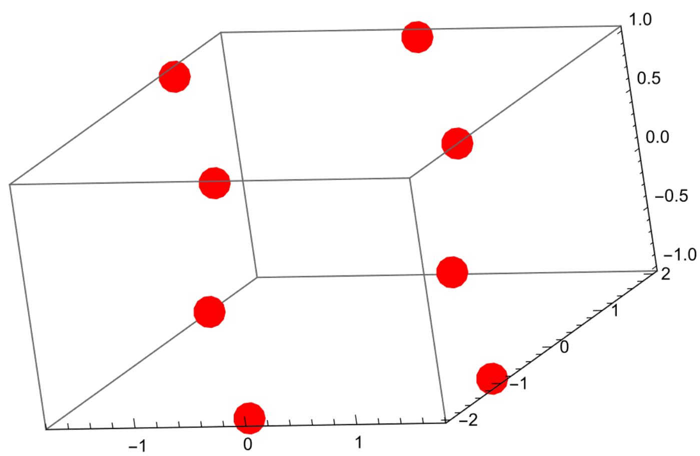
```
Print[c]
```

$\{\{0, 2, 1\}, \{-\sqrt{3}, 1, 1\}, \{\sqrt{3}, -1, 1\}, \{0, -2, 1\}, \{0, 2, -1\}, \{-\sqrt{3}, 1, -1\}, \{\sqrt{3}, -1, -1\}, \{0, -2, -1\}\}$

In[53]:=

```
Graphics3D[{Red, PointSize[0.05], Point[c]}, Axes → True, ViewProjection → "Orthographic"]
```

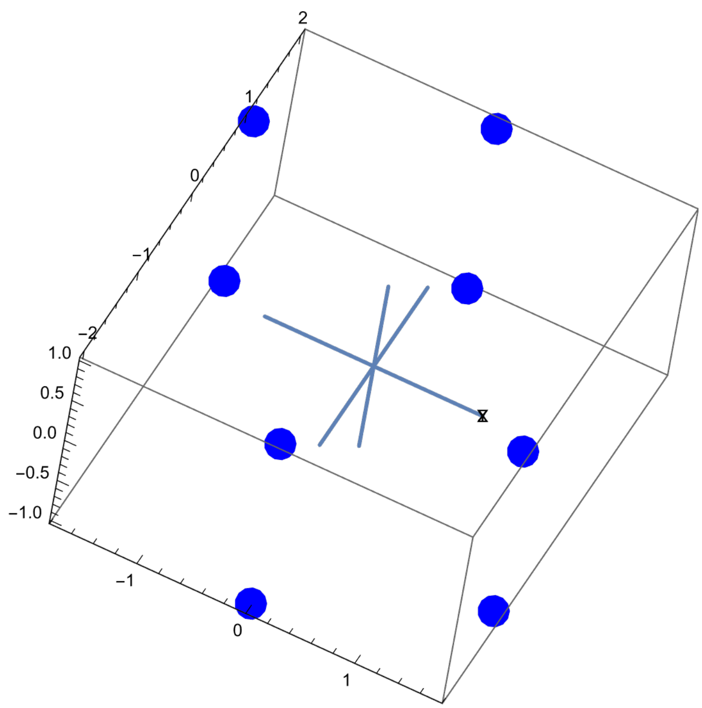
Out[53]=



The mouse cursor can be used to move around the direction of the view of the graph.

In[54]:=
```
Show[Graphics3D[{Blue, PointSize[0.05], Point[c]},
   Axes → True, ViewProjection → "Orthographic"],
  ParametricPlot3D[{t, 0, 0}, {t, -1, 1}], Graphics3D[Text["X", {xval, 0, 0}]],
  ParametricPlot3D[{0, t, 0}, {t, -1, 1}], Graphics3D[Text["Y", {xval, 0, 0}]],
  ParametricPlot3D[{0, 0, t}, {t, -1, 1}], Graphics3D[Text["Z", {xval, 0, 0}]]]
```

Out[54]=



The center of mass is located at vector rcm. Calculating the sum over the masses shows the rcm is at the origin.

In[55]:=
```
rcm = Total[c]
```

Out[55]= {0, 0, 0}

In[56]:=
```
im = IdentityMatrix[3]
```

Out[56]= {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

Calculating Moment of Inertia Tensor and displaying it in matrix form

In[57]:=
```
moi = Total[Table[Norm[c[[i]]] * im - TensorProduct[c[[i]], c[[i]]], {i, 1, 8}]]
```

Out[57]= $\{\{-12+8\sqrt{5}, 4\sqrt{3}, 0\}, \{4\sqrt{3}, -20+8\sqrt{5}, 0\}, \{0, 0, -8+8\sqrt{5}\}\}$

In[58]:=
```
MatrixForm[moi]
```

Out[58]//MatrixForm=

$$\begin{pmatrix} -12+8\sqrt{5} & 4\sqrt{3} & 0 \\ 4\sqrt{3} & -20+8\sqrt{5} & 0 \\ 0 & 0 & -8+8\sqrt{5} \end{pmatrix}$$

Calculating the eigenvalues and eigenvectors of the inertia tensor which are the principal moments of inertia and the principal axes.

In[59]:=
```
Eigenvalues[moi]
```

Out[59]= $\left\{8\times\left(-1+\sqrt{5}\right),\ 8\times\left(-1+\sqrt{5}\right),\ 8\times\left(-3+\sqrt{5}\right)\right\}$

In[60]:=
```
Eigenvectors[moi]
```

Out[60]= $\left\{\{0,\ 0,\ 1\},\ \left\{\sqrt{3},\ 1,\ 0\right\},\ \left\{-\frac{1}{\sqrt{3}},\ 1,\ 0\right\}\right\}$

In[61]:=
```
Show[Graphics3D[{Blue, PointSize[0.05], Point[c]},
  Axes → True, ViewProjection → "Orthographic"],
 ParametricPlot3D[{t, 0, 0}, {t, -1, 1}], Graphics3D[Text["X", {xval, 0, 0}]],
 ParametricPlot3D[{0, t, 0}, {t, -1, 1}], Graphics3D[Text["Y", {xval, 0, 0}]],
 ParametricPlot3D[{0, 0, t}, {t, -1, 1}], Graphics3D[Text["Z", {xval, 0, 0}]],
 ParametricPlot3D[{0, 0, 1} * t, {t, -1, 1}, PlotStyle → Blue],
 ParametricPlot3D[{Sqrt[3], 1, 0} * t, {t, -1, 1}, PlotStyle → Blue],
 ParametricPlot3D[{- 1 / Sqrt[3], 1, 0} * t, {t, -1, 1}, PlotStyle → Blue]]
```

Out[61]=