In[ ]:=
```
Remove ["Global`*"]
```

Part 1: We begin with the scalar field f(x,y) = (x^2 + y^2)/2. We then use the Grad function to find the gradient of the field. The expected value is vector xi + yj and the return of the function is exactly as expected.

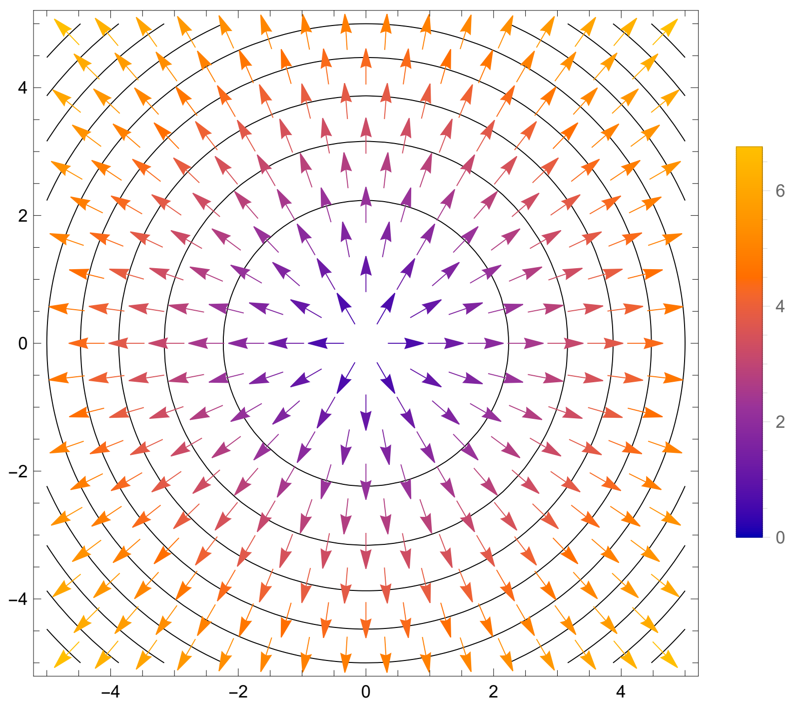
In[ ]:=
```
f = (x^2 + y^2) / 2
```

Out[ ]= $\frac{1}{2}\left(x^2+y^2\right)$

In[ ]:=
```
v = Grad[f, {x, y}]
```

Out[ ]= $\{x, y\}$

In[ ]:=
```
Show[ContourPlot[f, {x, -5, 5}, {y, -5, 5}, ContourShading → None],
 VectorPlot[v, {x, -5, 5}, {y, -5, 5}, PlotLegends → Automatic]]
```

Out[ ]= 

We then plot our vector value accordingly and it returns the graph above showing the direction of the gradient and the intensity radiating from the center outwards.

In[ ]:=
```
d = Div[v, {x, y}]
```

Out[ ]= 2

The divergence of the field is expected to be 2 as taking the partial derivative of the x part of the vector field with respect to x yields 1 and is added to the partial derivative of the y part of the vector field with respect to y totaling 2.

In[ ]:=
```
c = Curl[Flatten[{v, 0}], {x, y, z}]
```
Out[ ]= {0, 0, 0}

Calculating the curl gives the expected value of 0i - 0j + 0k which matches our value.

Part 2: we are giving a new scalar field f(x,y,z) = -1/sqrt(x^2 + y^2 + z^2). We then calculate the gradient again and turn the new vector field from 3d to 2d by replacing the z component with 0.

In[ ]:=
```
f2 = -1 / Sqrt[x^2 + y^2 + z^2]
```
Out[ ]= $$-\frac{1}{\sqrt{x^2+y^2+z^2}}$$

In[ ]:=
```
v2 = Grad[f2, {x, y, z}]
```
Out[ ]= $$\left\{\frac{x}{\left(x^2+y^2+z^2\right)^{3/2}}, \frac{y}{\left(x^2+y^2+z^2\right)^{3/2}}, \frac{z}{\left(x^2+y^2+z^2\right)^{3/2}}\right\}$$

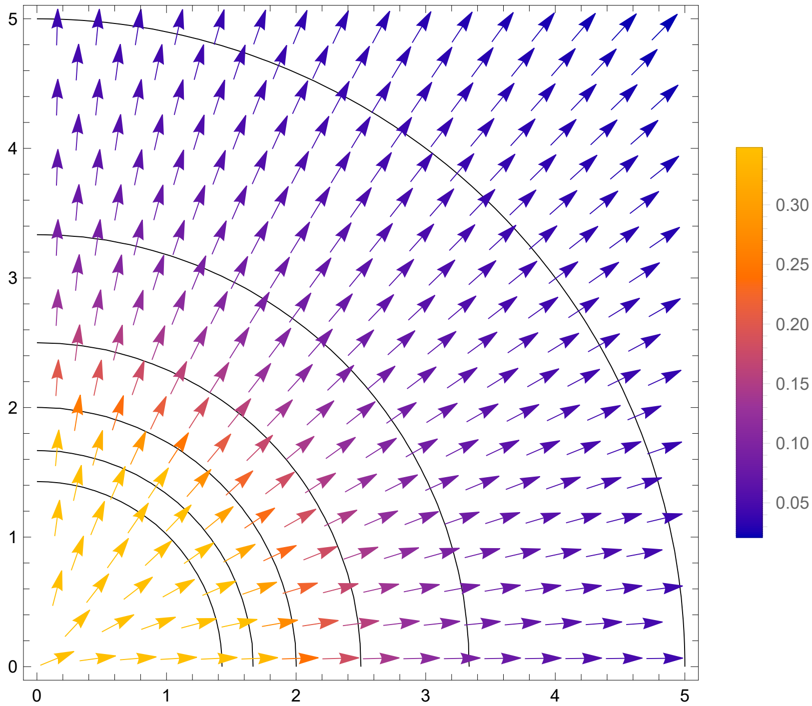
In[ ]:=
```
f2d = f2 /. z → 0
```
Out[ ]= $$-\frac{1}{\sqrt{x^2+y^2}}$$

In[ ]:=
```
v2d = v2 /. z → 0
```
Out[ ]= $$\left\{\frac{x}{\left(x^2+y^2\right)^{3/2}}, \frac{y}{\left(x^2+y^2\right)^{3/2}}, 0\right\}$$

In[ ]:=
```
v2d1 = {v2d[[1]], v2d[[2]]}
```
Out[ ]= $$\left\{\frac{x}{\left(x^2+y^2\right)^{3/2}}, \frac{y}{\left(x^2+y^2\right)^{3/2}}\right\}$$

In[∘]:=
```
Show[ContourPlot[f2d, {x, 0, 5}, {y, 0, 5}, ContourShading → None],
 VectorPlot[v2d1, {x, 0, 5}, {y, 0, 5}, PlotLegends → Automatic]]
```

Out[∘]=



Plotting this new vector field in the first quadrant only yields the graph above. Similar to the first graph but the intensity is now opposite direction.

In[∘]:=
```
d2 = Simplify[Div[v2, {x, y, z}]]
```

Out[∘]= 0

Calculating the divergence using the 3d version of the vector gives 0 which is unexpected.

In[∘]:=
```
tpc = ToPolarCoordinates[v2d]
```

Out[∘]= $$\left\{\sqrt{\frac{x^2}{\left(x^2+y^2\right)^3}+\frac{y^2}{\left(x^2+y^2\right)^3}},\ \text{ArcCos}\left[\frac{x}{\left(x^2+y^2\right)^{3/2}\sqrt{\frac{x^2}{\left(x^2+y^2\right)^3}+\frac{y^2}{\left(x^2+y^2\right)^3}}}\right],\ \text{ArcTan}\left[\frac{y}{\left(x^2+y^2\right)^{3/2}},\ 0\right]\right\}$$

In[∘]:=
```
tpc2 = Simplify[tpc /. {x → r Cos[θ], y → r Sin[θ]}, Assumptions → r > 0]
```

Out[∘]= $$\left\{\frac{1}{r^2},\ \text{ArcCos}[\text{Cos}[\theta]],\ \text{ArcTan}[\text{Sin}[\theta],\ 0]\right\}$$

Using the ToPolarCoordinates function, to turn the vector without the z component in polar coordinates, we find that the values do not yield zero. The fields describe their motion.

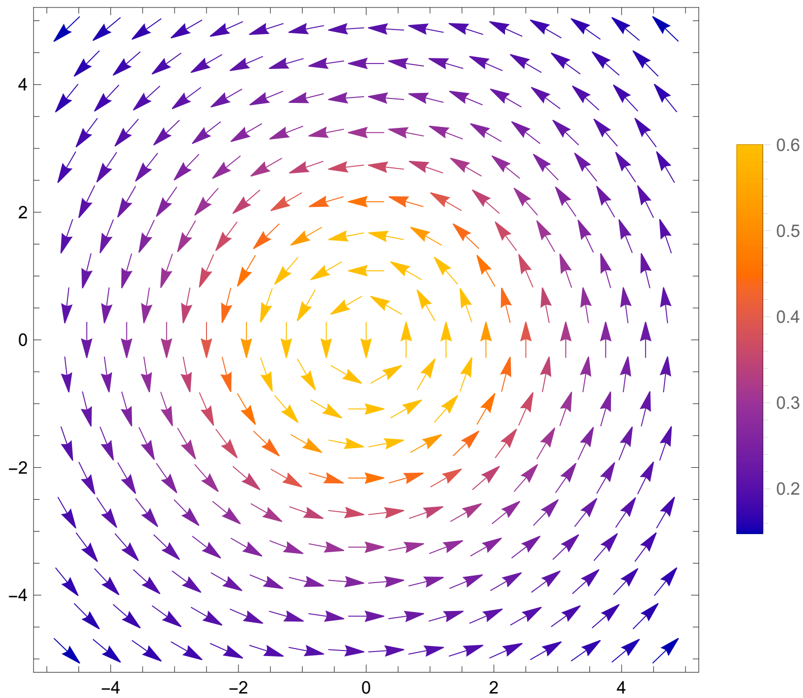
Part 3: we are given the vector field v(x,y) + (iy + jx)/(x^2 + y^2). We then graph the field and find the curl.

In[∘]:=
```
v3 = {-y, x, 0} / (x^2 + y^2)
```

Out[∘]= $$\left\{-\frac{y}{x^2+y^2},\ \frac{x}{x^2+y^2},\ 0\right\}$$

In[∘]:= `Show[VectorPlot[v3, {x, -5, 5}, {y, -5, 5}, PlotLegends → Automatic]]`

Out[∘]=



The plot shows the movement in the shape of a circle.

In[∘]:= `v3`

Out[∘]= $\left\{-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}\right\}$

In[∘]:= `c3 = Simplify[Curl[Flatten[v3, 0], {x, y, z}]]`

Out[∘]= {0, 0, 0}

The curl gives the surprising value of 0,0,0

In[∘]:= `tpc3 = ToPolarCoordinates[v3]`

Out[∘]= $\left\{\sqrt{\frac{x^2}{\left(x^2+y^2\right)^2}+\frac{y^2}{\left(x^2+y^2\right)^2}}, \text{ArcCos}\left[-\frac{y}{\left(x^2+y^2\right)\sqrt{\frac{x^2}{\left(x^2+y^2\right)^2}+\frac{y^2}{\left(x^2+y^2\right)^2}}}\right], \text{ArcTan}\left[\frac{x}{x^2+y^2}, 0\right]\right\}$

In[∘]:= `tpc4 = Simplify[tpc3 /. {x → r Cos[θ], y → r Sin[θ]}, Assumptions → r > 0]`

Out[∘]= $\left\{\frac{1}{r}, \text{ArcCos}[-\text{Sin}[\theta]], \text{ArcTan}[\text{Cos}[\theta], 0]\right\}$