In[37]:= `Remove["Global`*"]`

Using fourier decomposition to show shape of string fixed at x = 0 and x = L as a function of time.

In[38]:= `s = Sin[n Pi x / L]`

Out[38]= $\text{Sin}\left[\frac{n\pi x}{L}\right]$

In[39]:= `y1 = -4 h x / L (*line from 0 - L/4*)`

Out[39]= $-\frac{4hx}{L}$

In[40]:= `y2 = (8 h x / L) - 4 h / L (*line from L/4 - 3L/4*)`

Out[40]= $-\frac{4h}{L}+\frac{8hx}{L}$

In[41]:= `y3 = (-4 h x / L) + 4 h / L (*line from 3L/4 - L*)`

Out[41]= $\frac{4h}{L}-\frac{4hx}{L}$

In[42]:= `f1 = y1 s (*Function for line 1*)`

Out[42]= $-\frac{4hx\,\text{Sin}\left[\frac{n\pi x}{L}\right]}{L}$

In[43]:= `f2 = y2 s (*function for line 2*)`

Out[43]= $\left(-\frac{4h}{L}+\frac{8hx}{L}\right)\text{Sin}\left[\frac{n\pi x}{L}\right]$

In[44]:= `f3 = y3 s (*function for line 3*)`

Out[44]= $\left(\frac{4h}{L}-\frac{4hx}{L}\right)\text{Sin}\left[\frac{n\pi x}{L}\right]$

In[45]:= `Bn1 = Integrate[f1, {x, 0, L / 4}] (*Coefficients for fourier sine series of function 1*)`

Out[45]= $\frac{hL\left(n\pi\,\text{Cos}\left[\frac{n\pi}{4}\right]-4\,\text{Sin}\left[\frac{n\pi}{4}\right]\right)}{n^2\pi^2}$

In[46]:= `Bn2 = Integrate[f2, {x, L / 4, 3 L / 4}]`
`(*Coefficients for fourier sine series of function 2*)`

Out[46]= $\frac{1}{n^2\pi^2}2h\left((-2+L)\,n\pi\,\text{Cos}\left[\frac{n\pi}{4}\right]+(2-3L)\,n\pi\,\text{Cos}\left[\frac{3n\pi}{4}\right]+4L\left(-\text{Sin}\left[\frac{n\pi}{4}\right]+\text{Sin}\left[\frac{3n\pi}{4}\right]\right)\right)$

In[47]:= `Bn3 = Integrate[f3, {x, 3 L / 4, L}] (*Coefficients for fourier sine series of function 3*)`

Out[47]= $\frac{1}{n^2\pi^2}h\left(-\left((-4+3L)\,n\pi\,\text{Cos}\left[\frac{3n\pi}{4}\right]\right)+4\times(-1+L)\,n\pi\,\text{Cos}[n\pi]+4L\left(\text{Sin}\left[\frac{3n\pi}{4}\right]-\text{Sin}[n\pi]\right)\right)$

In[48]:=
```
Bn = 2 / L (Bn1 + Bn2 + Bn3) (*Total coefficients for fourier sine series*)
```

Out[48]=
$$\frac{1}{L}2\left(\frac{h L\left(n\pi \mathrm{Cos}\left[\frac{n\pi}{4}\right]-4\,\mathrm{Sin}\left[\frac{n\pi}{4}\right]\right)}{n^2\pi^2}+\frac{1}{n^2\pi^2}2h\left((-2+L)\,n\pi\,\mathrm{Cos}\left[\frac{n\pi}{4}\right]+(2-3L)\,n\pi\,\mathrm{Cos}\left[\frac{3n\pi}{4}\right]+4L\left(-\mathrm{Sin}\left[\frac{n\pi}{4}\right]+\mathrm{Sin}\left[\frac{3n\pi}{4}\right]\right)\right)+\frac{1}{n^2\pi^2}h\left(-\left((-4+3L)\,n\pi\,\mathrm{Cos}\left[\frac{3n\pi}{4}\right]\right)+4\times(-1+L)\,n\pi\,\mathrm{Cos}[n\pi]+4L\left(\mathrm{Sin}\left[\frac{3n\pi}{4}\right]-\mathrm{Sin}[n\pi]\right)\right)\right)$$

In[49]:=
```
uxt = Sum[Bn Sin[n Pi x / L] Cos[ n Pi v t / L], {n, 1, 5}]
  (*shape of string with 5 terms in expansion*)
```

Out[49]=
$$\frac{1}{L}2\left(\frac{hL\left(-2\sqrt{2}+\frac{\pi}{\sqrt{2}}\right)}{\pi^2}+\frac{2h\left(-\frac{(2-3L)\pi}{\sqrt{2}}+\frac{(-2+L)\pi}{\sqrt{2}}\right)}{\pi^2}+\frac{h\left(2\sqrt{2}L-4\times(-1+L)\pi+\frac{(-4+3L)\pi}{\sqrt{2}}\right)}{\pi^2}\right)\mathrm{Cos}\left[\frac{\pi t v}{L}\right]\mathrm{Sin}\left[\frac{\pi x}{L}\right]+\frac{2\left(-\frac{5hL}{\pi^2}+\frac{h(-4L+8\times(-1+L)\pi)}{4\pi^2}\right)\mathrm{Cos}\left[\frac{2\pi t v}{L}\right]\mathrm{Sin}\left[\frac{2\pi x}{L}\right]}{L}+\frac{1}{L}2\left(\frac{hL\left(-2\sqrt{2}-\frac{3\pi}{\sqrt{2}}\right)}{9\pi^2}+\frac{2h\left(\frac{3\times(2-3L)\pi}{\sqrt{2}}-\frac{3\times(-2+L)\pi}{\sqrt{2}}\right)}{9\pi^2}+\frac{h\left(2\sqrt{2}L-12\times(-1+L)\pi-\frac{3\times(-4+3L)\pi}{\sqrt{2}}\right)}{9\pi^2}\right)\mathrm{Cos}\left[\frac{3\pi t v}{L}\right]\mathrm{Sin}\left[\frac{3\pi x}{L}\right]+\frac{1}{L}2\left(-\frac{hL}{4\pi}+\frac{h(-4\times(2-3L)\pi-4\times(-2+L)\pi)}{8\pi^2}+\frac{h(16\times(-1+L)\pi+4\times(-4+3L)\pi)}{16\pi^2}\right)\mathrm{Cos}\left[\frac{4\pi t v}{L}\right]\mathrm{Sin}\left[\frac{4\pi x}{L}\right]+\frac{1}{L}2\left(\frac{hL\left(2\sqrt{2}-\frac{5\pi}{\sqrt{2}}\right)}{25\pi^2}+\frac{2h\left(\frac{5\times(2-3L)\pi}{\sqrt{2}}-\frac{5\times(-2+L)\pi}{\sqrt{2}}\right)}{25\pi^2}+\frac{h\left(-2\sqrt{2}L-20\times(-1+L)\pi-\frac{5\times(-4+3L)\pi}{\sqrt{2}}\right)}{25\pi^2}\right)\mathrm{Cos}\left[\frac{5\pi t v}{L}\right]\mathrm{Sin}\left[\frac{5\pi x}{L}\right]$$

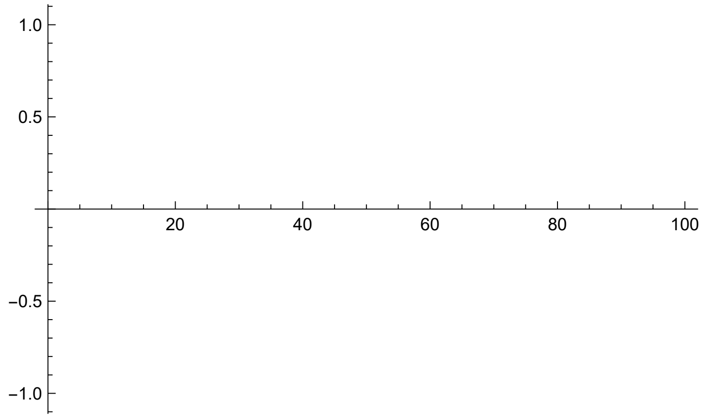
In[50]:=
```
uxt1 = Simplify[%]
```

Out[50]=
$$\frac{1}{15L\pi^2}h\left(60\times\left(-2+3\sqrt{2}\right)\times(-1+L)\,\pi\,\mathrm{Cos}\left[\frac{\pi t v}{L}\right]\mathrm{Sin}\left[\frac{\pi x}{L}\right]+60\,(L\,(-3+\pi)-\pi)\,\mathrm{Cos}\left[\frac{2\pi t v}{L}\right]\mathrm{Sin}\left[\frac{2\pi x}{L}\right]+\pi\left(-20\times\left(2+3\sqrt{2}\right)\times(-1+L)\,\mathrm{Cos}\left[\frac{3\pi t v}{L}\right]\mathrm{Sin}\left[\frac{3\pi x}{L}\right]+15\times(-4+5L)\,\mathrm{Cos}\left[\frac{4\pi t v}{L}\right]\mathrm{Sin}\left[\frac{4\pi x}{L}\right]-12\times\left(2+3\sqrt{2}\right)\times(-1+L)\,\mathrm{Cos}\left[\frac{5\pi t v}{L}\right]\mathrm{Sin}\left[\frac{5\pi x}{L}\right]\right)\right)$$

Plot initial shape

In[51]:=
```
Plot[uxt1, {t, 0, 100}]
```

Out[51]=



In[52]:=

Problem 4. Fourier transform

In[53]:=
```
fx = C (x^2 - a^2)^2
```

Out[53]= $C\left(-a^2+x^2\right)^2$