

In[53]:= **Remove["Global`\*"]**

A thin disk of charge  $q$  and radius  $R$  lies in a plane perpendicular to the  $z$ -axis. Every point on the charged thin ring of radius  $r$  and thickness  $dr$  is at the same distance  $p = \sqrt{r^2 + z^2}$  from any point on the  $z$ -axis. The electric potential due to the thin ring at a point  $z$  along the axis is then  $dV = k dq/p$  where  $dq$  is the charge on the ring  $k = 1$  in Gaussian units or  $k = 1/4 \pi \epsilon_0$  in SI units.

Calculate the potential  $V(z)$  from the entire disk. It will be useful to define the planar charge density  $\sigma = q / (\pi R^2)$ .

In[54]:= **dV = (k dq) / p**

$$\text{Out[54]} = \frac{dq k}{p}$$

In[55]:= **p = Sqrt[r^2 + z^2]**

$$\text{Out[55]} = \sqrt{r^2 + z^2}$$

In[56]:= **dq = 2 Pi r sigma**  
**sigma = q / (Pi R^2)**

$$\text{Out[56]} = 2 \pi r \sigma$$

$$\text{Out[57]} = \frac{q}{\pi R^2}$$

In[58]:= **dV = (k dq) / p**

$$\text{Out[58]} = \frac{2 k q r}{R^2 \sqrt{r^2 + z^2}}$$

In[59]:= **V = Integrate[dV, {r, 0, R}, Assumptions -> {z > 0, R > 0}]**

$$\text{Out[59]} = \frac{2 k q \left( -z + \sqrt{R^2 + z^2} \right)}{R^2}$$

Find the electric field  $E(z) = -dV/dz$

In[60]:= **Ez = - D[V, z]**

$$\text{Out[60]} = - \frac{2 k q \left( -1 + \frac{z}{\sqrt{R^2 + z^2}} \right)}{R^2}$$

$E(z)$  as  $z$  goes to zero

In[61]:= **Limit[Ez, z -> 0]**

$$\text{Out[61]} = \frac{2 k q}{R^2}$$

$E(z)$  as  $z$  goes to infinity

```
In[62]:= (Limit[Ez z^2, z → Infinity]) / z^2
```

```
Out[62]= 
$$\frac{k q}{z^2}$$

```