Lab 2

PHYS 2502

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In[53]:=
```
Remove["Global`*"]
```

In[54]:=
```
$Assumptions = {L > 0, z > 0, Q > 0}
```

Out[54]= $\{L > 0, z > 0, Q > 0\}$

The goal is to find the electric potential of a uniformly charged flat square plate, with side length L and total charge Q, which lies in xy plane centered at origin. We must find V(z) along the z-axis.

Electric potential at a point p from some charge q is given by V(r) = kq/r, where r is the distance between p and q and k is Coulomb's constant.

In[55]:=
```
Vr = k * q / r
```

Out[55]= $\frac{k\,q}{r}$

In[56]:=
```
r = Sqrt[x^2 + y^2 + z^2]
```

Out[56]= $\sqrt{x^2 + y^2 + z^2}$

To integrate over the surface of the plate, we must assume continuous charge distribution and define surface charge density s which is the distribution of charge over the surface area of the square plate. Since the charges are uniformly distributed, we can write s as

In[57]:=
```
s = Q / (L^2)
```

Out[57]= $\frac{Q}{L^2}$

Where Q is the total charge and L^2 is the area.

In[58]:=
```
dA = dx * dy
dq = s * dA
```

Out[58]= dx dy

Out[59]= $\frac{dx\,dy\,Q}{L^2}$

In[60]:=
```
dV = (k * dq) / r
```

Out[60]= $\frac{dx\,dy\,k\,Q}{L^2\sqrt{x^2 + y^2 + z^2}}$

Since we're integrating with respect to x and y, we can rewrite dV to take out dxdy.

In[61]:= `dV = dV /. {dq → s}`

Out[61]= $$\frac{k\,Q}{L^2\,\sqrt{x^2+y^2+z^2}}$$

In[62]:= `V = Integrate[dV, {x, -L / 2, L / 2}, {y, -L / 2, L / 2}]`

Out[62]= $$\frac{1}{L^2}2\,k\,Q\left(-L\,\text{Log}\left[L^2+4\,z^2\right]+2\,L\,\text{Log}\left[L+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right]+z\left(3\,\pi-2\,i\,\text{Log}\left[-L^2-(1+i)\,z\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right]+2\,i\,\text{Log}\left[-L^2-(1-i)\,z\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right]\right)\right)$$

In[63]:= `Ez = - D[V, z]`

Out[63]= $$-\frac{1}{L^2}2\,k\,Q\left(3\,\pi-\frac{8\,L\,z}{L^2+4\,z^2}+\frac{4\,\sqrt{2}\,L\,z}{\sqrt{L^2+2\,z^2}\left(L+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)}+z\left(-\left(\left(2\,i\left((-1-i)\,z\left(2+\frac{2\,\sqrt{2}\,z}{\sqrt{L^2+2\,z^2}}\right)-(1+i)\times\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right)\right)\Big/\left(-L^2-(1+i)\,z\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right)\right)+\left(2\,i\left((-1+i)\,z\left(2+\frac{2\,\sqrt{2}\,z}{\sqrt{L^2+2\,z^2}}\right)-(1-i)\times\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right)\right)\Big/\left(-L^2-(1-i)\,z\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right)\right)-2\,i\,\text{Log}\left[-L^2-(1+i)\,z\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right]+2\,i\,\text{Log}\left[-L^2-(1-i)\,z\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right]\right)$$

In[64]:= `Es = Simplify[Ez]`

Out[64]= $$-\frac{1}{L^2}2\,k\,Q\left(-2\,i\,\text{Log}\left[-L^2-(1+i)\,z\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right]+\left(i\left(L^2+4\,z^2\right)\left(\sqrt{2}\,L^4+4\,L\,z^2\left(\sqrt{2}\,z+\sqrt{L^2+2\,z^2}\right)+8\,z^3\left(\sqrt{2}\,z+\sqrt{L^2+2\,z^2}\right)+L^3\left(2\,\sqrt{2}\,z+\sqrt{L^2+2\,z^2}\right)+2\,L^2\,z\left(3\,\sqrt{2}\,z+2\,\sqrt{L^2+2\,z^2}\right)\right)\left(3\,\pi+2\,i\,\text{Log}\left[-L^2-(1-i)\,z\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right]\right)\right)\Big/\left(\sqrt{L^2+2\,z^2}\left(L+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\left(i\,L^2+(1+i)\,z\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right)\left(L^2+(1+i)\,z\left(2\,z+\sqrt{2}\,\sqrt{L^2+2\,z^2}\right)\right)\right)\right)$$

In[65]:=
```
(Limit[Es z^2, z → Infinity]) / z^2
```

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[65]= $\frac{k\,Q}{z^2}$

In[66]:=
```
Series[Es, z → 0]
```

Out[66]= $\frac{2\,k\,\pi\,Q}{L^2} + O[z]^1$

In[67]:=
```
Normal[%]
```

Out[67]= $\frac{2\,k\,\pi\,Q}{L^2}$