In[ ]:=
```
Remove ["Global`*"]
```

Remove: There are no symbols matching "Global`*".

In[ ]:=
```
ClearAll
```

Out[ ]=
```
ClearAll
```

In[ ]:=
```
$Assumptions = {m > 0, g > 0, y0 > 0, v0 > 0, b > 0, vt > 0};
```

The goal of this lab is to use Mathematica to solve the differential equation of the vertical position and speed of a mass that falls from rest and is subject to linear drag force.

First I set up my vertical speed equation
F = ma : ma = mv'
-mg - bv = mv'
rearranging the equation to get
v' + (b/m)v + g = 0
this gives me a first order differential equation for the speed
with this i can then set up another equation for y remembering that v' = y'' and v = y'
y'' + (b/m) y' + g = 0
this then gives me a second order homogenous DE for the y position.
I will now use the solve function to solve for my y(t)

In[ ]:=

In[ ]:=
```
sol1 = DSolve[{y''[t] == -g - (b / m) y'[t], y'[0] == v0, y[0] == y0}, y[t], t];
y = (Simplify[y[t] /. sol1[[1]]])
```

Out[ ]=
$$\frac{g\,m\left(m - e^{-\frac{b\,t}{m}}\,m - b\,t\right) + b\left(m\left(v0 - e^{-\frac{b\,t}{m}}\,v0\right) + b\,y0\right)}{b^2}$$

Now i want to see what happens when b = 0

In[ ]:=
```
Limit[y, b → 0]
```

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[ ]=
$$-\frac{g\,t^2}{2} + t\,v0 + y0$$

Now I'll use my solution to find the velocity v. v = y' so i take the derivative of my y with respect to t then take the limit as v0 goes to 0 to find v.

In[ ]:=

In[ ]:=
```
velocity = Simplify[Limit[D[y, t], v0 → 0]]
```

Limit: Warning: Assumptions that involve the limit variable are ignored.

Out[ ]=
$$\frac{\left(-1 + e^{-\frac{b t}{m}}\right) g m}{b}$$

The velocity when v0 = 0 matches the one from class.

In[ ]:=

Now we want to find the time it takes for the mass to go to y0 = 0 and back down. which means we will want 2 equations. one for when the mass is going up and when its coming back down.

In[ ]:=
```
vup = 1 / (-g - (b / m) v )
vdown = 1 / (-g + (b / m) v)
```

Out[ ]=
$$\frac{1}{-g - \frac{b v}{m}}$$

Out[ ]=
$$\frac{1}{-g + \frac{b v}{m}}$$

In[ ]:=
```
dt = 1
tup = Integrate[dt, {t, 0, t1}] == Integrate[vup, {v, v0, 0}]
tdown = Integrate[dt, {t, t1, t2}] == Integrate[vdown, {v, 0, vt}]
```

Out[ ]= 1

I used my velocity as the mass goes up and down to relate to the time. I then use my two time equations and relate them to find the final time.

Out[ ]=
$$t1 == -\frac{m \operatorname{Log}\left[\frac{g m}{g m + b v0}\right]}{b}$$

Out[ ]=
$$-t1 + t2 == \boxed{\frac{m \operatorname{Log}\left[1 - \frac{b vt}{g m}\right]}{b} \quad \text{if } b\, vt < g m}$$

In[ ]:=

My final solution is solved by using my t equations and finding t2 which should be the final time. its not in the correct format so i use the replace function to have it in the correct format then i simplify to clean it up.

In[ ]:=
```
sol = Solve[tdown, t2]
```

Out[ ]=
$$\left\{\left\{t2 \to \boxed{\frac{b\, t1 + m \operatorname{Log}\left[1 - \frac{b vt}{g m}\right]}{b} \quad \text{if } b < \frac{g m}{vt}}\right\}\right\}$$

In[∘]:= `t2 = t2 /. sol[[1]]`

Out[∘]= $\frac{b\, t1 + m \,\text{Log}\left[1 - \frac{b\, vt}{g\, m}\right]}{b}$ if $b < \frac{g\, m}{vt}$

In[∘]:= `tf = Simplify[t2 /. t1 → (-m Log[(g m) / (g m + b v0)]) / b]`

Out[∘]= $\frac{m \,\text{Log}\left[\frac{(g\, m + b\, v0)\,(g\, m - b\, vt)}{g^2\, m^2}\right]}{b}$ if $b\, vt < g\, m$

Now i have to expand my final time equation about the b value. i used the series function and set the highest order to 5 and the first value of my output is indeed missing the b. And with v0 = 0 the first term, accurately describe time in relation to acceleration and velocity

In[∘]:= `Series[tf, {b, 0, 5}]`

Out[∘]=

$$\frac{v0 - vt}{g} + \frac{\left(-v0^2 - vt^2\right) b}{2\, g^2\, m} + \frac{(v0 - vt)\left(v0^2 + v0\, vt + vt^2\right) b^2}{3\, g^3\, m^2} + \frac{\left(-v0^4 - vt^4\right) b^3}{4\, g^4\, m^3} + \frac{\left(v0^5 - vt^5\right) b^4}{5\, g^5\, m^4} + \frac{\left(-v0^6 - vt^6\right) b^5}{6\, g^6\, m^5} + O[b]^6$$