```
Remove["Global`*"]
```

Remove: There are no symbols matching "Global`*".

The total mechanical energy of a simple pendulum, with length l and mass m, is given here as TE. θ is the angle formed by the motion of the pendulum and θt = dθ/dt

```
In[5]:= $Assumptions = {l > 0, g > 0, θ > 0, θ < Pi, θn > 0, θn < Pi, m > 0};
```

The solve function is used to find dθ

```
In[6]:= TE = (1/2) (m l^2 (θt)^2) + m g l (1 - Cos[θ]) == m g l (1 - Cos[θn])

sol = Solve[TE, θt] [[2]]
```

Out[6]= $\frac{1}{2} l^2 m\, \theta t^2 + g\, l\, m\, (1 - \mathrm{Cos}[\theta]) == g\, l\, m\, (1 - \mathrm{Cos}[\theta n])$

Out[7]= $\left\{\theta t \to \sqrt{2} \sqrt{\frac{g}{l}} \sqrt{\mathrm{Cos}[\theta] - \mathrm{Cos}[\theta n]} \text{ if } \theta < \theta n\right\}$

Dummy variable is created to transform the expression

```
In[8]:= b = θt /. sol
```

Out[8]= $\sqrt{2} \sqrt{\frac{g}{l}} \sqrt{\mathrm{Cos}[\theta] - \mathrm{Cos}[\theta n]} \text{ if } \theta < \theta n$

Simplify function is used to find the simplest form of the expression

```
In[9]:= dT = Simplify[(1/b) / (2 Pi Sqrt[l/g])]
```

Out[9]= $\frac{1}{2 \sqrt{2}\, \pi \sqrt{\mathrm{Cos}[\theta] - \mathrm{Cos}[\theta n]}} \text{ if } \theta < \theta n$

Using the integrate function to find the period of oscillation in relation to dt

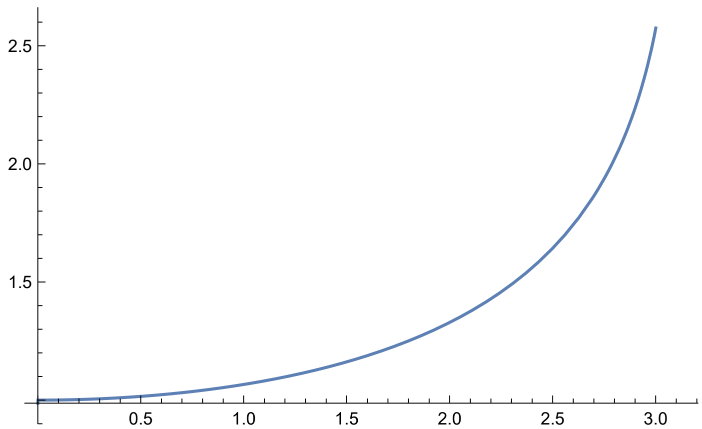
```
In[10]:= T = 4 Integrate[dT, {θ, 0, θn}]
```

Out[10]= $\frac{2 \sqrt{2}\, \mathrm{EllipticF}\left[\frac{\theta n}{2}, \mathrm{Csc}\left[\frac{\theta n}{2}\right]^2\right]}{\pi \sqrt{1 - \mathrm{Cos}[\theta n]}}$

Plotting the newly integrated function, its shown in the graph that as θn goes to pi, the graph goes to infinity

```
Plot[T, {θn, 0, Pi}]
```

Out[11]=



Numerical Integration

```
In[ ]:= NIntegrate[T, {θn, 0, Pi}]
Out[ ]= 4.37688
```

Using the substitute sin(θ/2) = Au to find a new integral for the period as a function of θn

```
In[ ]:= Tθn = 2 / Pi Integrate[(1 / Sqrt[1 - u^2]) × (1 / Sqrt[1 - A^2 u^2]), {u, 0, 1}]
```

Out[ ]= $\frac{2\,\text{EllipticK}[A^2]}{\pi}$ if $\text{Re}[A^2] \leq 1 \,||\, A^2 \notin \mathbb{R}$

Series function is used to expand to the lowest non zero order

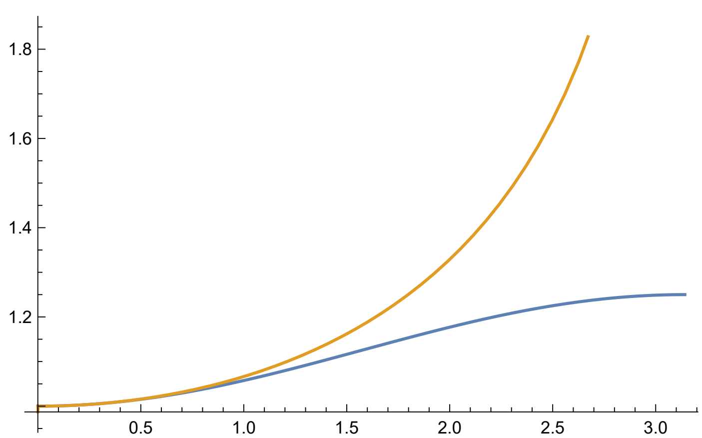
```
In[ ]:= eT = Series[Tθn, {A, 0, 2}] /. A → Sin[θn / 2] // Normal
```

Out[ ]= $1 + \frac{1}{4}\,\text{Sin}\left[\frac{\theta n}{2}\right]^2$

The new function(blue) is plotted on the same graph as the old one(orange) which shows the expansion only works for θn less than one

```
In[ ]:= Plot[{eT, T}, {θn, 0, Pi}]
```

Out[ ]=



In[ ]:=