Lab 7

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In[ ]:= `Remove["Global`*"]`

Part one: ContourPlot the function then form 2D vector field and plot field on top of contour plot. Then calculate gradient.

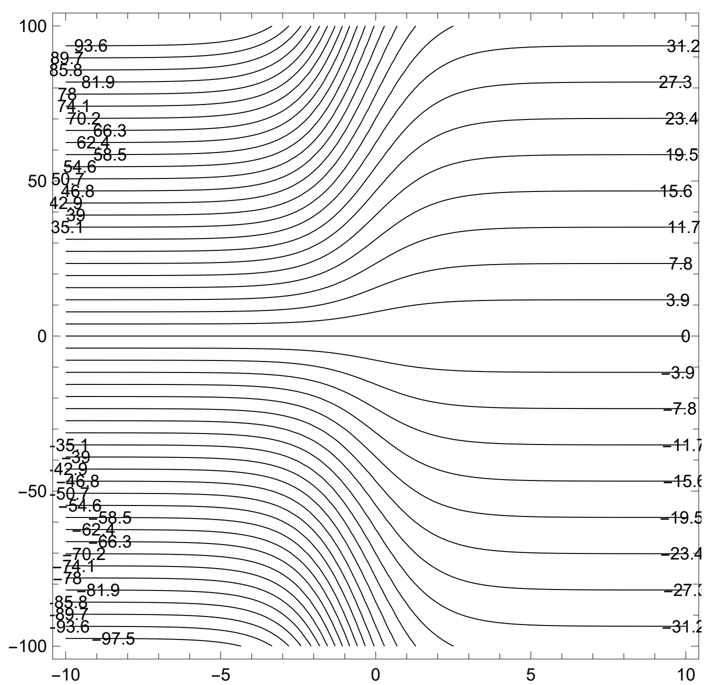
In[ ]:= `f = y * ((Exp[-x] + 1) / (Exp[-x] + 3))`

Out[ ]= $$\frac{\left(1+e^{-x}\right) y}{3+e^{-x}}$$

Plot with no shading and contours labeled

In[ ]:= `P1 = ContourPlot[f, {x, -10, 10}, {y, -100, 100}, PlotRange → All, Contours → 50, ContourShading → None, ContourLabels → True]`

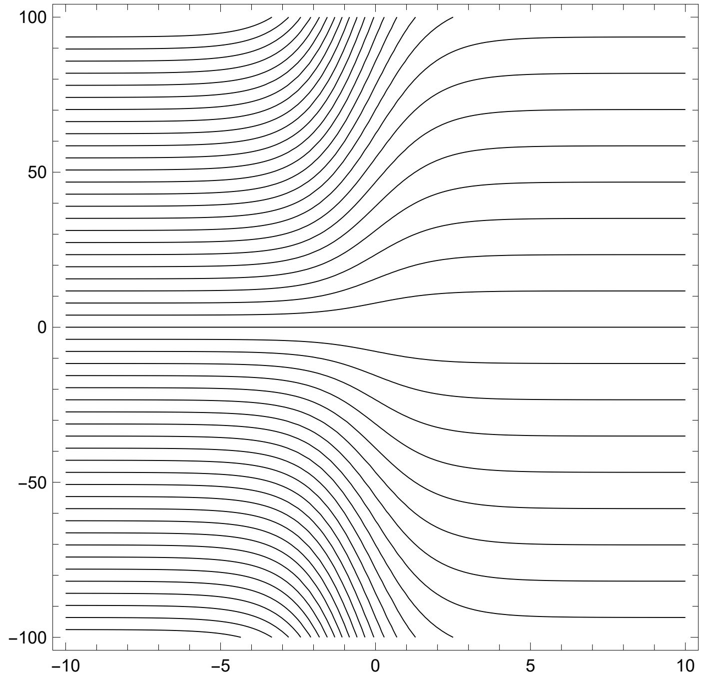
Out[ ]=



Plot of function with no contour labels

In[∘]:=
```
P1a = ContourPlot[f, {x, -10, 10}, {y, -100, 100},
  PlotRange → All, Contours → 50, ContourShading → None]
```

Out[∘]=

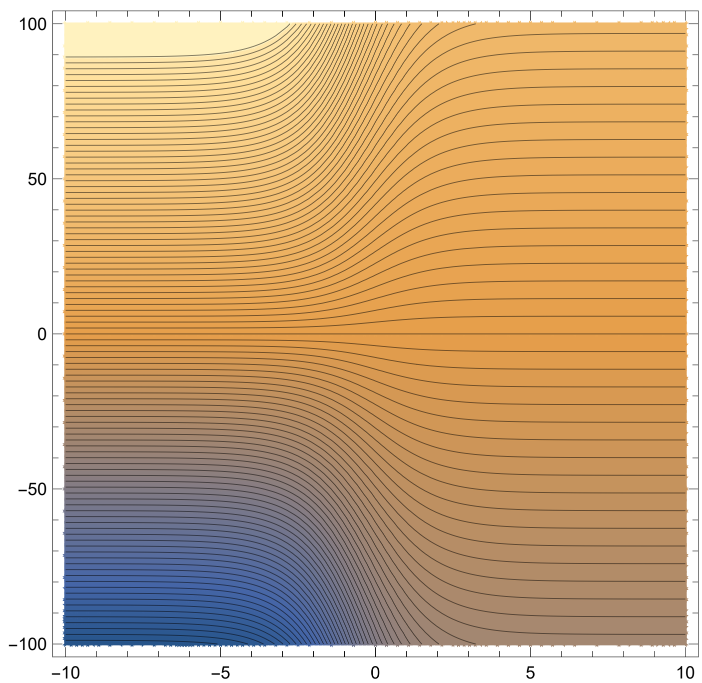


In[∘]:=

Plot with shading and no contour labels

In[∘]:=
```
P2 = ContourPlot[f, {x, -10, 10}, {y, -100, 100},
  PlotRange → All, Contours → 100, ContourShading → True]
```

Out[∘]=

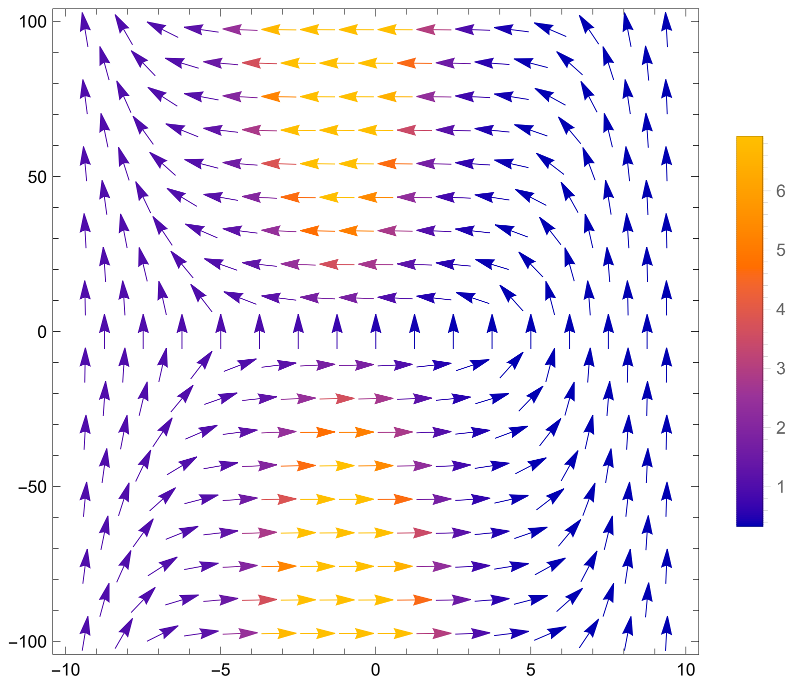


2D vector field (x,y) of the function plus its plot on top of P1.

In[∘]:=
```
v = Grad[f, {x, y}]
```

Out[∘]=
$$\left\{\frac{e^{-x}\left(1+e^{-x}\right)y}{\left(3+e^{-x}\right)^{2}}-\frac{e^{-x}\,y}{3+e^{-x}},\ \frac{1+e^{-x}}{3+e^{-x}}\right\}$$

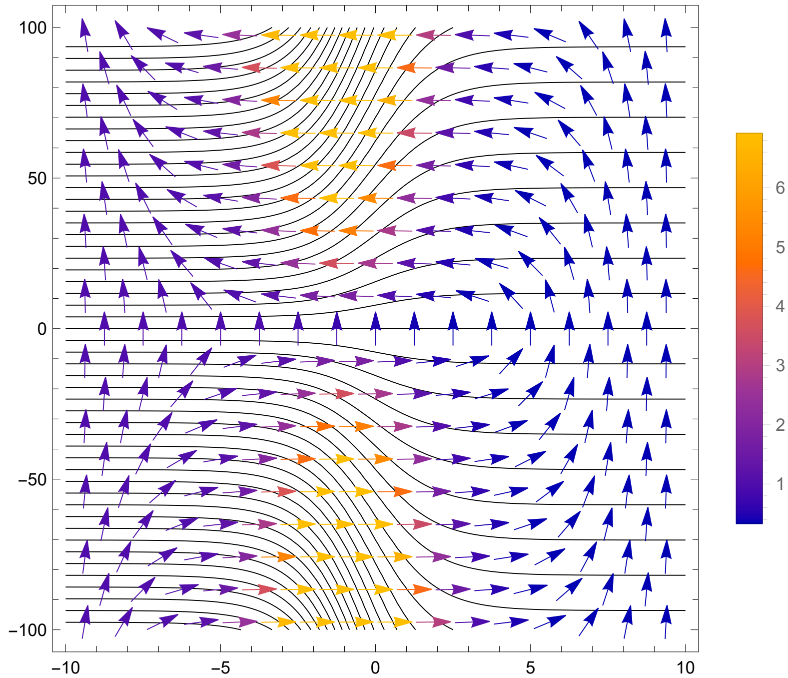
In[∘]:=
```
P3 = VectorPlot[v, {x, -10, 10}, {y, -100, 100}, PlotLegends → Automatic]
```

Out[∘]=


Combination of plot of original function and plot of vector field

In[ ]:= **Show[P1a, P3]**

Out[ ]=



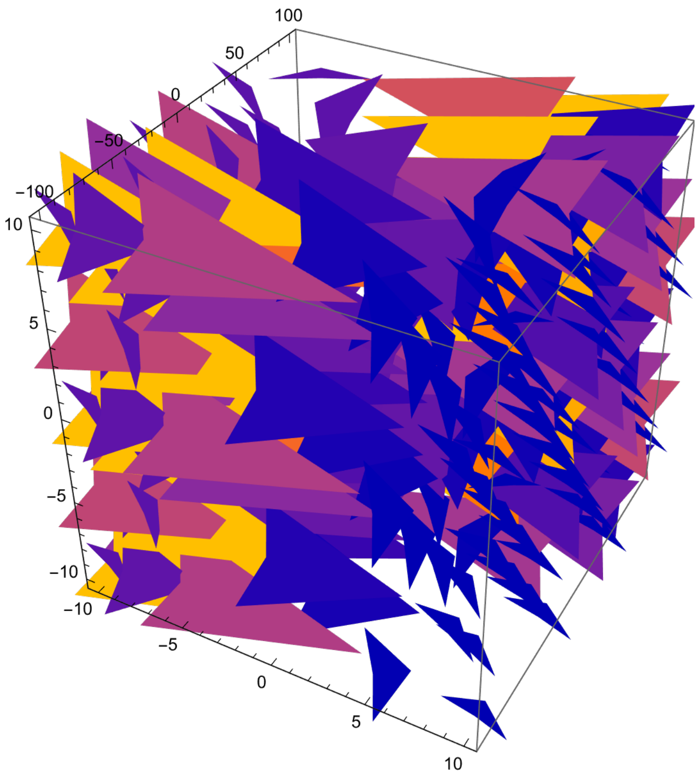
Converting 2D vector field to 3D

In[ ]:= **v1 = Grad[f, {x, y, z}]**

Out[ ]= $\left\{\frac{e^{-x}\left(1+e^{-x}\right) y}{\left(3+e^{-x}\right)^{2}}-\frac{e^{-x} y}{3+e^{-x}}, \frac{1+e^{-x}}{3+e^{-x}}, 0\right\}$

In[ ]:=
```
VectorPlot3D[v1, {x, -10, 10}, {y, -100, 100}, {z, -10, 10}, VectorMarkers → "Arrow"]
```

Out[ ]=



In[ ]:=
```
Curl[v1, {x, y, z}]
```

Out[ ]= {0, 0, 0}

This makes sense because the curl of a gradient is always zero.

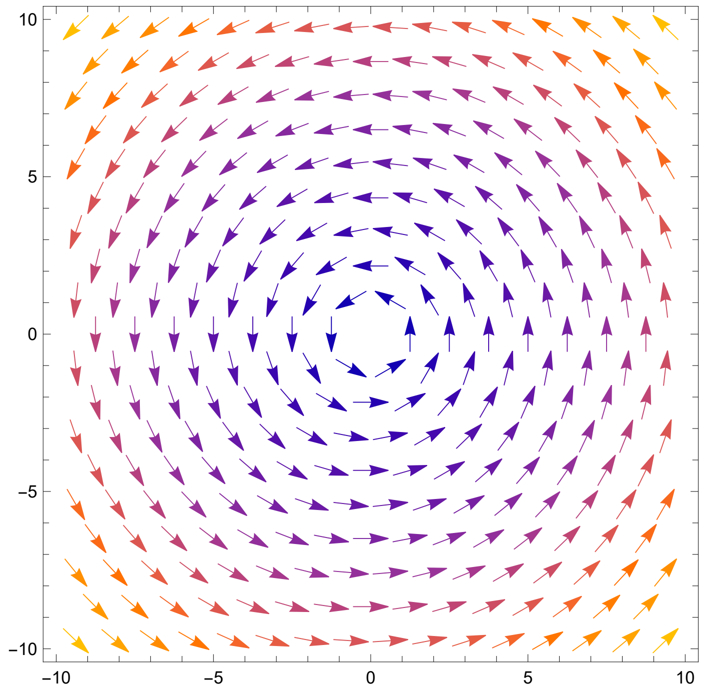
Part Two: Construct vector plot of the field then find curl and make a contour plot of curl with vector plot. Then find divergence of curl

In[ ]:=
```
v2 = {-y * r, x * r}
r = Sqrt[x^2 + y^2]
```

Out[ ]= $\{-r\, y, r\, x\}$

Out[ ]= $\sqrt{x^2 + y^2}$

In[ ]:= `Pv = VectorPlot[v2, {x, -10, 10}, {y, -10, 10}]`

Out[ ]=


In[ ]:= `w = Curl[v2, {x, y}]`

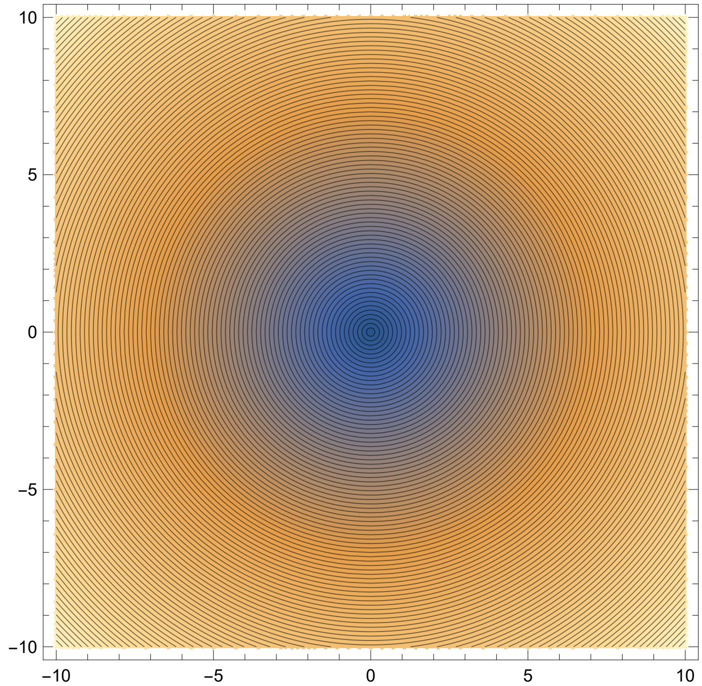
Out[ ]= $$\frac{x^2}{\sqrt{x^2+y^2}}+\frac{y^2}{\sqrt{x^2+y^2}}+2\sqrt{x^2+y^2}$$

w has to point in the

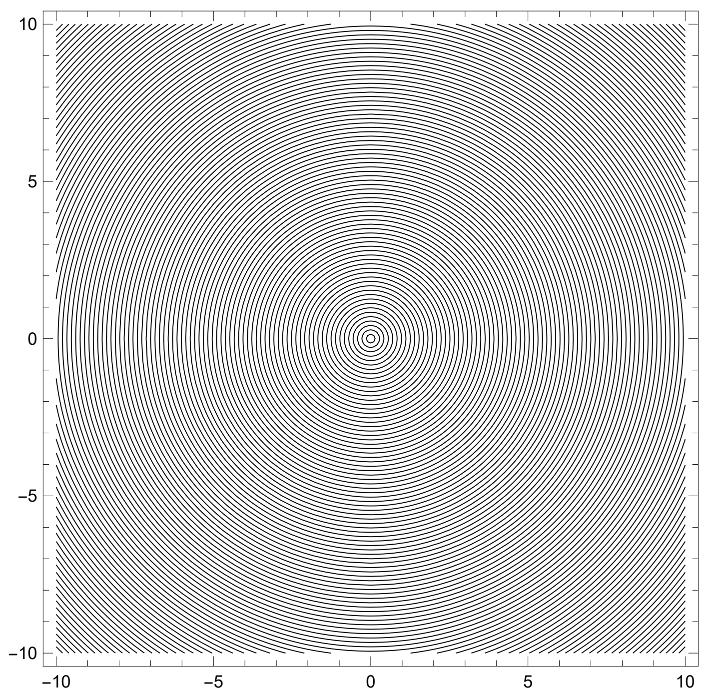
In[ ]:= `magw = Norm[w]`

Out[ ]= $$\text{Norm}\left[\frac{x^2}{\sqrt{x^2+y^2}}+\frac{y^2}{\sqrt{x^2+y^2}}+2\sqrt{x^2+y^2}\right]$$

In[ ]:= `Pws = ContourPlot[magw, {x, -10, 10}, {y, -10, 10}, Contours → 100]`

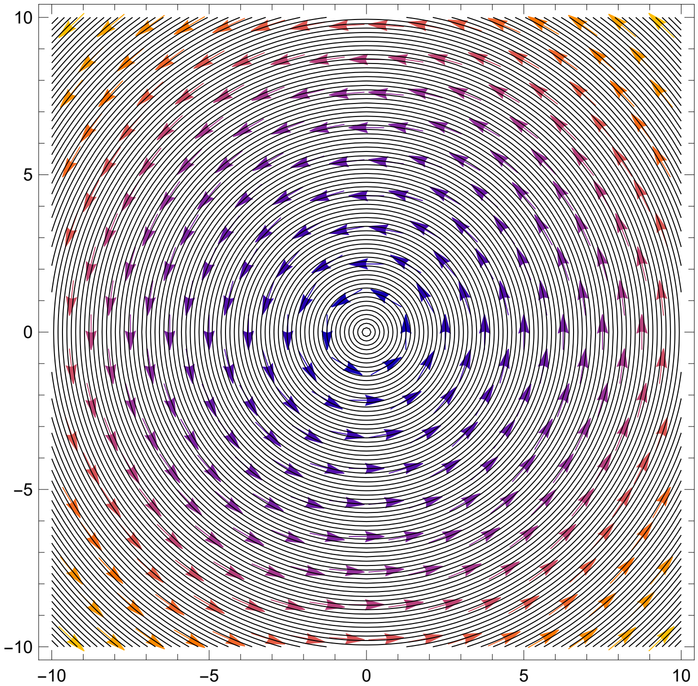
Out[ ]=


In[ ]:= `Pw = ContourPlot[magw, {x, -10, 10}, {y, -10, 10}, Contours → 100, ContourShading → None]`

Out[ ]=


In[∘]:= `Show[Pv, Pw]`

Out[∘]=



In[∘]:= `Grad[w, {x, y}]`

Out[∘]= $$\left\{-\frac{x^3}{\left(x^2+y^2\right)^{3/2}}-\frac{x\,y^2}{\left(x^2+y^2\right)^{3/2}}+\frac{4\,x}{\sqrt{x^2+y^2}},\ -\frac{x^2\,y}{\left(x^2+y^2\right)^{3/2}}-\frac{y^3}{\left(x^2+y^2\right)^{3/2}}+\frac{4\,y}{\sqrt{x^2+y^2}}\right\}$$

Part Three: Plot vector field then find curl

In[∘]:= `v3 = {-y / r^2, x / r^2}`

Out[∘]= $$\left\{-\frac{y}{x^2+y^2},\ \frac{x}{x^2+y^2}\right\}$$

In[∘]:= `w1 = Curl[v3, {x, y}]`

Out[∘]= $$-\frac{2\,x^2}{\left(x^2+y^2\right)^2}-\frac{2\,y^2}{\left(x^2+y^2\right)^2}+\frac{2}{x^2+y^2}$$

In[∘]:= `Simplify[w1]`

Out[∘]= `0`