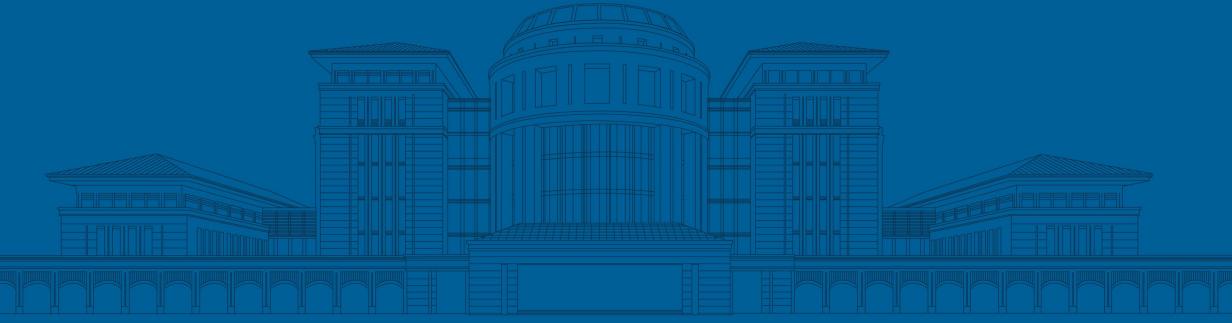


# Group Report

speaker: Xiongyi Li

7th Dec 2023



### Outline

- Deep Reinforcement Learning
  - Part III
  - Part IV



# Part III Policy-Based Reinforce Learning

- Revison
- Policy gradient
- Baseline
- High-level skills
- Continue control



#### Revision

• Discounted return(折扣回报):

$$U_t = R_t + R_{t+1} + R_{t+2} + R_{t+3} + \dots + R_n.$$

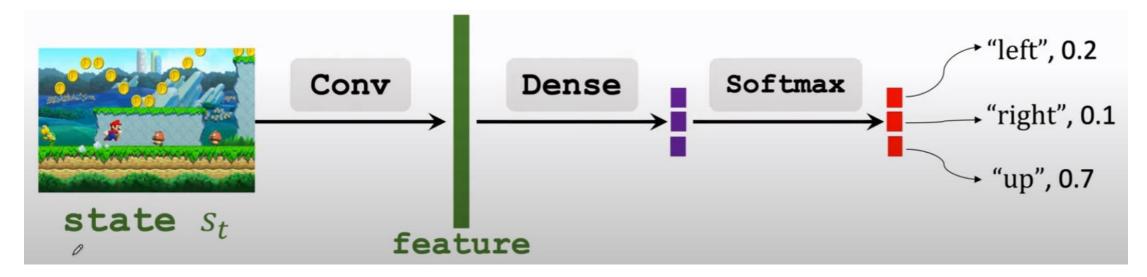
• Action-value function(动作价值函数):

$$Q_{\pi}(s_t, a_t) = \mathbb{E}_{S_{t+1}, A_{t+1}, \dots, S_n, A_n} \left[ U_t \, \middle| \, S_t = s_t, A_t = a_t \right].$$

• State-value function(状态价值函数):

$$V_{\pi}(s) = \mathbb{E}_{A \sim \pi} [Q_{\pi}(s, A)].$$

- Policy Network  $\pi(a|s;\theta)$
- To approximate  $\pi(a|s)$





• 
$$V_{\pi}(s) = \mathbb{E}_{A \sim \pi} [Q_{\pi}(s, A)] = \sum_{a} \pi(a|st; \theta) Q_{\pi}(st, a)$$

•  $\theta$  dicide the value of  $V\pi(s)$ 

$$J(\boldsymbol{\theta}) = \mathbb{E}_S [V_{\pi}(S)].$$
  $\underset{\boldsymbol{\theta}}{\max} J(\boldsymbol{\theta}).$ 

- Policy gradient ascent
- $\theta = \theta + \beta \frac{\partial V(s,\theta)}{\partial \theta}$



- Algorithm:
- Oberseve s<sub>t</sub>
- Random action  $a_t \leftarrow \pi(\cdot|st;\theta)$
- Compute  $q_t \approx Q_{\pi}(st, at)$
- $G(a_t, \theta_t) = q_t \frac{\partial \ln(\pi(a_t|s_t; \theta))}{\partial \theta} |_{\theta = \theta t}$
- $\theta_{t+1} = \theta_t + \beta G(a_t, \theta_t)$

#### Reinforce

- Get  $s_1, a_1, r_1$ ;  $s_2, a_2, r_2$ ; ....  $s_t, a_t, r_t$
- Monte Calro:  $u_t = Q_{\pi}(s_t, a_t)$

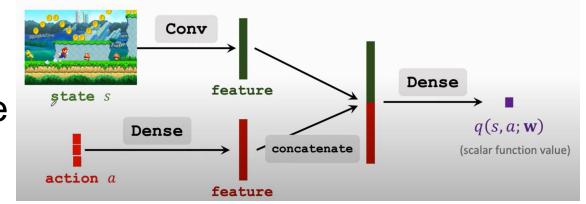
#### **Actor-Critic**

Conv Dense Softmax "right", 0.2

"up", 0.7

State  $S_t$  feature

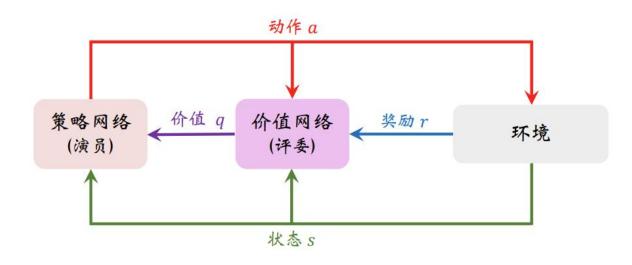
- $V_{\pi}(s) = \sum_{a} \pi(a|s) Q_{\pi}(s,a)$
- Actor:  $\pi(a|s;\theta)$  approximate  $\pi(a|s)$
- Critic:  $q(s, a; \omega)$  approximate  $Q_{\pi}(s, a)$
- $V_{\pi}(s) = \sum_{a} \pi(a|s;\theta) q(s,a;\omega)$
- Update  $\theta$  to increase V(s,  $\theta$ ,  $\omega$ )
- Update  $\omega$  to make q more accurate





#### **Actor-Critic**

- Algorithm:
- Oberseve s<sub>t</sub>
- Random action  $a_t \leftarrow \pi(\cdot|st;\theta)$
- from a<sub>t</sub> to observe S<sub>t+1</sub> & r<sub>t</sub>
- Update θ by TD
- Update  $\omega$  by Policy Gradient





### Policy Gradient with Baseline

• 
$$\frac{\partial V(s,\theta)}{\partial \theta} = \mathsf{IE}_{\mathsf{A}} \left[ \frac{\partial \ln[\pi(\alpha|s;\theta)]}{\partial \theta} Q_{\pi}(s,\alpha) \right]$$

• 
$$\mathsf{IE}_{\mathsf{A-}\pi}[\frac{\partial \ln[\pi(A|s;\theta)]}{\partial \theta} \ b]$$
 (b is independent of A)

$$= \mathsf{IE}_{\mathsf{A-}\pi} \left[ \frac{\partial \ln[\pi(A|S;\theta)]}{\partial \theta} \right] * \mathsf{b}$$

=b \* 
$$\sum_{a} \pi(a|s;\theta) \frac{\partial \ln[\pi(a|s;\theta)]}{\partial \theta}$$

$$=b * \sum_{a} \frac{\partial \pi(a|s;\theta)}{\partial \theta} = b * \frac{\partial \sum_{a} \pi(a|s;\theta)}{\partial \theta} = 0$$



### Policy Gradient with Baseline

• 
$$\frac{\partial V(s,\theta)}{\partial \theta} = IE_A \left[ \frac{\partial \ln \left[ \pi(a|s;\theta) \right]}{\partial \theta} \left( Q_\pi(s,a) - b \right) \right]$$

- During Monte Calro approximation, b can reduce variance and speed up convergence
- Often b = 0 or  $V_{\pi}(s_t)$   $V_{\pi}(s) = \mathbb{E}_{A \sim \pi}[Q_{\pi}(s, A)].$

#### Reinforce with Baseline

• G(a<sub>t</sub>) = 
$$\frac{\partial \ln(\pi(a_t|s_t;\theta))}{\partial \theta}$$
 (Q<sub>\pi</sub>(st, at) -V<sub>\pi</sub>(s<sub>t</sub>))  
=  $\frac{\partial \ln(\pi(a_t|s_t;\theta))}{\partial \theta}$  (u<sub>t</sub>-V<sub>\pi</sub>(s<sub>t</sub>))

- Algorithm:
- Oberseve s<sub>t</sub>
- Random action  $a_t \leftarrow \pi(\cdot | s_t; \theta)$
- Compute  $q_t \approx Q_{\pi}(s_t, a_t)$

• 
$$G(a_t, \theta_t) = q_t \frac{\partial \ln(\pi(a_t|s_t; \theta))}{\partial \theta}|_{\theta = \theta t}$$

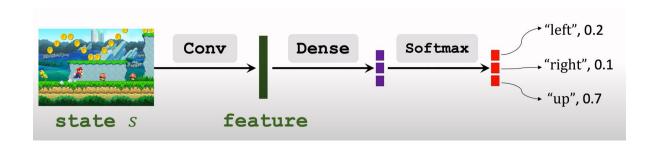
• 
$$\theta_{t+1} = \theta_t + \beta G (a_t, \theta_t)$$

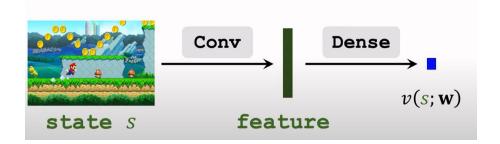
• Use neural network v(s;w) to approximate  $V_{\pi}(s_t)$ 

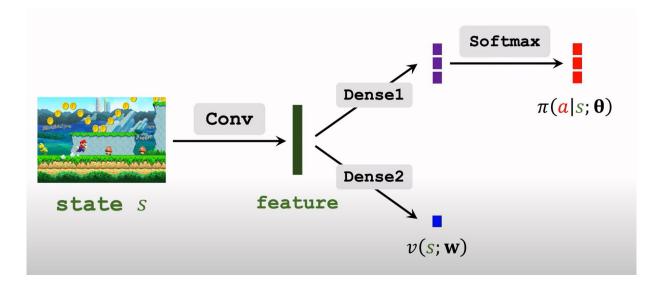
• G(a<sub>t</sub>) = 
$$\frac{\partial \ln(\pi(a_t|S_t;\theta))}{\partial \theta}$$
 ( $u_t$ -v(s;w))

3 approximation ※

### Policy and Value Network









### Updating

- Policy gradient:  $\theta = \theta + \beta \frac{\partial \ln(\pi(\alpha_t|s_t;\theta))}{\partial \theta} (u_t v(s;w))$
- Value network:
- Prediction error:  $\delta_t = v(s_t; w) u_t$
- Gradient:  $\frac{\partial \delta_t^2/2}{\partial w} = \delta_t \frac{\partial \mathbf{v}(\mathbf{st;w})}{\partial \theta}$
- Gradient descent:  $w = w \alpha \delta_t \frac{\partial v(st;w)}{\partial \theta}$



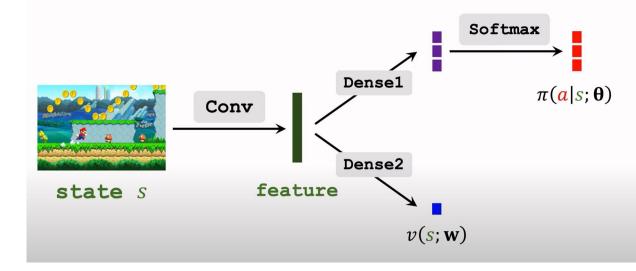
# Advantage Actor-Critic (A2C)

- Observe a transition(s<sub>t</sub>,s<sub>t</sub>,r<sub>t</sub>,s<sub>t+1</sub>)
- TD target:  $y_t = rt + \gamma v(s_{t+1}; w)$
- TD error:  $\delta_t = v(s_t; w) y_t$
- Update the policy network

$$\theta = \theta - \beta \frac{\partial \ln(\pi(\alpha_t|s_t;\theta))}{\partial \theta} \delta_t$$

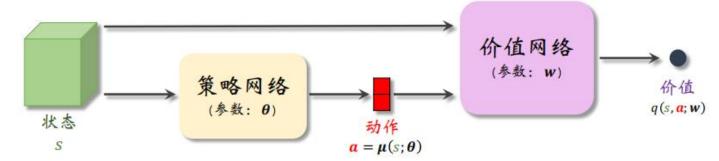
Update the value network

$$\mathbf{w} = \mathbf{w} - \alpha \delta_t \frac{\partial \mathbf{v}(\mathbf{s}_t; \mathbf{w})}{\partial \theta}$$





• Deterministic policy gradient(确定策略梯度)



- Transition( $s_t, s_t, r_t, s_{t+1}$ )
- Value network:  $q_t = q(s_t, a_t; w)$
- $q_{t+1} = q(s_{t+1}, a'_{t+1}; w), a'_{t+1} = \pi(s_{t+1}; \theta)$
- TD error:  $\delta_t = q_t (r_t + \gamma q_{t+1})$
- Update:  $w = w \alpha \delta_t \frac{\partial q(s_t, a_t; w)}{\partial w}$

- Policy network
- Goal: increase  $q(s,a;w) \& a = \pi(s;\theta)$

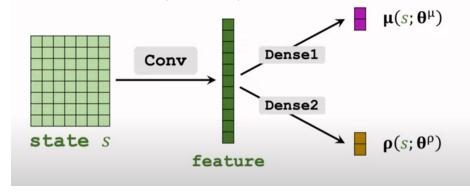
• DPG: 
$$g = \frac{\partial q(s,\pi(s;\theta);w)}{\partial \theta} = \frac{\partial a}{\partial \theta} \frac{\partial q(s,a;w)}{\partial a}$$

• Gradient ascent:  $\theta = \theta + \beta g$ 

- Policy function:  $\pi(a \mid s) = \frac{1}{\sqrt{6.28} \cdot \sigma(s)} \cdot \exp\left(-\frac{[a \mu(s)]^2}{2 \cdot \sigma^2(s)}\right)$ .
- action a is d-dim

$$\pi(\mathbf{a}|s) = \prod_{i=1}^{d} \frac{1}{\sqrt{6.28} \,\sigma_i} \cdot \exp\left(-\frac{(\mathbf{a}_i - \mu_i)^2}{2\sigma_i^2}\right).$$

- use a neural network  $\mu(s; \vartheta^{\mu})$  to approximate  $\mu(s)$
- use a neural network  $\rho(s; \vartheta^{\rho})$  to approximate  $\rho = \ln \sigma^2$



- Observe state s.
- Compute mean and log variance using the neural network:

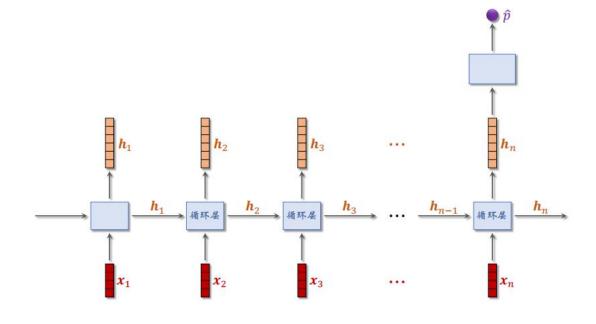
$$\widehat{\mu} = \mu(s; \theta^{\mu})$$
 and  $\widehat{\rho} = \rho(s; \theta^{\rho})$ .

- Compute  $\hat{\sigma}_i^2 = \exp(\hat{\rho}_i)$ , for all  $i = 1, \dots, d$ .
- Randomly sample action a by

$$\mathbf{a}_i \sim \mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$$
, for all  $i = 1, \dots, d$ .

### Recurrent neural network(RNN)

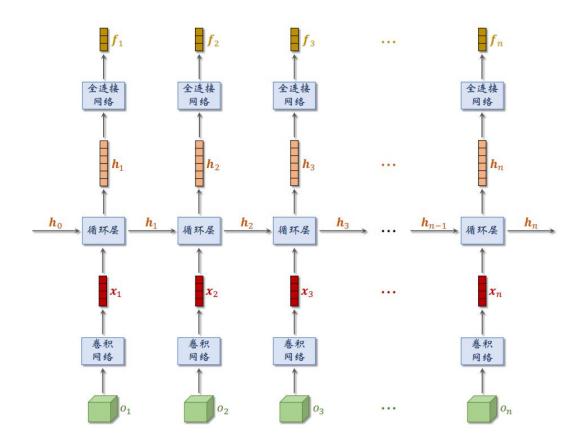
```
egin{array}{lll} (oldsymbol{x}_1) & \Longrightarrow & oldsymbol{h}_1, \ (oldsymbol{x}_1, oldsymbol{x}_2) & \Longrightarrow & oldsymbol{h}_2, \ (oldsymbol{x}_1, oldsymbol{x}_2, oldsymbol{x}_3) & \Longrightarrow & oldsymbol{h}_3, \ dots & & dots \ (oldsymbol{x}_1, oldsymbol{x}_2, oldsymbol{x}_3, \cdots, oldsymbol{x}_{n-1}) & \Longrightarrow & oldsymbol{h}_{n-1}, \ (oldsymbol{x}_1, oldsymbol{x}_2, oldsymbol{x}_3, \cdots, oldsymbol{x}_{n-1}, oldsymbol{x}_n) & \Longrightarrow & oldsymbol{h}_{n-1}, \ (oldsymbol{x}_1, oldsymbol{x}_2, oldsymbol{x}_3, \cdots, oldsymbol{x}_{n-1}, oldsymbol{x}_n) & \Longrightarrow & oldsymbol{h}_{n-1}, \ (oldsymbol{x}_1, oldsymbol{x}_2, oldsymbol{x}_3, \cdots, oldsymbol{x}_{n-1}, oldsymbol{x}_n) & \Longrightarrow & oldsymbol{h}_{n-1}, \ (oldsymbol{x}_1, oldsymbol{x}_2, oldsymbol{x}_3, \cdots, oldsymbol{x}_{n-1}, oldsymbol{x}_n) & \Longrightarrow & oldsymbol{h}_{n-1}, \ (oldsymbol{x}_1, oldsymbol{x}_2, oldsymbol{x}_3, \cdots, oldsymbol{x}_{n-1}, oldsymbol{x}_n) & \Longrightarrow & oldsymbol{h}_{n-1}, \ (oldsymbol{x}_1, oldsymbol{x}_2, oldsymbol{x}_2, oldsymbol{x}_2, \cdots, oldsymbol{x}_{n-1}, oldsymbol{x}_n) & \Longrightarrow & oldsymbol{h}_{n-1}, \ (oldsymbol{x}_1, oldsymbol{x}_2, oldsymbol{x}_2, oldsymbol{x}_2, \cdots, oldsymbol{x}_{n-1}, oldsymbol{x}_n) & \Longrightarrow & oldsymbol{h}_{n-1}, \ (oldsymbol{x}_2, oldsymbol{x}_2, oldsymbol{x}_2, oldsymbol{x}_2, \cdots, oldsymbol{x}_{n-1}, oldsymbol{x}_n) & \Longrightarrow & oldsymbol{h}_n. \end{array}
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### **RNN**

RNN as policy network





### Behaviour cloning

- Goal: mimic human's action to make a random policy network  $\pi(a|s;\theta)$  or a certain policy network  $\mu(s;\theta)$
- Data set:  $\mathcal{X} = \{(s_1, a_1), \dots, (s_n, a_n)\}.$
- s: state; s: action(human)



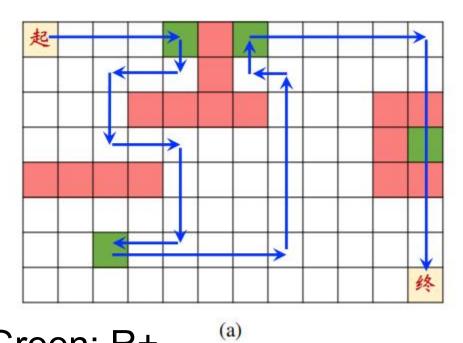
### Inverse reinforcement learning

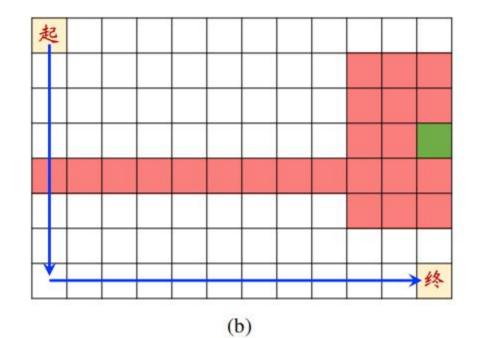
IRL setting: interact with environment without knowing what the reward is;unknown policy provided by human but can be used Goal: to make a policy network  $\pi(a|s;\theta)$  and mimic the unknown policy

专家的策略 逆向强化学习 学出的奖励 强化学习 学出的策略 
$$R(s,a; \rho)$$
  $\pi(a|s,\theta)$ 



### **IRL**





• Green: R+

• Red: R-



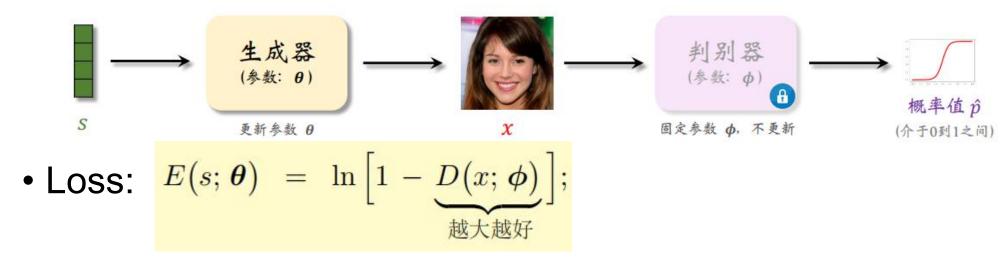
#### **IRL**

- Use learned reward function to train a policy network  $\pi(a|s;\theta)$
- Learned reward function: R(s, a; ρ)
- From the trajectory  $(s_1,a_1;s_2,a_2;s_3,a_3....)$
- $r_t = R(s_t, a_t; \rho)$
- $\theta = \theta + \beta \sum_{t=1}^{n} \gamma^{t-1} u_t \frac{\partial ln(\pi(a|s;\theta))}{\partial \theta}$  (reinforce)



### Generative adversarial imitation learning

Generative adversarial network (GAN)

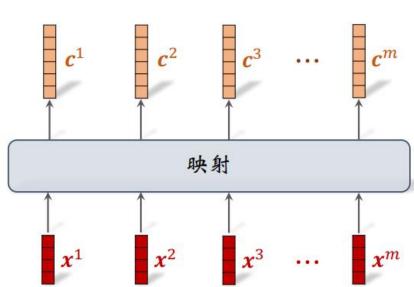


- Output  $p = D(x; \varphi)$
- Update:  $\theta = \theta \beta \frac{\partial E(s;\theta)}{\partial \theta}$



#### Attention

- Self-attention layer
- Solve two questions
- 1.m is uncertain and the parameter c
- 2.Ci is related to all input x





#### Attention

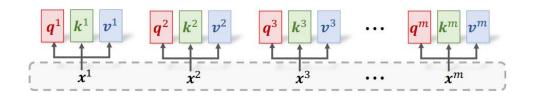


图 17.2: 首先把  $x^i$  映射到三元组  $(q^i, k^i, v^i)$ , $\forall i = 1, \cdots, m$ 。

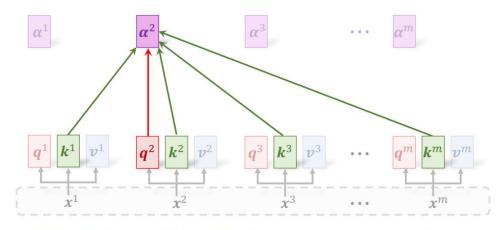


图 17.3: 然后用  $q^i$  和  $(k^1, \dots, k^m)$  计算权重向量  $\alpha^i \in \mathbb{R}^m$ ,  $\forall i = 1, \dots, m$ 。

$$\boldsymbol{\alpha}^i = \operatorname{softmax} \left( \langle \boldsymbol{q}^i, \boldsymbol{k}^1 \rangle, \langle \boldsymbol{q}^i, \boldsymbol{k}^2 \rangle, \cdots, \langle \boldsymbol{q}^i, \boldsymbol{k}^m \rangle \right),$$

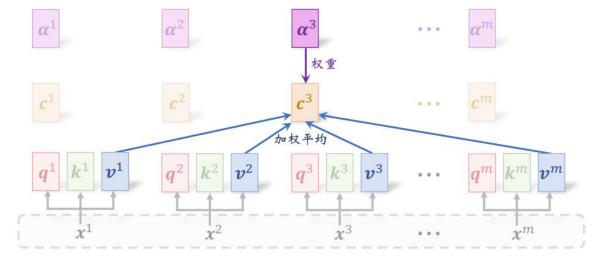


图 17.4: 最后用  $\alpha^i$  和  $(v^1, \dots, v^m)$  计算输出向量  $c^i \in \mathbb{R}^{d_{\text{out}}}, \ \forall i = 1, \dots, m$ 。



#### Attention

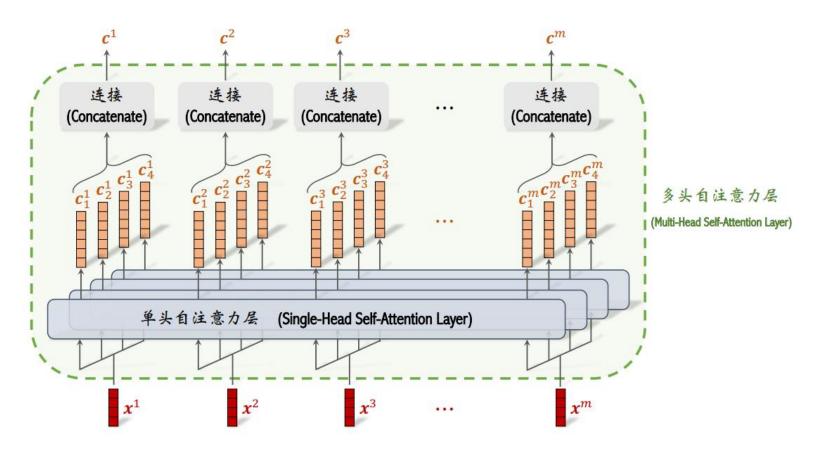


图 17.6: 这个例子中,多头自注意力层由 l=4 个单头自注意力层组成。



#### **Thank You!**

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