

Image Segmentation

Image segmentation [Haralick&Shapiro]

An image segmentation is the partition of an image into a set of nonoverlapping regions whose union is the entire image. The purpose of segmentation is to decompose the image into parts that are meaningful with respect to a particular application....

It is very difficult to tell a computer program what constitutes a "meaningful" segmentation. Instead, general segmentation procedures tend to obey the following rules.

1

Image Segmentation

Rules:

1. Regions of an image segmentation should be uniform and homogeneous with respect to some characteristics, such as grey level or texture.
2. Region interiors should be simple and without many holes
3. Adjacent regions of a segmentation should have different values with respect to the characteristic on which they are uniform.
4. Boundaries of each segment should be simple, not ragged, and must be spatially accurate.

uB **Image Segmentation**

Formulation

Image = Region R

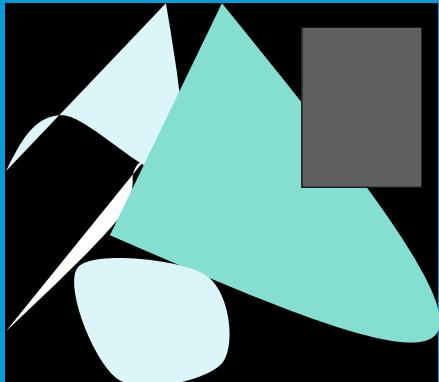
$$R = \bigcup_{i=1}^n R_i$$

R_i is connex

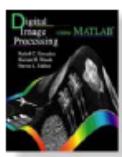
$$R_i \cap R_j = \emptyset \text{ for } i \neq j$$

$P(R_i) = \text{True}$

$P(R_i \cap R_j) = \text{False for } i \neq j$



uB **Image Segmentation**



González, Rafael C., Woods, Richard E., Eddins, Steven L.
(cop. 2004). *Digital image processing using Matlab*.
Prentice Hall.

<http://homepages.inf.ed.ac.uk/rbf/CVonline/>

Compendium Contents
Additional Vision Educational Resources
Vision Related Books including Online Books and Book Support Sites.

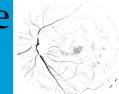
Introduction to image segmentation

- The purpose of image segmentation is to partition an image into *meaningful* regions with respect to a particular application
- The segmentation is based on measurements taken from the image and might be *greylevel, colour, texture, depth or motion*

5

Introduction to image segmentation

- Usually image segmentation is an initial and vital step in a series of processes aimed at overall image understanding
- Applications of image segmentation include
 - Identifying objects in a scene for object-based measurements/recognition
 - Identifying objects in a moving scene for *object-based video compression (MPEG4)*
 - Identifying objects which are at different distances from a sensor using depth measurements from a laser range finder.



6

Introduction to image segmentation

- Example 1

- Segmentation based on greyscale
- Very simple ‘model’ of greyscale leads to inaccuracies in object labelling



7

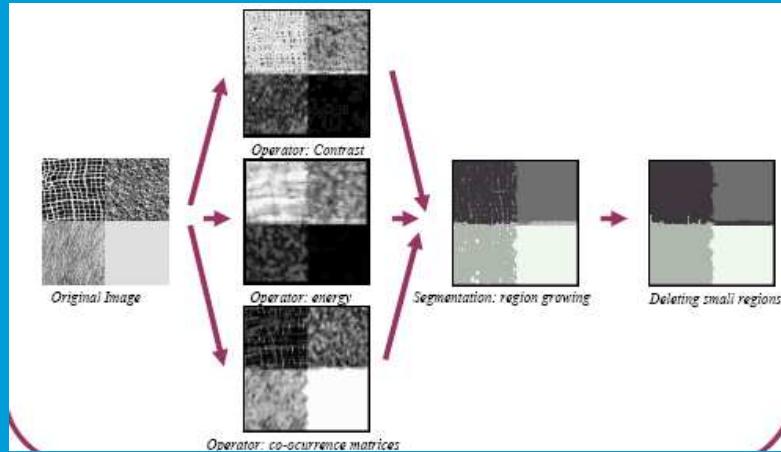
Introduction to image segmentation

- Example 2

- Segmentation based on texture
- Enables object surfaces with varying patterns of grey to be segmented

8

Introduction to image segmentation



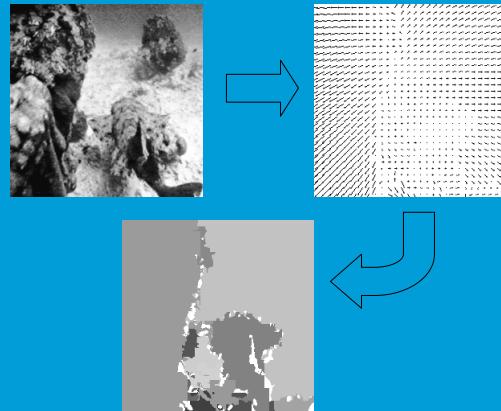
9

Introduction to image segmentation

- Example 3
 - Segmentation based on motion
 - The main difficulty of motion segmentation is that an intermediate step is required to estimate an *optical flow field*
 - The segmentation must be based on this estimate and not, in general, the true flow

10

Introduction to image segmentation



11

Introduction to image segmentation

- Example 4
 - Segmentation based on depth
 - This example shows a range image, obtained with a laser range finder
 - A segmentation based on the range (the object distance from the sensor) is useful in guiding mobile robots

12

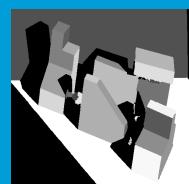
Introduction to image segmentation

Original image



Range image

Segmented image



13

Segmentation

- Process of identifying structure in 2D & 3D images
- Output may be
 - labeled pixels
 - edge map
 - set of contours

Evaluation

How to **evaluate**? The common practice is the comparison with
manual segmentation (called **groundtruth**):

- Using real images → hard work, non objective task
- Using synthetic images → other problems! That not are real

Two typical measures:

- **Contour Criteria:** To compare obtained region limits vs. original image contours
- **Region Criteria:** Area intersection (obtained region and original image region). Pixels belonging to a object and classified as a background (and vice versa)

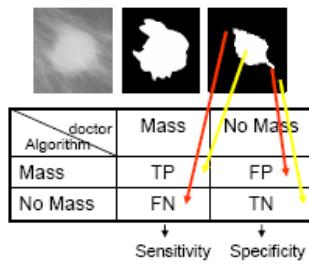
Evaluation

Oversegmentation / Undersegmentation



Evaluation

ROC (Receiver Operated Curve): comparing the ground truth with the algorithm result

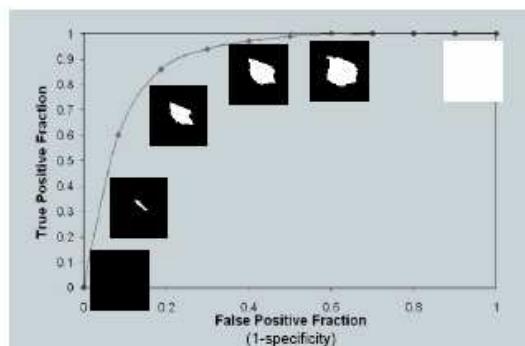


$$\text{sensitivity} = \frac{\text{number of True Positives}}{\text{number of True Positives} + \text{number of False Negatives}},$$

$$\text{specificity} = \frac{\text{number of True Negatives}}{\text{number of True Negatives} + \text{number of False Positives}}$$

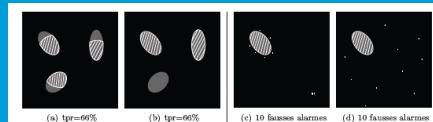
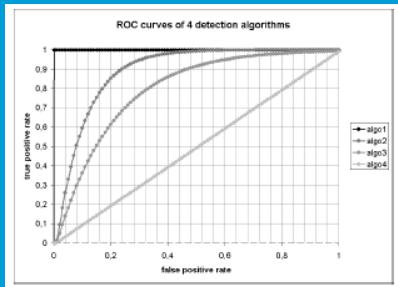
Evaluation

ROC (Receiver Operated Curve): comparing the hand segmented with the algorithm result



AUC

Evaluation



?????

Approaches

- Region Based approach
 - Region Growing
 - Split and merge
- Clustering based methods
 - Threshold
 - methods Kmeans
 - Hierarchical clustering, Graph Cut....
- Edge/Boundary based
 - Contours/boundary surface
 - Deformable warping
 - Deformable registration to atlases

An initial classification:

- Unsupervised segmentation



- Purposive segmentation



Region Growing

- A simple approach to image segmentation is to start from some pixels (seeds) representing distinct image regions and to grow them, until they cover the entire image
- For region growing we need a rule describing a growth mechanism and a rule checking the homogeneity of the regions after each growth step

Region Growing

- The growth mechanism – at each stage k and for each region $R_i(k)$, $i = 1, \dots, N$, we check if there are unclassified pixels in the 8-neighbourhood of each pixel of the region border
- Before assigning such a pixel x to a region $R_i(k)$, we check if the region homogeneity:
 $P(R_i(k) \cup \{x\}) = \text{TRUE}$, is valid

Region Growing

- The arithmetic mean m and standard deviation $s.d$ of a class R_i having n pixels:

$$M = (1/n) \sum_{(r,c) \in R_i} I(r,c)$$

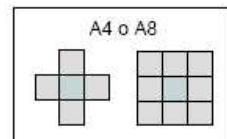
$$s.d = \sqrt{\frac{1}{n} \sum_{(r,c) \in R_i} [I(r,c) - M]^2}$$

Can be used to decide if the merging of the two regions R_1, R_2 is allowed, if

$|M_1 - M_2| < (k)s.d(i)$, $i = 1, 2$, two regions are merged

Region Growing

```
PROGRAM Region_Growing
Mark all the pixels as not considered
FOR all the pixels (x,y) of the image DO
    IF pixel(x,y) no considered THEN
        Begin statistics new region Ri
        Mark pixel(x,y) as a considered
        Explore(x,y)
        Increase the number of seeds
    ENDIF
```



```
ACTION Explore(x,y)
WHILE ((x',y') is an adjacent pixel respect to (x,y) not considered)
and ((x',y') belongs to actual region) DO
    Mark pixel(x',y') as a considered
    Recompute statistics of region Ri
    Explore(x',y')
```

Aggregation Criteria:
intensity
If f(x,y) - μ _{Ri} ≤ Δ

Region Growing

3	5	7	3	4	2	1	
2	4	9	10	22	9	3	
3	5	12	11	15	10	3	
5	6	11	9	17	19	1	
2	3	11	12	18	16	2	
3	6	8	10	18	9	5	
4	6	7	8	3	3	1	

Region Growing

3	5	7	3	4	2	1	
2	4	9	10	22	9	3	
3	5	12	11	15	10	3	
5	6	11	9	17	19	1	
2	3	11	12	18	16	2	
3	6	8	10	18	9	5	
4	6	7	8	3	3	1	

Region Growing

3	5	7	3	4	2	1	
2	4	9	10	22	9	3	
3	5	12	11	15	10	3	
5	6	11	9	17	19	1	
2	3	11	12	18	16	2	
3	6	8	10	18	9	5	
4	6	7	8	3	3	1	

Region Growing

3	5	7	3	4	2	1	
2	4	9	10	22	9	3	
3	5	12	11	15	10	3	
5	6	11	9	17	19	1	
2	3	11	12	18	16	2	
3	6	8	10	18	9	5	
4	6	7	8	3	3	1	

Region Growing

3	5	7	3	4	2	1	
2	4	9	10	22	9	3	
3	5	12	11	15	10	3	
5	6	11	9	17	19	1	
2	3	11	12	18	16	2	
3	6	8	10	18	9	5	
4	6	7	8	3	3	1	

Split / Merge

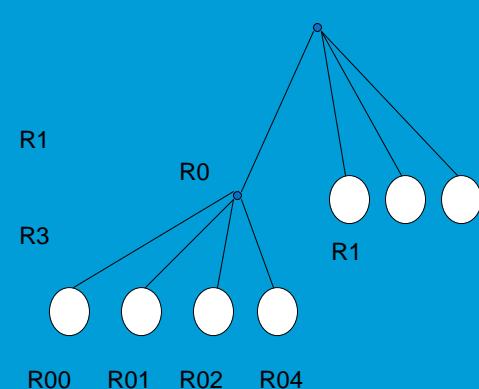
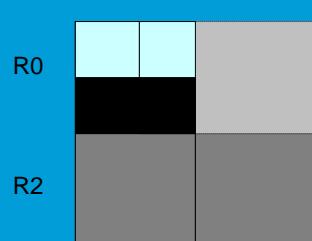
- The opposite approach to region growing is region shrinking (splitting).
- It is a top-down approach and it starts with the assumption that the entire image is homogeneous
- If this is not true , the image is split into four sub images
- This splitting procedure is repeated recursively until we split the image into homogeneous regions

Split / Merge

- If the original image is square $N \times N$, having dimensions that are powers of 2 ($N = 2^n$):
- All regions produced by the splitting algorithm are squares having dimensions $M \times M$, where M is a power of 2 as well ($M=2^m, M \leq n$).
- Since the procedure is recursive, it produces an image representation that can be described by a tree whose nodes have four sons each
- Such a tree is called a Quadtree.

Split / Merge

Quadtree



Split / Merge

- Splitting techniques disadvantage, they create regions that may be adjacent and homogeneous, but not merged.
- Split and Merge method – It is an iterative algorithm that includes both splitting and merging at each iteration:

Split / Merge

- If a region R is inhomogeneous ($P(R) = \text{False}$) then is split into four sub regions
- If two adjacent regions R_i, R_j are homogeneous ($P(R_i \cup R_j) = \text{TRUE}$), they are merged
- The algorithm stops when no further splitting or merging is possible

Split / Merge

- The split and merge algorithm produces more compact regions than the pure splitting algorithm

Split / Merge

0	1	0	0	7	7	7	7
1	0	2	2	7	7	7	7
0	2	2	2	7	7	7	7
4	4	2	2	7	7	7	7
0	0	1	1	3	3	7	7
1	1	2	2	3	7	7	7
2	4	3	0	5	7	7	7
2	3	3	5	5	0	7	7

Image initiale

0	1	0	0	7	7	7	7
1	0	2	2	7	7	7	7
0	2	2	2	7	7	7	7
4	4	2	2	7	7	7	7
0	0	1	1	3	3	7	7
1	1	2	2	3	7	7	7
2	4	3	0	5	7	7	7
2	3	3	5	5	0	7	7

Split 1

0	1	0	0	7	7	7	7
1	0	2	2	7	7	7	7
0	2	2	2	7	7	7	7
4	4	2	2	7	7	7	7
0	0	1	1	3	3	7	7
1	1	2	2	3	7	7	7
2	4	3	0	5	7	7	7
2	3	3	5	5	0	7	7

Split 2

0	1	0	0	7	7	7	7
1	0	2	2	7	7	7	7
0	2	2	2	7	7	7	7
4	4	2	2	7	7	7	7
0	0	1	1	3	3	7	7
1	1	2	2	3	7	7	7
2	4	3	0	5	7	7	7
2	3	3	5	5	0	7	7

Split 3

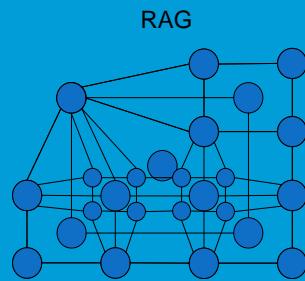
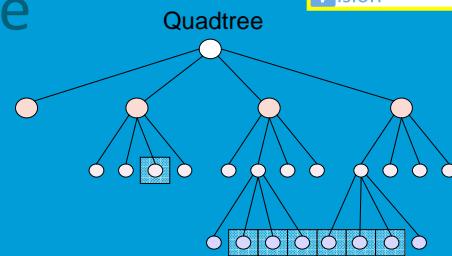
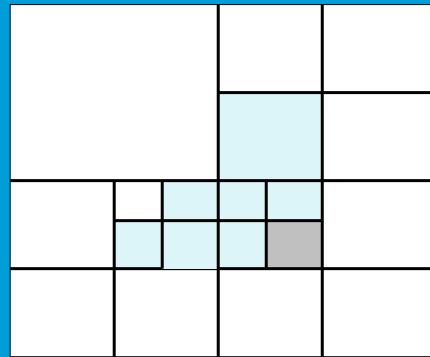


Homogeneity = Criteria based on variance

Split / Merge

Region Adjacency Graph

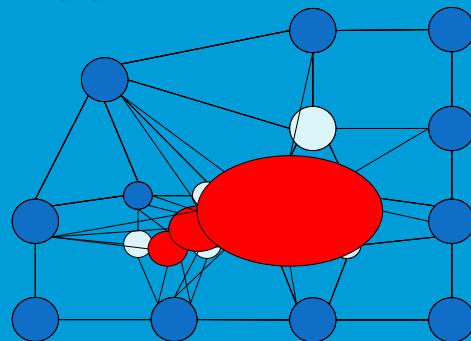
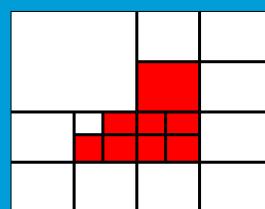
- Connect adjacent regions
- Edges = measures of the homogeneity difference



Split / Merge

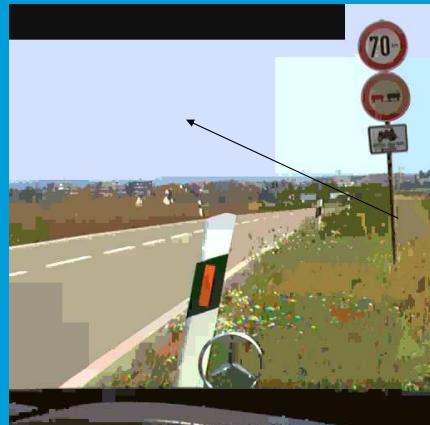
MERGE:

- Each node of the RAG is examined.
- If a neighboring node is at a distance lower than a threshold then the two nodes are merged in the RAG.
- Procedures stop when no further merging is possible.



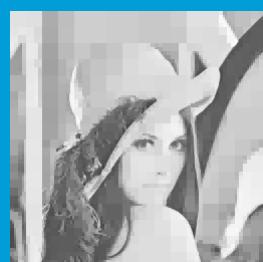
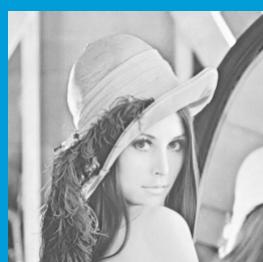
Split / Merge

Split & Merge



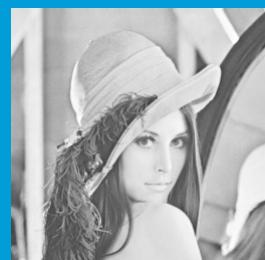
Original

Results – Region Split





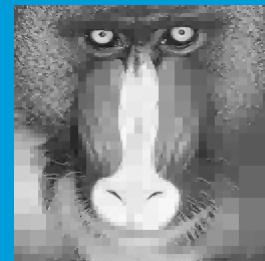
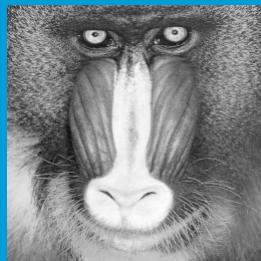
Results – Region Split and Merge



Results – Region growing



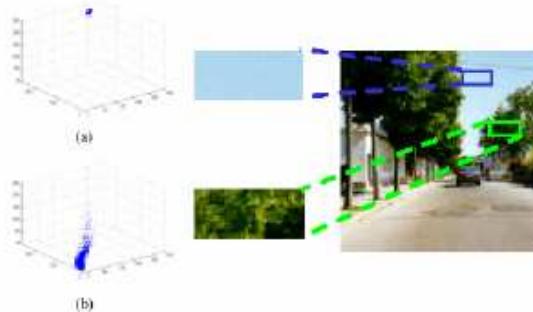
Results – Region Split



Clustering Based techniques

Image segmentation [Haralick&Shapiro]

The difference between image segmentation and clustering is that in clustering, the grouping is done in measurement space: In image segmentation, the grouping is done in spatial domain of the image.





Clustering Based techniques



- Cluster: grouping of pixels considering 1 or more features
- Feature selection: Feature Space (R,G and B, in the previous Figure)
- Similar features... in a cluster
- Features significantly different between different clusters
- The spatial distribution of pixels in the image is not considered (feature space is considered)
- Shape of clusters: compact but, spherical, ellipsoidal, elongated, ...
- Ownership of a pixel to a cluster depends on a proximity measure (distance). Grouping Methodology !!!
- Results may be subjective...
- Key point: number of clusters of an image ??



- Number of possible clusterings Let $X=\{\underline{x}_1, \underline{x}_2, \dots, \underline{x}_N\}$.

In how many ways the N points can be assigned into m groups?

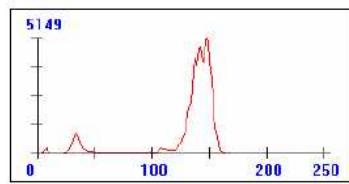
- Example $S(N, m) = \frac{1}{m!} \sum_{i=0}^m (-1)^{m-i} \binom{m}{i} i^N$

$$S(15,3) = 2375101$$

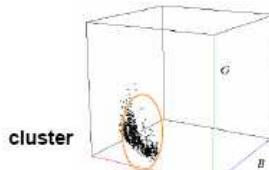
$$S(20,4) = 45232115901$$

$$S(100,5) = 10^{68} !!$$

Clustering en 1D = Histogram analysis → thresholding

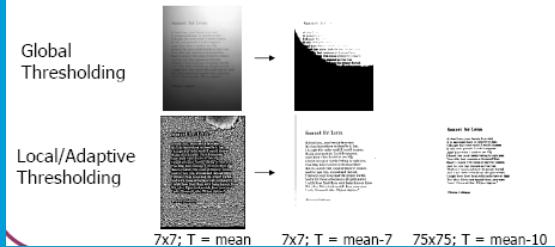


Region characterization
based on colour R.G.B



Clustering Based techniques : Threshold

- In fixed or global thresholding, the threshold value is held constant throughout the image
- A variation could uses two or more thresholds



Clustering Based techniques : Threshold

$$g(x, y) = \begin{cases} 0 & \text{si } f(x, y) \leq T \\ 1 & \text{si } f(x, y) > T \end{cases}$$

With

$$T = T[(x, y), p(x, y), f]$$

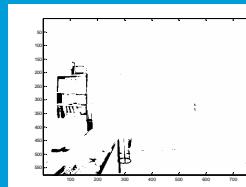
Pixel coordinates



Local properties

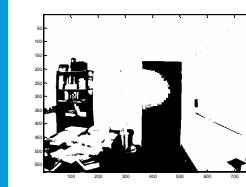


Clustering Based techniques : Threshold

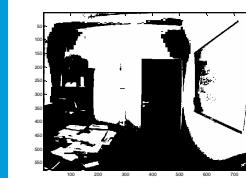


Master in
Computer
Vision

$$T = 64$$



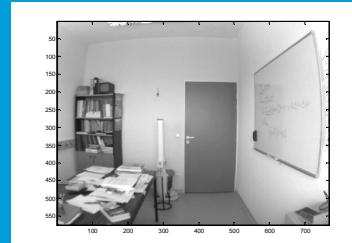
$$T = 127$$



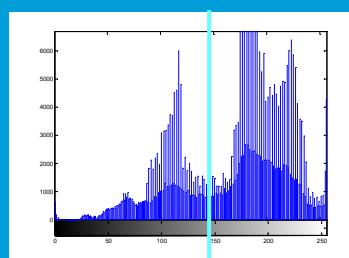
$$T = 180$$



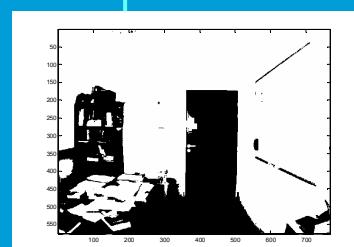
Clustering Based techniques : Threshold



$$T = 150$$

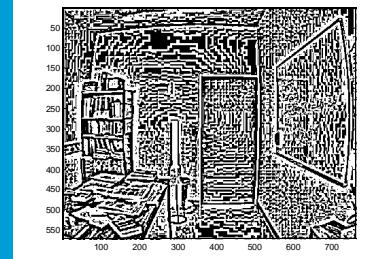
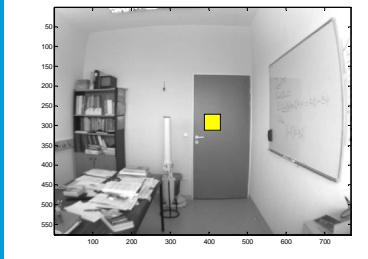


Master in
Computer
Vision



Clustering Based techniques : Threshold

$$T = T[p(x, y), f(x, y)]$$



Clustering Based techniques : Threshold – optimal method

- Methods able to find the optimal threshold:
- Isodata, Peak and valley, Otsu, p-tile,...
- What happens when appearing more than two modes: Ohlander,...

Clustering Based techniques : Threshold – ISODATA

- **ISODATA** (Iterative Self-Organising Data Analysis Technique Algorithm)

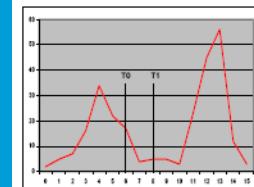
Initially the histogram is segmented in 2 parts. Then, we compute the mean value associated to each part of the histogram m_1, m_2 . With these values we compute a new threshold doing $T = (m_1 + m_2) / 2$. We repeat the process until convergence

- **ISODATA** step by step:

- 1) Choose the initial threshold $T_i = T_0$ (median histogram, maximum gray level...)
- 2) Divide the image in two groups R_1 and R_2 using T_i
- 3) Compute the mean of the gray level for both parts m_1 and m_2
- 4) Select the new threshold $T_{i+1} = (m_1 + m_2) / 2$
- 5) Repeat steps 2-4 until $T_i = T_{i-1}$

Clustering Based techniques : Threshold – ISODATA

- Example:
 $H(z) = [2 \ 5 \ 7 \ 16 \ 34 \ 22 \ 17 \ 4 \ 5 \ 5 \ 5 \ 3 \ 23 \ 45 \ 56 \ 12 \ 3]$
- $H(z) =$
- z has values from 0 to 15 (4 bits)
- 1) $T_0 = \text{position(maximum)} / 2 = 13 / 2 = 6$
- (we could also do $T_0 = \text{mean of the histogram } T_0 = \sum z * H(z) / \sum H(z) \Rightarrow T_0 = 8.85$)
- 2) $m_1 = \sum z * H(z) / \sum H(z)$ for $z = 0$ to 6 = 4.02
- $m_2 = \sum z * H(z) / \sum H(z)$ for $z = 7$ to 15 = 12.03
- $T_1 = (m_1 + m_2) / 2 = 8.03$
- 3) $m_1 = \sum z * H(z) / \sum H(z)$ for $z = 0$ to 8 = 4.31
- $m_2 = \sum z * H(z) / \sum H(z)$ for $z = 9$ to 15 = 12.31
- $T_2 = (m_1 + m_2) / 2 = 8.31$



Clustering Based techniques : Threshold – Peak and Valley Method

1. Assign to Z_1 the gray level z which is the **maximum** of $H(z)$

2. Assign to Z_2 the gray level that **maximises** $g(z) = |Z_1 - z| * H(z)$

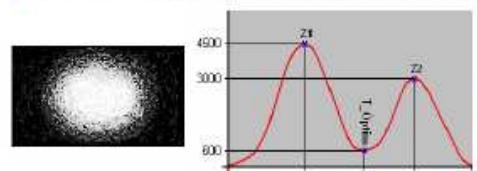
This is called to maximise the peakiness, the difference between peaks and valley

3. Assign the threshold T to the gray level that is the **minimum of $H(z)$** going:

- from $z = Z_1 + 1$ to $Z_2 - 1$, if $Z_2 > Z_1$
- from $z = Z_2 + 1$ to $Z_1 - 1$, if $Z_2 < Z_1$

Clustering Based techniques : Threshold – Peak and Valley Method

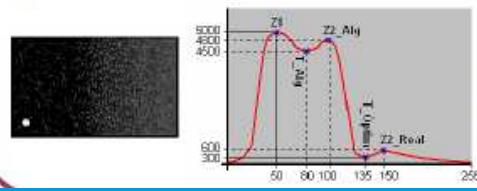
✓ It works when the histogram has 2 well defined pics



Computing Z_2 :

$$\begin{aligned} g(z) &= |Z_1 - z| * H(z) \\ Z_2 \rightarrow g(z)_{\max} &= \\ &= |90 - 190| * 3.000 = 300.000 \end{aligned}$$

✗ If 1 peak is very small we can have problems computing Z_2



Computing Z_2 :

$$\begin{aligned} g(z) &= |Z_1 - z| * H(z) \\ Alg \rightarrow |50 - 100| * 4.800 &= 240.000 \\ Real \rightarrow |50 - 150| * 600 &= 60.000 \end{aligned}$$

Clustering Based techniques : Threshold – Otsu

- Any threshold separates the histogram into 2 groups with each group having its own statistics (mean, variance)
- The homogeneity of each group is measured by the *within group variance*
- The optimum threshold is that threshold which minimizes the within group variance thus maximizing the homogeneity of each group

Clustering Based techniques : Threshold – Otsu

- Let group o (object) be those pixels with greylevel $\leq T$
- Let group b (background) be those pixels with greylevel $> T$
- The prior probability of group o is $p_o(T)$
- The prior probability of group b is $p_b(T)$

Clustering Based techniques : Threshold – Otsu

- The following expressions can easily be derived for prior probabilities of object and background

$$p_o(T) = \sum_{i=0}^T P(i)$$

$$p_b(T) = \sum_{i=T+1}^{255} P(i)$$

$$P(i) = h(i)/N$$

- where $h(i)$ is the histogram of an N pixel image

Clustering Based techniques : Threshold – Otsu

- The mean and variance of each group are as follows :

$$\mu_o(T) = \sum_{i=0}^T i P(i) / p_o(T)$$

$$\mu_b(T) = \sum_{i=T+1}^{255} i P(i) / p_b(T)$$

$$\sigma_o^2(T) = \sum_{i=0}^T [i - \mu_o(T)]^2 P(i) / p_o(T)$$

$$\sigma_b^2(T) = \sum_{i=T+1}^{255} [i - \mu_b(T)]^2 P(i) / p_b(T)$$

Clustering Based techniques : Threshold – Otsu

- The within group variance is defined as :

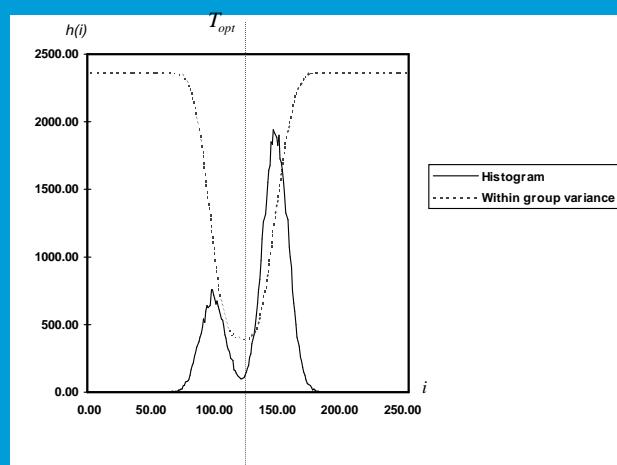
$$\sigma_w^2(T) = \sigma_o^2(T)p_o(T) + \sigma_b^2(T)p_b(T)$$

- We determine the optimum T by minimizing this expression with respect to T

- Only requires 256 comparisons for an 8-bit greylevel image

63

Clustering Based techniques : Threshold – Otsu



64

Clustering Based techniques : Threshold – Ohlander

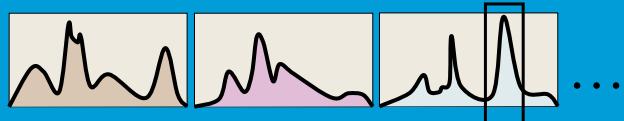
- Region based algorithm using thresholding technique
- Based on exploiting the chromatic information by constructing colour and hue histograms
- Regions are split recursively based upon histogram analysis
- Picture is thresholded at its most clearly separated peak

Clustering Based techniques : Threshold – Ohlander

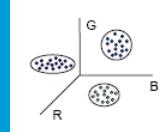
Ohlander (1975); Ohta, Kanade,Sakai (1980)



Initial image

Réinjection of
sufficient size
regionsSuppression
of the
extracted
regionRetroprojection of
the Histogram
window

Clustering Based techniques : K means



K-means algorithm

- Successive generation of clusters, minimizing a cost function J , obtaining the “best” clusters
- - “a priori” knowledge of: # of clusters, ownership of every pixel to only one cluster C_j (minimum distance to the centroid ϕ_j)

$$\phi_j = \frac{1}{N} \sum_{x_i \in C_j} X_i \quad d(X_i, \phi_j) = \|X_i - \phi_j\|$$

Clustering Based techniques : K means

- Cost Function J :

$$J = \sum_{i=1}^N \sum_{j=1}^K u_{ij} \times d(x_i, \phi_j) \quad \text{where } \phi_j \text{ is the centroid of cluster } C_j$$

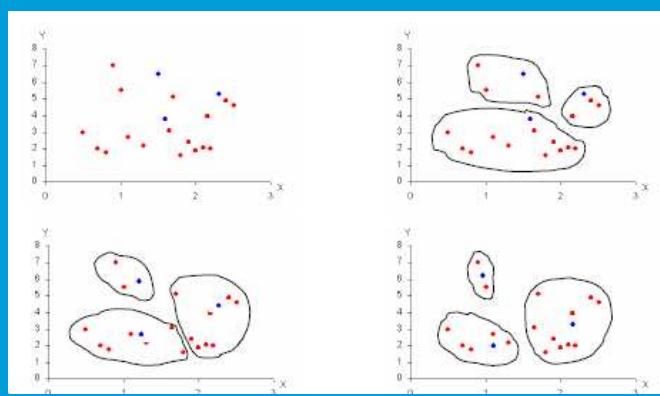
- - U_{ij} is a coefficient related to the ownership of a pixel i to a cluster j :
- $U_{ij} = 1$, if d is the minimum distance
- $U_{ij} = 0$, otherwise

Clustering Based techniques : K means

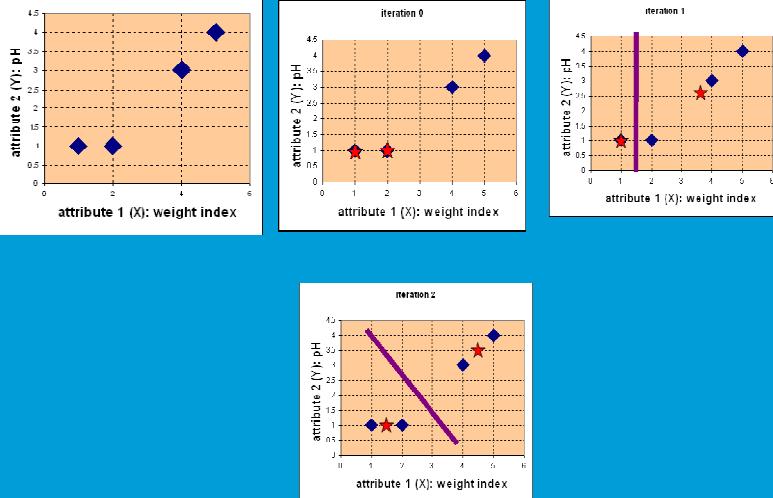
- 4 steps, iterative algorithm:
 - **Phase 1**
 - K is known
 - Determination of the initial centers of each cluster ϕ_j , $j = 1 \dots K$
 - **Phase 2**
 - Each pixel x_i is assigned to its nearest cluster C_j
 - **Phase 3**
 - Compute the “new” centers of the clusters ϕ_j
 - **Phase 4**
 - If any pixel has changed the cluster (phase 2) then go to phase 2,
else the algorithm finish.
- Final Goal minimize

$$J = \sum_{i=1}^N \sum_{j=1}^K u_{ij} * d(X_i, \phi_j)$$

Clustering Based techniques : K means



Clustering Based techniques : K means



Clustering Based techniques : Hierarchical clustering

- In the beginning each pixel of the image constitute a cluster. Step by step the algorithm merges the clusters until a final condition is reached: only one cluster for the whole image.

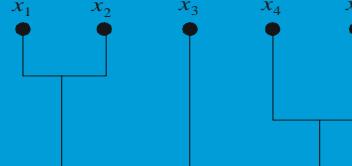
Clustering Based techniques : Hierarchical clustering

$$D(X) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 5 & 4 \\ 6 & 5 \\ 6.5 & 6 \end{bmatrix}$$

- **Example 1:** Let $X = \{\underline{x}_1, \underline{x}_2, \underline{x}_3, \underline{x}_4, \underline{x}_5\}$, with $\underline{x}_1 = [1, 1]^T, \underline{x}_2 = [2, 1]^T, \underline{x}_3 = [5, 4]^T, \underline{x}_4 = [6, 5]^T, \underline{x}_5 = [6.5, 6]^T$.

$$P(X) = \begin{bmatrix} 0 & 1 & 5 & 6.4 & 7.4 \\ 1 & 0 & 4.2 & 5.7 & 6.7 \\ 5 & 4.2 & 0 & 1.4 & 2.5 \\ 6.4 & 5.7 & 1.4 & 0 & 1.1 \\ 7.4 & 6.7 & 2.5 & 1.1 & 0 \end{bmatrix}$$

{ $\{\underline{x}_1\}, \{\underline{x}_2\}, \{\underline{x}_3\}, \{\underline{x}_4\}, \{\underline{x}_5\}$ }
{ $\{\underline{x}_1, \underline{x}_2\}, \{\underline{x}_3\}, \{\underline{x}_4\}, \{\underline{x}_5\}$ }
{ $\{\underline{x}_1, \underline{x}_2\}, \{\underline{x}_3, \underline{x}_4\}, \{\underline{x}_5\}$ }
{ $\{\underline{x}_1, \underline{x}_2\}, \{\underline{x}_3, \underline{x}_4, \underline{x}_5\}$ }
{ $\{\underline{x}_1, \underline{x}_2, \underline{x}_3, \underline{x}_4, \underline{x}_5\}$ }



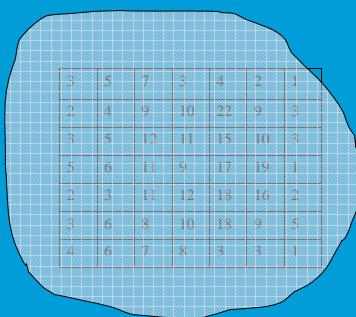
Regions detection

- Regions are bounded by edges
 - detection of closed curves
 - linking all edge points
 - iterative line fitting
 - finding lines or circles or ... with the Hough transform
- Classification
- Region growing algorithm
- Split and Merge algorithm
- Other approaches :
 - snakes, deformable contours, ...

Deformable Surfaces

3	5	7	3	4	2	1
2	4	9	10	22	9	3
3	5	12	11	15	10	3
5	6	11	9	17	19	1
2	3	11	12	18	16	2
3	6	8	10	18	9	5
4	6	7	8	3	3	1

Deformable Surfaces



Deformable Surfaces

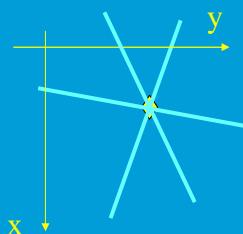
3	5	7	3	4	2	1
2	4	9	10	22	9	3
3	5	12	11	15	10	
5	6	11	9	17	19	1
2	3	11	12	18	16	2
3	6	8	10	18	9	
4	6	7	8	3	3	1

Deformable Surfaces

3	5	7	3	4	2	1
2	4	9	10	22	9	3
3	5	12	11	15	10	
5	6	11	9	17	19	1
2	3	11	12	18	16	2
3	6	8	10	18	9	5
4	6	7	8	3	3	1

Image Plane

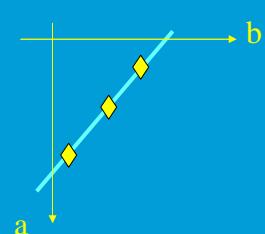
$$y = ax + b$$



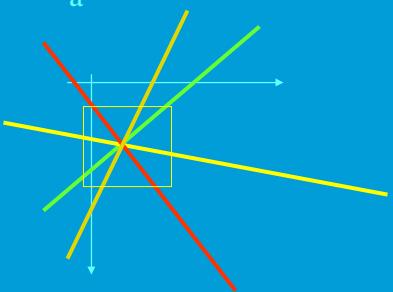
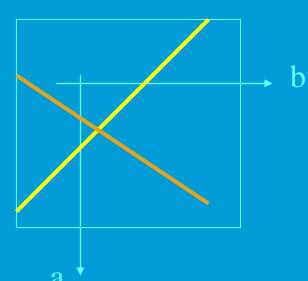
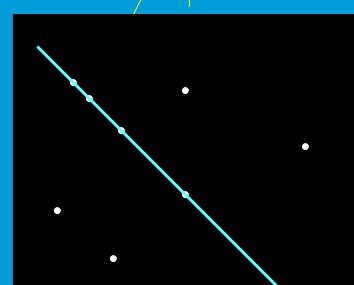
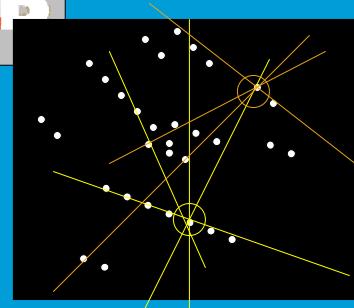
a et b fixed

Hough parameters Plane

$$b = -xa + y$$



x et y fixed



ALGORITHM

1) Grid for the (a,b) space

2) For each cell $A(u,v)=0$

3) for each image point

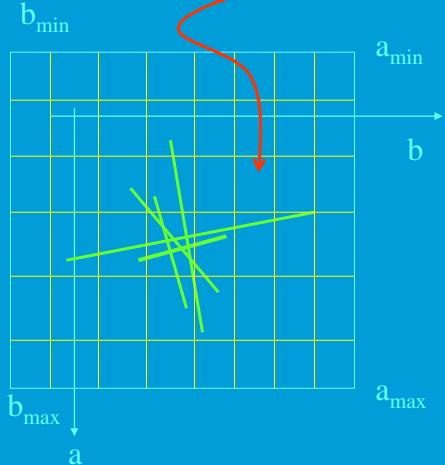
a) Choose $a \in [a_{\min}, a_{\max}]$

b) Calculate $b = -xa + y$

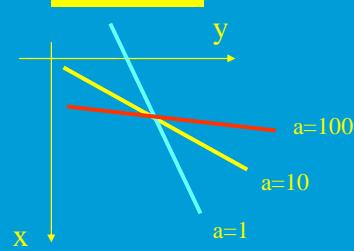
c) Calculate the cell (u,v)

d) $A(u,v) = A(u,v) + 1$

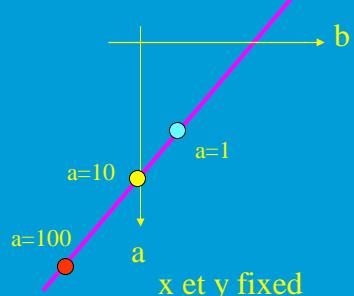
4) For every cell (u,v) , if $A(u,v) > \text{Threshold}$, this cell represents a line

cell $(u,v) \Rightarrow$ accumulator


$$y = ax + b$$



$$b = -xa + y$$

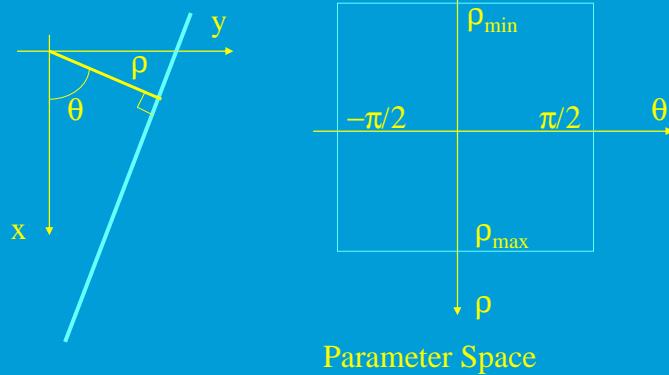

Grid is not possible for all slopes (∞)

uB

Solution :

Master in
Computer
Vision

$$y = ax + b \rightarrow x \cos(\theta) + y \sin(\theta) = \rho$$



Parameter Space

uB

$$x \cos(\theta) + y \sin(\theta) = \rho$$

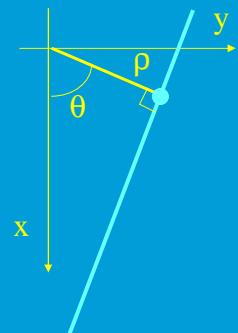
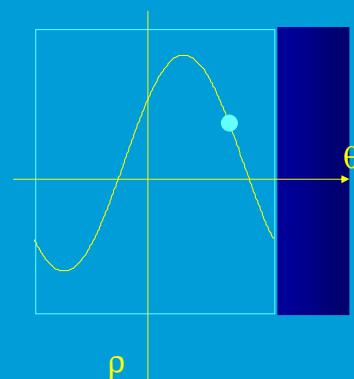
Master in
Computer
Vision

Image Space



Parameter Space

uB

Example of transformed images

parameters space $[a,b]$
t.q. $y=ax+b$

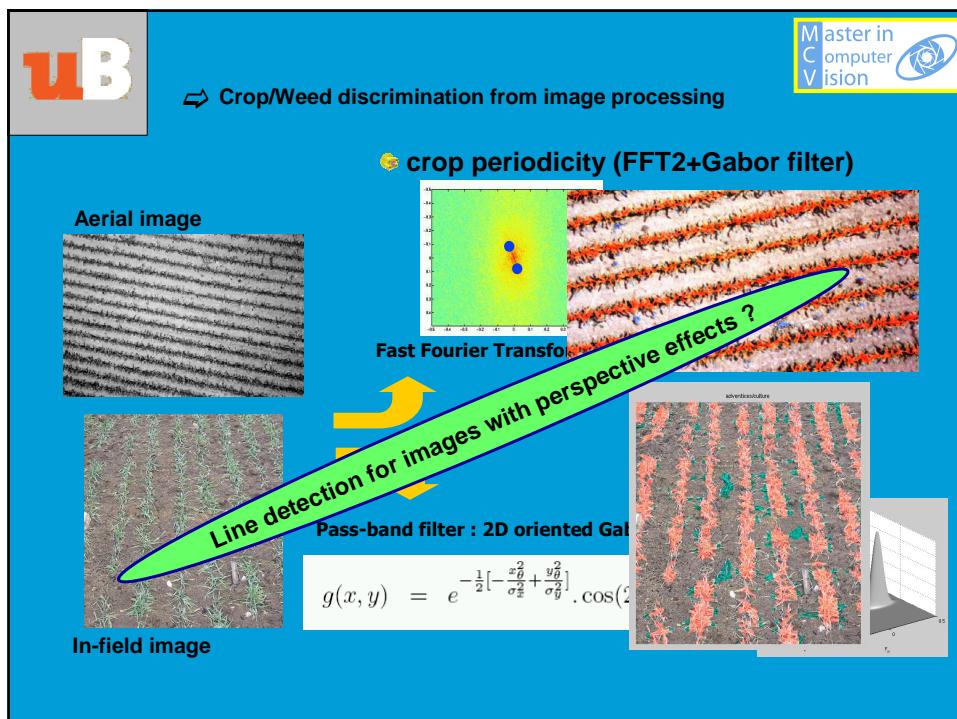
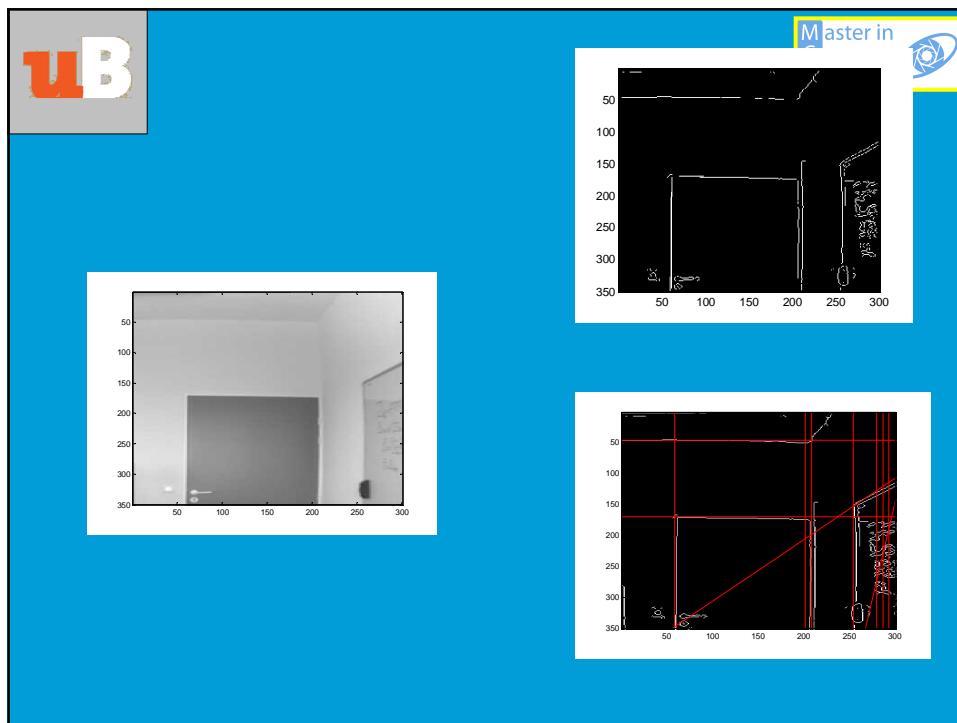
parameters space $[\rho,\theta]$ s.as
 $\rho=x\cos(\theta)+y\sin(\theta)$

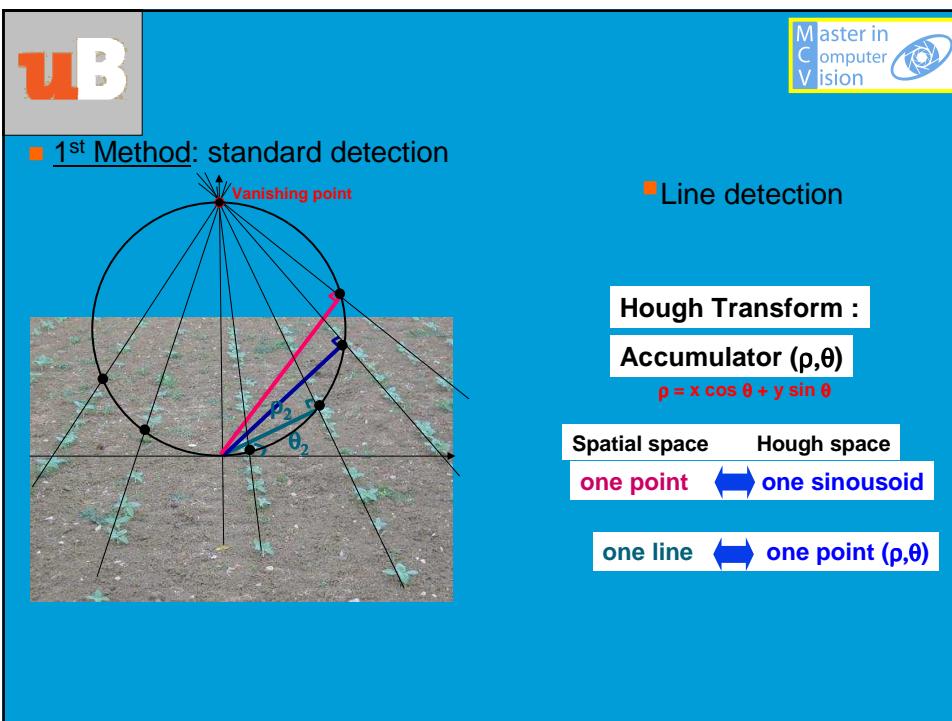
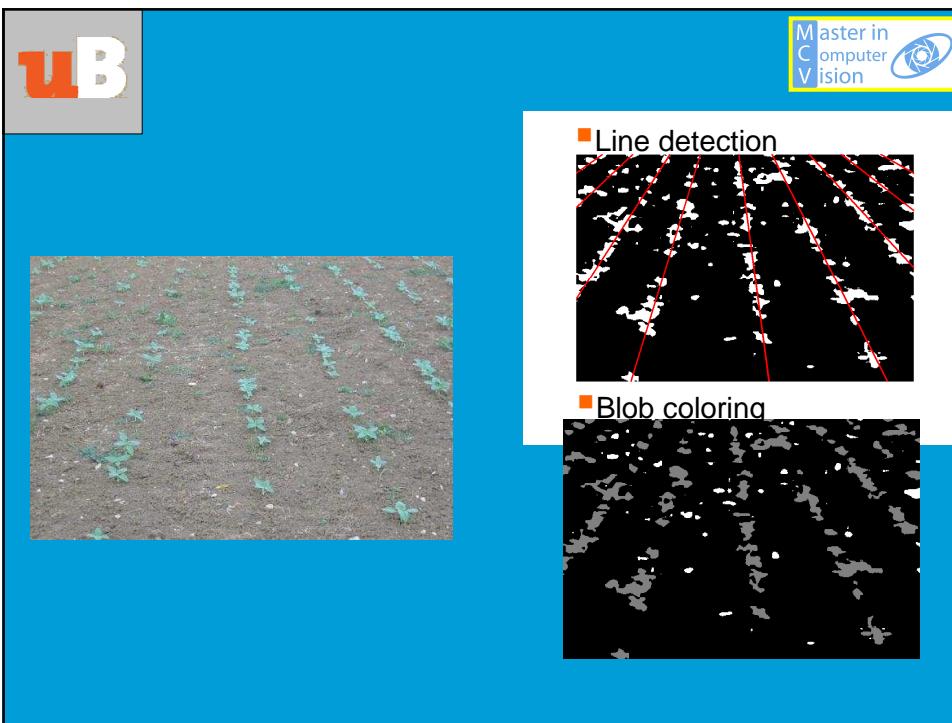
The first plot shows a square parameter space for a linear transformation $y = ax + b$. A diagonal line segment is shown within the square, with points a_0 and b_0 marked on the axes. The second plot shows a square parameter space for a polar transformation $\rho = x\cos(\theta) + y\sin(\theta)$. A curved boundary is shown within the square, with points ρ_0 and θ_0 marked on the axes.

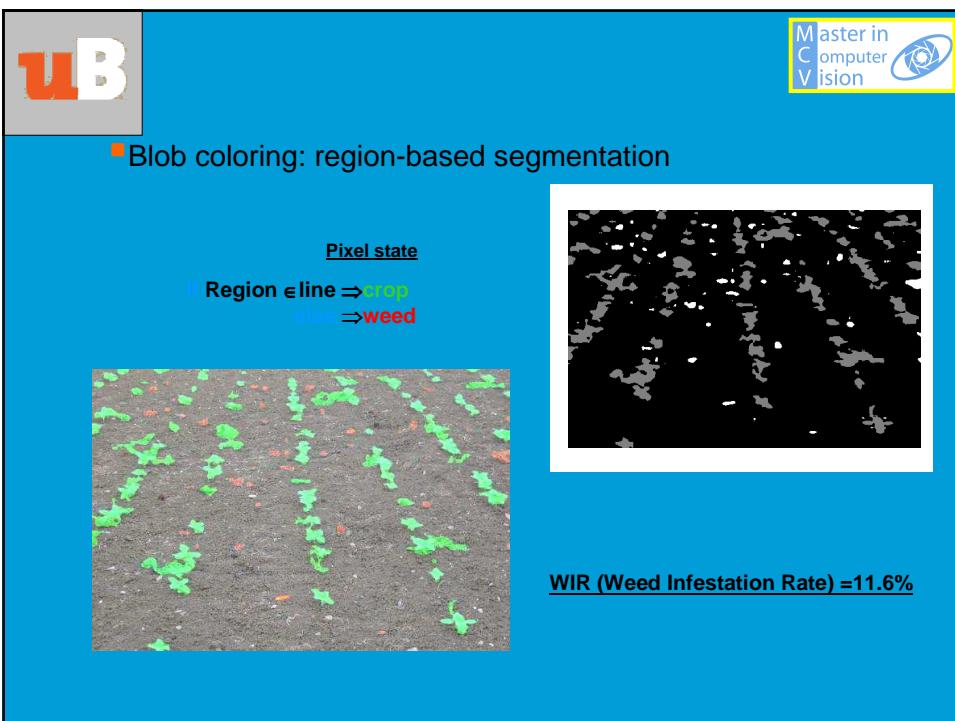
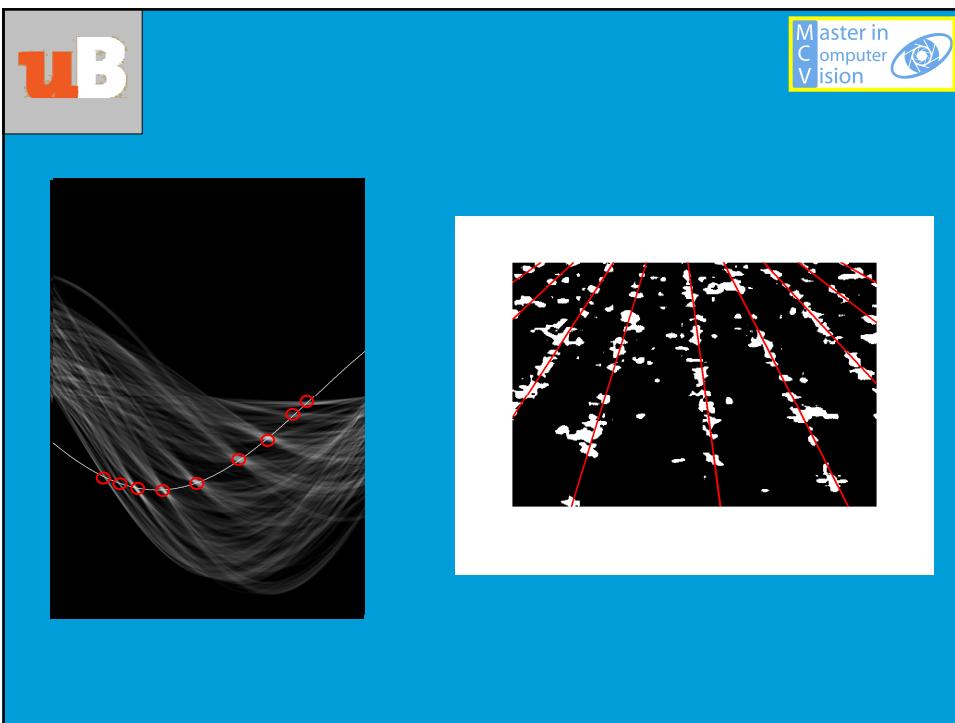
uB

Another example with 5 lines

A small square icon containing five intersecting lines is shown above a larger image. The larger image shows a distorted, curved surface with several parallel lines drawn on it, representing the transformed space of the original intersecting lines.







Connex components labeling

1	1	1	0	1	1	0	1
1	1	0	1	1	0	1	1
1	0	1	1	0	1	1	1
1	1	1	0	1	1	1	1
0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

image after edge
detection and
thresholding

1	1	1	0	2	2	2	0	3
1	1	0	4	2	0	5	3	3
1	0	6	2	0	7	3	3	3
1	1	1	0	8	3	3	3	3
0	0	0	9	3	3	3	3	3
10	10	10	3	3	3	3	3	3
10	10	10	3	3	3	3	3	3
10	10	3	3	3	3	3	3	3

image of labels after
the first browsing

1	1	1	0	1	1	0	3
1	1	0	1	1	0	3	3
1	0	1	1	0	3	3	3
1	1	1	0	3	3	3	3
0	0	0	3	3	3	3	3
3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3
3	3	3	3	3	3	3	3

image of labels
after update

1	2	3	4	5	6	7	8	9	10
			2	3	2	3	3	3	3
1		1		1					