

# Multi-Robot Path Planning Using Nash Equilibrium based Game Theory

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**Abstract**—With, human life becomes more and more reliant on robots, accurate path planning of the robot is essential. The utility and significance of a multi-robot system are determined by its ability to reach the desired location and complete the given task. Therefore, the proposed approach using game theory for multi robot path planning holds great potential for improving the efficiency and safety of robotic systems in a variety of applications. The proposed system consists of two robots that will follow a predetermined path. To select the robot, the electrical battery level and mechanical robustness are considered, and the best performing robot will be selected. Based on the performance scores for the battery level and the score for mechanical robustness a Game Theory algorithm is implemented based on Nash equilibrium to achieve optimal path planning, which treats each robot as a player in a game. A Nash equilibrium is a specific condition of Game Theory where no robot can improve its outcome by changing its strategy. The work is implemented on two robots in the laboratory.

**Index Terms**—Multi-robot, Path Planning, Game Theory, Nash equilibrium, Strategies, PyCharm.

## I. INTRODUCTION

Robotics are now widely used in a variety of fields including medicine, agriculture, industries, and military applications. They are often utilized in hazardous environments where human safety is a concern, and in crucial environments, where precise operations are necessary. In such scenarios, robot path planning becomes essential, as it allows the robot to navigate from its starting position to its target position efficiently while avoiding any obstacles in its path. In multi-robot path planning selecting the most suitable robot for a specific task is crucial. The best-performing robot without any defect or error is selected based on certain strategies.

A game theory algorithm can be used to plan its path in a known environment with static obstacles. Game theory, with its core concept of Nash equilibrium, is particularly useful in multi-robot path planning, as it allows to predict how players will behave and interact with each other while making decisions concurrently. By considering other robots' actions, it becomes possible to anticipate what they will do, leading to optimal outcomes for all robots.

The major contributions are discussed which include:

- The proposed approach is demonstrated through a multi robot workspace exploration, where each robot is dis-

guised by replicating the process and applying it to them.

- This paper presents a solution for multi robot path planning using with simple Nash equilibrium unlike complicated methods like matrix method or Shapely value method in Game theory. By using different strategies, the best-performing robot will be selected and will achieve its goal with maximum efficiency.

This paper is organized into five sections. In Section II, related work done in multi-robot path planning using game theory is discussed. Section III provides an overview of the complete work with a system overview and the strategies employed for the proposed work are also explained in detail. Section IV provides a detailed description of the implementation of the proposed approach, and the simulation results are presented and discussed.

Finally, Section V summarizes the main findings of the work and discusses its implications for future research.

## II. RELATED WORK

The work related to path planning using Game Theory and other related methods s discussed in this section.

In [1], this paper proposes a cooperative game theory approach for multi-robot path planning. The approach involves decomposing the problem into sub-tasks, each of which is assigned to a robot. The robots work cooperatively to achieve the overall goal of finding an optimal path while avoiding collisions. The proposed approach is evaluated using simulations, and the results show that it is effective in reducing the time taken to plan paths while ensuring collision avoidance. The authors also compared their approach with other multi robot path planning algorithms and showed that it outperforms them in terms of efficiency and optimality.

According to [2], this paper presents a real-time game theoretic planner for autonomous two-player drone racing. The ultimate objective is to create a planner capable of optimizing drone trajectories and strategies in competitive racing settings. The suggested method simulates the interactions of the drones as a non-cooperative game, considering their dynamics, relative locations, and access control inputs. The research demonstrates that the game-theoretic planner out performs previous techniques through comprehensive simulations.

and real-world testing, displaying superior maneuverability, flexibility to opponents' moves, and quicker lap times. The proposed solution enhances competitiveness and performance in autonomous two-player drone racing. Another modification

In [3], the game-theoretic framework, specifically utilizing the concept of repeated games, for multi agent navigation in a multi-story parking garage. The approach employs the Iterative Prisoner's Dilemma algorithm, a well-known algorithm in game theory, to model the interactions between the agents. Each agent aims to minimize its parking time while considering the actions of other agents. The suggested framework enables intelligent decision-making and optimization of parking efficiency in a complex parking garage environment by iteratively solving the game and updating tactics. Simulations and experiments confirm the efficacy of this game-theoretic strategy based on the Iterative Prisoner's Dilemma algorithm.

In [4], this paper presents a solution for multi-robot path planning using a deep reinforcement learning DQN algorithm. The approach combines deep learning and reinforcement learning to enable robots to develop optimum path-planning strategies. The approach allows robots to adapt to dynamic situations, avoid collisions, and optimize their trajectories to achieve specified objectives by estimating Q-values and utilizing neural networks. Experiment findings show that the suggested technique is successful in enhancing efficiency and scalability in multi-robot route planning scenarios.

In [5], this paper discusses lane detection research that used a Convolutional Neural Network (CNN) technique. The authors offer a strategy that entails precisely building and optimizing the CNN architecture for the goal of finding lanes. To properly interpret visual data and recognize lane markers, the algorithm is likely to include layers such as convolutional layers, pooling layers, and fully connected layers. The study intends to increase the accuracy and reliability of lane identification algorithms, hence advancing autonomous driving systems.

According to [6], this paper presents a system design for addressing the challenges of multi-robot task allocation and path planning. The proposed approach combines job distribution and path planning algorithms to enhance efficiency and resource use. The system efficiently allocates tasks to robots and arranges their trajectories by considering aspects like job priority, robot capabilities, and resource restrictions. This leads to increased overall performance and shortened task completion times.

In this analysis, a decentralized method for task and route planning in multi-robot systems is presented. The proposed methodology enhances the system's overall performance and coordination by enabling individual robots to make choices independently using local information and sparse communication. The algorithm effectively allocates jobs and plans paths while considering both local and global limits.

It does this by taking into account the interdependence between tasks and the dynamics of the robots. The decentralized strategy's success is demonstrated by experimental findings, which also illustrate how it has the potential to improve the capabilities of multi-robot systems in a variety of applications. [7]

In [8], this work presents a game theory-based clustering-based method for multi-robot task distribution. The suggested approach makes use of clustering techniques to allocate jobs to robots while considering their strategic relationships and inter dependencies. The method tries to accomplish effective job allocation in dynamic contexts by optimizing resource utilization and performance. Through simulations and studies, the approach's efficacy is confirmed, demonstrating enhanced task allocation effectiveness and overall performance.

In [9], a game theory model for multi-robot collaboration in Industry 4.0 situations is presented in this work. The model tries to tackle the problem of multi-robot coordination and resource allocation optimization in complicated industrial contexts. The concept makes it possible for each robot to make intelligent decisions by framing robot interactions as a cooperative game that considers both its own utility and the utilities of other robots. Experimental assessments show that the suggested paradigm improves coordination, resource allocation, and overall productivity in Industry 4.0 environments.

In [10], this paper introduces a game-theoretic utility tree framework for multi-robot cooperative pursuit strategies. The approach proposed for cooperative pursuit strategy. The GUT model is based on game theory and represents the interactions between the pursuers and the evader as a tree. The nodes in the tree represent the different states of the game, and the edges represent the possible actions that can be taken by the pursuers and the evader. The utilities of the nodes in the tree are calculated using a game-theoretic approach, and these utilities are used to determine the best course of action for the pursuers. The authors evaluate the performance of the GUT model using simulations and real-robot experiments, and the results show that the GUT model can effectively organize cooperation strategies.

In [11], this paper addresses the problem of multi-robot path planning in a static environment. In the proposed approach, pathways for numerous robots moving from their starting places to their objective positions while avoiding collisions are efficiently found using algorithmic techniques such as graph-based methods or heuristic search. The technique seeks to reduce path length and increase effectiveness. The study offers a method to effectively navigate numerous robots in a static environment without dynamic barriers by utilizing the selected algorithm. The usefulness of the suggested strategy in establishing optimal pathways for multi-robot navigation is supported by experimental findings and performance assessments.

In [12], the simultaneous localization and mapping (SLAM) method is used to construct an indoor self-mapping pill dispenser. The gadget scans and creates real-time maps of interior settings while precisely localizing itself inside the map using sensors and perception algorithms. The use of SLAM enables the gadget to precisely find pill compartments, navigate its surroundings autonomously, and administer medication. The gadget is effective at automating pill distribution procedures, as shown in the trial assessments, making it a potential option for indoor pill management systems.

### III. MULTIROBOT SYSTEM

The multi-robot path planning system considers a known environment consisting of static obstacles arranged in a  $5 \times 5$  gridmap. In contrast to conventional algorithms like A\*, Dijkstra, a game theory-based algorithm offers greater flexibility in choosing various strategies. This system requires inputs such as the current location and battery level of two robots, their respective energy consumption per step, the mechanical robustness of the robot. Depending on the robot level, either RA or RB is selected to play the game. The number of obstacles present between the two robots is also considered before selecting the algorithm.

Game theory is a mathematical framework that analyzes the strategic interactions, decision-making of rational individuals in competitive and in cooperative situations. It provides a systematic way to study the behavior of players, predict their choices, and understand the resulting outcomes.

Here are some key concepts and components of game theory:

- **Players:** In game theory, players are the participants or decision-makers involved in the game. In this work, there are two players namely Robot A and Robot B
- **Strategies:** Strategies are the plans or courses of action available to players. Each player selects a strategy based on their goals, beliefs, and understanding of the game. The strategy chosen by a player determines their actions in the game and influences the overall outcome.
- **Payoffs:** Payoffs represent the benefits or utilities that players receive based on the outcome of the game. Payoffs capture the preferences and objectives of players and guide their decision-making.
- **Game Matrix:** A game matrix, also known as a payoff matrix or strategic form, is a representation of the possible strategies and corresponding payoffs for each player in a game.
- **Solution Methods:** Game theory provides various solution methods to determine the optimal strategies and outcomes in games. These include dominant strategy equilibrium, mixed strategy equilibrium, backward induction, iterated elimination of dominated strategies, and more. Solution methods help analyze games and predict the likely behavior of players.

These advantages make game theory a valuable tool for understanding and analyzing strategic interactions, facilitating optimal decision-making, resolving conflicts, designing policies, and managing risks. Its applications are broad and can be found in various fields where strategic considerations are important. Once the robot is selected, a game is played using the game theory Nash equilibrium approach. The obstacle is also considered as a player in the game. The payoff matrix is constructed based on the distance between the robot and the obstacle. The path is planned based on the payoff, and the robot moves toward the destination while avoiding obstacles. The proposed system provides an efficient solution for multirobot path planning in a static environment as shown Fig.1

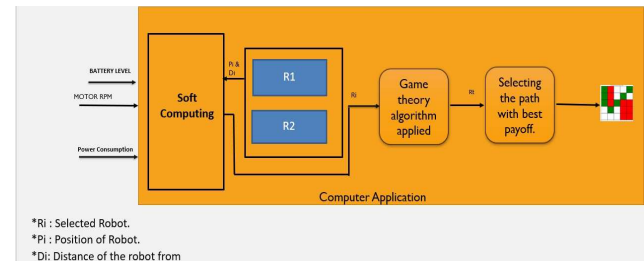


Fig. 1. System Overview

#### A. Nash equilibrium

Nash equilibrium is a concept in game theory where best outcome of a game is one in which there is no incentive to stray from the initial strategy. When the Nash equilibrium is reached, it shows that no player can unilaterally change his or her decision to improve the reward. It can also be called "no regrets", meaning that once a decision has been made, the player will not guess it based on the outcome. In most cases, Nash equilibrium is reached over time. The game is modeled as a two-player game between the robot and the obstacle, where the robot aims to reach the destination while avoiding collisions with the obstacle.

The payoff function for the game is defined as follows:

- If the robot collides with the obstacle, the robot receives a negative payoff (-1).
- If the robot reaches the destination without colliding with the obstacle, the robot receives a positive payoff (+1).
- The robot and the obstacle select their actions (i.e., diagonal, straight)
- If the robot and the obstacle select the same action, there is a collision, and the robot receives a negative payoff (-1).

If the robot and the obstacle select different actions, the robot moves to its selected location and the obstacle moves to the random location. The Nash equilibrium is calculated by finding the optimal strategy for both players such that neither player has the incentive to change their strategy.

In this case, the Nash equilibrium is achieved when the robot selects the action that maximizes its expected payoff given the obstacle's strategy, and the obstacle selects the action that minimizes the robot's expected payoff given the robot's strategy.

#### IV. STRATEGIES USED

Let  $RX$  represent the robots engaged in path planning and  $X$  represent either robot A or robot B. The two strategies chosen for the work are electrical parameter namely battery level and mechanical robustness of each robot. The battery level is further improved as an associated performance score.

##### A. Strategy 1

level for each robot should be taken. The consumed battery  $RX_{cbl}$  is the quantity of battery utilized by the robot to go from the start to the end point. Consumed Battery for Robot A as given in Equation 1,

$$RA_{cbl} = RA_{bc} * \left( \frac{RA_d}{RA_s} \right) \quad (1)$$

Consumed Battery for Robot B as given in Equation 2,

$$RB_{cbl} = RB_{bc} * \left( \frac{RB_d}{RB_s} \right) \quad (2)$$

With Equation (1) and (2) the discharge battery level is calculated. Discharge battery level for Robot A as given in Equation 3,

$$RA_{dbl} = RA_{bl} - RA_{cbl} \quad (3)$$

Discharge battery level for Robot B as given in Equation 4,

$$RB_{dbl} = RB_{bl} - RB_{cbl} \quad (4)$$

To select Robot A and Robot B, the performance score is developed for each robot. Performance Score for Robot A as given in Equation 5,

$$RA_{psr} = RA_v * RA_a / (RA_w * (1 + \frac{RA_{ob}}{10})) \quad (5)$$

Performance Score for Robot B as given in Equation 6,

$$RB_{psr} = RB_v * RB_a / (RB_w * (1 + \frac{RB_{ob}}{10})) \quad (6)$$

Equation (5) and (6) show the performance of each robot based on the parameters. This factor  $RX_{ob}$  by 10 is added to reduce the performance score based on the number of obstacles. The larger the number of the obstacle, the smaller the performance score will be. The division by 10 is just an arbitrary scaling factor to adjust the impact of the obstacle on the performance score. These equations are based on various parameters defined in the choice for strategies namely velocity  $RX_v$ , acceleration  $RX_a$ , weight  $RX_w$  and the number of obstacles  $RX_{ob}$  is also considered in the expression of performance score evaluation. Finally, in (7) and (8) the robot scores  $RX_{sr}$  are defined using the distance  $RX_d$ , discharged battery level of the robot  $RX_{dbl}$  and the performance score of each robot  $RA_{psr}$ .

Robot Score for Robot A as given in Equation 7,

$$RA_{sr} = RA_{psr} * \left( \frac{RA_{dbl}}{RA_d} \right) \quad (7)$$

Robot Score for Robot B as given in Equation 8,

$$RB_{sr} = RB_{psr} * \left( \frac{RB_{dbl}}{RB_d} \right) \quad (8)$$

To determine the best-performing robot, the higher score from the Robots is taken and compared. Based on the above calculation the higher performance scores of Robot A and Robot B are calculated, and it determines which robot has a higher score. To select the best-performing robot, assume the parameters like Voltage  $R_v$ , Battery consumption  $R_{bc}$ , Battery level, Speed  $R_s$ , Acceleration  $R_a$ , and weight  $R_w$ . Before considering this strategy the distance  $R_d$ , position  $R_p$ , and the number of obstacles Robin the path were also added by playing the game between two robots the battery consumption  $R_{bc}$  per step for  $RX$  is 0.05 To calculate the discharged battery level, the consumed battery based on the higher score, the robot will be selected and the game is played between the obstacle and the selected robot.

##### B. Strategy 2

The mechanical robustness of the robot is arrived by a score which is based on the wheel movement and the response of the robot to different terrains like smooth, uneven and rough. A value is assigned based on the two parameters from 1 to 5 namely  $RX$  (where  $x$  indicates robot A or B). A low value indicates that the robot is incapable of proper movement. A value of 5 means its wheel movement is smooth and it can run on all terrains.

C. GT algorithm based on Nash Equilibrium The overall algorithm is explained as follows:

- Let's  $S_i$  be the set of all possible strategies for player  $i=1, \dots, N$  (set of robots). Here two strategies are considered for two robots. Hence  $S_i = 4$ .
- Let's  $s^* (s_i^*, s_{-i}^*)$  be a strategy profile, a set consisting of one set of strategies for each player. The strategy profile has two strategies.
- $s^*$  denotes the  $N-1$  strategies of all the players except  $i$ .
- Lets  $U_i (s_i^*, s_{-i}^*)$  be the player  $i$ 's payoff as a function of the strategies. Based on the two strategies, namely the robot score and the mechanical robustness the respective payoffs are utilized in a game that incorporates Nash equilibrium.

The outcome of the game indicates that there is no better solution that can be obtained for the respective strategy concerned and with the chosen robot.

#### V. SIMULATION RESULT

The Multi-robot path planning algorithm is simulated using PyCharm on a Computer (HP ENVY x360 16GB RAM). The algorithm is tested in a  $5 \times 5$  grid environment for different values of the two strategies. A few test cases are explained. A. Test case 1 In Test Case 1, based on the robot parameter namely robot voltage  $RX_v$ , the initial battery level  $RX_b$ , battery consumption  $RX_{bc}$ , robot speed  $RX_s$ , acceleration  $RX_a$ , robot weight  $RX_w$  for both robots are initially observed. The respective performance score is calculated from equation (7) and (8). The score of  $RA_{sr}$  is 6.583 and  $RA_{ob}$  is 5,  $RB_{sr}$  8.64 and the  $RB_{ob}$  is 4.

The game is played as explained in Section IV C. The Robot B is designated for implementing the path planning. The simulation output is shown in Fig.2 for seven obstacles. The Discharge battery level of robot  $BRB_{dbl}$  is added for upcoming path planning.

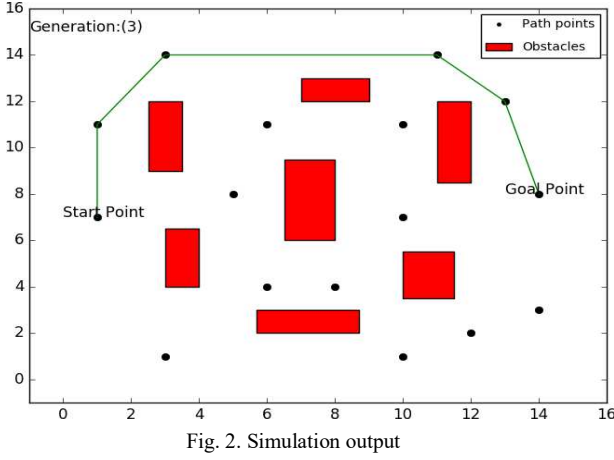


Fig. 2. Simulation output

### C. Test case 3

In test case 3 the robots are similar scores  $RX_{sr}$ , which implies that the robots perform the same way. However, the second strategy namely mechanical robustness is higher for robot A namely 4 compared to the value of 2 for Robot B. Hence, Robot A is chosen. The simulated output with seven obstacles is shown for Robot A in Fig.4.

### D. Test case 4

In this scenario, Robot A score  $RA_{sr}$  is 7.15 and Robot B score  $RB_{sr}$  is 5.33. Robot A obstacle is 7 and Robot B obstacle is 4. Here Robot A outperforms Robot B in terms of its robot score, but Robot B has fewer obstacles to navigate.

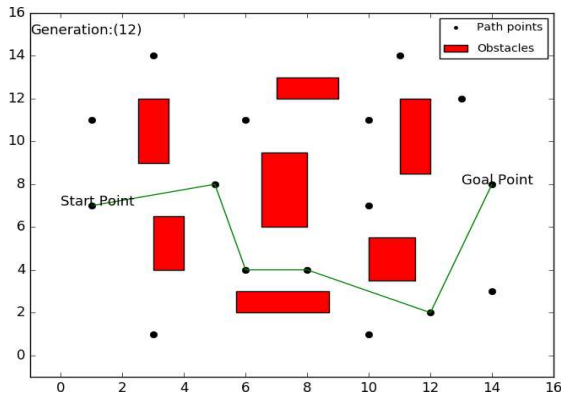


Fig. 3. Simulation output

However, in this case, minimizing disruptions and collisions are more important, Robot B would be selected due to its lower obstacle count. The decision ultimately hinges on the specific priorities and trade-offs involved in the given situation, considering factors such as the significance of performance versus obstacle avoidance and any additional criteria or constraints. The various test cases explained are tabulated in Table I for better analysis.

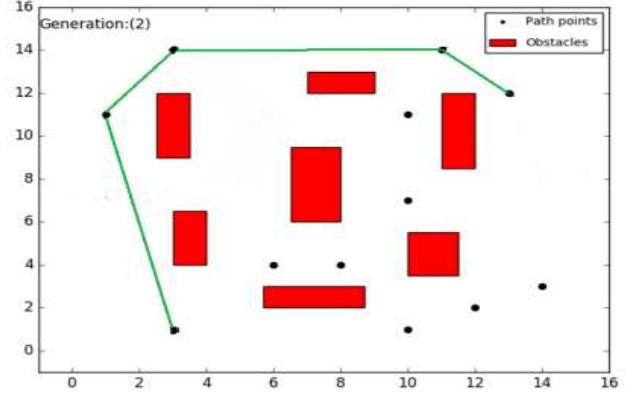


Fig. 4 Simulation output

The effect of start and end points on the algorithm is also explored. It is observed that this does not affect the robot navigation in any way.

TABLE I  
TEST CASES FOR THE SYSTEM

Test Case	Robot score		Obstacle		Selected Robot	Start	End
	$RA_{sr}$	$RB_{sr}$	$RA_{ob}$	$RB_{ob}$			
1	6.58	8.64	5	4	B	(0,3)	(4,4)
2	6.1	4.12	4	7	A	(0,3)	(4,4)
3	5.33	5.33	4	4	X	(2,1)	(4,4)
4	7.15	5.33	7	4	B	(0,0)	(4,4)

## VI. CONCLUSION AND FUTURE SCOPE

In conclusion, to explore efficient path planning the game theory algorithm is used with different strategies. The analysis shows that these techniques can effectively coordinate the movements of multiple robots while avoiding collisions and optimizing performance. The effectiveness of the system through simulations and various test cases demonstrates its potential to revolutionize path planning in robotics, offering a robust and adaptable solution for real-world applications. Work is being carried out using iRobot's in the laboratory. However, there is still scope for improvement in the system. One area for future research is the development of machine learning algorithms that can adapt to changing environments and handle more complex scenarios by incorporating real-time feedback from multiple robots and recording their performance parameters.

## VII. REFERENCES

- [1] P. Supriya, T. N. Padmanabha Nambiar, and S. B. Jasna, "Co-operative Game Theory Approach for Multi Robotic Path Planning," International Journal of Advanced Science and Technology, vol. 29, no. 3, pp. 4592-4608, 2020
- [2] R. Spica, E. Cristofalo, Z. Wang, E. Montijano and M. Schwager, "A Real Time Game Theoretic Planner for Autonomous Two-Player Drone Racing," in IEEE Transactions on Robotics, vol. 36, no. 5, pp. 1389-1403, Oct. 2020, doi:10.1109/TRO.2020.2994881.
- [3] Gindullina, S. Mortag, M. Dudin and L. Badia, "Multi-Agent Navigation of a Multi-Storey Parking Garage via Game Theory," 2021 IEEE 22nd International Symposium on a World of Wireless, Mobile and

- Multimedia Networks (WoWMoM), Pisa, Italy, 2021, pp. 280-285.
- [4] Yang, Yang, Li Juntao, and Peng Lingling. "Multi-robot path planning based on a deep reinforcement learning DQN algorithm." *CAAI Transactions on Intelligence Technology* 5.3 (2020): 177-183.
  - [5] Aravind, H., P. Sivraj, and K. I. Ramachandran. "Design and Optimization of CNN for Lane Detection." 2020 11th International Conference on Computing, Communication and Networking Technologies (ICCCNT). IEEE, 2020.
  - [6] Fan, Y., Deng, F., Shi, X. (2020, July). Multi-robot task allocation and path planning system design. In 2020 39th Chinese Control Conference (CCC) (pp. 4759-4764). IEEE.
  - [7] Chen, Y., Rosolia, U., Ames, A. D. (2021). Decentralized task and path planning for multi-robot systems. *IEEE Robotics and Automation Letters*, 6(3), 4337-4344
  - [8] Martin, J. G., Muros, F. J., Maestre, J. M., Camacho, E. F. (2023). Multirobot task allocation clustering based on game theory. *Robotics and Autonomous Systems*, 161, 104314
  - [9] Gindullina, Elvina, et al. "A game theory model for multi robot cooperation in industry 4.0 scenarios." 2021 IEEE Asia Pacific Conference on Circuit and Systems (APCCAS). IEEE, 2021
  - [10] Yang, Qin, and Ramviyas Parasuraman. "Game-theoretic Utility Tree for Multi-Robot Cooperative Pursuit Strategy." *ISR Europe 2022; 54th International Symposium on Robotics*. VDE, 2022.
  - [11] Nair, Sandeep S., and P. Supriya. "Multi Robotic Path Planning in a Static Environment." 2022 4th International Conference on Inventive Research in Computing Applications (ICIRCA). IEEE, 2022
  - [12] Ganga, et al. "Indoor Self-Mapping Pill Dispensing Device Using SLAM Algorithm." 2021 6th International Conference on Communication and Electronics Systems (ICCES). IEEE, 2021