Coordinated Cooperative Control of Mobile Manipulators

Víctor Andaluz*, V. T. L. Rampinelli‡, Flavio Roberti*† and Ricardo Carelli*†
*Instituto de Automática, Universidad Nacional de San Juan, Argentina
†Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina.
{vandaluz,froberti,rcarelli}@inaut.unsj.edu.ar
‡Universidade Federal do Espírito Santo, Vitória, ES, Brasil
vinicius.rampinelli@ele.ufes.br

Abstract- **This paper presents a multi-layer scheme for coordinated cooperative control of mobile manipulators, for transporting a common object. Each layer works as an independent module, dealing with a specific part of the problem of coordination and cooperation, thus giving more flexibility to the system. A methodology to avoid obstacles in the trajectory of any mobile manipulator is designed based on the concept of mechanical impedance of the interaction robots-environment, without deforming the virtual structure and maintaining its desired trajectory. Stability is proved by using Lyapunov's method. Simulation results show a good performance of the proposed controller as proved by the theoretical design.**

I. INTRODUCTION

Mobile manipulator is nowadays a widespread term that refers to robots built by a robotic arm mounted on a mobile platform. This kind of system, which is usually characterized by a high degree of redundancy, combines the manipulability of a fixed-base manipulator with the mobility of a wheeled platform. Such systems allow the most usual missions of robotic systems which requiere both locomotion and manipulation abilities. Coordinated control of multiple mobile manipulators have attracted the attention of many researchers [1,2,3,4]. The interest in such systems stems from the capability for carrying out complex and dexterous tasks which cannot be simply made using a single robot. Moreover, multiple small mobile manipulators are also more appropriate for realizing several tasks in the human environments than a large and heavy mobile manipulator from a safety point of view.

Main coordination schemes for multiple mobile manipulators in the literature are:

1) Leader–follower control for mobile manipulator, where one or a group of mobile manipulators play the role of a leader, which track a preplanned trajectory, and the rest of the mobile manipulators form the follower group which move in conjunction with the leader mobile manipulators [2, 5, 6]. In [7], a leader-follower type formation control is designed for a group of mobile manipulators. To overcome parameter uncertainty in the model of the robot, a decentralized control law is applied to individual robots, in which an adaptive NN is used to model robot dynamics online.
2) Hybrid position–force control by decentralized/centralized scheme, where the position of the object is controlled in a certain direction of the workspace, and the internal force of the object is controlled in a small range of the origin [1, 3, 8]. In [9], robust adaptive controls of multiple mobile manipulators carrying a common object in a cooperative manner have been investigated with unknown inertia parameters and disturbances. At first, a concise dynamics consisting of the dynamics of mobile manipulators and the geometrical constraints between the end-effectors and the object is developed for coordinated multiple mobile manipulators. In [10] coupled dynamics are presented for two cooperating mobile robotic manipulators manipulating an object with relative motion in the presence of uncertainties and external disturbances. Centralized robust adaptive controls are introduced to guarantee the motion and force trajectories of the constrained object. A simulation study to the decentralized dynamic control for a robot collective consisting of nonholonomic wheeled mobile manipulators is performed in [11], by tracking the trajectories of the load, where two reference signals are used for each robot, one for the mobile platform and another for end-effector of the manipulating arm.

In this paper, we propose a novel method for centralized coordinated cooperative control of multiple nonholonomic wheeled mobile manipulators. Also, it is worth noting that, differently to the work in [11], we use a single reference for the end-effector of the robot mobile manipulator.

Although centralized control approaches present intrinsic problems, like the difficulty to sustain the communication between the robots and the limited scalability, they have technical advantages when applied to control a group of robots with defined geometric formations. Therefore, there still exists significant interest in their use. As an example, in [12] a centralized multi-robot system is proposed for an entrapment/escorting mission, where the escorted agent is kept in the centroid of a polygon of n sides, surrounded by n robots

978-1-4244-9066-0/11/$26.00 ©2011 IEEE

positioned in the vertices of the polygon. Another task for which it is important to keep a formation during navigation is large-objects transportation, because the load has a fixed geometric form. Another recent work dealing with centralized formation control is [13], where a control approach based on a virtual structure, called Cluster Space Control, is presented. There, the positioning control is carried out considering the centroid of a geometric structure corresponding to a three-robot formation.

In this paper, the proposed strategy conceptualizes the *n*-mobile manipulators system as a single group, and the desired motions are specified as a function of cluster attributes, such as position, orientation, and geometry. These attributes guide the selection of a set of independent system state variables suitable for specification, control, and monitoring. The control is based on a virtual 3-dimensional structure, where the position control (or tracking control) is carried out considering the centroid of the upper side of a geometric structure (shaped as a prism) corresponding to a three-mobile manipulators formation. Moreover, it is utilized the concept of manipulability measure of the mobile manipulators to maintain maximum manipulability during the execution of the task. The concept of the manipulability ellipsoid was originally introduced by Yoshikawa [14] and in [15] an analysis of the concept of manipulability applied to mobile manipulator systems is presented.

In addition, we propose a methodology to avoid obstacles in the trajectory of any mobile manipulator based on the concept of mechanical impedance of the interaction robots-environment, without deforming the virtual structure and maintaining its desired trajectory. It is considered that the obstacle is placed at a maximum height that does not interfere with the workspace, so that the arm of the mobile manipulators can follow the desired trajectory even when the platform is avoiding the obstacle.

This paper is organized as follows. In Section II it is obtained the mobile manipulator kinematic model. Section III presents the multi-layer control scheme for the coordinated and cooperative control of mobile manipulators and presents the forward and inverse kinematics transformation necessary for the control scheme. Also, the design of the controller and an analysis of the system's stability are developed. The simulation results are presented and discussed in Section IV. Finally, conclusions and future work are given in Section V.

II. Mobile Manipulator System

The mobile manipulator configuration is defined by a vector $\mathbf{q}$ of n independent coordinates, called *generalized coordinates of the mobile manipulator*, where $\mathbf{q} = [q_1 \quad q_2 \quad \dots \quad q_n]^T = [\mathbf{q}_p^T \quad \mathbf{q}_a^T]^T$ where $\mathbf{q}_a$ represents the generalized coordinates of the arm, and $\mathbf{q}_p$ the generalized coordinates of the mobile platform. We notice that $n = n_a + n_p$, where n_a and n_p are respectively the dimensions of the generalized spaces associated to the robotic arm and to the mobile platform. The configuration $\mathbf{q}$ is an element of the mobile manipulator *configuration space*; denoted by $\mathcal{N}$. The location of the end-effector of the mobile manipulator is given by the $m-$dimensional vector $\mathbf{h} = [h_1 \quad h_2 \quad \dots \quad h_m]^T$ which define the position and the orientation of the end-effector of the mobile manipulator in $\mathcal{R}$. Its m coordinates are the *operational coordinates of the mobile manipulator*. The set of all locations constitutes the *mobile manipulator operational space*, denoted by $\mathcal{M}$.

The location of the mobile manipulator end-effector can be defined in different ways according to the task, i.e., it can be considered only the position of the end-effector or both its position and its orientation. .

A. Mobile Manipulator Kinematic Model

The *kinematic model of a mobile manipulator* gives the location of the end-effector $\mathbf{h}$ as a function of the robotic arm configuration and the platform location (or its operational coordinates as functions of the robotic arm generalized coordinates and the mobile platform operational coordinates).

$$f: \ \mathcal{N}_a \text{ x } \mathcal{M}_p \ \rightarrow \ \mathcal{M}$$
$$\left(\mathbf{q}_a, \mathbf{q}_p\right) \ \mapsto \ \mathbf{h} = f\left(\mathbf{q}_a, \mathbf{q}_p\right)$$

where, $\mathcal{N}_a$ is the *configuration space* of the robotic arm, $\mathcal{M}_p$ is the *operational space of the platform*.

The *instantaneous kinematic model of a mobile manipulator* gives the derivative of its end-effector location as a function of the derivatives of both the robotic arm configuration and the location of the mobile platform,

$$\dot{\mathbf{h}} = \frac{\partial f}{\partial \mathbf{q}}\left(\mathbf{q}_a, \mathbf{q}_p\right)\mathbf{v}$$

where, $\dot{\mathbf{h}} = [\dot{h}_1 \quad \dot{h}_2 \quad \dots \quad \dot{h}_m]^T$ is the vector of the end-effector velocity, $\mathbf{v} = [v_1 \quad v_2 \quad \dots \quad v_{\delta_n}]^T = [v_p^T \quad v_a^T]^T$ is the control vector of mobility of the mobile manipulator. Its dimension is $\delta_n = \delta_{n_p} + \delta_{n_a}$, where δ_{n_p} and δ_{n_a} are respectively the dimensions of the control vector of mobility associated to the robotic arm and the mobile platform. Now, after replacing $\mathbf{J}(\mathbf{q}) = \frac{\partial f}{\partial \mathbf{q}}\left(q_a, q_p\right)$ in the above equation, we obtain

$$\dot{\mathbf{h}}(t) = \mathbf{J}(\mathbf{q})\mathbf{v}(t)$$

where, $\mathbf{J(q)}$ is the Jacobian matrix that defines a linear mapping between the vector of the mobile manipulator velocities $\mathbf{v}(t)$ and the vector of the end-effector velocity $\dot{\mathbf{h}}(t)$. In [16] the kinematic model for a mobile manipulator 6-DOF is presented.

The *i-th* mobile manipulator work team members can be represented by the following kinematic model,

$$\dot{\mathbf{h}}_i(t) = \mathbf{J}_i(\mathbf{q}_i)\mathbf{v}_i(t) \tag{1}$$

where, $\mathbf{h}_i(t)=[h_{i1} \quad h_{i2} \quad \dots \quad h_{im}]^T$, $\dot{\mathbf{h}}_i(t)=[\dot{h}_{i1} \quad \dot{h}_{i2} \quad \dots \quad \dot{h}_{im}]^T$, $\mathbf{q}_i(t)=[\mathbf{q}_{ip}^T \quad \mathbf{q}_{ia}^T]^T$ and $\mathbf{v}_i(t)=[v_{ip}^T \quad v_{ia}^T]^T$.

III. CONTROL SCHEME

Fig. 1, shows the Multi-layer control Scheme of the coordinated and cooperative control of mobile manipulators which is taken into account in this article.

Each layer works as an independent module, dealing with a specific part of the problem of coordinated and cooperative control, and such control scheme includes a basic structure defined by the control layer, the robots layer and the environment layer.

Above such layers are the planning layers, namely the off-line planning layer and the on-line planning layer. The first one is responsible for setting up the initial conditions, thus generating the trajectory of the object to be tracked, and for establishing the desired structure. The on-line planning layer is capable of changing the references in order to make the formation react to the environment, e. g., to modify the trajectory. The Control Layer is responsible for generating the control signals to be sent to the mobile manipulators working as a team in order to reach the desired values established by the planning layers. The Minimal Norm Kimematic Controller is responsible for generating the control signals to the end-effector of the robots. The robot layer, represents the mobile manipulators (or work team), and finally, the environment layer represents all objects surrounding the mobile manipulators, including the mobile manipulators themselves.

B. Kinematic Transformation

The proposed coordinated and cooperative control method consists of three mobile manipulators, and is based on creating a regular or irregular prism defined by the position of the end-effector of each mobile manipulator.

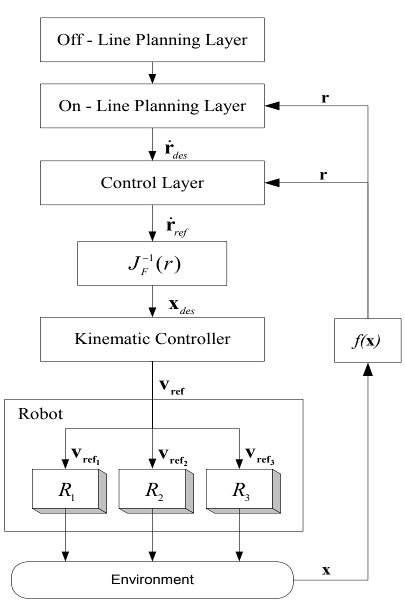

Fig. 1. Multi-layer control

The location of the upper side of the prism in the plane *X-Y* of the global framework is defined by $P_F=[x_F \quad y_F \quad \psi_F]$, where $(x_F \quad y_F)$ represents the position of its centroid, and ψ_F represents its orientation with respect to the global *Y*-axis. The structure shape of the prism (regular or irregular) is defined by $S_F=[p_F \quad q_F \quad \beta_F \quad z_{1F} \quad z_{2F} \quad z_{3F}]$, where, p_F represents the distance between $\mathbf{h}_1$ and $\mathbf{h}_2$, q_F the distance between $\mathbf{h}_1$ and $\mathbf{h}_3$, β_F the angle formed by $\mathbf{h}_2\hat{\mathbf{h}}_1\mathbf{h}_3$, and $(z_{1F} \quad z_{2F} \quad z_{3F})$ represents the height of the upper side of the prism. This situation is illustrated in Fig. 2.

The relationship between the prism pose-orientation-shape and the end-effector positions of the mobile manipulators, is given by the forward and inverse kinematics transformation, i.e., $\mathbf{r}=f(\mathbf{x})$ and $\mathbf{x}=f^{-1}(\mathbf{r})$, where $\mathbf{r}=[\mathbf{P_F} \quad \mathbf{S_F}]^T$ and $\mathbf{x}=[\mathbf{h}_1^T \quad \mathbf{h}_2^T \quad \mathbf{h}_3^T]^T$.

The forward kinematic transformation $f(.)$, as shown in Fig. 2, is given by

$$\mathbf{P_F}=\begin{bmatrix} \dfrac{x_1+x_2+x_3}{3} \\ \dfrac{y_1+y_2+y_3}{3} \\ \arctan\dfrac{\frac{2}{3}x_1-\frac{1}{3}(x_2+x_3)}{\frac{2}{3}y_1-\frac{1}{3}(y_2+y_3)} \end{bmatrix}^T ; \quad \mathbf{S_F}=\begin{bmatrix} \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \\ \sqrt{(x_1-x_3)^2+(y_1-y_3)^2} \\ \arccos\dfrac{p_F^2+q_F^2-r_F^2}{2p_Fq_F} \\ z_{1F} \\ z_{2F} \\ z_{3F} \end{bmatrix}^T$$

where, $r_F=\sqrt{(x_2-x_3)^2+(y_2-y_3)^2}$. In turn, for the inverse kinematic transformation $f^{-1}(.)$, two representations are possible, depending on the disposition of the mobile manipulators in the prism shape (clockwise or counter-clockwise). Such disposition can be referred to as $R_1R_2R_3$ or $R_1R_3R_2$ sequence. Considering the first possibility $\mathbf{x}=f^{-1}_{R_1R_2R_3}(\mathbf{r})$ is given by,

$$\mathbf{x}=\begin{bmatrix}\mathbf{h}_1\\ \mathbf{h}_2\\ \mathbf{h}_3\end{bmatrix}=\begin{bmatrix} x_F+\frac{2}{3}h_F\sin\psi_F \\ y_F+\frac{2}{3}h_F\cos\psi_F \\ z_{1F} \\ x_F+\frac{2}{3}h_F\sin\psi_F-p_F\sin(\alpha+\psi_F) \\ y_F+\frac{2}{3}h_F\cos\psi_F-p_F\cos(\alpha+\psi_F) \\ z_{2F} \\ x_F+\frac{2}{3}h_F\sin\psi_F+q_F\sin(\beta_F-\alpha-\psi_F) \\ y_F+\frac{2}{3}h_F\cos\psi_F-q_F\cos(\beta_F-\alpha-\psi_F) \\ z_{3F} \end{bmatrix}$$

where, $h_F=\sqrt{\frac{1}{2}\left(p_F^2+q_F^2-\frac{1}{2}r_F^2\right)}$ represents the distance between the end-effector $\mathbf{h}_1$ and the point in the middle of the segment $\overline{\mathbf{h}_2\mathbf{h}_3}$, passing through $(x_F \quad y_F)$, and $\alpha=\arccos\dfrac{p_F^2+h_F^2-\frac{1}{4}r_F^2}{2p_Fh_F}$.

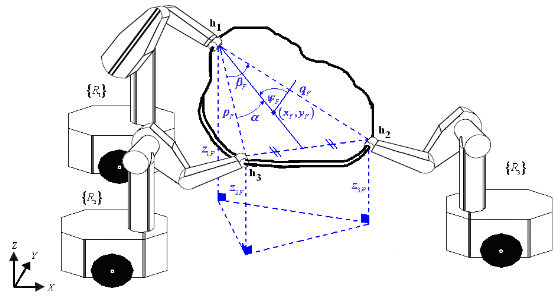

Fig. 2. Structure variables

Fig. 3 shows the control structure proposed in this paper for the coordinated cooperative control of mobile manipulators. Taking the time derivative of the forward and the inverse kinematics transformations we can obtain the relationship between the $\mathbf{x}$ and $\mathbf{r}$ velocities, represented by the Jacobian matriz $\mathbf{J_F}$, which is given by $\dot{\mathbf{r}} = \mathbf{J_F}(\mathbf{x})\dot{\mathbf{x}}$ and its inverse, $\dot{\mathbf{x}} = \mathbf{J}^{-1}_{\mathbf{F}}(\mathbf{r})\dot{\mathbf{r}}$. Where, $\mathbf{J_F}(\mathbf{x}) = \dfrac{\partial \mathbf{r}_{fx1}}{\partial \mathbf{x}_{ex1}}$ and $\mathbf{J}^{-1}_{\mathbf{F}}(\mathbf{r}) = \dfrac{\partial \mathbf{x}_{ex1}}{\partial \mathbf{r}_{fx1}}$ with $e, f = 1, 2.., 9$.

C. Structure Controller

The Control Layer receives from the upper layer the desired formation pose and shape $\mathbf{r}_d = [\mathbf{P}_{\mathbf{F}_d} \quad \mathbf{S}_{\mathbf{F}_d}]^T$ and its desired variations $\dot{\mathbf{r}}_d = [\dot{\mathbf{P}}_{\mathbf{F}_d} \quad \dot{\mathbf{S}}_{\mathbf{F}_d}]^T$. It generates the pose and shape variation references $\dot{\mathbf{r}}_{ref} = [\dot{\mathbf{P}}_{\mathbf{F}_{ref}} \quad \dot{\mathbf{S}}_{\mathbf{F}_{ref}}]^T$, where the subscripts *d* and *ref* represent the desired and reference signals, respectively. Defining the formation error as $\tilde{\mathbf{r}} = \mathbf{r}_d - \mathbf{r}$ the proposed formation control law is

$$\dot{\mathbf{r}}_{ref} = \dot{\mathbf{r}}_d + \mathbf{D}\tilde{\mathbf{r}} \tag{2}$$

where, $\mathbf{D}$ is a definite positive diagonal matrix that weigh the error vector $\tilde{\mathbf{r}}$.

Now, considering the inverse kinematics transformations and (2), it is obtained

$$\dot{\mathbf{x}}_{ref} = \mathbf{J}^{-1}_{\mathbf{F}}\dot{\mathbf{r}}_{ref} \tag{3}$$

where (3) represents the desired references vector for each ones end-effectors of the work team members.

Let us consider a difference $\delta_{\mathbf{r}}$ between the desired and the real formation variations, such as $\delta_{\mathbf{r}} = \dot{\mathbf{r}}_{ref} - \dot{\mathbf{r}}$. Then, the closed loop system equation can be written as

$$\dot{\tilde{\mathbf{r}}} + \mathbf{D}\tilde{\mathbf{r}} = \delta_{\mathbf{r}} \tag{4}$$

For the stability analysis the following Lyapunov candidate function is considered $V(\tilde{\mathbf{r}}) = \frac{1}{2}\tilde{\mathbf{r}}^{\mathbf{T}}\tilde{\mathbf{r}}$. Its time derivative on the trajectories of the system is

$$\dot{V}(\tilde{\mathbf{r}}) = \tilde{\mathbf{r}}^{\mathbf{T}}\delta_{\mathbf{r}} - \tilde{\mathbf{r}}^{\mathbf{T}}\mathbf{D}\tilde{\mathbf{r}}$$

A sufficient condition for $\dot{V}(\tilde{\mathbf{h}})$ to be negative definite is

$$\left|\tilde{\mathbf{r}}^{\mathbf{T}}\mathbf{D}\tilde{\mathbf{r}}\right| > \left|\tilde{\mathbf{r}}^{\mathbf{T}}\delta_{\mathbf{r}}\right|$$

$$\|\tilde{\mathbf{r}}\| > \frac{\|\delta_{\mathbf{r}}\|}{\lambda_{\min}(\mathbf{D})}$$

thus implying that the error $\tilde{\mathbf{r}}$ is bounded by,

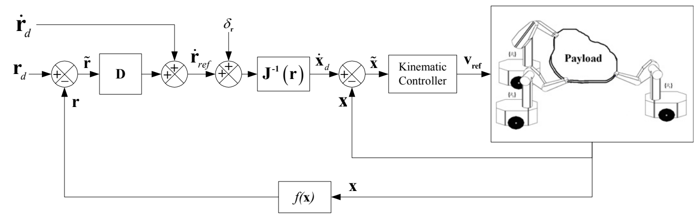

Fig. 3. Control system block diagram

$$\|\tilde{\mathbf{r}}\| \le \frac{\|\delta_{\mathbf{r}}\|}{\lambda_{\min}(\mathbf{D})} \tag{5}$$

where, $\lambda_{\min}(\mathbf{D})$ is the minimun eigenvalues of a matrix $\mathbf{D}$.

Assuming perfect velocity tracking $\delta_{\mathbf{r}} = 0$ then $\dot{V}(\tilde{\mathbf{r}}) < 0$ it can now be concluded that the error vector $\tilde{\mathbf{r}} \to 0$ asymptotically. On the other hand, if $\delta_{\mathbf{r}} \neq 0$ and represents a bounded velocity error not converging to zero, then the formation error $\|\tilde{\mathbf{r}}\|$ is ultimately bounded by $\|\delta_{\mathbf{r}}\| / \lambda_{\min}(\mathbf{D})$.

D. Kinematic Controller

To obtain the control actions $\mathbf{v}_i$ that correspond to the end-effector of the *i-th* mobile manipulator, the right pseudo-inverse Jacobian matrix $\mathbf{J}_i$ is used

$$\mathbf{v}_i = \mathbf{J}^{\#}_i\dot{\mathbf{h}}_i \tag{6}$$

where, $\mathbf{J}^{\#}_i = \mathbf{W}^{-i}_i\mathbf{J}^T_i(\mathbf{J}_i\mathbf{W}^{-1}_i\mathbf{J}^T_i)^{-1}$, being $\mathbf{W}_i$ a definite positive matrix that weighs the control actions of the system,

$$\mathbf{v}_i = \mathbf{W}^{-i}_i\mathbf{J}^T_i(\mathbf{J}_i\mathbf{W}^{-1}_i\mathbf{J}^T_i)^{-1}\dot{\mathbf{h}}_i$$

The following control law is proposed for the mobile manipulator system. It is based on a minimal norm solution, which means that, at any time, the mobile manipulator will attain its navigation target with the smallest number of possible movements,

$$\mathbf{v_{ci}} = \mathbf{J}^{\#}_i\left(\dot{\mathbf{h}}_{\mathbf{d}_i} + \mathbf{L}_{\mathbf{K}_i}\tanh\left(\mathbf{L}^{-1}_{\mathbf{K}_i}\mathbf{K}_i\,\tilde{\mathbf{h}}_i\right)\right) + \left(\mathbf{I}_i - \mathbf{J}^{\#}_i\mathbf{J}_i\right)\mathbf{L}_{\mathbf{D}_i}\tanh\left(\mathbf{L}^{-1}_{\mathbf{D}_i}\mathbf{D}_i\,\Lambda_i\right) \tag{7}$$

where, $\dot{\mathbf{h}}_{\mathbf{d}} = [\dot{h}_{1d} \quad \dot{h}_{2d} \quad \ldots \quad \dot{h}_{md}]^T$ is the desired velocities vector of the end-effector $\mathbf{h}_i$, $\tilde{\mathbf{h}}_i$ is the vector of control errors defined as $\tilde{\mathbf{h}}_i = [\mathbf{h}_{di} - \mathbf{h}_i]^T$, $\mathbf{K}_i$, $\mathbf{D}_i$, $\mathbf{L}_{\mathbf{K}_i}$ and $\mathbf{L}_{\mathbf{D}_i}$ are definite positive diagonal matrices that weigh the error vector $\tilde{\mathbf{h}}_i$ and vector Λ_i. The second term represents the projection on the null space of the Jacobian matrix $\mathbf{J}_i$, where Λ_i is an arbitrary vector which contains the velocities associated to the mobile manipulator. Therefore, any value given to Λ_i will have effects on the internal structure of the manipulator only, and will not affect the final control of the end-effector at all. By using this term, different secondary control objectives can be achieved effectively, as described below.

In order to include an analytical saturation of velocities in the mobile manipulator system, the tanh(.) function, which limits the error in $\tilde{\mathbf{h}}_i$ and the magnitude of vector Λ_i, is proposed. The expressions $\tanh(\mathbf{L}^{-1}_{\mathbf{K}_i}\mathbf{K}_i\,\tilde{\mathbf{h}}_i)$ and $\tanh(\mathbf{L}^{-1}_{\mathbf{D}_i}\mathbf{D}_i\,\Lambda_i)$ denote a component by component operation.

The behaviour of the control error of the end-effector $\tilde{\mathbf{h}}_i$ is now analyzed assuming perfect velocity tracking $\mathbf{v}_i \equiv \mathbf{v}_{ci}$. By substituting (7) in (6) it is obtained

$$\dot{\tilde{\mathbf{h}}}_i + \mathbf{L}_{\mathbf{K}_i} \tanh(\mathbf{L}^{-1}_{\mathbf{K}_i} \mathbf{K}_i \, \tilde{\mathbf{h}}_i) = \mathbf{0} \quad (8)$$

For the stability analysis the following Lyapunov candidate function is considered $V(\tilde{\mathbf{h}}_i) = \frac{1}{2}\tilde{\mathbf{h}}_i^{\mathrm{T}}\tilde{\mathbf{h}}_i$. Its time derivative on the trajectories of the system is,

$$\dot{V}(\tilde{\mathbf{h}}_i) = -\tilde{\mathbf{h}}_i^{\mathrm{T}}\mathbf{L}_{\mathbf{K}_i} \tanh(\mathbf{L}^{-1}_{\mathbf{K}_i} \mathbf{K}_i \, \tilde{\mathbf{h}}_i) < 0$$

which implies that the equilibrium on the closed-loop (8) is asymptotically stable, thus the position error of the end-effector verifies $\tilde{\mathbf{h}}_i(t) \to \mathbf{0}$ asymptotically.

E. Secondary control objectives

A mobile manipulator is defined as redundant because has more degrees of freedom than are required to achieve the desired end-effector motion. The redundancy of such mobile manipulators can be effectively used for the achievement of an additional performance such as: avoiding obstacles in the workspace and singular configuration, or to optimize various performance criterions. In this work both the manipulability control and obstacle avoidance for each mobile manpulator are considered.

Manipulability.- Much research has been done in order to quantify the ability of a manipulator, seen from the dynamics and kinematics point of view, [14,15]. A global representative measure of manipulation ability can be obtained by considering the volume of this ellipsoid which is proportional to the quantity w called the *manipulability measure,* $w = \sqrt{\det(\mathbf{J}(\mathbf{q})\mathbf{J}^T(\mathbf{q}))}$.

Obstacle Avoindance.- Fig. 4 shows the obstacle avoidance scheme for the *i-th* mobile manipulator. It is proposed a method in which the angular velocity and longitudinal velocity of the mobile platform will be affected by a fictitious repulsion force. Such force will depend on the angle α, which represents the incidence angle on the obstacle, and distance d to the obstacle. This way, the following control laws are proposed:

$$u_{iobs} = Z^{-1}\left(k_{uobs}(r-d)[\pi/2 - |\alpha|]\right)$$

$$\omega_{iobs} = Z^{-1}\left(k_{\omega obs}(r-d)sign(\alpha)[\pi/2 - |\alpha|]\right)$$

where, r is the radius which determines the distance at which the obstacle starts to be avoided, k_{uobs} and $k_{\omega obs}$ are positive adjustment gains, the sign function allows defining to which side the obstacle is to be avoided and Z represents the mechanical impedance characterizing the robot-environment interaction, which is considered as $Z = Is^2 + Bs + K$ with I, B and K being positive constants representing, respectively, the effect of the inertia, the damping and the elasticity.

Taking into account the maximum manipulability and obstacle avoidance, vector Λ_i is defined in this way,

$$\Lambda_i = \begin{bmatrix} -u_{iobs} & \omega_{iobs} & \theta_{i1d} - \theta_{i1} & \theta_{i2d} - \theta_{i2} & \ldots & \theta_{inad} - \theta_{ina} \end{bmatrix}^T$$

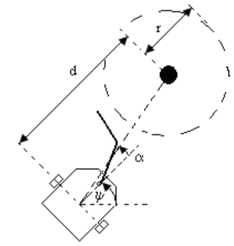

Fig. 4. Obstacle evasion scheme for the *i-th* mobile manipulator

where, $\theta_{ijd} - \theta_{ij}$ $(j = 1, 2, .., na)$ is the configuration error of the *i-th* mobile manipulator, so when minimizing this index, the manipulator joints will be pulled to the desired θ_{ijd} values that maximize manipulability.

IV. Simulation Results

In order to assess and discuss the performance of the proposed coordinated and cooperative controller, we developed a simulation platform for mobile manipulators with Matlab interface. This is an online simulator, which allows users to view three-dimensional environment navigation of mobile manipulators.

The simulation consists in a team of three robots tracking a desired trajectory while carrying a payload cooperatively. It is also considered the existence of four obstacles, which have a maximum height that does not interfere with the robotic arms. That is, the obstacles only affect the platform navigation. The positions of the arm joints that maximize the arms's manipulability are obtained through numeric simulation. This way, the joint angles should be, Robot_1: $\theta_{1d} = 0[\mathrm{rad}]$, $\theta_{2d} = -0.6065[\mathrm{rad}]$, $\theta_{3d} = 1.2346[\mathrm{rad}]$. Robot_2 and Robot_3: $\theta_{1d} = 0[\mathrm{rad}]$, $\theta_{2d} = 0.6065[\mathrm{rad}]$, and $\theta_{3d} = -1.2346[\mathrm{rad}]$.

The desired trajectory for the centroid is described by, $x_{F_d} = 0.2\,t + 3.56$, $y_{F_d} = 2\sin(0.1\,t) + 3$ and $\psi_{F_d} = \frac{\pi}{2} + \gamma$, where $\gamma = \arctan\left(\frac{dy_{Fd}}{dt} \Big/ \frac{dx_{Fd}}{dt}\right)$.

The desired virtual structure is described by, $p_{F_d} = 1.6\,[\mathrm{m}]$, $q_{F_d} = 1.8\,[\mathrm{m}]$ y $\beta_{F_d} = 1.165\,[\mathrm{rad}]$, $z_{1F_d} = 0.3\,[\mathrm{m}]$ y $z_{2F_d} = z_{3F_d} = 0.35\,[\mathrm{m}]$.

Fig. 5 shows the stroboscopic movement on the X-Y-Z space. It can be seen that the proposed controller works correctly, where three mobile manipulators work in coordinated and cooperative form, for transporting a common object. It can be noticed in Fig. 5 that there are three triangles of different colors representing the upper side of a virtual structure. The yellow triangle represents the shape-position of the virtual structure that describes the end-effector of the mobile manipulator robots, while the pink triangle represents the location and shape of the upper side of the desired virtual structure, and orange triangle indicates that both the position-shape of the desired structure and position-shape of the structure described by the robots are equal. Fig. 6 shows that the errors $\tilde{\mathbf{r}} \to 0$ asymptotically. Figs. 7 show the longitudinal velocity and the rotational velocity of the three mobile robots, respectively,

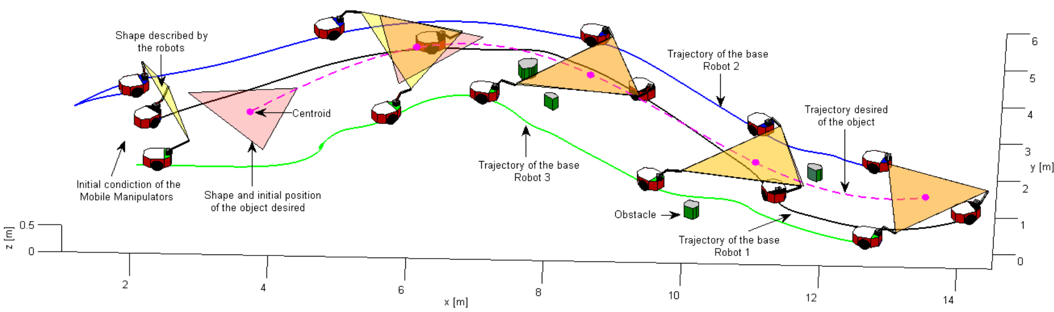

Fig. 5. Coordinated cooperative control of three mobile manipulators transporting a common object.

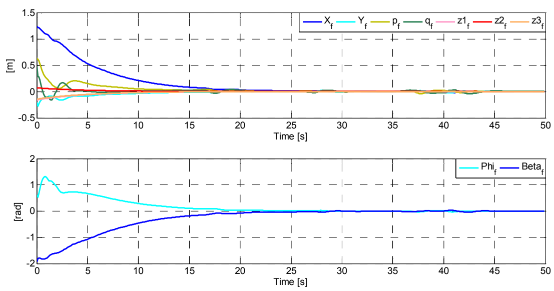

Fig. 6. Errors of control of shape-pose of the object desired

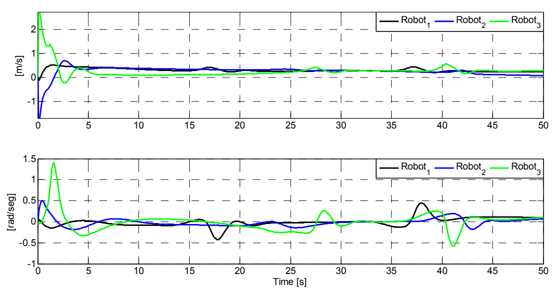

Fig. 7. Velocity of the three mobile robots

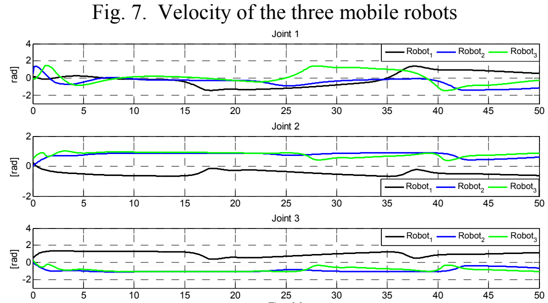

Fig. 8. Joint position of the three robot arms

while Fig. 8 shows the evolution of the joints of the robotic arms for each mobile manipulator.

V. CONCLUSIONS AND FUTURE WORK

A multi-layer scheme for coordinated cooperative control of mobile manipulators, for transporting a common object was presented in this paper. Stability of the system have been analytically proved. The results, which were obtained by simulation, show a good performance of the proposed control system. For future work, we will investigate the integration of the proposed method considering the adaptive dynamic compensation of each one of the robots.

REFERENCES

[1] 0. Khatib, K. Yokoi, K. Chang, D. Ruspini, R. Holmberg, and A. Casal "Vehicle/Arm Coordination and Multiple Mobile Manipulator", *Intelligent Robots and Systems*, pp. 546 - 553 vol.2, 1996.

[2] Fujii M., Inamura W., Murakami H. and Tanaka K., "Cooperative Control of Multiple Mobile Robots Transporting a Single Object with Loose Handling", *IEEE International Conference on Robotics and Biomimetics*, pp. 816-822, 2007.

[3] H. G. Tanner, S. Loizou, and K. J. Kyriakopoulos, "Nonholonomic navigation and control of cooperating mobile manipulators," *IEEE Trans. Robot. Autom.*, vol. 19, no. 1, pp. 53–64, Feb. 2003.

[4] Yasuhisa Hirata, Youhei Kume, Zhi-Dong Wang, Kszuhiro Kosiige, "Coordinated Motion Control of Multiple Mobile Manipulators based on Virtual 3-D Caster", *International ConFerence on Robotics,Intelligent Systcms and Signal Processing*, pp. 19-24, October 2003.

[5] Y. Hirata, Y. Kume, T. Sawada, Z.Wang, and K. Kosuge, "Handling of an object by multiple mobile manipulators in coordination based on casterlike dynamics," *in Proc. IEEE Int. Conf. Robot. Autom.*, vol. 26, pp. 807–812, 2004.

[6] Thomas G. Sugar and Vijay Kumar, "Control of Cooperating Mobile Manipulators", *IEEE Transactions on Robotics and Automation*, vol. 18, no. 1, pp. 94-103, 2002.

[7] Xin Chen and Yangmin Li, "Cooperative Transportation by Multiple Mobile Manipulators Using Adaptive NN Control", *International Joint Conference on Neural Networks,* pp. 4193-4200, 2006.

[8] Y. Yamamoto, Y. Hiyama, and A. Fujita, "Semi-autonomous reconfiguration of wheeled mobile robots in coordination," *in Proc. IEEE Int. Conf.Robot. Autom.*, pp. 3456–3461, 2004.

[9] Zhijun Lia, b, Shuzhi Sam Gea, , and Zhuping Wanga, "Robust adaptive control of coordinated multiple mobile manipulators", *Mechatronics Volume 18, Issues 5-6*, Pages 239-250, June 2008.

[10] Zhijun Li, Pey Y. Tao, Shuzhi S. Ge, Martin Adams, and Wijerupage S. W., "Robust Adaptive Control of Cooperating Mobile Manipulators With Relative Motion", *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 39, no. 1, pp. 103-116, 2009.

[11] Hao Su and Venkat Krovi, "Decentralized Dynamic Control of a Nonholonomic Mobile Manipulator Collective: a Simulation Study", *ASME Dynamic Systems and Control Conference,* pp.1-8, USA 2008.

[12] G. Antonelli, F. Arrichiello, and S. Chiaverini, "The Entrapment/Escorting Mission," *Robotics & Automation Magazine, IEEE*, vol. 15, no. 1, pp. 22–29, 2008.

[13] I. Mas, O. Petrovic, and C. Kitts, "Cluster space specification and control of a 3-robot mobile system," *IEEE International Conference on Robotics and Automation - ICRA*, pp. 3763–3768, 2008.

[14] T. Yoshikawa, "Dynamic manipulability of robot manipulators" *International Journal of Robotic Systems,* 2(2), pp. 113-124, 1985.

[15] B. Bayle and J.-Y. Fourquet, "Manipulability Analysis for Mobile Manipulators", *IEEE International Conference on Robots & Automation,* pp. 1251-1256, 2001.

[16] Andaluz V. Roberti Flavio, Carelli Ricardo, "Robust Control with Redundancy Resolution and Dynamic Compensation for Mobile Manipulators", *IEEE-ICIT Internacional Conference on Industrial Technology*, pp. 1449-1454, 2010.