

形式化方法 实验小作业5 ROCQ

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Lemma homework1

```
1 Lemma homework1: forall A, ~~~ A → ~ A.
2 Proof.
3   intros.
4   intro H0.
5   apply H.
6   intro H1.
7   apply H1.
8   exact H0.
9 Qed.
```

The screenshot shows the Coq proof assistant interface. On the left, the code up to step 8 is shown. On the right, the proof state is displayed:

Proof

Main 1 Shelved 0 Given up 0

Goal 1

A, B : Prop
T : Type
P : T → Prop

(1 / 1) —

forall A0 : Prop, ~ ~ ~ A0 → ~ A0

The screenshot shows the Coq proof assistant interface. On the left, the code up to step 18 is shown. On the right, the proof state is displayed:

Proof

Main 1 Shelved 0 Given up 0

Goal 1

A, B : Prop
T : Type
P : T → Prop
A0 : Prop
H : ~ ~ ~ A0

(1 / 1) —

~ A0

The screenshot shows the Coq proof assistant interface. On the left, the code up to step 19 is shown. On the right, the proof state is displayed:

Goal 1

A, B : Prop
T : Type
P : T → Prop
A0 : Prop
H : ~ ~ ~ A0
H0 : A0

(1 / 1) —

False

```

215
216 Lemma homework1: forall A, ~~~ A -> ~ A.
217 Proof.
218   intros.
219   intro H0.
220   apply H.
221   intro H1.
222   apply H1.
223   exact H0.
224 Qed.

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $A0 : \text{Prop}$
 $H : \sim \sim \sim A0$
 $H0 : A0$
(1 / 1) —————
 $\sim \sim A0$

```

215
216 Lemma homework1: forall A, ~~~ A -> ~ A.
217 Proof.
218   intros.
219   intro H0.
220   apply H.
221   intro H1.
222   apply H1.
223   exact H0.
224 Qed.
225

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $A0 : \text{Prop}$
 $H : \sim \sim \sim A0$
 $H0 : A0$
 $H1 : \sim A0$
(1 / 1) —————
False

```

215
216 Lemma homework1: forall A, ~~~ A -> ~ A.
217 Proof.
218   intros.
219   intro H0.
220   apply H.
221   intro H1.
222   apply H1.
223   exact H0.
224 Qed.
225

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $A0 : \text{Prop}$
 $H : \sim \sim \sim A0$
 $H0 : A0$
 $H1 : \sim A0$
(1 / 1) —————
A0

```

215
216 Lemma homework1: forall A, ~~~ A -> ~ A.
217 Proof.
218   intros.
219   intro H0.
220   apply H.
221   intro H1.
222   apply H1.
223   exact H0.
224 Qed.
225

```

⌚ There are no more subgoals
Messages

```

212 Variables A B : Prop.
213 Variable T : Type.
214 Variable P : T -> Prop .
215
216 Lemma homework1: forall A, ~~~ A -> ~ A.
217 Proof.
218   intros.
219   intro H0.
220   apply H.
221   intro H1.
222   apply H1.
223   exact H0.
224 Qed.

```

Proof
Not in proof mode
Messages
homework1 is defined

Lemma homework2

```
1 Lemma homework2: A \vee B → ~ (¬ A /\ ¬ B).
2 Proof.
3   intro Hor.
4   intro Hand.
5   destruct Hor as [HA | HB].
6   destruct Hand as [HNA HNB].
7   apply HNA.
8   exact HA.
9   destruct Hand as [HNA HNB].
10  apply HNB.
11  exact HB.
12 Qed.
```

```
226 Lemma homework2: A \vee B → ~ (¬ A /\ ¬ B).
227 Proof.
228   intro Hor.
229   intro Hand.
230   destruct Hor as [HA | HB].
231   destruct Hand as [HNA HNB].
232   apply HNA.
233   exact HA.
234   destruct Hand as [HNA HNB].
235   apply HNB.
236   exact HB.
237 Qed.
```

Goal 1
A, B : Prop
T : Type
P : T → Prop
(1 / 1) —
A \vee B → ~ (¬ A /\ ¬ B)

Messages

```
226 Lemma homework2: A \vee B → ~ (¬ A /\ ¬ B).
227 Proof.
228   intro Hor.
229   intro Hand.
230   destruct Hor as [HA | HB].
231   destruct Hand as [HNA HNB].
232   apply HNA.
233   exact HA.
234   destruct Hand as [HNA HNB].
235   apply HNB.
236   exact HB.
237 Qed.
```

Goal 1
A, B : Prop
T : Type
P : T → Prop
Hor : A \vee B
(1 / 1) —
~ (¬ A /\ ¬ B)

Messages

```
226 Lemma homework2: A \vee B → ~ (¬ A /\ ¬ B).
227 Proof.
228   intro Hor.
229   intro Hand.
230   destruct Hor as [HA | HB].
231   destruct Hand as [HNA HNB].
232   apply HNA.
233   exact HA.
234   destruct Hand as [HNA HNB].
235   apply HNB.
236   exact HB.
237 Qed.
```

Goal 1
A, B : Prop
T : Type
P : T → Prop
Hor : A \vee B
Hand : ~ A /\ ¬ B
(1 / 1) —
False

Messages

```

226 Lemma homework2: A \vee B -> ~ (~ A /\ ~ B).
227 Proof.
228 | intro Hor.
229 | intro Hand.
230 | destruct Hor as [HA | HB].
231 | destruct Hand as [HNA HNB].
232 | apply HNA.
233 | exact HA.
234 | destruct Hand as [HNA HNB].
235 | apply HNB.
236 | exact HB.
237 Qed.
238

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $HA : A$
 $Hand : \sim A /\ \sim B$

(1 / 2) —————

False

Goal 2
(2 / 2) —————

False

```

226 Lemma homework2: A \vee B -> ~ (~ A /\ ~ B).
227 Proof.
228 | intro Hor.
229 | intro Hand.
230 | destruct Hor as [HA | HB].
231 | destruct Hand as [HNA HNB].
232 | apply HNA.
233 | exact HA.
234 | destruct Hand as [HNA HNB].
235 | apply HNB.
236 | exact HB.
237 Qed.
238

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $HA : A$
 $HNA : \sim A$
 $HNB : \sim B$

(1 / 2) —————

False

Goal 2
(2 / 2) —————

False

```

226 Lemma homework2: A \vee B -> ~ (~ A /\ ~ B).
227 Proof.
228 | intro Hor.
229 | intro Hand.
230 | destruct Hor as [HA | HB].
231 | destruct Hand as [HNA HNB].
232 | apply HNA.
233 | exact HA.
234 | destruct Hand as [HNA HNB].
235 | apply HNB.
236 | exact HB.
237 Qed.
238

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $HA : A$
 $HNA : \sim A$
 $HNB : \sim B$

(1 / 2) —————

A

Goal 2
(2 / 2) —————

False

```

226 Lemma homework2: A \vee B -> ~ (~ A /\ ~ B).
227 Proof.
228 | intro Hor.
229 | intro Hand.
230 | destruct Hor as [HA | HB].
231 | destruct Hand as [HNA HNB].
232 | apply HNA.
233 | exact HA.
234 | destruct Hand as [HNA HNB].
235 | apply HNB.
236 | exact HB.
237 Qed.
238

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $HB : B$
 $Hand : \sim A /\ \sim B$

(1 / 1) —————

False

Messages

```

226 Lemma homework2: A \vee B -> \sim (\sim A /\ \sim B).
227 Proof.
228   intro Hor.
229   intro Hand.
230   destruct Hor as [HA | HB].
231   destruct Hand as [HNA HNB].
232   apply HNA.
233   exact HA.
234   destruct Hand as [HNA HNB].  

235   apply HNB.
236   exact HB.
237 Qed.

```

```

226 Lemma homework2: A \vee B -> \sim (\sim A /\ \sim B).
227 Proof.
228   intro Hor.
229   intro Hand.
230   destruct Hor as [HA | HB].
231   destruct Hand as [HNA HNB].
232   apply HNA.
233   exact HA.
234   destruct Hand as [HNA HNB].  

235   apply HNB.
236   exact HB.
237 Qed.
238

```

```

226 Lemma homework2: A \vee B -> \sim (\sim A /\ \sim B).
227 Proof.
228   intro Hor.
229   intro Hand.
230   destruct Hor as [HA | HB].
231   destruct Hand as [HNA HNB].
232   apply HNA.
233   exact HA.
234   destruct Hand as [HNA HNB].  

235   apply HNB.
236   exact HB.
237 Qed.
238

```

Lemma homework3

```

1 Lemma homework3: (\sim exists x, P x) -> forall x, \sim P x.
2 Proof.
3   intro HNexists.
4   intro x0.
5   intro HPx0.
6   apply HNexists.
7   exists x0.
8   exact HPx0.
9 Qed.

```

```

239 Lemma homework3: (~ exists x, P x) -> forall x, ~ P x.
240 Proof.
241   intro HNexists.
242   intro x0.
243   intro HPx0.
244   apply HNexists.
245   exists x0.
246   exact HPx0.
247 Qed.

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
(1 / 1) —
 $\sim (\exists x : T, P x) \rightarrow \forall x : T, \sim P x$

```

239 Lemma homework3: (~ exists x, P x) -> forall x, ~ P x.
240 Proof.
241   intro HNexists.
242   intro x0.
243   intro HPx0.
244   apply HNexists.
245   exists x0.
246   exact HPx0.
247 Qed.

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $\text{HNexists} : \sim (\exists x : T, P x)$
(1 / 1) —
 $\forall x : T, \sim P x$

```

239 Lemma homework3: (~ exists x, P x) -> forall x, ~ P x.
240 Proof.
241   intro HNexists.
242   intro x0.|.
243   intro HPx0.
244   apply HNexists.
245   exists x0.
246   exact HPx0.
247 Qed.

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $\text{HNexists} : \sim (\exists x : T, P x)$
 $x0 : T$
(1 / 1) —
 $\sim P x0$

```

239 Lemma homework3: (~ exists x, P x) -> forall x, ~ P x.
240 Proof.
241   intro HNexists.
242   intro x0.
243   intro HPx0.|.
244   apply HNexists.
245   exists x0.
246   exact HPx0.
247 Qed.

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $\text{HNexists} : \sim (\exists x : T, P x)$
 $x0 : T$
 $\text{HPx0} : P x0$
(1 / 1) —
 False

```

239 Lemma homework3: (~ exists x, P x) -> forall x, ~ P x.
240 Proof.
241   intro HNexists.
242   intro x0.
243   intro HPx0.
244   apply HNexists.|.
245   exists x0.
246   exact HPx0.
247 Qed.

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $\text{HNexists} : \sim (\exists x : T, P x)$
 $x0 : T$
 $\text{HPx0} : P x0$
(1 / 1) —
 $\exists x : T, P x$

```

239 Lemma homework3: (~ exists x, P x) -> forall x, ~ P x.
240 Proof.
241   intro HNexists.
242   intro x0.
243   intro HPx0.
244   apply HNexists.
245   exists x0.|.
246   exact HPx0.
247 Qed.

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $\text{HNexists} : \sim (\exists x : T, P x)$
 $x0 : T$
 $\text{HPx0} : P x0$
(1 / 1) —
 $P x0$

```

239 Lemma homework3: (~ exists x, P x) -> forall x, ~ P x.
240 Proof.
241   intro HNexists.
242   intro x0.
243   intro HPx0.
244   apply HNexists.
245   exists x0.
246   exact HPx0.
247 Qed.

```

Goal 1
 $\textcircled{S} \text{ There are no more subgoals}$

Messages

```

239 Lemma homework3: ( $\exists$  x, P x)  $\rightarrow$   $\forall$  x,  $\neg$  P x.
240 Proof.
241   intro HExists.
242   intro x0.
243   intro HPx0.
244   apply HExists.
245   exists x0.
246   exact HPx0.
247 Qed.

```

homework3 is defined

Lemma homework4

```

1 Lemma homework4: A  $\rightarrow$   $\neg$ A.
2 Proof.
3   intro HA.
4   intro HNA.
5   apply HNA.
6   exact HA.
7 Qed.

```

```

249 Lemma homework4: A  $\rightarrow$   $\neg\neg$ A.
250 Proof.
251   intro HA.
252   intro HNA.
253   apply HNA.
254   exact HA.
255 Qed.

```

Goal 1
A, B : Prop
T : Type
P : T \rightarrow Prop
(1 / 1) —————
A \rightarrow $\neg\neg$ A

```

249 Lemma homework4: A  $\rightarrow$   $\neg\neg$ A.
250 Proof.
251   intro HA.
252   intro HNA.
253   apply HNA.
254   exact HA.
255 Qed.
256

```

Goal 1
A, B : Prop
T : Type
P : T \rightarrow Prop
HA : A
(1 / 1) —————
 $\neg\neg$ A

```

249 Lemma homework4: A  $\rightarrow$   $\neg\neg$ A.
250 Proof.
251   intro HA.
252   intro HNA.
253   apply HNA.
254   exact HA.
255 Qed.
256

```

Goal 1
A, B : Prop
T : Type
P : T \rightarrow Prop
HA : A
HNA : \neg A
(1 / 1) —————
False

```

249 Lemma homework4: A  $\rightarrow$   $\neg\neg$ A.
250 Proof.
251   intro HA.
252   intro HNA.
253   apply HNA.
254   exact HA.
255 Qed.
256

```

Goal 1
A, B : Prop
T : Type
P : T \rightarrow Prop
HA : A
HNA : \neg A
(1 / 1) —————
A

```

249 Lemma homework4: A -> ~A.
250 Proof.
251   intro HA.
252   intro HNA.
253   apply HNA.
254   exact HA.
255 Qed.

```

Messages

There are no more subgoals

```

249 Lemma homework4: A -> ~A.
250 Proof.
251   intro HA.
252   intro HNA.
253   apply HNA.
254   exact HA.
255 Qed.

```

Messages

homework4 is defined

Lemma homework5

```

1 Lemma homework5: (A \/\ ~A) -> (~A -> A).
2 Proof.
3   intro Hor.
4   intro HNNA.
5   destruct Hor as [HA | HNA].
6   - exact HA.
7   - exfalso. apply HNNA. exact HNA.
8 Qed.

```

```

257 Lemma homework5: (A \/\ ~A) -> (~A -> A).
258 Proof.
259   intro Hor.
260   intro HNNA.
261   destruct Hor as [HA | HNA].
262   - exact HA.
263   - exfalso. apply HNNA. exact HNA.
264 Qed.

```

Goal 1

A, B : Prop
T : Type
P : T -> Prop
(1 / 1) —
A \/\ ~A -> ~ ~A -> A

```

257 Lemma homework5: (A \/\ ~A) -> (~A -> A).
258 Proof.
259   intro Hor.
260   intro HNNA.
261   destruct Hor as [HA | HNA].
262   - exact HA.
263   - exfalso. apply HNNA. exact HNA.
264 Qed.

```

Goal 1

A, B : Prop
T : Type
P : T -> Prop
Hor : A \/\ ~A
(1 / 1) —
~ ~A -> A

```

257 Lemma homework5: (A \/\ ~A) -> (~A -> A).
258 Proof.
259   intro Hor.
260   intro HNNA.
261   destruct Hor as [HA | HNA].
262   - exact HA.
263   - exfalso. apply HNNA. exact HNA.
264 Qed.

```

Goal 1

A, B : Prop
T : Type
P : T -> Prop
Hor : A \/\ ~A
HNNA : ~ ~A
(1 / 1) —
A

```

257 Lemma homework5: (A \vee ~A) -> (~~A -> A).
258 Proof.
259   intro Hor.
260   intro HNNA.
261   destruct Hor as [HA | HNA].
262   - exact HA.
263   - exfalso. apply HNNA. exact HNA.
264 Qed.
265
266 End Homework.
267
268

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $HNA : \sim A$
 $\text{HNNA} : \sim \sim A$
 $(1 / 2)$ —————
 A

Goal 2
 $(2 / 2)$ —————
 A

```

257 Lemma homework5: (A \vee ~A) -> (~~A -> A).
258 Proof.
259   intro Hor.
260   intro HNNA.
261   destruct Hor as [HA | HNA].
262   - exact HA.
263   - exfalso. apply HNNA. exact HNA.
264 Qed.

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $HA : A$
 $\text{HNNA} : \sim \sim A$
 $(1 / 1)$ —————
 A

```

257 Lemma homework5: (A \vee ~A) -> (~~A -> A).
258 Proof.
259   intro Hor.
260   intro HNNA.
261   destruct Hor as [HA | HNA].
262   - exact HA.
263   - exfalso. apply HNNA. exact HNA.
264 Qed.

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $HA : A$
 $\text{HNNA} : \sim \sim A$
 $(1 / 1)$ —————
 A

Messages

```

257 Lemma homework5: (A \vee ~A) -> (~~A -> A).
258 Proof.
259   intro Hor.
260   intro HNNA.
261   destruct Hor as [HA | HNA].
262   - exact HA.
263   - exfalso. apply HNNA. exact HNA.
264 Qed.
265

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $HNA : \sim A$
 $\text{HNNA} : \sim \sim A$
 $(1 / 1)$ —————
 A

```

257 Lemma homework5: (A \vee ~A) -> (~~A -> A).
258 Proof.
259   intro Hor.
260   intro HNNA.
261   destruct Hor as [HA | HNA].
262   - exact HA.
263   - exfalso. apply HNNA. exact HNA.
264 Qed.
265

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $HNA : \sim A$
 $\text{HNNA} : \sim \sim A$
 $(1 / 1)$ —————
 False

```

257 Lemma homework5: (A \vee ~A) -> (~~A -> A).
258 Proof.
259   intro Hor.
260   intro HNNA.
261   destruct Hor as [HA | HNA].
262   - exact HA.
263   - exfalso. apply HNNA. exact HNA.
264 Qed.
265

```

Goal 1
 $A, B : \text{Prop}$
 $T : \text{Type}$
 $P : T \rightarrow \text{Prop}$
 $HNA : \sim A$
 $\text{HNNA} : \sim \sim A$
 $(1 / 1)$ —————
 $\sim A$

```
256 Lemma homework5: (A \/\ ~A) -> (~~A -> A).  
257 Proof.  
258   intro Hor.  
259   intro HNNA.  
260   destruct Hor as [HA | HNA].  
261   - exact HA.  
262   - exfalso. apply HNNA. exact HNA.  
263  
264 Qed.  
265
```

The screenshot shows a proof assistant interface with a code editor on the left and a message panel on the right. The code editor contains a Coq script for proving De Morgan's law. The message panel displays a success message: "There are no more subgoals".

```
257 Lemma homework5: (A \/\ ~A) -> (~~A -> A).  
258 Proof.  
259   intro Hor.  
260   intro HNNA.  
261   destruct Hor as [HA | HNA].  
262   - exact HA.  
263   - exfalso. apply HNNA. exact HNA.  
264 Qed.  
265
```

The screenshot shows a proof assistant interface with a code editor on the left and a message panel on the right. The code editor contains the same Coq script as the previous screenshot. The message panel displays a message: "homework5 is defined".