4.2.2.

1\. 1. Maximize $z = 1 + 2x_1 + 3x_2 + 4x_3$ satisfying the constraints:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &\leq 10 \\ x_1 - x_3 &\leq 3 \\ -x_2 + 2x_3 &\leq 5 \end{aligned}$$

where $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

slack form

$$y_1 = 10 - x_1 - 2x_2 - 3x_3$$

$$y_2 = 3 - x_1 + x_3$$

$$y_3 = 5 + x_2 - 2x_3$$

$$z = 1 + 2x_1 + 3x_2 + 4x_3$$

Now increase x_1 as much as possible while $x_2 = x_3 = 0$

$$y_1 = 10 - x_1 \geqslant 0$$

$$y_2 = 3 - x_1 \geqslant 0 \quad \Rightarrow 0 \leq x_1 \leq 3$$

$$y_3 = 5 \geqslant 0$$

The highest allowed value for $x_1 = 3$, then $y_2 = 0$

Now swap x_1 and y_2

$$x_1 = 3 - y_2 + x_3$$

$$y_1 = 7 + y_2 - 2x_2 - 4x_3$$

$$y_3 = 5 + x_2 - 2x_3$$

$$z = 7 - 2y_2 + 3x_2 + 6x_3$$

Now increase x_2 while $y_2 = x_3 = 0$

$$x_1 = 3 \geq 0$$
$$y_1 = 7 - 2x_2 \geq 0 \quad \Rightarrow \quad 0 \leq x_2 \leq \frac{7}{2}$$
$$y_3 = 5 + x_2 \geq 0$$

The highest allowed value for x_2 is $\frac{7}{2}$, then $y_1 = 0$

Now swap y_1 and x_2

$$x_2 = \frac{7}{2} - \frac{1}{2}y_1 + \frac{1}{2}y_2 - 2x_3$$
$$x_1 = 3 - y_2 + x_3$$
$$y_3 = \frac{17}{2} - \frac{1}{2}y_1 + \frac{1}{2}y_2 - 4x_3$$

$$Z = \frac{35}{2} - \frac{3}{2}y_1 - \frac{1}{2}y_2$$

无可增的 non-basic variable

故 Z 最大为 $\frac{35}{2}$，当 $x_1 = 3$，$x_2 = \frac{7}{2}$, $x_3 = 0$ 时取得

2. Fine values $x, y \geq 0$ satisfying

2.

$$
\begin{aligned}
x - y &\leq -3 \\
2x + y &\leq 7 \\
-x - 2y &\leq -8
\end{aligned}
$$

Introduce z for maximizing $-z$ $(z \geq 0)$

$$
\begin{aligned}
x - y - z &\leq -3 \\
2x + y - z &\leq 7 \\
-x - 2y - z &\leq -8
\end{aligned}
$$

Slack form

$$
\begin{aligned}
y_1 &= -3 - x + y + z \\
y_2 &= 7 - 2x - y + z \\
y_3 &= -8 + x + 2y + z
\end{aligned}
$$

$b_3 = -8$ is the most negative

Swap y_3 and z

$$
\begin{aligned}
z &= 8 - x - 2y + y_3 \\
y_1 &= 5 - 2x - y + y_3 \\
y_2 &= 15 - 3x - 3y + y_3
\end{aligned}
$$

此时可使用 Simplex Method

最大化 $-8 + x + 2y - y_3$

Now increase x while $y = y_3 = 0$

$$z = 8 - x \geq 0$$

$$y_1 = 5 - 2x \geq 0 \quad \Rightarrow \quad 0 \leq x \leq \frac{5}{2}$$

$$y_2 = 15 - 3x \geq 0$$

当 $x = \frac{5}{2}$ 时，$y_1 = 0$

Swap x and y_1

$$x = \frac{5}{2} - \frac{1}{2}y_1 - \frac{1}{2}y + \frac{1}{2}y_3$$

$$z = \frac{11}{2} - \frac{3}{2}y + \frac{1}{2}y_3 + \frac{1}{2}y_1$$

$$y_2 = \frac{15}{2} - \frac{3}{2}y + \frac{3}{2}y_1 - \frac{1}{2}y_3$$

最大化 $-\frac{11}{2} + \frac{3}{2}y - \frac{1}{2}y_3 - \frac{1}{2}y_1$

Now increase y while $y_3 = y_1 = 0$

$$x = \frac{5}{2} - \frac{1}{2}y \geq 0$$

$$z = \frac{11}{2} - \frac{3}{2}y \geq 0 \quad \Rightarrow \quad y \leq \frac{11}{3}$$

$$y_2 = \frac{15}{2} - \frac{3}{2}y \geq 0$$

当 $y = \frac{11}{3}$ 时，$z = 0$

此时 $-z = 0$ 为最优

$$x = \frac{5}{2} - \frac{1}{2}\cdot 0 - \frac{1}{2}\cdot\frac{11}{3} + \frac{1}{2}\cdot 0$$

$= \frac{2}{3}$，satisfiable 且解为 $x = \frac{2}{3}$，$y = \frac{11}{3}$

4.2.3.

1. Construct a formula in CNF based on the following truth table:

1.

p	q	r	ϕ	
1	1	1	0	√
1	1	0	1	
1	0	1	0	√
0	1	1	1	
1	0	0	0	√
0	1	0	0	√
0	0	1	1	
0	0	0	0	√

有5个0，构造5个clause

$$(\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge$$

$$(p \vee \neg q \vee r) \wedge (p \vee q \vee r)$$

2. Apply algorithm HORN to each of these Horn formulas:

1. $(p \wedge q \wedge s \to \perp) \wedge (q \wedge r \to p) \wedge (\top \to s)$
2. $(p_5 \to p_{11}) \wedge (p_2 \wedge p_5 \to p_{13}) \wedge (\top \to p_5) \wedge (p_5 \wedge p_{11} \to \perp)$
3. $(\top \to q) \wedge (\top \to s) \wedge (w \to \perp) \wedge (p \wedge q \wedge s \to \perp) \wedge (v \to s) \wedge (\top \to s) \wedge (r \to p)$

① Marked: $\top$ s return 'satisfiable'

② Marked: $\top$ p_5 p_{11} $\perp$ return 'unsatisfiable'

③ Marked: $\top$ q s return 'satisfiable'