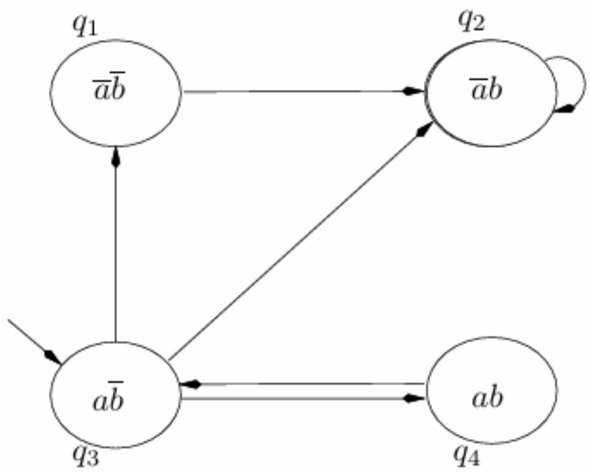


Figure 3.39. A model $\mathcal{M}$.

2. Consider the system of Figure 3.39. For each of the formulas ϕ:
   (a) $\mathrm{G}\, a$
   (b) $a \mathrel{\mathrm{U}} b$
   (c) $a \mathrel{\mathrm{U}} \mathrm{X}\,(a \wedge \neg b)$
   (d) $\mathrm{X}\, \neg b \wedge \mathrm{G}\,(\neg a \vee \neg b)$
   (e) $\mathrm{X}\,(a \wedge b) \wedge \mathrm{F}\,(\neg a \wedge \neg b)$
      (i) Find a path from the initial state q_3 which satisfies ϕ.
      (ii) Determine whether $\mathcal{M}, q_3 \vDash \phi$.

LTL

2、(a) (i) $q_3 \to q_4 \to q_3 \to q_4 \to \cdots$

(ii) $\mathcal{M}, q_3 \vDash \mathrm{G}\, a$ does not hold

进入 q_1 或 q_2 则不满足

(b) (i) $q_3 \to q_2 \to q_2 \to q_2 \to \cdots$

(ii) $\mathcal{M}, q_3 \vDash a \mathrel{\mathrm{U}} b$ does not hold

反例：$q_3 \to q_1 \to q_2 \to \cdots$

(c) (i) $q_3 \to q_4 \to q_3 \to \cdots$

(ii) $\mathcal{M}, q_3 \vDash a \mathrel{\mathrm{U}} \mathrm{X}\,(a \wedge \neg b)$ does not hold

需至少隔一步后再次回到 q_3

若 $q_3 \to q_2 \to q_2 \to \cdots$ 则不成立

(d) (i) $q_3 \to q_1 \to q_2 \to q_2 \to \cdots$

(ii) $\mathcal{M}, q_3 \vDash X \neg b \wedge G(\neg a \vee \neg b)$ does not hold

反例: $q_3 \to q_4 \to \cdots$

(e) (i) $q_3 \to q_4 \to q_3 \to q_1 \to \cdots$

(ii) $\mathcal{M}, q_3 \vDash X(a \wedge b) \wedge F(\neg a \wedge \neg b)$ does not hold

反例: $q_3 \to q_1 \to \cdots$

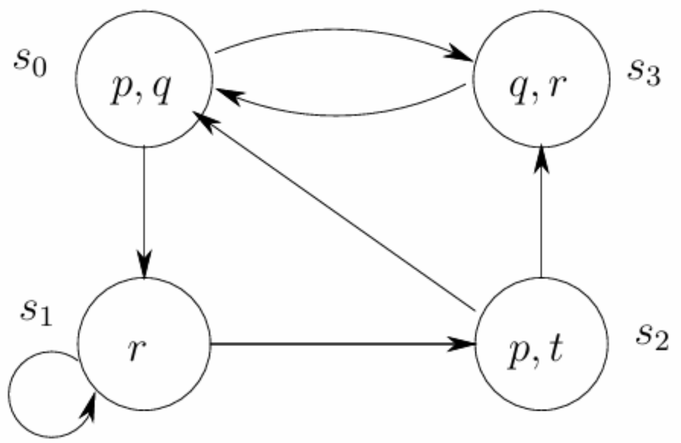


Figure 3.41. Another model with four states.

8. Consider the model $\mathcal{M}$ in Figure 3.41. Check whether $\mathcal{M}, s_0 \vDash \phi$ and $\mathcal{M}, s_2 \vDash \phi$ hold for the CTL formulas ϕ:
   (a) $\mathrm{AF}\, q$
   (b) $\mathrm{AG}\, (\mathrm{EF}\, (p \vee r))$
   (c) $\mathrm{EX}\, (\mathrm{EX}\, r)$
   (d) $\mathrm{AG}\, (\mathrm{AF}\, q)$.

CTL

8. (a) $\mathcal{M}, s_0 \vDash AF\, q$ holds

s_0本身包含q, all paths starting from s_0 hit a q.

$\mathcal{M}, s_2 \vDash AF\, q$ holds

s_2进入s_0或s_3, 二者都含q

(b) $\mathcal{M}, s_0 \models AG(EF(p \vee r))$ holds

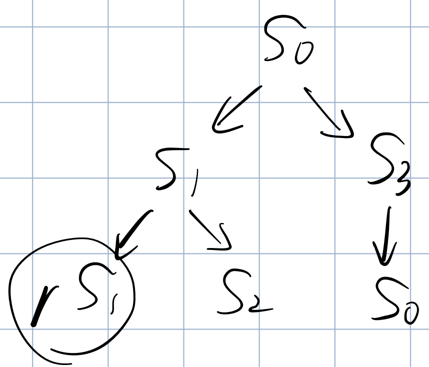
$p \vee r$ 对每个状态都成立

则上式必定成立

$\mathcal{M}, s_2 \models AG(EF(p \vee r))$ holds

同上理

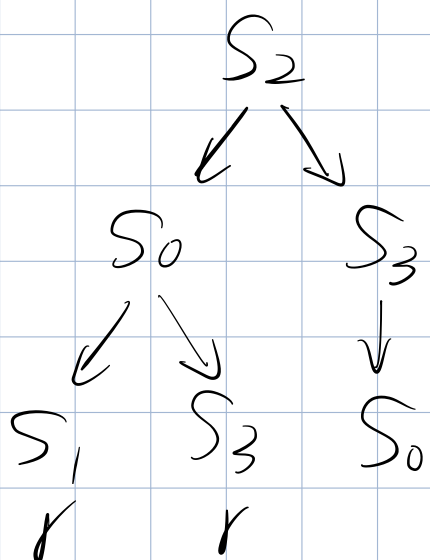
(c) $\mathcal{M}, s_0 \models EX(EX\, r)$ holds



存在 $s_0 \to s_1 \to s_1 \to \cdots$ 这条 path

满足

$\mathcal{M}, s_2 \models EX(EX\, r)$ holds



同上理

(d) $\mathcal{M}, s_0 \models AG(AF\, q)$ does not hold

$\mathcal{M}, s_2 \models AG(AF\, q)$ does not hold

即每条path上的每个状态 s_r 都要满足 $M, s_r \models AF\,q$

反例：$\cdots \to s_1 \to s_1 \to s_1 \to \cdots$（一直留在 s_1）

此条路径上的这几个 s_1 状态及后续的所有 s_1 状态都不满足条件.