a. 1. $((p \to q) \to p) \to p$

p	$\to$	q	$\to$	p	$\to$	p
T	T	T	T	T	T	T
T	F	F	T	T	T	T
F	T	T	F	F	T	F
F	T	F	F	F	T	F

2. $(p \wedge q) \to (p \vee q)$

p	$\wedge$	q	$\to$	p	$\vee$	q
T	T	T	T	T	T	T
T	F	F	T	T	T	F
F	F	T	T	F	T	T
F	F	F	T	F	F	F

3. $(p \to q) \vee (p \to \neg q)$

p	$\to$	q	$\vee$	p	$\to$	$\neg$	q
T	T	T	T	T	F	F	T
T	F	F	T	T	T	T	F
F	T	T	T	F	T	F	T
F	T	F	T	F	T	T	F

4. $((p \vee q) \to r) \to ((p \to r) \vee (q \to r))$

p	$\vee$	q	$\to$	r	$\to$	p	$\to$	r	$\vee$	q	$\to$	r
T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	T	F	F	T	T	F	F	F	T	F	F
T	T	F	T	T	T	T	T	T	T	F	T	T
T	T	F	F	F	T	T	F	F	T	F	T	F

F	T	T	T	T	T	F	T	T	T	T	T	T
F	T	T	F	F	T	F	T	F	T	T	F	F
F	F	F	T	T	T	F	T	T	T	F	T	T
F	F	F	T	F	T	F	T	F	T	F	T	F

1.

(a) $\forall x(P(x) \to A(m,x))$

(b) $\exists x(P(x) \wedge A(x,m))$

(c) $A(m,m)$

(d) $\neg\exists x(S(x) \to \forall y(L(y) \wedge B(x,y)))$

(e) $\neg\exists y(L(y) \to \forall x(S(x) \wedge B(x,y)))$

(f) $\neg\exists y(L(y) \to \exists x(S(x) \wedge B(x,y)))$

2. (a) 对于 $P(x,z) \to P(z,x)$

当 $x=0$ 时，$P(x,z) \to T$

$P(z,x) \to \perp$

故上式恒假，M不满足 ϕ

(b) 对于 $P(x,z) \to P(z,x)$

当 $x\neq 0$ 时，$P(x,z) \to T$

$P(z,x) \to \perp$

故上式恒假，$\mathcal{M}$不满足ϕ

(c) $\forall x$，取 $x=y=z$，则 $P(x,y)\wedge P(z,y)\wedge(P(x,z)\rightarrow P(z,x))$ 为真

故$\mathcal{M}$满足ϕ

3.1

1. Prove the validity of the following sequents:

1. $(p\wedge q)\wedge r, s\wedge t\vdash q\wedge s$
2. $q\rightarrow r\vdash(p\rightarrow q)\rightarrow(p\rightarrow r)$
3. $p\wedge\neg p\vdash\neg(r\rightarrow q)\wedge(r\rightarrow q)$

①

1	$(p\wedge q)\wedge r$	premise
2	$s\wedge t$	premise
3	$p\wedge q$	$\wedge e_1$ 1
4	q	$\wedge e_2$ 3
5	s	$\wedge e_1$ 2
6	$q\wedge s$	$\wedge i$ 4,5

②

1	$q\rightarrow r$	premise
2	$p\rightarrow q$	assumption
3	p	assumption
4	q	$\rightarrow e$ 2,3
5	r	$\rightarrow e$ 1,4
6	$p\rightarrow r$	$\rightarrow i$ 3-5

7	$(p \to q) \to (p \to r)$	$\to i$ 2-6

③

1	$p \land \neg p$	premise
2	p	$\land e_1$ 1
3	$\neg p$	$\land e_2$ 1
4	$\bot$	$\neg e$ 2,3
5	$r \to q$	$\bot e$ 4
6	$\neg(r \to q)$	$\bot e$ 4
7	$\neg(r \to q) \land (r \to q)$	$\land i$ 6,5

2. Prove the validity of the following sequents in predicate logic, where P, and Q have arity 1, and S has arity 0 (a 'propositional atom'):

1. $\exists x(S \to Q(x)) \vdash S \to \exists x\, Q(x)$
2. $\forall x P(x) \to S \vdash \exists x(P(x) \to S)$
3. $\neg\forall x \neg P(x) \vdash \exists x P(x)$

①

1		$\exists x(S \to Q(x))$	premise
2	x_0	$S \to Q(x_0)$	assumption
3		S	assumption
4		$Q(x_0)$	$\to e$ 2,3
5		$\exists x\, Q(x)$	$\exists x\, i$ 4
6		$S \to \exists x\, Q(x)$	$\to i$ 3-5
7		$S \to \exists x\, Q(x)$	$\exists x\, e$ 1, 2-6

②

1		$\forall x P(x) \to S$	premise
2	x_0	$P(x_0) \to S$	$\forall x$ e 1
3		$\exists x (P(x) \to S)$	$\exists x$ i 2

③

1		$\neg \forall x \neg P(x)$	premise
2	x_0	$\neg\neg P(x_0)$	$\forall x$ e 1
3		$P(x_0)$	$\neg\neg$e 2
4		$\exists x P(x)$	$\exists x$ i 3