

4.

作业 4: Practice in program, compute

- $post^\#(x := x + 1, x < 100)$
- $post^\#(x := x + 1, x < 100 \wedge y = 100)$

Example

```

1 x:=0; y:= 0;
2 while (x<100)
3 {
4     x := x+1;
5     y := y+1;
6 }
7 assert(y = 100);

```

- Predicate set  $\mathcal{P} = \{x < 100, y = 100\}$

$$\textcircled{1} \quad sp(x := x+1, x < 100) \Leftrightarrow \exists x_0. x_0 < 100 \wedge x = x_0 + 1$$

$$\text{可知 } x < 101 \not\Rightarrow x < 100$$

$$x < 101 \not\Rightarrow x \geq 100 \quad \text{故 } b_1 = *$$

$$\text{对于 } p_2 \text{ 无约束, 故 } b_2 = *$$

$$\text{故 } post^\#(x := x+1, x < 100) = [*, *]$$

$$\textcircled{2} \quad sp(x := x+1, x < 100 \wedge y = 100)$$

$$\Leftrightarrow \exists x_0, y_0. x_0 < 100 \wedge y_0 = 100 \wedge x = x_0 + 1 \wedge y = 100$$

$$\Leftrightarrow x < 101 \wedge y = 100$$

$$\text{同 } \textcircled{1} \text{ 理 } b_1 = *$$

$$b_2 = 1$$

$$\text{故 } post^\#(x := x+1, x < 100 \wedge y = 100) = [*, 1]$$

## Program Abstraction

CFA (Predicate Abstraction)

CFA (Original)

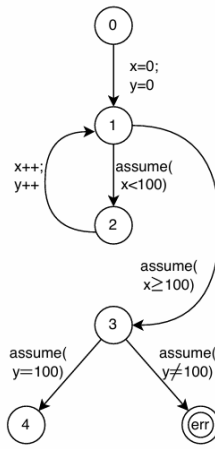
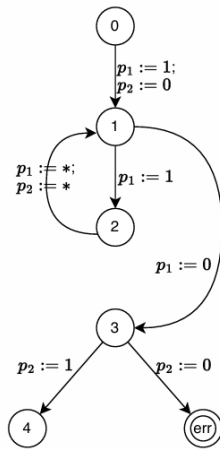
- state space

- From  $6 * 2^{32} * 2^{32}$  to  $6 * 2 * 2$

问: How to translate the program into a boolean program?

作业 5: Translate statements in CFA

- $1 \rightarrow 2$
- $2 \rightarrow 1$



## Example

```

1 x:=0; y:= 0;
2 while (x<100)
3 {
4     x := x+1;
5     y := y+1;
6 }
7 assert(y = 100);
    
```

- Predicate set  $\mathcal{P} = \{\underbrace{x < 100}_{p_1}, \underbrace{y = 100}_{p_2}\}$

①  $1 \rightarrow 2$

$S: \text{assume}(x < 100)$

$$\text{wp}(\text{assume}(x < 100), x < 100) \Leftrightarrow T$$

$$\text{wp}(\text{assume}(x < 100), y = 100) \Leftrightarrow y = 100, b_2 \Rightarrow (y = 100)$$

$\neg b_2 \Rightarrow \neg (y = 100)$   
无需翻译

$b_1 := \text{true}$

②  $2 \rightarrow 1$

$S: x := x+1; y := y+1$

对  $x := x+1$

$$\text{wp}(x := x+1, x < 100) \Leftrightarrow x < 99, b_1 \neq (x < 99)$$

$$\text{wp}(x := x+1, y = 100) \Leftrightarrow y = 100, b_2 \Rightarrow (y = 100)$$

$\neg b_2 \Rightarrow \neg (y = 100)$

$$\neg b_1 \Rightarrow \neg (x < 99)$$

if ( !b1 )    b1 := false

else    b1 := \*

无需翻译

对  $y := y + 1$

$wp(y := y + 1, x < 100) \Leftrightarrow x < 100$  ,     $b_1 \Rightarrow (x < 100)$   
 $\neg b_1 \Rightarrow \neg(x < 100)$  无需翻译

$wp(y := y + 1, y = 100) \Leftrightarrow y = 99$  ,     $b_2 \Rightarrow \neg(y = 99)$

$\neg b_2 \neq (y = 99)$

if ( b2 )    b2 := false

else    b2 := \*

$\neg b_2 \neq \neg(y = 99)$