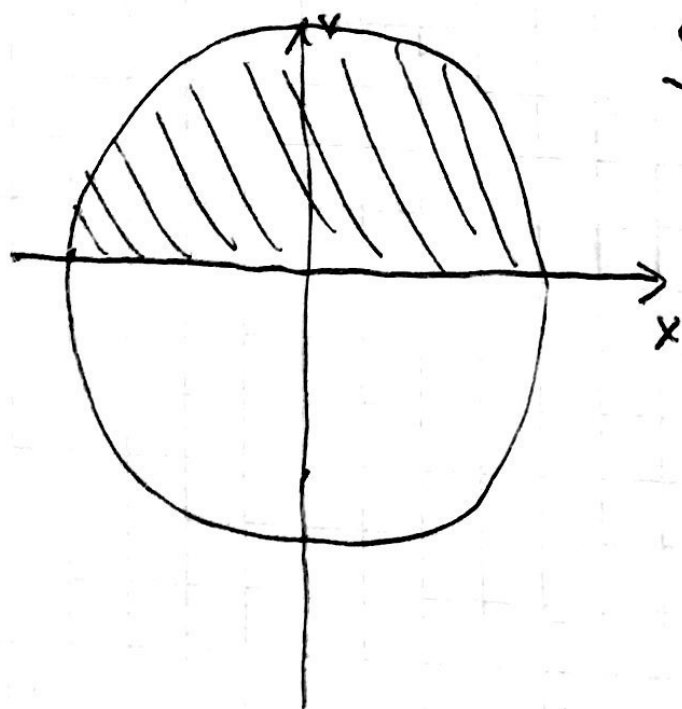# Самостійна робота

N2

$$\int 2|x|\,dx = \begin{cases} \int 2x\,dx, & x \geqslant 0 \\ \int -2x\,dx, & x < 0 \end{cases}, \rightarrow \begin{cases} x^2, & x \geqslant 0 \\ -x^2, & x < 0 \end{cases} \rightarrow |x^2|$$

N3

$$\int_{-4}^{4} \sqrt{16 - x^2}\,dx =$$

$$* \quad \sqrt{16 - x^2} = y$$

$$y^2 + x^2 = 16.(4^2)$$

$$S = \pi z^2 \cdot \frac{1}{2} = \pi \cdot 16 \cdot \frac{1}{2} = \boxed{8\pi}$$

$$f(x) = \frac{1}{\sqrt{x}} - 1$$

$$F(x) = \int \frac{1}{\sqrt{x}} - 1 \, dx = \int \frac{dx}{\sqrt{x}} - \int 1 \, dx = \boxed{2\sqrt{x} - x + C}$$

$$-4 = 2 \cdot \sqrt{9} - 9 + C$$

$$-4 = 6 - 9 + C$$

$$-4 = -3 + C$$

$$C = -1$$

$$\boxed{F(x) = 2\sqrt{x} - x - 1}$$

№7

$$\int_0^4 \frac{dx}{\sqrt{2x+1}} = \frac{1}{2} \cdot 2\sqrt{2x+1} \Big|_0^4 = \sqrt{2x+1} \Big|_0^4 = \sqrt{8+1} - \sqrt{0+1} =$$

$$= 3 - 1 = \boxed{2}$$

№8

$$\int_{-\pi}^{\pi} x \cos x \, dx$$

$$f(-x) = -f(x)$$

Оскільки $-x \cos x = -x \cdot \cos(-x) \Rightarrow f(x) = f(-x)$

Готово ф-я непарна $\longrightarrow \int\limits_{-\pi}^{\pi} x\cos x\, dx = 0.$

$$N9$$

$f(x) = -4\cos x.$

$$\boxed{F(x) = -4 \cdot \sin x + C}$$

$$N10$$

$f(x) = \cos^2 x$

$$\int \cos^2 x\, dx = \int \frac{1+\cos 2x}{2}\, dx = \int \frac{1}{2}\, dx + \int \frac{\cos 2x}{2}\, dx =$$

$$= \boxed{\frac{1}{2}x + \frac{\sin 2x}{4} + C}$$

$$N12$$

$$\int\limits_{0}^{\frac{\pi}{2}} \sin 2x \cos 2x\, dx = \int\limits_{0}^{\frac{\pi}{2}} 2\sin x \cos x \cdot (2\cos^2 x - 1)$$

$$N13$$

$$\int\limits_{0}^{2} e^{\frac{1}{2}x}\, dx = 2e^{\frac{1}{2}x}\Big|_{0}^{2} = 2e^1 - 2e^0 = \boxed{2e - 2}$$

$v(t) = 2t + 1$

$S'(t) = v(t)$

$S(t) = t^2 + t + c$

$3 = 1 + 1 + c$

$c = 1$

$\underline{S(t) = t^2 + t + 1}$

№17

$$\int\limits_{\frac{\pi}{20}}^{\frac{\pi}{10}} \sin\left(10x + \frac{\pi}{3}\right) dx = \frac{1}{10}\left(-\cos\left(10x + \frac{\pi}{3}\right)\right) = -\frac{\cos 10x \, \cos\frac{\pi}{3} + \sin}{10}$$

$$-\frac{\sin 10x \, \sin\frac{\pi}{3}}{10} = \frac{\cos 10x - \sqrt{3}\,\sin 10x}{20} \Bigg|_{\frac{\pi}{20}}^{\frac{\pi}{10}} \; \overset{(=)}{}$$

$$\overset{(=)}{} \frac{\sqrt{3}\cdot\sin\pi - \cos\pi}{20} - \frac{\sqrt{3}\,\sin\frac{\pi}{2} - \cos\frac{\pi}{2}}{20} =$$

$$= \frac{0\sqrt{3} - (-1)}{20} - \frac{\sqrt{3} - 0}{20} = \frac{1}{20} - \frac{\sqrt{3}}{20} = -\frac{\sqrt{3} - 1}{20}.$$

$\circledast$

$$\overset{4}{\cancel{80}}(\sqrt{3} + 1) \cdot \left(-\frac{\sqrt{3} - 1}{\cancel{20}}\right) = 4\cdot(-(3 - 1)) = 4\cdot(-2) = \boxed{-8}$$

$$\int_{0}^{a} \cos \frac{x}{2}\, dx, \quad a \in \mathbb{R}$$

$$\int \cos \frac{x}{2}\, dx = 2 \sin \frac{x}{2}$$

$$\cancel{\ast}\ 2 \sin \frac{0}{2} = 0$$

$$\Downarrow$$

$$\cancel{\ast}\ 2 \sin \frac{x}{2} \ (min)$$

$$\overline{}$$

$$\sin \frac{x}{2} \ (min) = -1$$

$$\Downarrow$$

$$\frac{x}{2} = \frac{3\pi}{2} + 2\pi \cdot n, \quad n \in \mathbb{Z}$$

$$\boxed{x = 3\pi + 4\pi n, \quad n \in \mathbb{Z}}$$

$$\boxed{a = 3\pi}$$

$$2 \sin \frac{x}{2} \Big|_{0}^{3\pi} = 2 \sin \frac{3\pi}{2} - 2 \sin 0 = -2 - 0 = \boxed{-2}$$