MARKOV RANDOM FIELDS

Machine Learning

IIIy CSE IDP and IDDMP

Refer : Stephen Marsland

Motivation

1) Undirected Structure
   - In Bayesian networks each edge has a direction; removing arrows yields no parents/children hierarchy.

2) Conditional Independence Simplified
   - Two nodes are independent given a third if every path between them passes through that third node.

3) Markov Property
   - The state of a node depends only on its immediate neighbours.
   - All other nodes are conditionally independent given those neighbours.

4) Inference Complexity
   - Despite this locality, exact inference in general MRFs remains #P-hard.

Image Denoising Example

Problem Setup

Observed noisy binary image $I_{xi,xj} \in \{-1,1\}$

True "ideal" image $I^0_{xi,xj}$ to be recovered

Assumptions

Small noise $\Rightarrow$ strong correlation: $I_{xi,xj}$ correlates with $I^0_{xi,xj}$

Local patches are homogeneous $\Rightarrow$ pixels correlate with immediate neighbours (e.g. $I_{xi,xj}$ with $I_{xi+1,xj}$, $I_{xi,xj-1}$).

MRF Justification

Homogeneity $\rightarrow$ large regions of constant pixel value.

Given its neighbours, each pixel is conditionally independent of all others.

Outcome

Enforces smoothness and denoises by leveraging local pixel agreements.

Physical Origins & Energy-Based Inference

Ising Model Roots

1. Developed by physicists to describe atoms in a chain, each "spin" being +1 or –1.
2. Stable configurations correspond to minimum overall energy.

Energy Formulation for Image Pairs

3. **Data term:** low energy when noisy pixel $I_{xi,xj}$ matches true pixel $I^{0}_{xi,xj}$
4. **Smoothness term:** low energy when neighbouring pixels $I_{xi,xj}$ and $I_{xi\pm1,xj\pm1}$ agree.
5. **Final energy equation:**

$$E(I, I') = -\zeta \sum_{i,j}^{N} I_{x_i,x_j} I_{x_i\pm1,x_j\pm1} - \eta \sum_{i,j=1}^{N} I_{x_i,x_j} I'_{x_i,x_j}$$

ζ and η are parameters

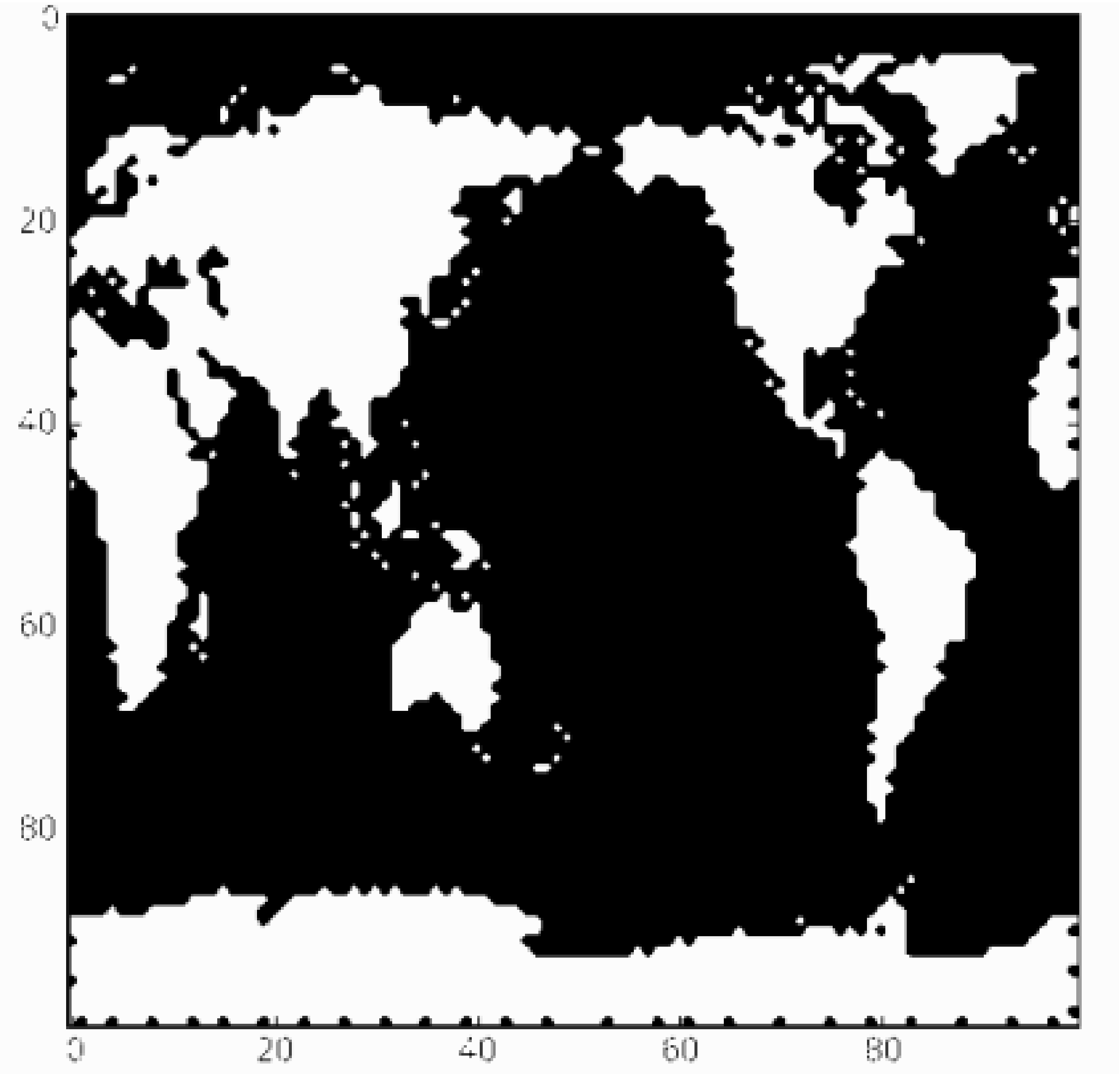
Physical Origins & Energy-Based Inference (cont.)

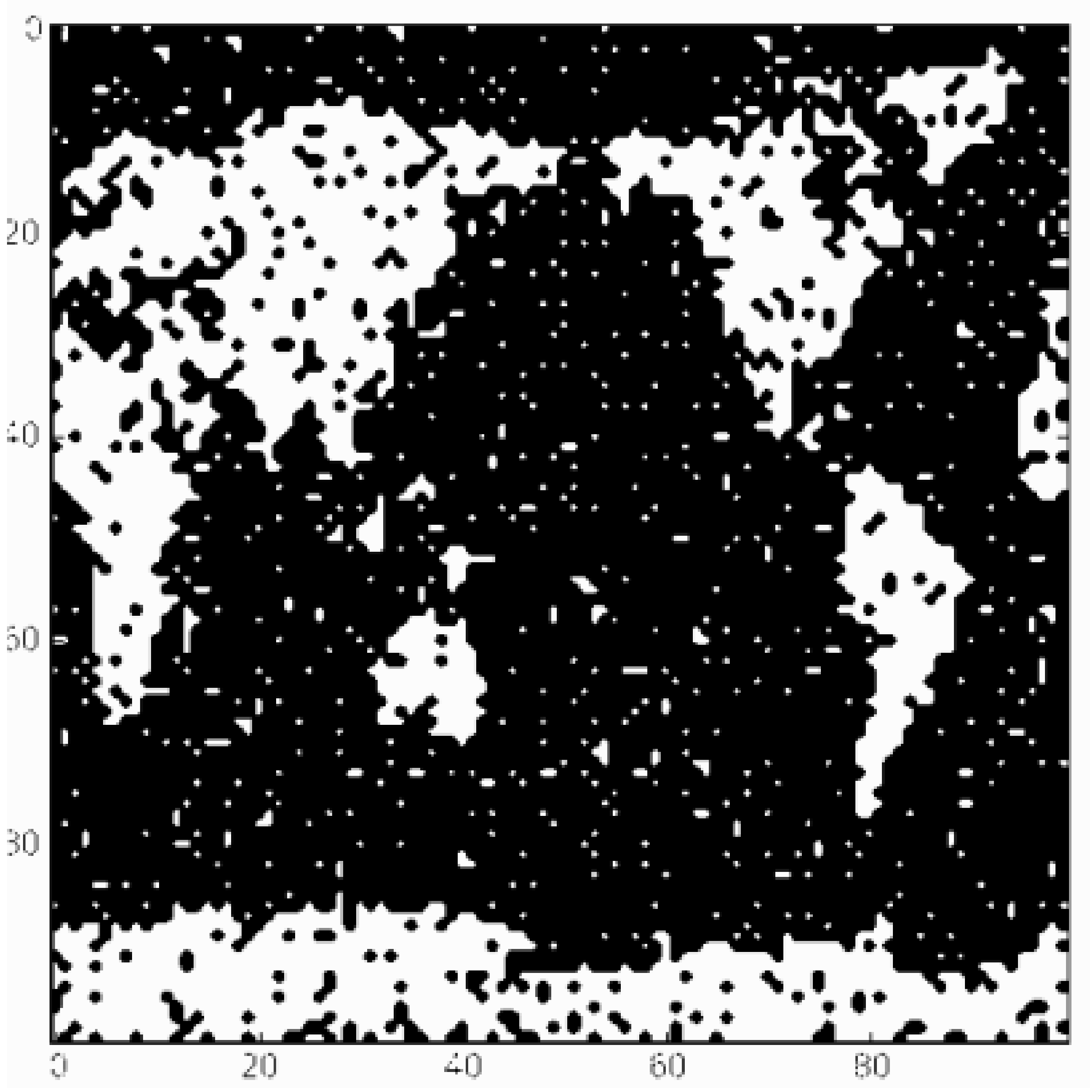
1. **Iterative Update Algorithm**
    a. Start with the noisy image I and ideal image I^0
    b. For each pixel $I_{xi,xj}$:
        i. Compute total energy if it's set to +1 vs. −1.
        ii. Choose the spin that yields lower energy.
    c. Repeat—either in random order or raster scan—until no pixel flips.

2. **Result**

Dramatically reduces noise (e.g. from 10% down to <1%), at the cost of some detail (see next slide for Figure 16.5)

MRF Image Denoising Algo. Results (Figure 16.5)





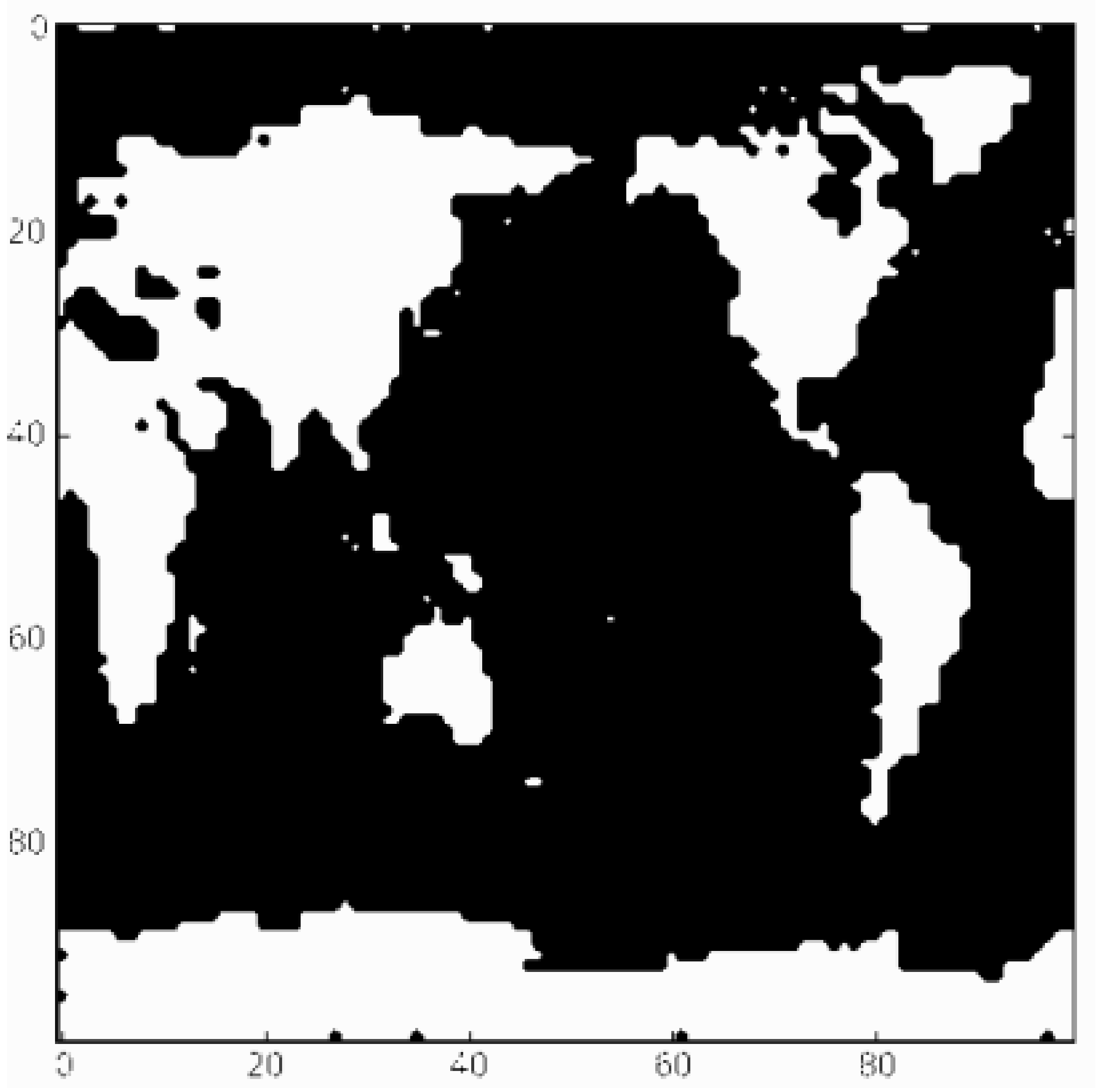


FIGURE 16.5 Using the MRF image denoising algorithm with $\eta = 2.1, \zeta = 1.5$ on a map of the world (*top left*) corrupted by 10% uniformly distributed random noise (*top right*) gives the image below which has about 1% error, although it has smoothed out the edges of all the continents.

MRF Image Denoising Algorithm

The Markov Random Field Image Denoising Algorithm

- Given a noisy image I and an original image I', together with parameters η, ζ:
- Loop over the pixels of image I:
  - compute the energies with the current pixel being -1 and 1
  - pick the one with lower energy and set its value in I accordingly

Summary & Conclusion

Key Concepts Recap

1. Markov Random Fields are undirected graphical models emphasizing local dependencies.
2. Conditional independence is determined by graph separation—no directed arrows needed.
3. The Markov property confines each node's dependence to its immediate neighbours.

Energy-Based Framework

4. Energy terms encode fidelity to observed data (data term) and smoothness across neighbours.
5. Inference proceeds by iteratively flipping pixel states to minimize total energy.

Practical Outcomes

6. Despite #P-hard exact inference, simple local updates yield effective denoising (e.g. noise reduction from 10% to <1%).
7. Widely applied in image restoration and other spatial data tasks.

Take-Away Message

8. MRFs blend physics-inspired energy minimization with probabilistic graphical modeling.
9. Offer flexible, powerful tools for leveraging local structure, at the cost of computational complexity