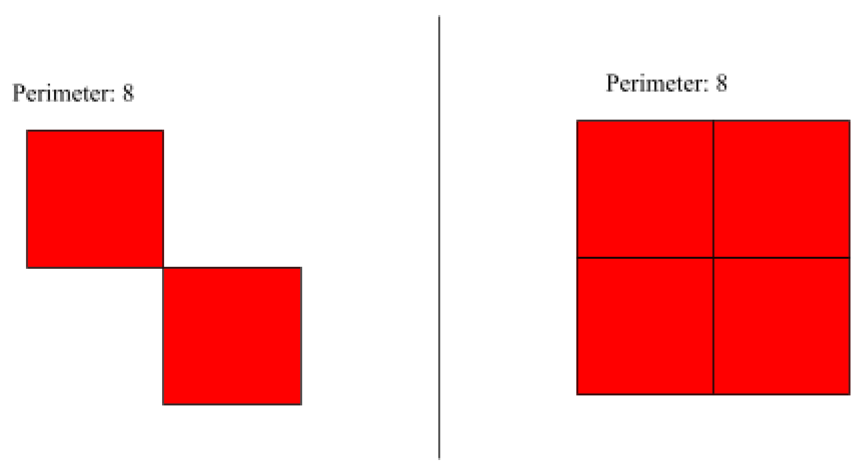
1. The maximum number of students that can be placed in the classroom which will never result in all students becoming infected is 4. This can be proved by the inductive proof below:

   Each square has 4 edges on it, and we will be counting the edges which have an infection on 1 side, we will let P(i) represent the number of edges. For a fully infected set of students, the perimeter will be equal to 4n (as can be proved by adding the edges of the 5x5 board which equals 20).

   Assuming a base case of P(0) < 4n, we can prove this base case by noting that since we start with less than n infected students, and since each cell has at most 4 edges on it, the perimeter must be less than 4n proving the base case true.

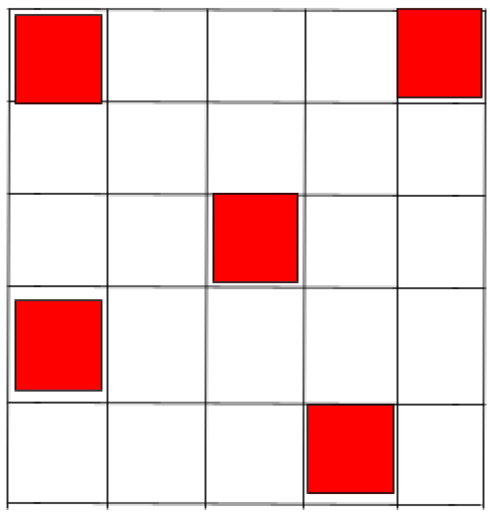
   Moving on to the inductive step, which claims that P(i) < 4n => P(i+1) < 4n, an assumption is made that for a given i value, the perimeter is less than 4n (as per the base case above). Now the argument is that if i goes from i to i + 1, the perimeter will not increase (which would result in all students becoming infected being impossible). This is true if a new cell is infected by being adjacent to 2 already infected cells, as at least 2 edges are lost and at least 2 edges are gained at the same time (as per the diagram below):



   Thus, the inductive step is proven true and the perimeter will not increase regardless of how many cells become infected resulting in all students becoming infected being impossible with 4 or less students.

2. The minimum number of initially infected students required for it to be possible to get all students infected is 5. This is because as per the proof from part 1, the maximum number of students which make it impossible to get all students infected is 4, meaning 1 more will make it now possible.

3. Yes, like so:



4. Considering a n x n grid with n^2 students, the way my answers would change is that the maximum number of students which would never result in all students becoming infected would be n-1, and the minimum number of students required to make it possible for all students to become infected would be n students.

5. It does not matter if n is even or odd, so long as it is a positive integer. The only impact this would have is if the grid changes size, then there would be a different breakpoint at which it would be possible to have all students become infected as per the proof from part 1.

6. As per the proof from part 1, the geometric property which never changes about the set of infected students is the overall perimeter of all of the infected squares. This means that the number of edges that border the infected region remains constant as the infection spreads.