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Prove that for any starting configuration C, and any sequence of moves S of any length, if S is repeated enough times, you will eventually return to C.

Proof:

Consider a state space where each state represents a configuration. Let C be the starting configuration. Let S be some sequence of moves that can transform one configuration to another. If we assume that there exists a sequence of moves S such that, when repeated indefinitely, it never returns to the initial configuration C, this implies that there is an infinite path in the state space that avoids C. Let P be this infinite path, starting from C. Since S is repeated indefinitely, P must also be infinite as well. Now if we consider all possible configurations reachable from C using the available moves in S, since the path P is infinite, it must visit the same configuration more than once due to the finite number of possible configurations. By the Pigeonhole Principle, there exists a repeated configuration in P. Let C2 be the first repeated configuration encountered in P. This means there exists a cycle from C2 back to itself, formed by the repeated moves. This cycle contradicts our assumption that S never returns to C. Thus, the original statement is proven true by contradiction as S cannot both return to C and not return to C at the same time.