Задача трёх тел

Вариант описания 1

$$\begin{pmatrix} x_1(t) & v_{x1}(t) & y_1(t) & v_{y1}(t) \\ x_2(t) & v_{x2}(t) & y_2(t) & v_{y2}(t) \\ x_3(t) & v_{x3}(t) & y_3(t) & v_{y3}(t) \end{pmatrix}$$

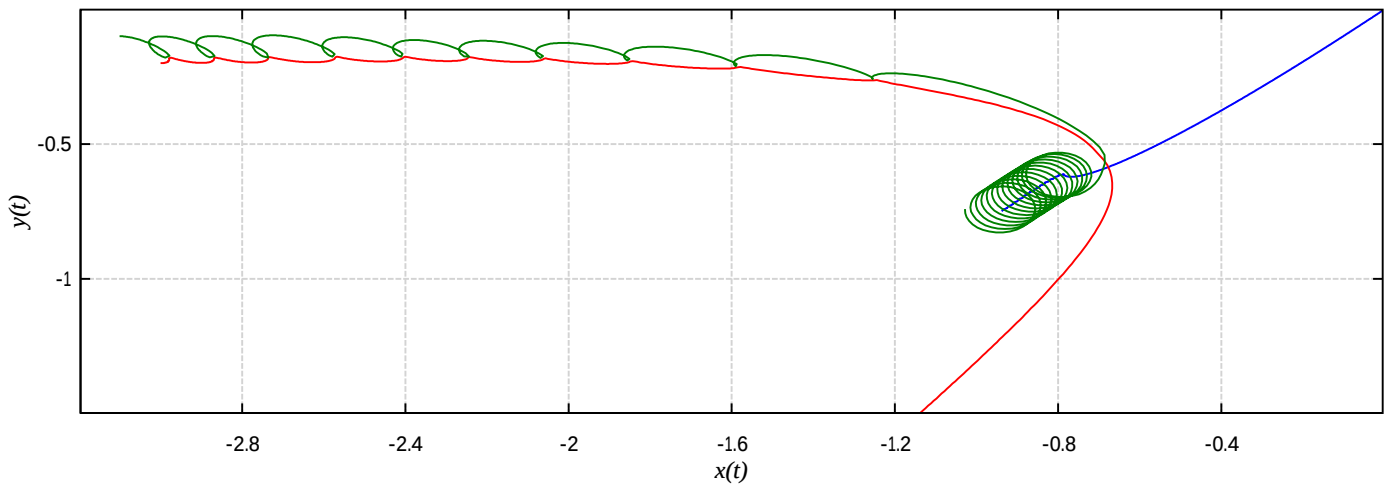
$\mathrm{appVersion}(4) = \text{"1.2.9018.0"}$

$G := 1 \quad m_1 := 30 \quad m_2 := 2 \quad m_3 := 0.5 \quad t_{end} := 1$

funvec

$$\begin{pmatrix}
x_1(0) = 0 & x_2(0) = -3 & x_3(0) = -3.1 & y_1(0) = 0 & y_2(0) = -0.2 & y_3(0) = -0.1 \\
v_{x1}(0) = -1 & v_{x2}(0) = 1 & v_{x3}(0) = 2 & v_{y1}(0) = -1 & v_{y2}(0) = 0 & v_{y3}(0) = 0 \\
\frac{d}{dt}x_1(t) = v_{x1}(t) & \frac{d}{dt}v_{x1}(t) = \frac{G\cdot m_2\cdot(x_2(t)-x_1(t))}{\sqrt{(x_2(t)-x_1(t))^2+(y_2(t)-y_1(t))^2}^3} + \frac{G\cdot m_3\cdot(x_3(t)-x_1(t))}{\sqrt{(x_3(t)-x_1(t))^2+(y_3(t)-y_1(t))^2}^3} & & & & \\
\frac{d}{dt}y_1(t) = v_{y1}(t) & \frac{d}{dt}v_{y1}(t) = \frac{G\cdot m_2\cdot(y_2(t)-y_1(t))}{\sqrt{(x_2(t)-x_1(t))^2+(y_2(t)-y_1(t))^2}^3} + \frac{G\cdot m_3\cdot(y_3(t)-y_1(t))}{\sqrt{(x_3(t)-x_1(t))^2+(y_3(t)-y_1(t))^2}^3} & & & & \\
\frac{d}{dt}x_2(t) = v_{x2}(t) & \frac{d}{dt}v_{x2}(t) = \frac{G\cdot m_1\cdot(x_1(t)-x_2(t))}{\sqrt{(x_2(t)-x_1(t))^2+(y_2(t)-y_1(t))^2}^3} + \frac{G\cdot m_3\cdot(x_3(t)-x_2(t))}{\sqrt{(x_3(t)-x_2(t))^2+(y_3(t)-y_2(t))^2}^3} & & & & \\
\frac{d}{dt}y_2(t) = v_{y2}(t) & \frac{d}{dt}v_{y2}(t) = \frac{G\cdot m_1\cdot(y_1(t)-y_2(t))}{\sqrt{(x_2(t)-x_1(t))^2+(y_2(t)-y_1(t))^2}^3} + \frac{G\cdot m_3\cdot(y_3(t)-y_2(t))}{\sqrt{(x_3(t)-x_2(t))^2+(y_3(t)-y_2(t))^2}^3} & & & & \\
\frac{d}{dt}x_3(t) = v_{x3}(t) & \frac{d}{dt}v_{x3}(t) = \frac{G\cdot m_1\cdot(x_1(t)-x_3(t))}{\sqrt{(x_3(t)-x_1(t))^2+(y_3(t)-y_1(t))^2}^3} + \frac{G\cdot m_2\cdot(x_2(t)-x_3(t))}{\sqrt{(x_3(t)-x_2(t))^2+(y_3(t)-y_2(t))^2}^3} & & & & \\
\frac{d}{dt}y_3(t) = v_{y3}(t) & \frac{d}{dt}v_{y3}(t) = \frac{G\cdot m_1\cdot(y_1(t)-y_3(t))}{\sqrt{(x_3(t)-x_1(t))^2+(y_3(t)-y_1(t))^2}^3} + \frac{G\cdot m_2\cdot(y_2(t)-y_3(t))}{\sqrt{(x_3(t)-x_2(t))^2+(y_3(t)-y_2(t))^2}^3} & & & &
\end{pmatrix}$$

$M := \mathrm{Rkadapt}(\mathrm{funvec}, t_{end}, 1100)$



Вариант описания 2

ClearAll(1) = 1

$x1(t)$ $y1(t)$

$x2(t)$ $y2(t)$

$x3(t)$ $y3(t)$

$G := 1$ $m_1 := 30$ $m_2 := 2$ $m_3 := 0.5$ $t_{end} := 1$

funvec

$$x1(0) = 0 \quad x1'(0) = -1 \quad x2(0) = -3 \quad x2'(0) = 1 \quad x3(0) = -3.1 \quad x3'(0) = 2$$

$$y1(0) = 0 \quad y1'(0) = -1 \quad y2(0) = -0.2 \quad y2'(0) = 0 \quad y3(0) = -0.1 \quad y3'(0) = 0$$

$$x1''(t) = \frac{G \cdot m_2 \cdot (x2(t) - x1(t))}{\sqrt{(x2(t) - x1(t))^2 + (y2(t) - y1(t))^2}^3} + \frac{G \cdot m_3 \cdot (x3(t) - x1(t))}{\sqrt{(x3(t) - x1(t))^2 + (y3(t) - y1(t))^2}^3}$$

$$y1''(t) = \frac{G \cdot m_2 \cdot (y2(t) - y1(t))}{\sqrt{(x2(t) - x1(t))^2 + (y2(t) - y1(t))^2}^3} + \frac{G \cdot m_3 \cdot (y3(t) - y1(t))}{\sqrt{(x3(t) - x1(t))^2 + (y3(t) - y1(t))^2}^3}$$

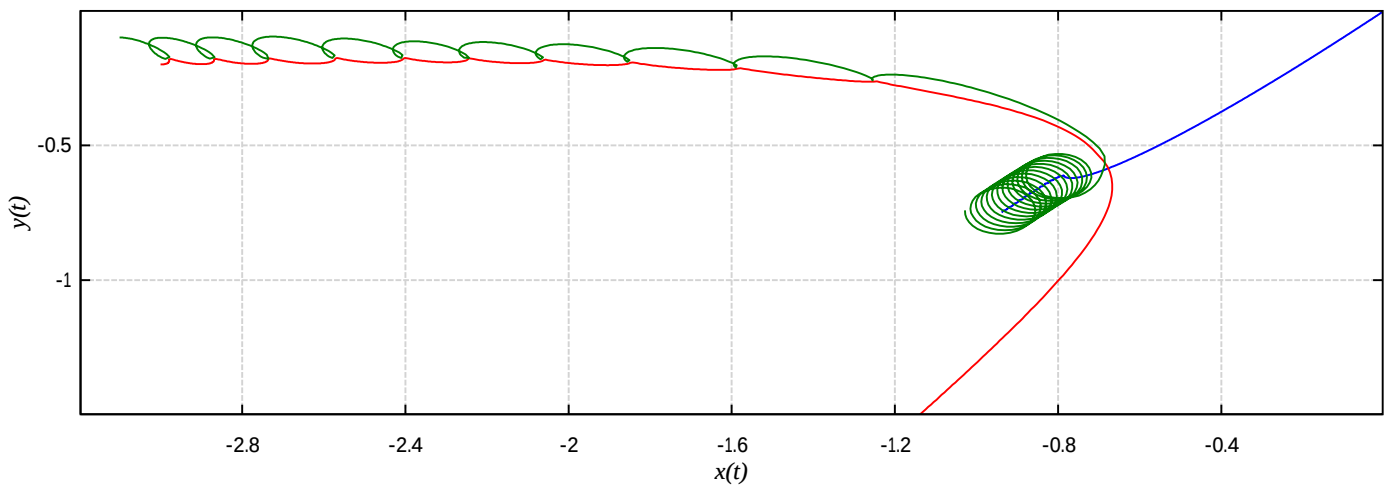
$$x2''(t) = \frac{G \cdot m_1 \cdot (x1(t) - x2(t))}{\sqrt{(x2(t) - x1(t))^2 + (y2(t) - y1(t))^2}^3} + \frac{G \cdot m_3 \cdot (x3(t) - x2(t))}{\sqrt{(x3(t) - x2(t))^2 + (y3(t) - y2(t))^2}^3}$$

$$y2''(t) = \frac{G \cdot m_1 \cdot (y1(t) - y2(t))}{\sqrt{(x2(t) - x1(t))^2 + (y2(t) - y1(t))^2}^3} + \frac{G \cdot m_3 \cdot (y3(t) - y2(t))}{\sqrt{(x3(t) - x2(t))^2 + (y3(t) - y2(t))^2}^3}$$

$$x3''(t) = \frac{G \cdot m_1 \cdot (x1(t) - x3(t))}{\sqrt{(x3(t) - x1(t))^2 + (y3(t) - y1(t))^2}^3} + \frac{G \cdot m_2 \cdot (x2(t) - x3(t))}{\sqrt{(x3(t) - x2(t))^2 + (y3(t) - y2(t))^2}^3}$$

$$y3''(t) = \frac{G \cdot m_1 \cdot (y1(t) - y3(t))}{\sqrt{(x3(t) - x1(t))^2 + (y3(t) - y1(t))^2}^3} + \frac{G \cdot m_2 \cdot (y2(t) - y3(t))}{\sqrt{(x3(t) - x2(t))^2 + (y3(t) - y2(t))^2}^3}$$

$M := \text{Rkadapt}(\text{funvec}, t_{end}, 1100)$



Вариант описания 3

ClearAll(1) = 1

$$\begin{matrix} x1(t) & y1(t) \\ x2(t) & y2(t) \\ x3(t) & y3(t) \end{matrix}$$

$G := 1$ $m_1 := 30$ $m_2 := 2$ $m_3 := 0.5$ $t_{end} := 1$

$$\Delta x_{21} := x2(t) - x1(t) \quad \Delta x_{12} := -\Delta x_{21} \quad \Delta y_{21} := y2(t) - y1(t) \quad \Delta y_{12} := -\Delta y_{21}$$
$$\Delta x_{31} := x3(t) - x1(t) \quad \Delta x_{13} := -\Delta x_{31} \quad \Delta y_{31} := y3(t) - y1(t) \quad \Delta y_{13} := -\Delta y_{31}$$
$$\Delta x_{32} := x3(t) - x2(t) \quad \Delta x_{23} := -\Delta x_{32} \quad \Delta y_{32} := y3(t) - y2(t) \quad \Delta y_{23} := -\Delta y_{32}$$

funvec

$$r_{21} := \sqrt{\Delta x_{21}^{\,2} + \Delta y_{21}^{\,2}} \qquad r_{31} := \sqrt{\Delta x_{31}^{\,2} + \Delta y_{31}^{\,2}} \qquad r_{32} := \sqrt{\Delta x_{32}^{\,2} + \Delta y_{32}^{\,2}}$$

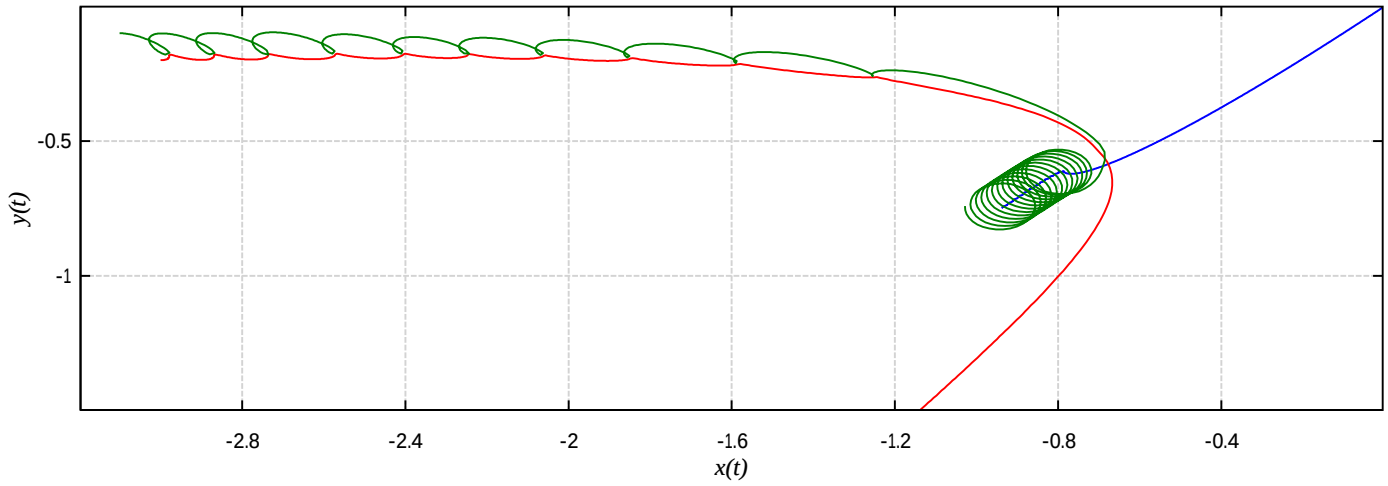
$$x1(0) = 0 \quad x1'(0) = -1 \quad x2(0) = -3 \quad x2'(0) = 1 \quad x3(0) = -3.1 \quad x3'(0) = 2$$
$$y1(0) = 0 \quad y1'(0) = -1 \quad y2(0) = -0.2 \quad y2'(0) = 0 \quad y3(0) = -0.1 \quad y3'(0) = 0$$

$$x1''(t) = \frac{G \cdot m_2 \cdot \Delta x_{21}}{r_{21}^{\,3}} + \frac{G \cdot m_3 \cdot \Delta x_{31}}{r_{31}^{\,3}} \qquad y1''(t) = \frac{G \cdot m_2 \cdot \Delta y_{21}}{r_{21}^{\,3}} + \frac{G \cdot m_3 \cdot \Delta y_{31}}{r_{31}^{\,3}}$$

$$x2''(t) = \frac{G \cdot m_1 \cdot \Delta x_{12}}{r_{21}^{\,3}} + \frac{G \cdot m_3 \cdot \Delta x_{32}}{r_{32}^{\,3}} \qquad y2''(t) = \frac{G \cdot m_1 \cdot \Delta y_{12}}{r_{21}^{\,3}} + \frac{G \cdot m_3 \cdot \Delta y_{32}}{r_{32}^{\,3}}$$

$$x3''(t) = \frac{G \cdot m_1 \cdot \Delta x_{13}}{r_{31}^{\,3}} + \frac{G \cdot m_2 \cdot \Delta x_{23}}{r_{32}^{\,3}} \qquad y3''(t) = \frac{G \cdot m_1 \cdot \Delta y_{13}}{r_{31}^{\,3}} + \frac{G \cdot m_2 \cdot \Delta y_{23}}{r_{32}^{\,3}}$$

$M := \text{Rkadapt}(\text{funvec}, t_{end}, 1100)$



Вариант описания 4

ClearAll(1) = 1

$\Delta x_{21} := x2(t) - x1(t)$ $\Delta x_{12} := -\Delta x_{21}$ $\Delta y_{21} := y2(t) - y1(t)$ $\Delta y_{12} := -\Delta y_{21}$

$\Delta x_{31} := x3(t) - x1(t)$ $\Delta x_{13} := -\Delta x_{31}$ $\Delta y_{31} := y3(t) - y1(t)$ $\Delta y_{13} := -\Delta y_{31}$

$\Delta x_{32} := x3(t) - x2(t)$ $\Delta x_{23} := -\Delta x_{32}$ $\Delta y_{32} := y3(t) - y2(t)$ $\Delta y_{23} := -\Delta y_{32}$

$$F(n\#, u\#, v\#) := \left| \begin{array}{l} s := \text{var2str}\left(\dfrac{G \cdot m_u \cdot \Delta n_{uv}}{\sqrt{\Delta x_{uv}^{\,2} + \Delta y_{uv}^{\,2}}^{\,3}} \right) \\ s := \text{strrep}(s, \text{num2str}(u), \text{num2str}(u\#)) \\ s := \text{strrep}(s, \text{num2str}(v), \text{num2str}(v\#)) \\ \text{str2num}(\text{strrep}(s, \text{num2str}(n), \text{num2str}(n\#))) \end{array} \right.$$

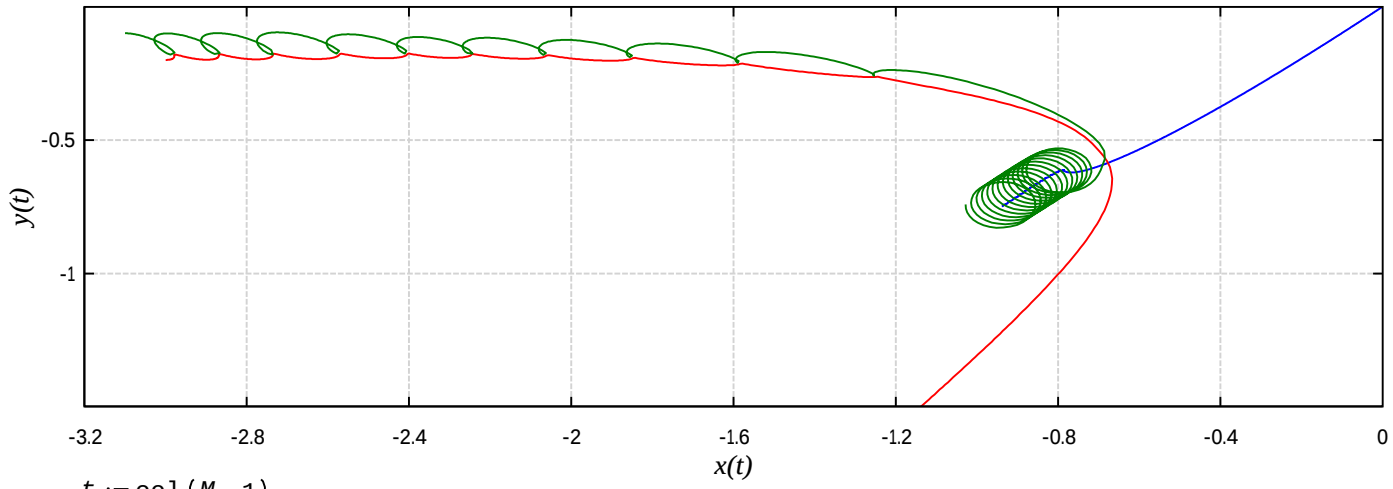
$$\left\{ \begin{array}{ll} x1(t) & y1(t) \\ x2(t) & y2(t) \\ x3(t) & y3(t) \end{array} \right.$$

funvec

$G := 1$ $m_1 := 30$ $m_2 := 2$ $m_3 := 0.5$ $t_{end} := 1$

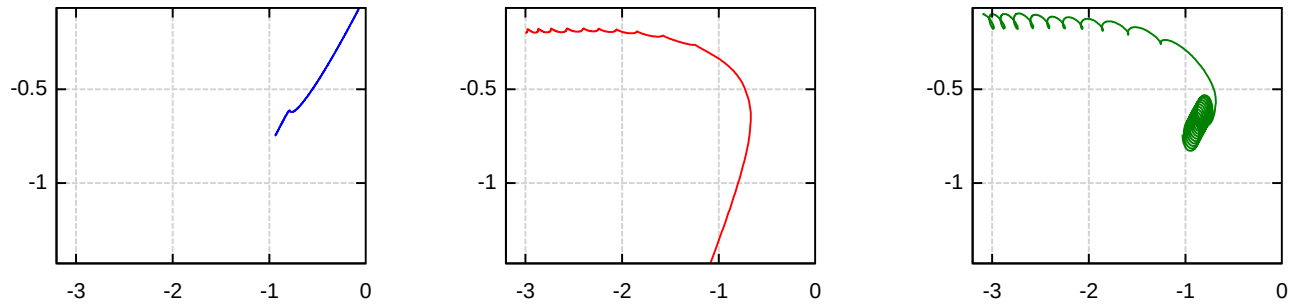
$$\left\{ \begin{array}{llllll} x1(0) = 0 & x1'(0) = -1 & x2(0) = -3 & x2'(0) = 1 & x3(0) = -3.1 & x3'(0) = 2 \\ y1(0) = 0 & y1'(0) = -1 & y2(0) = -0.2 & y2'(0) = 0 & y3(0) = -0.1 & y3'(0) = 0 \\ x1''(t) = F(x, 2, 1) + F(x, 3, 1) & & & y1''(t) = F(y, 2, 1) + F(y, 3, 1) & & \\ x2''(t) = F(x, 1, 2) + F(x, 3, 2) & & & y2''(t) = F(y, 1, 2) + F(y, 3, 2) & & \\ x3''(t) = F(x, 1, 3) + F(x, 2, 3) & & & y3''(t) = F(y, 1, 3) + F(y, 2, 3) & & \end{array} \right.$$

$M := \text{Rkadapt}(\text{funvec}, t_{end}, 1100)$



$t := \text{col}(M, 1)$

$m_1 = 30$ $m_2 = 2$ $m_3 = 0.5$



$\begin{bmatrix} x1(t) \\ y1(t) \end{bmatrix}$ $\begin{bmatrix} x2(t) \\ y2(t) \end{bmatrix}$ $\begin{bmatrix} x3(t) \\ y3(t) \end{bmatrix}$

Вариант описания 5

ClearAll(1) = 1

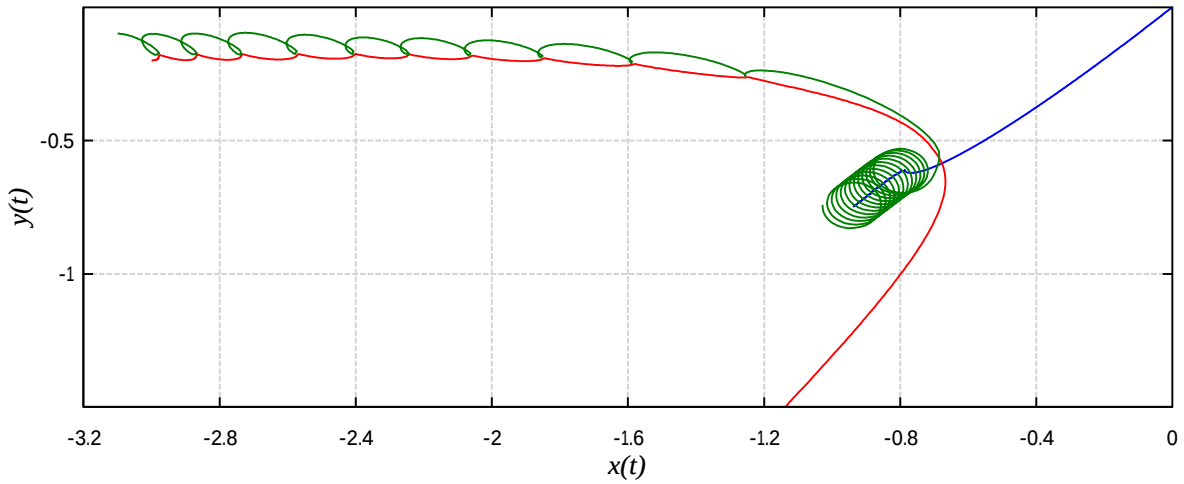
$$\begin{cases} x1(t) & x2(t) & x3(t) \\ y1(t) & y2(t) & y3(t) \end{cases} \qquad \begin{vmatrix} x := R_{[1..3]} \\ y := R_{[4..6]} \end{vmatrix} \qquad A(R, i, j) := \frac{G \cdot M_i \cdot (R_i - R_j)}{\text{norme}([x_i \; y_i] - [x_j \; y_j])^3}$$

R

$G := 1$ $m_1 := 30$ $m_2 := 2$ $m_3 := 0.5$ $t_{end} := 1$ $M := \text{eval}(\text{stack}(m_1, m_2, m_3))$

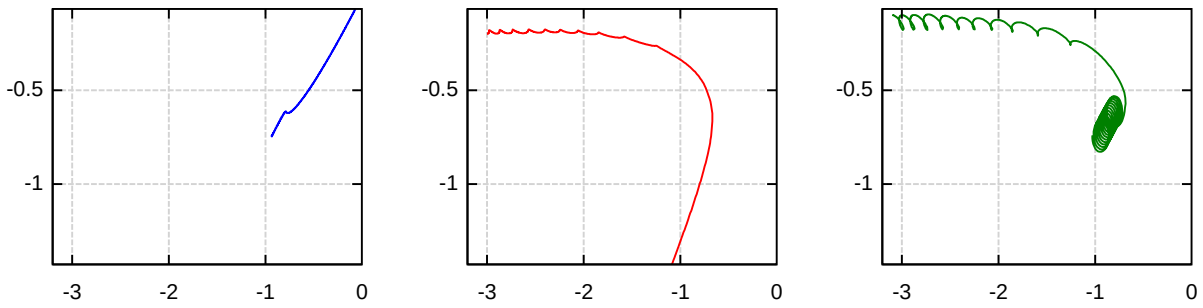
$$\begin{cases} x1''(t) = A(x,2,1) + A(x,3,1) & x1(0) = 0 & x1'(0) = -1 \\ x2''(t) = A(x,1,2) + A(x,3,2) & x2(0) = -3 & x2'(0) = 1 \\ x3''(t) = A(x,1,3) + A(x,2,3) & x3(0) = -3.1 & x3'(0) = 2 \\ y1''(t) = A(y,2,1) + A(y,3,1) & y1(0) = 0 & y1'(0) = -1 \\ y2''(t) = A(y,1,2) + A(y,3,2) & y2(0) = -0.2 & y2'(0) = 0 \\ y3''(t) = A(y,1,3) + A(y,2,3) & y3(0) = -0.1 & y3'(0) = 0 \end{cases}$$

M := Rkadapt(R, t_{end}, 1100)



$t := \text{col}(M, 1)$

$m_1 = 30$ $m_2 = 2$ $m_3 = 0.5$



$$\begin{bmatrix} x1(t) \\ y1(t) \end{bmatrix} \qquad \begin{bmatrix} x2(t) \\ y2(t) \end{bmatrix} \qquad \begin{bmatrix} x3(t) \\ y3(t) \end{bmatrix}$$

$$\text{var2str}\left(\begin{bmatrix} x1(t) \\ y1(t) \end{bmatrix}\right) = \text{"mat(x1(~t),y1(~t),2,1)"}$$

Вариант описания 6

ClearAll(1) = 1

$$\begin{cases} x1(t) & x2(t) & x3(t) \\ y1(t) & y2(t) & y3(t) \end{cases}$$

R

$$\begin{vmatrix} x := R_{[1..3]} \\ y := R_{[4..6]} \end{vmatrix}$$

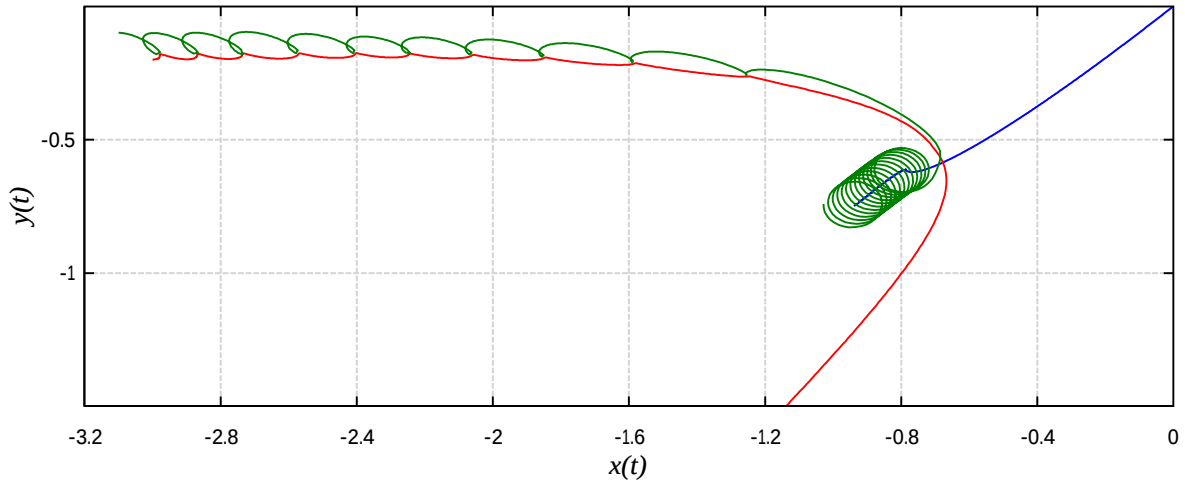
$$a(p, i, j) := := \frac{G \cdot M_i \cdot (p_i - p_j)}{\text{norme}\left(\begin{bmatrix} x_i & y_i \end{bmatrix} - \begin{bmatrix} x_j & y_j \end{bmatrix}\right)^3}$$

$$\Sigma a(p, j) := \begin{vmatrix} \text{for } i \in [1\ 2\ 3] \\ a_i := a(p, i, j) \\ \sum a \end{vmatrix}$$

$G := 1$ $m_1 := 30$ $m_2 := 2$ $m_3 := 0.5$ $t_{end} := 1$ $M := \text{stack}(m_1, m_2, m_3)$

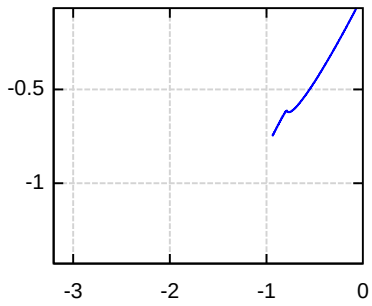
$$\begin{cases} x1''(t) = \Sigma a(x, 1) & x1(0) = 0 & x1'(0) = -1 \\ x2''(t) = \Sigma a(x, 2) & x2(0) = -3 & x2'(0) = 1 \\ x3''(t) = \Sigma a(x, 3) & x3(0) = -3.1 & x3'(0) = 2 \\ y1''(t) = \Sigma a(y, 1) & y1(0) = 0 & y1'(0) = -1 \\ y2''(t) = \Sigma a(y, 2) & y2(0) = -0.2 & y2'(0) = 0 \\ y3''(t) = \Sigma a(y, 3) & y3(0) = -0.1 & y3'(0) = 0 \end{cases}$$

M := Rkadapt(R, t_{end}, 1100)



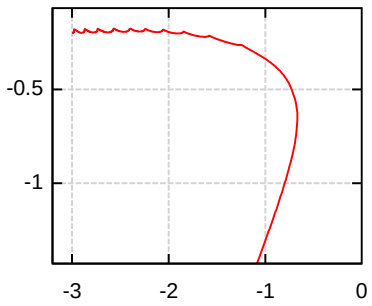
t := col(M, 1)

$m_1 = 30$



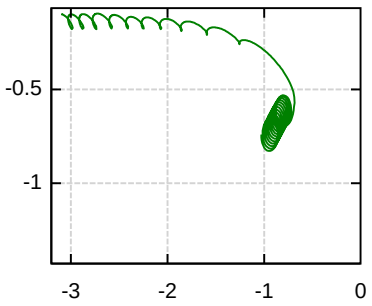
$$\begin{bmatrix} x1(t) \\ y1(t) \end{bmatrix}$$

$m_2 = 2$



$$\begin{bmatrix} x2(t) \\ y2(t) \end{bmatrix}$$

$m_3 = 0.5$



$$\begin{bmatrix} x3(t) \\ y3(t) \end{bmatrix}$$

$$\text{var2str}\left(\begin{bmatrix} x1(t) \\ y1(t) \end{bmatrix}\right) = \text{"mat(x1(~t),y1(~t),2,1)"}$$