1 Straightening

The core of the program is about utilizing of formula (2.10) from the paper

$$\begin{aligned} T_{g's',gs}(E) &= 2\pi\hbar\sqrt{\omega_{s'}\omega_s} \\ &\times \sum_{b,a=1}^{N}\sum_{n',n}\left(\mathbf{d}\cdot\mathbf{E}^{(s')}(\mathbf{r}_b)\right)^*_{n'm'_b}\left(\mathbf{d}\cdot\mathbf{E}^{(s)}(\mathbf{r}_a)\right)_{nm_a} \\ &\times\langle\ldots m'_{b-1},n',m'_{b+1}\ldots|\tilde{\hat{R}}(E)|\ldots m_{a-1},n,m_{a+1}\ldots\rangle \end{aligned} \tag{1}$$

for a chain of Λ-atoms ($F_0 = 1 \to F = 0$). In a particular case of a chain consisting of $m_0 = +1$-polarized atoms and a photon with $\sigma = -1$, formula is reduced to

$$\begin{aligned} T_{g's',gs}(E) = -2\pi\hbar\omega_s|d_0|^2\sum_{b,a=1}^{N}(-)^{m'_b}\left(E^{(\sigma',f'k)}_{-m'_b}(\mathbf{r}_b)\right)^* E^{(\sigma=-1,+k)}_{-1}(\mathbf{r}_a) \\ \times\langle\ldots(1,m'_{b-1}),(0,0)_b,(1,m'_{b+1})\ldots|\tilde{\hat{R}}(E)|\ldots(1,+1)_{a-1},(0,0)_a,(1,+1)_{a+1}\ldots\rangle \end{aligned} \tag{2}$$

Introducing the map $C(m_1,...m_p,...m_{N-1})$ such that

$$C(m_1,\ldots m_p,\ldots m_{N-1}) = \sum_{p=1}^{N-1}(1-m_p)\times 3^{p-1} \tag{3}$$

will perform the straightening:

$$(1,m_1),\ldots(0,0)_a,\ldots(1,m_N)\to a\times 3^{N-1} - C(m_1,\ldots m_{a-1},m_{a+1},\ldots m_N) \tag{4}$$

In case of Rayleigh channels (all atoms return to same states):

$$\begin{aligned} T_{\sigma',f'}(E) = 2\pi\hbar\omega_s|d_0|^2\sum_{b,a=1}^{N}\left(E^{(\sigma',f'k)}_{-1}(\mathbf{r}_b)\right)^* E^{(\sigma=-1,+k)}_{-1}(\mathbf{r}_a) \\ \times\langle b\times 3^{N-1}\,|\tilde{\hat{R}}(E)|\,a\times 3^{N-1}\rangle \end{aligned} \tag{5}$$

In case of Raman scattering which result in pseudospin projection change of only j-th atom to $m_j \neq +1$ (lacuna in j-th atom).

$$\begin{aligned} T^{m'_j}_{\sigma',f'}(E) = -2\pi\hbar\omega_s|d_0|^2\sum_{b,a=1}^{N}(-)^{m'_b}\left(E^{(\sigma',f'k)}_{-m'_b}(\mathbf{r}_b)\right)^* E^{(\sigma=-1,+k)}_{-1}(\mathbf{r}_a) \\ \times\langle b\times 3^{N-1} - (1-\delta_{j,b})(1-m'_j)\times 3^{j-1-\theta(j-b)}\,|\tilde{\hat{R}}(E)|\,a\times 3^{N-1}\rangle \end{aligned} \tag{6}$$

Due to absence of cylindrical symmetry, there are other channels of scattering where total angular momentum is not conserved. Moreover, the order of such contributions is $O(\sqrt{\chi})$, where χ is fiber susceptibility. Counting such channels is much more complicated.

2 The concept of neighbours

Upper straightening is useless for big chains of atoms due to fast growing of 3^{N-1} number. Thus, under consideration of small fiber susceptibility and fast decreasing of Raman part (from vacuum Green's function) we could assume that Raman diagrams are essential only in small cluster of nearest atoms - "neighbours" of atom.

$$(1,m_1=+1),\ldots(1,m_{a-b_{left}}),\ldots(0,0)_a,\ldots(1,m_{a+b_{right}})\ldots(1,m_N=+1)\rightarrow \\ a\times 3^{b_l+b_r}-C_a(m_{a-b_l},\ldots m_{a-1},m_{a+1},\ldots m_{a+b_r}) \tag{7}$$

$$T^{m'_j}_{\sigma',f'}(E)=-2\pi\hbar\omega_s|d_0|^2\sum_{b,a=1}^{N}(-)^{m'_b}\left(E^{(\sigma',\,f'k)}_{-m'_b}(\mathbf{r}_b)\right)^*E^{(\sigma=-1,\,+k)}_{-1}(\mathbf{r}_a) \\ \times\theta(b-j-b_l)\theta(b-j+b_r)\langle b\times 3^{b_r+b_l}-(1-\delta_{j,b})(1-m_j)\times 3^{j-1-\theta(j-b)}\,|\tilde{\hat{R}}(E)|\,a\times 3^{b_r+b_l}\rangle \tag{8}$$

Next step is to change variables in summation over b.

$$T^{m'_j}_{\sigma',f'}(E)=-2\pi\hbar\omega_s|d_0|^2\sum_{a=1}^{N}\sum_{b=j-b_l}^{j+b_r}(-)^{m'_b}\left(E^{(\sigma',\,f'k)}_{-m'_b}(\mathbf{r}_b)\right)^*E^{(\sigma=-1,\,+k)}_{-1}(\mathbf{r}_a) \\ \times\langle b\times 3^{b_r+b_l}-(1-\delta_{j,b})(1-m_j)\times 3^{j-1-\theta(j-b)}\,|\tilde{\hat{R}}(E)|\,a\times 3^{b_r+b_l}\rangle \tag{9}$$

It is natural to split it in two sums:

$$\begin{aligned}
T^{m'_j}_{\sigma',f'}(E) &= \tilde{T}^{m'_j}_{\sigma',f'}(E)+\delta T^{m'_j}_{\sigma',f'}(E)\\
\tilde{T}^{m'_j}_{\sigma',f'}(E) &= -2\pi\hbar\omega_s|d_0|^2(-)^{m'_j}\left(E^{(\sigma',\,f'k)}_{-m'_j}(\mathbf{r}_j)\right)^*\\
&\times \sum_{a=1}^{N}E^{(\sigma=-1,\,+k)}_{-1}(\mathbf{r}_a)\langle j\times 3^{b_r+b_l}\,|\tilde{\hat{R}}(E)|\,a\times 3^{b_r+b_l}\rangle\\
\delta T^{m'_j}_{\sigma',f'}(E) &= 2\pi\hbar\omega_s|d_0|^2\sum_{a=1}^{N}\sum_{b=j-b_l,b\neq j}^{j+b_r}\left(E^{(\sigma',\,f'k)}_{-1}(\mathbf{r}_b)\right)^*E^{(\sigma=-1,\,+k)}_{-1}(\mathbf{r}_a)\\
&\times \langle b\times 3^{b_r+b_l}-(1-m_j)\times 3^{b-j+b_l-\theta(j-b)}\,|\tilde{\hat{R}}(E)|\,a\times 3^{b_r+b_l}\rangle
\end{aligned} \tag{10}$$

And the dimension becomes $N\times 3^{b_r+b_l}$.