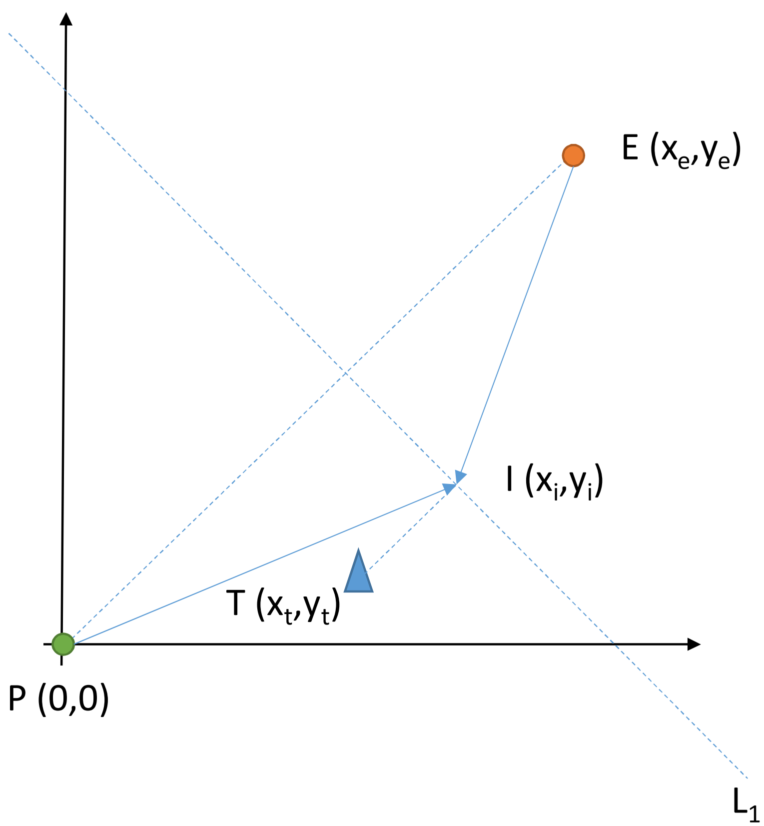
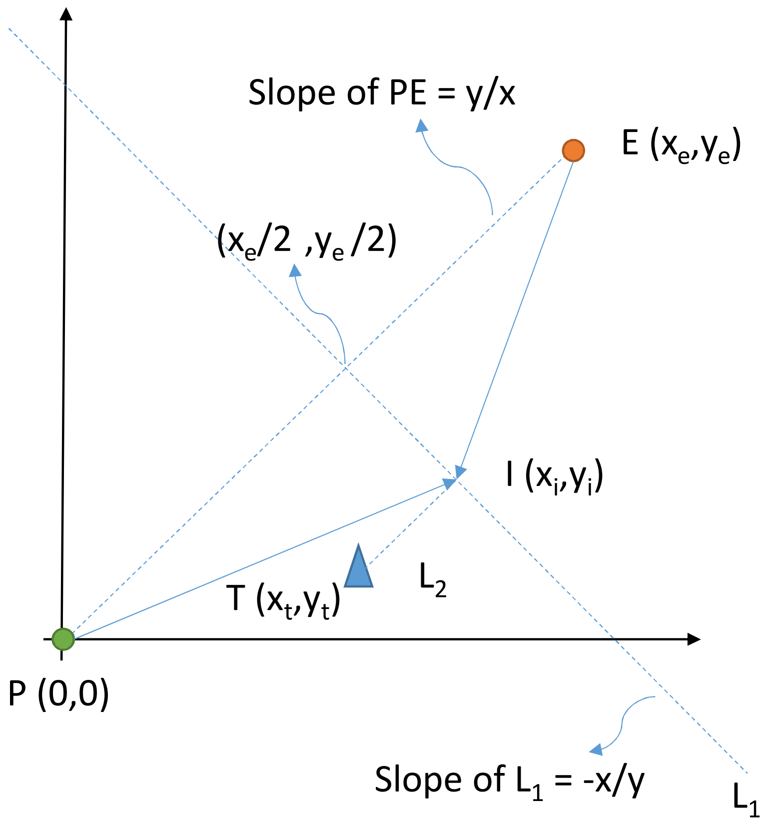
The Target Guarding Game with multiple evaders and single pursuer

Introduction



- E is the instantaneous position of the evader, P that of the pursuer and T that of the target. Both P and E move with equal velocities.
- E tries to capture T and P tries to capture E before T gets captured.
- The origin of the co-ordinate system is always taken to be the position of P.
- L_1 is the perpendicular bisector of PE. L_1 divides the space into dominance regions of P and E.
- If T lies in the dominance region of P, E heads to I (intercept point), which is the point on L_1 closest to T.
- In such a case, P also heads to I and wins the game by intercepting E.
- ***The aim is to find out the dynamics of E and I if P's path is known.***

The governing equations



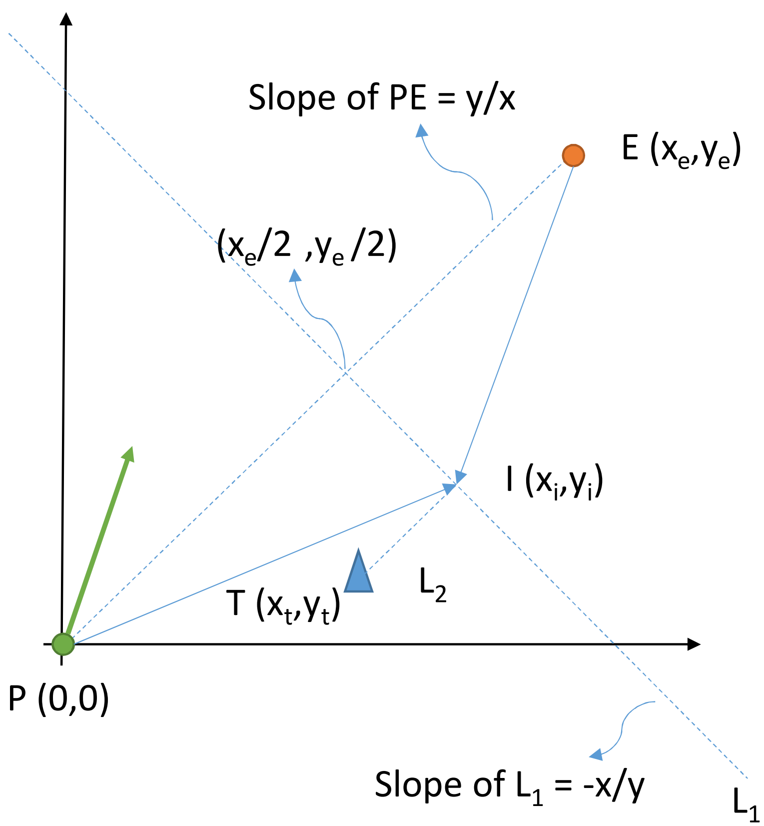
Let L_2 be the line TI. Hence point I can be found out by simultaneously solving for L_1 and L_2

$$x_i = \frac{x_e}{2} + \frac{y_e}{{x_e}^2 + {y_e}^2}(y_e x_t + x_e y_t) \qquad 1$$

$$y_i = \frac{y_e}{2} + \frac{x_e y_t}{{x_e}^2 + {y_e}^2}(x_e - y_e) \qquad 2$$

As the game progresses, all the above variables also become functions of time.

The governing equations



- If P doesn't take the path PI, E also will not take the path EI. E will always draw the perpendicular bisector and move to that point on the bisector closest to T. Thus point I will keep on changing, and E is always chasing I.
- ***The game can now be thought of as a game where E pursues I.***
- ***Thus, the pursuit equation between E and I is to be solved.***

Curves of pursuit

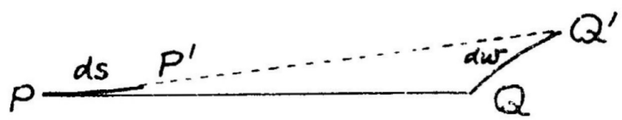


Fig. 1

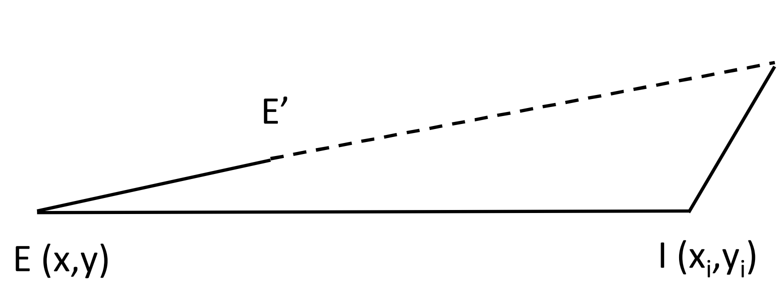
The point Q traverses an arbitrary track $Q(w)$ with speed $n = dw/dt$ and is pursued by the point P along the curve $P(s)$ with speed $m = ds/dt$. The ratio of corresponding arc lengths $ds/dw = m/n$ is arbitrary, but the velocity of pursuit dP/dt has always the same direction as the separation vector PQ.

Using rectangular coordinates, $P = (x,y)$ and $Q = (u,v)$, these differential conditions may be written

$$\frac{dx}{u-x} = \frac{dy}{v-y} = \frac{ds}{r} = \frac{mdw}{nr}, \qquad (1)$$

where $r^2 = (x-u)^2 + (y-v)^2$. For *uniform* pursuit (m/n = constant) Maupertuis suggests an equivalent geometric formulation: The curve QQ' is given; find the curve PP' such that any two of its tangents PQ and $P'Q'$ intercept an arc QQ' proportional to the arc PP'.

- Dynamics of pursuit curves were first studied in 1958 and have been solved for particular cases. (for example when the chased one follows a straight line or a circle)



$$\frac{dx}{x_i - x} = \frac{dy}{y_i - y}$$

The subscript 'e' is dropped from the evader co-ordinates to improve readability

Solving for the evader dynamics

From (3),

$$\frac{dy}{dx} = \frac{y_i - y}{x_i - x} \tag{4}$$

Equations (1) and (2) were derived assuming P at origin. Let P move along a straight line x=at, y=bt, t being the time. Thus, at any instance of time ‘t’, (1) and (2) will now change to:

$$x_i(x, y, t) = \frac{x}{2} + \frac{y}{x^2 + y^2}(yx_t + xy_t) + at \tag{5}$$

$$y_i(x, y, t) = \frac{y}{2} + \frac{xy_t}{x^2 + y^2}(x - y) + bt \tag{6}$$

Are the above equations correct? Is there anything fundamentally wrong in the derivations?

Solving for the evader dynamics

From (4), (5) and (6)

$$\frac{dy}{dx} = \frac{\frac{xy_t}{x^2+y^2}(x-y) + bt - \frac{y}{2}}{\frac{y}{x^2+y^2}(yx_t + xy_t) + at - \frac{x}{2}} \qquad 7$$

OR

$$\boxed{\frac{dy}{dx} = \frac{-y^3 - x^2y + 2(y_t + bt)x^2 + (2bt)y^2 - (2y_t)xy}{-x^3 - xy^2 + 2(x_t + at)y^2 + (2at)x^2 + (2y_t)xy}} \qquad 8$$

How is the above diff equation solved for y?