Fourierreihen
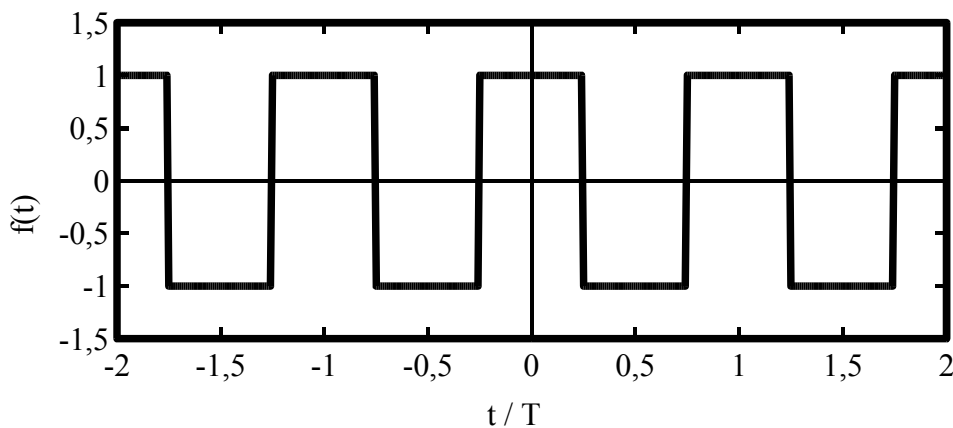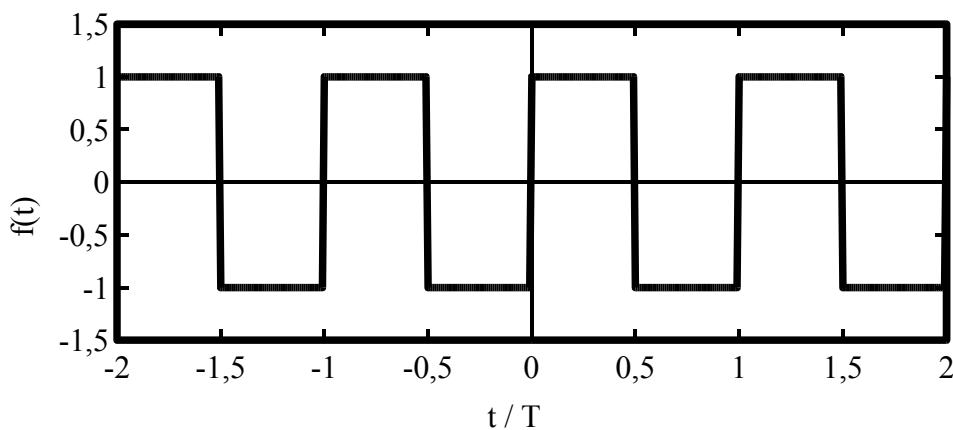

$$\overline{|f(t)|}=1$$
$$f_{\text{eff}}=1$$

$$f(t)=\frac{4}{\pi}\left(\frac{\cos\omega_1 t}{1}-\frac{\cos 3\omega_1 t}{3}+\frac{\cos 5\omega_1 t}{5}-\cdots\right)$$
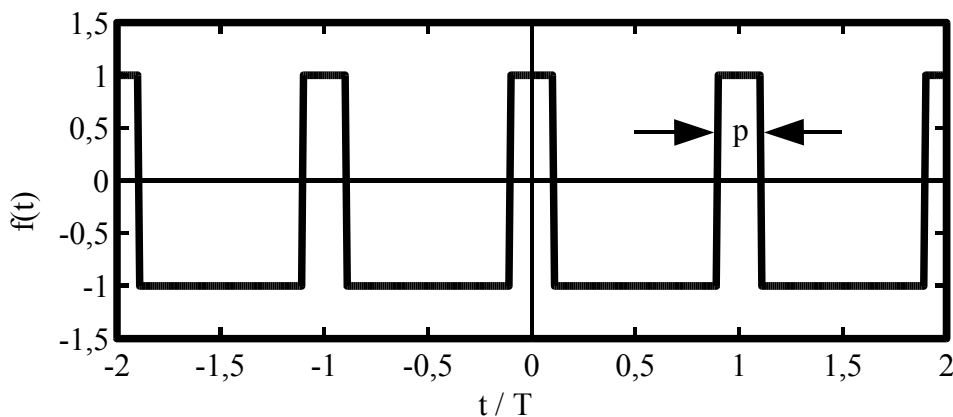
1.1

$$\overline{|f(t)|}=1$$
$$f_{\text{eff}}=1$$

$$f(t)=\frac{4}{\pi}\left(\frac{\sin\omega_1 t}{1}+\frac{\sin 3\omega_1 t}{3}+\frac{\sin 5\omega_1 t}{5}+\cdots\right)$$

1.2

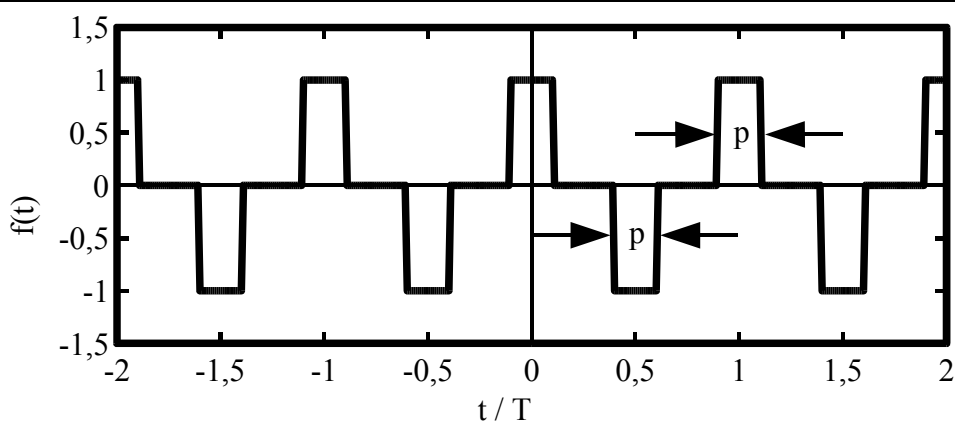$$\overline{|f(t)|}=1$$
$$f_{\text{eff}}=1$$

$$f(t)=2p-1+\frac{4}{\pi}\left(\frac{\sin\pi p}{1}\cos\omega_1 t+\frac{\sin 2\pi p}{2}\cos 2\omega_1 t+\frac{\sin 3\pi p}{3}\cos 3\omega_1 t+\cdots\right)$$
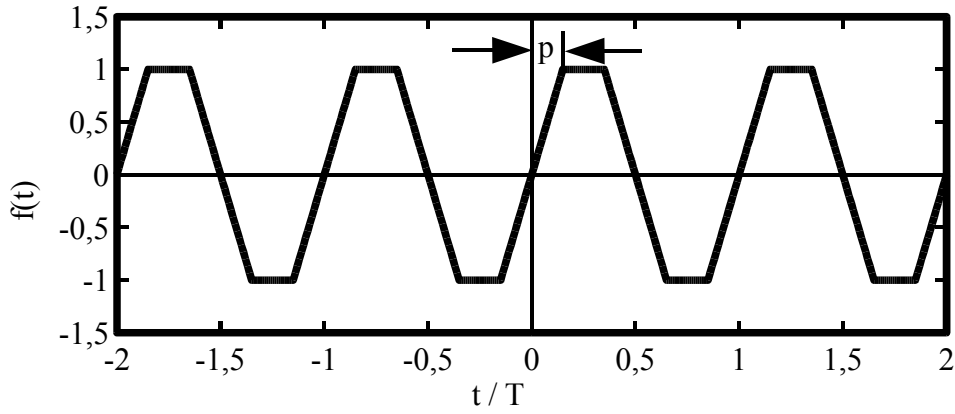
1.3

$$\overline{|f(t)|} = 2\,p$$

$$f_{\text{eff}} = \sqrt{2\,p}$$

$$f(t) = \frac{4}{\pi}\left(\frac{\sin \pi\,p}{1}\cos\omega_1 t + \frac{\sin 3\pi\,p}{3}\cos 3\,\omega_1 t + \frac{\sin 5\pi\,p}{5}\cos 5\,\omega_1 t + \cdots\right)$$
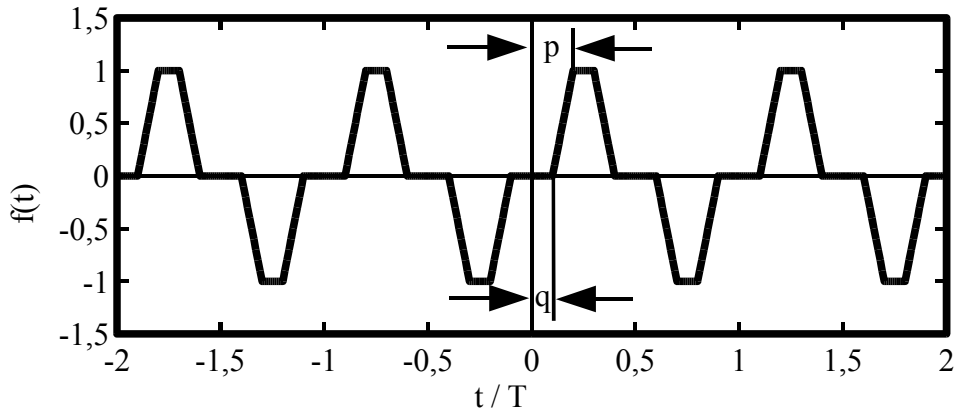
2.1

$$\overline{|f(t)|} = 1 - 2\,p$$

$$f_{\text{eff}} = \sqrt{1 - \frac{8}{3}\,p}$$

$$f(t) = \frac{2}{\pi^2\,p}\left(\frac{\sin 2\pi\,p}{1^2}\sin\omega_1 t + \frac{\sin 6\pi\,p}{3^2}\sin 3\,\omega_1 t + \frac{\sin 10\pi\,p}{5^2}\sin 5\,\omega_1 t + \cdots\right)$$
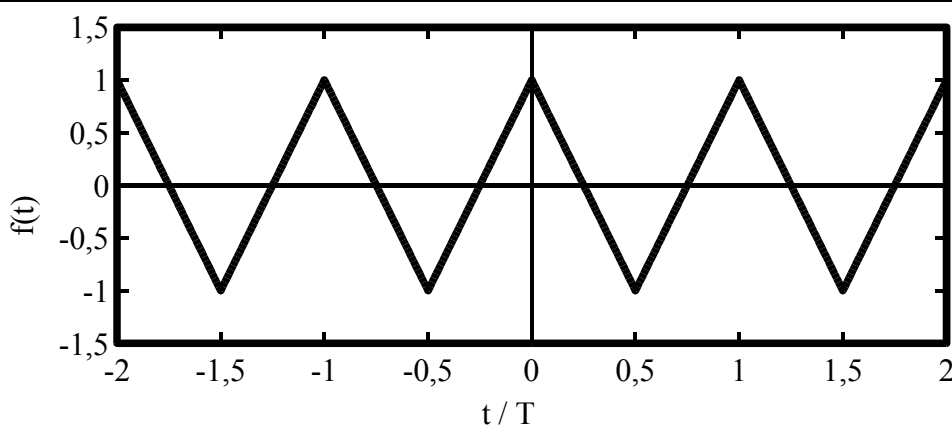
2.2

$$\overline{|f(t)|} = 1 - 2(p+q)$$

$$f_{\text{eff}} = \sqrt{1 - \frac{4}{3}(2\,p+q)}$$

$$f(t) = \frac{2}{\pi^2(p-q)}\left(\frac{\sin 2\pi\,p - \sin 2\pi\,q}{1^2}\sin\omega_1 t + \frac{\sin 6\pi\,p - \sin 6\pi\,q}{3^2}\sin 3\,\omega_1 t + \right.$$

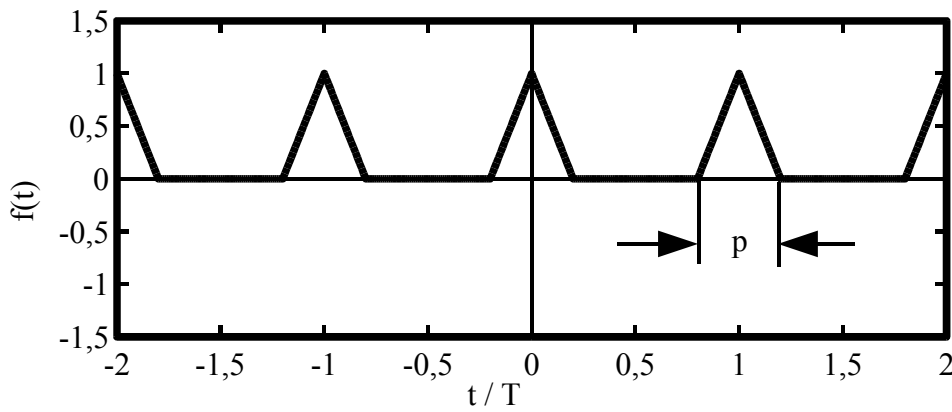$$\left. + \frac{\sin 10\pi\,p - \sin 10\pi\,q}{5^2}\sin 5\,\omega_1 t + \cdots\right)$$

2.3

$$\overline{|f(t)|} = \frac{1}{2}$$

$$f_{\text{eff}} = \frac{1}{\sqrt{3}}$$

$$f(t) = \frac{8}{\pi^2}\left(\frac{\cos\omega_1 t}{1^2} + \frac{\cos 3\omega_1 t}{3^2} + \frac{\cos 5\omega_1 t}{5^2} + \cdots\right)$$
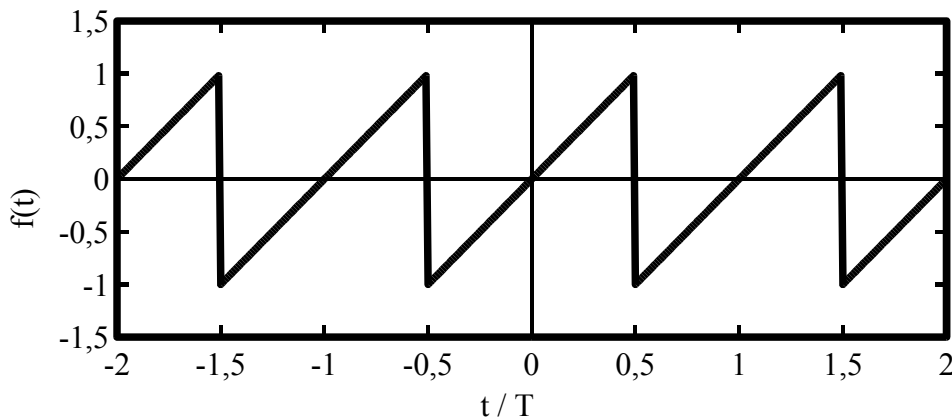
3.1

3.2

$$\overline{|f(t)|} = \frac{p}{2}$$

$$f_{\text{eff}} = \sqrt{\frac{p}{3}}$$

$$f(t) = \frac{p}{2} + \frac{4}{\pi^2 p}\left[\left(\sin\frac{\pi p}{2}\right)^2 \cos\omega_1 t + \left(\frac{1}{2}\sin\frac{2\pi p}{2}\right)^2 \cos 2\omega_1 t + \left(\frac{1}{3}\sin\frac{3\pi p}{2}\right)^2 \cos 3\omega_1 t + \cdots\right]$$
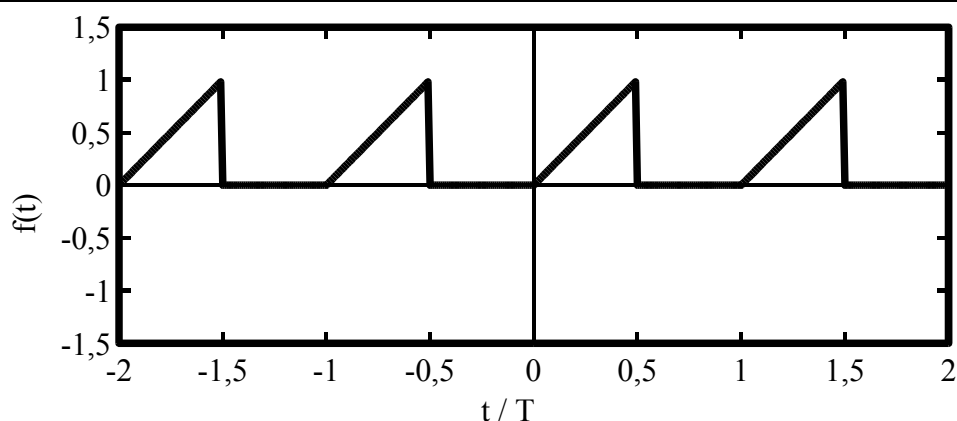
3.3

$$\overline{|f(t)|} = \frac{1}{2}$$

$$f_{\text{eff}} = \frac{1}{\sqrt{3}}$$

$$f(t) = \frac{2}{\pi}\left(\frac{\sin\omega_1 t}{1} - \frac{\sin 2\omega_1 t}{2} + \frac{\sin 3\omega_1 t}{3} - \cdots\right)$$

4.1

$$\overline{|f(t)|} = \frac{1}{4}$$

$$f_{\text{eff}} = \frac{1}{\sqrt{6}}$$

$$f(t) = \frac{1}{4} - \frac{2}{\pi^2} \left(\frac{\cos\omega_1 t}{1^2} + \frac{\cos 3\omega_1 t}{3^2} + \frac{\cos 5\omega_1 t}{5^2} + \cdots \right) + \frac{1}{\pi} \left(\frac{\sin\omega_1 t}{1} - \frac{\sin 2\omega_1 t}{2} + \frac{\sin 3\omega_1 t}{3} - \cdots \right)$$
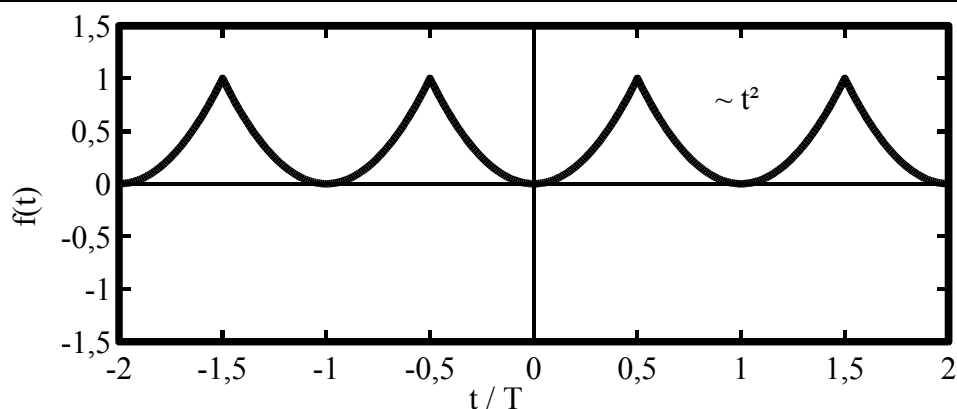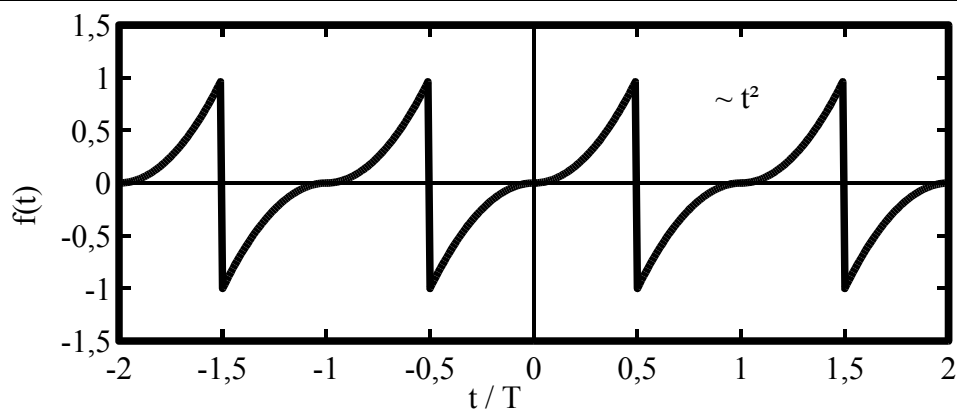

4.2

$\sim t^2$

$$\overline{|f(t)|} = \frac{1}{3}$$

$$f_{\text{eff}} = \frac{1}{\sqrt{5}}$$

$$f(t) = \frac{1}{3} - \frac{4}{\pi^2} \left(\frac{\cos\omega_1 t}{1^2} - \frac{\cos 2\omega_1 t}{2^2} + \frac{\cos 3\omega_1 t}{3^2} - \cdots \right)$$
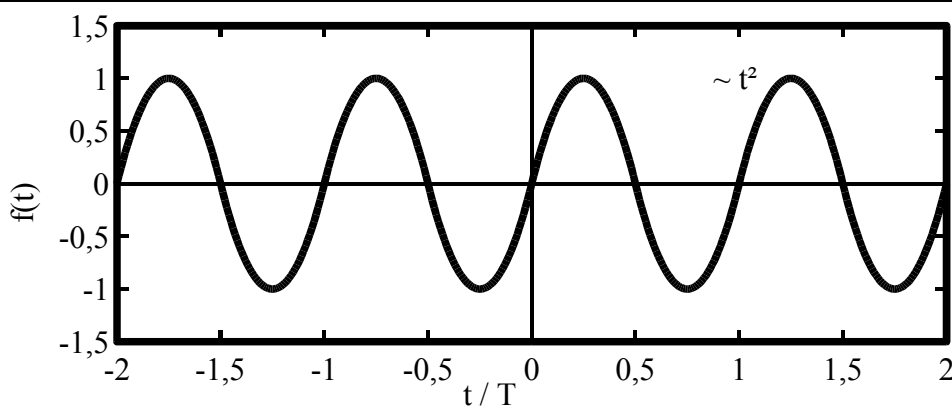

4.3

$\sim t^2$

$$\overline{|f(t)|} = \frac{1}{3}$$

$$f_{\text{eff}} = \frac{1}{\sqrt{5}}$$

$$f(t) = \frac{2}{\pi} \left(\frac{\sin\omega_1 t}{1} - \frac{\sin 2\omega_1 t}{2} + \frac{\sin 3\omega_1 t}{3} - \cdots \right) - \frac{8}{\pi^3} \left(\frac{\sin\omega_1 t}{1^3} + \frac{\sin 3\omega_1 t}{3^3} + \frac{\sin 5\omega_1 t}{5^3} + \cdots \right)$$
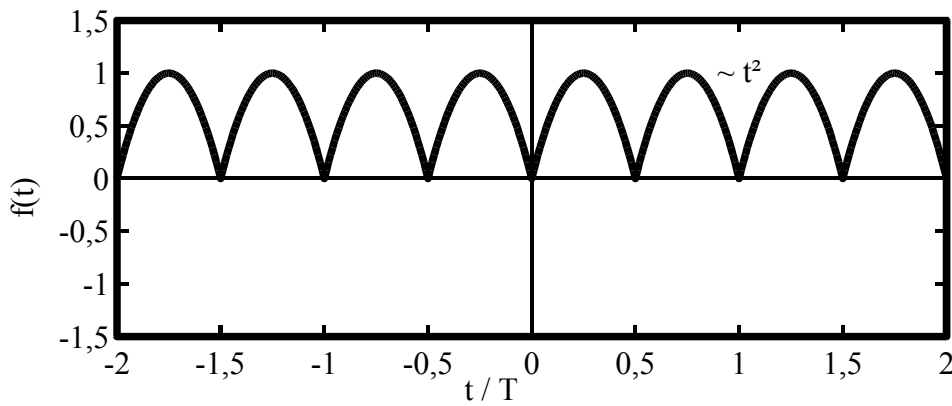
5.1

$$\overline{|f(t)|} = \frac{2}{3}$$

$$f_{\text{eff}} = \sqrt{\frac{8}{15}}$$

$$f(t) = \frac{32}{\pi^3}\left(\frac{\sin\omega_1 t}{1^3} + \frac{\sin 3\omega_1 t}{3^3} + \frac{\sin 5\omega_1 t}{5^3} + \cdots\right)$$
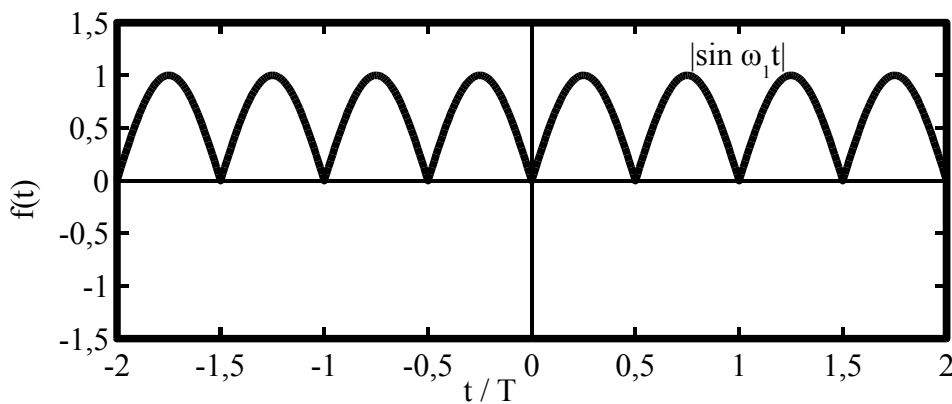
5.2

$$\overline{|f(t)|} = \frac{2}{3}$$

$$f_{\text{eff}} = \sqrt{\frac{8}{15}}$$

$$f(t) = \frac{2}{3} - \frac{4}{\pi^2}\left(\frac{\cos 2\omega_1 t}{1^2} + \frac{\cos 4\omega_1 t}{2^2} + \frac{\cos 6\omega_1 t}{3^2} + \cdots\right)$$
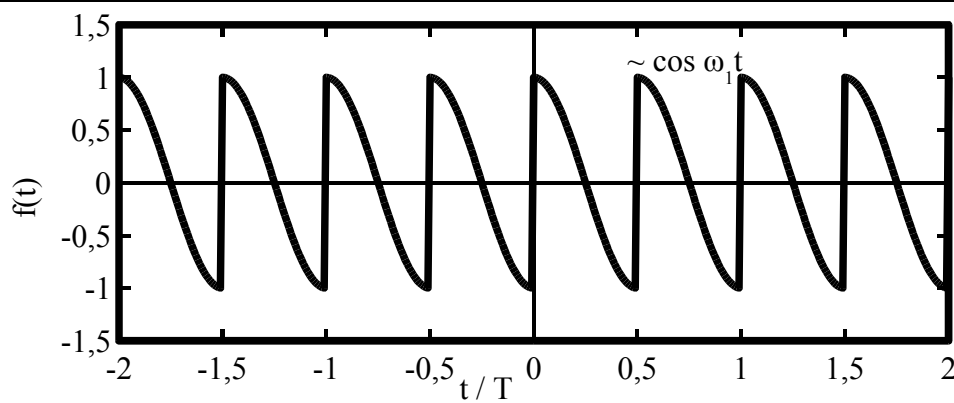
5.3

$$\overline{|f(t)|} = \frac{2}{\pi}$$

$$f_{\text{eff}} = \frac{1}{\sqrt{2}}$$

$$f(t) = \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{\cos 2\omega_1 t}{1\cdot 3} + \frac{\cos 4\omega_1 t}{3\cdot 5} + \frac{\cos 6\omega_1 t}{5\cdot 7} + \cdots\right)$$
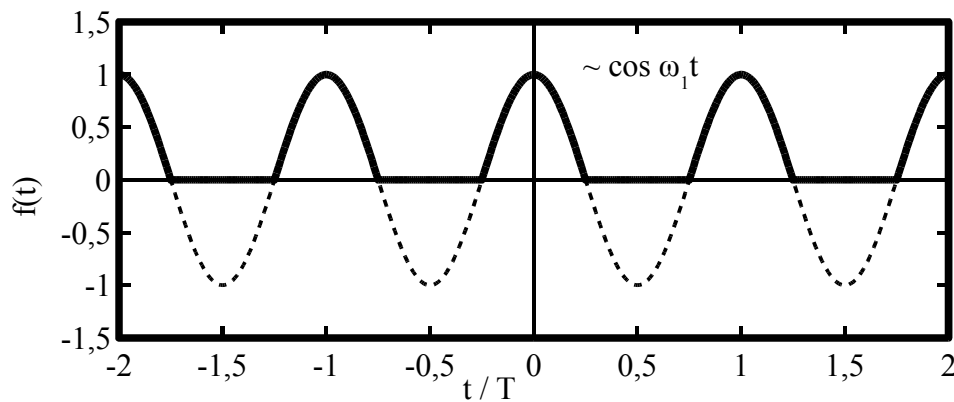
6.1

$$\overline{|f(t)|} = \frac{2}{\pi}$$

$$f_{\text{eff}} = \frac{1}{\sqrt{2}}$$

$$f(t) = \frac{4}{\pi}\left(\frac{2\sin 2\omega_1 t}{1\cdot 3} + \frac{4\sin 4\omega_1 t}{3\cdot 5} + \frac{6\sin 6\omega_1 t}{5\cdot 7} + \cdots\right)$$
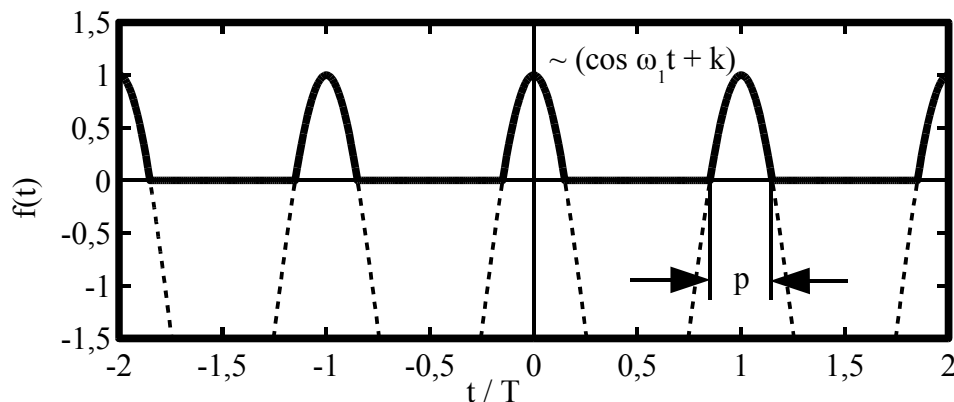

6.2

$$\overline{|f(t)|} = \frac{1}{\pi}$$

$$f_{\text{eff}} = \frac{1}{2}$$

$$f(t) = \frac{1}{\pi} + \frac{1}{2}\cos\omega_1 t + \frac{2}{\pi}\left(\frac{\cos 2\omega_1 t}{1\cdot 3} - \frac{\cos 4\omega_1 t}{3\cdot 5} + \frac{\cos 6\omega_1 t}{5\cdot 7} - \cdots\right)$$

6.3
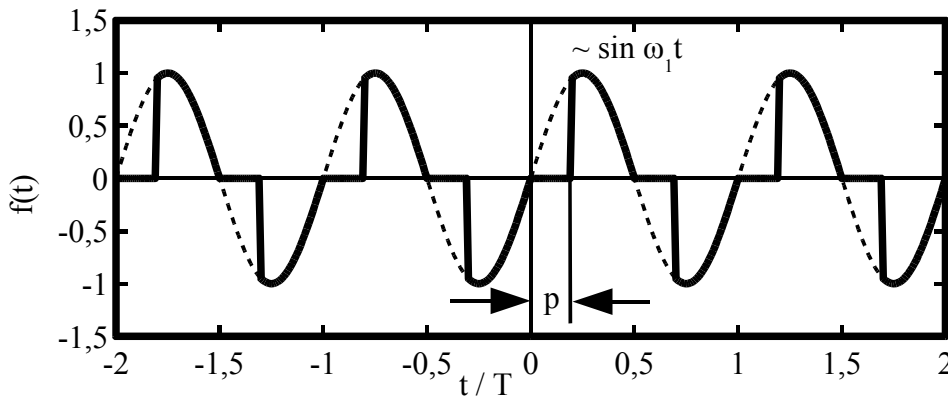
$$\overline{|f(t)|} = \frac{2p}{\pi}$$

$$f_{\text{eff}} = \sqrt{\frac{p}{2}}$$

$$f(t) = \frac{2p}{\pi} + \frac{4p}{\pi}\left(\frac{\cos\pi p}{1-(2p)^2}\cos\omega_1 t + \frac{\cos 2\pi p}{1-(4p)^2}\cos 2\omega_1 t + \frac{\cos 3\pi p}{1-(6p)^2}\cos 3\omega_1 t + \cdots\right)$$

7.1

$\sim \sin \omega_1 t$

$$\overline{|f(t)|} = \frac{2}{\pi} \cos^2(\pi p)$$

$$f_{\text{eff}} = \sqrt{\frac{1}{2} - p + \frac{\sin 4\pi p}{4\pi}}$$

$$f(t) = -\frac{2}{\pi}\left(\frac{\sin^2(2\pi p)}{2}\cos\omega_1 t + \frac{3\sin(2\pi p)\cdot\sin(6\pi p)+\cos(2\pi p)\cdot\cos(6\pi p)-1}{2\cdot 4}\cos 3\omega_1 t + \right.$$
$$\left. + \frac{5\sin(2\pi p)\cdot\sin(10\pi p)+\cos(2\pi p)\cdot\cos(10\pi p)-1}{4\cdot 6}\cos 5\omega_1 t + \cdots \right) +$$
$$+ \frac{2}{\pi}\left(\frac{\sin(4\pi p)-4\pi p+2\pi}{4}\sin\omega_1 t + \frac{3\sin(2\pi p)\cdot\cos(6\pi p)-\cos(2\pi p)\cdot\sin(6\pi p)}{2\cdot 4}\sin 3\omega_1 t + \right.$$
$$\left. + \frac{5\sin(2\pi p)\cdot\cos(10\pi p)-\cos(2\pi p)\cdot\sin(10\pi p)}{4\cdot 6}\sin 5\omega_1 t + \cdots \right)$$
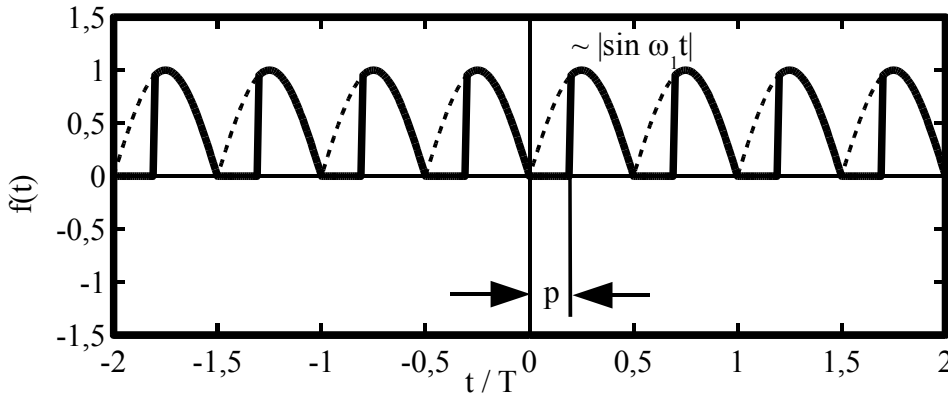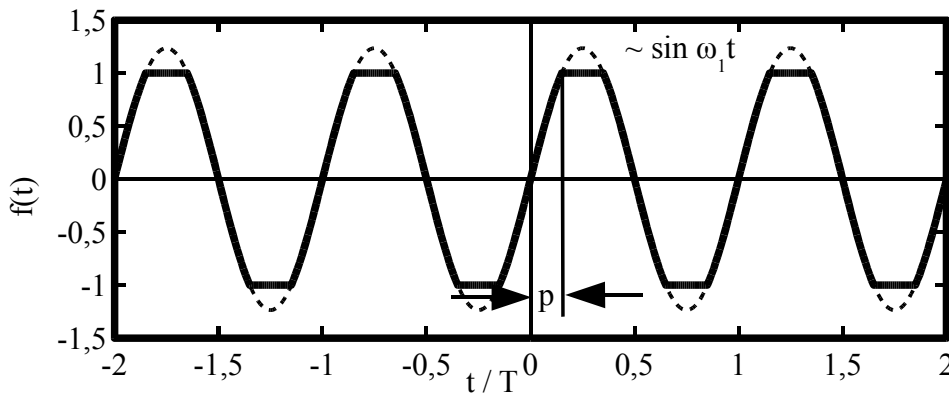

7.2

$\sim |\sin \omega_1 t|$

$$\overline{|f(t)|} = \frac{2}{\pi}\ \cos^2(\pi p)$$

$$f_{\text{eff}} = \sqrt{\frac{1}{2} - p + \frac{\sin 4\pi p}{4\pi}}$$

$$f(t) = \frac{2}{\pi}\cos^2(\pi p) - \frac{2}{\pi}\left(\frac{2\sin(2\pi p)\cdot\sin(4\pi p)+\cos(2\pi p)\cdot\cos(4\pi p)+1}{1\cdot 3}\cos 2\omega_1 t + \right.$$
$$+ \frac{4\sin(2\pi p)\cdot\sin(8\pi p)+\cos(2\pi p)\cdot\cos(8\pi p)+1}{3\cdot 5}\cos 4\omega_1 t +$$
$$\left. + \frac{6\sin(2\pi p)\cdot\sin(12\pi p)+\cos(2\pi p)\cdot\cos(12\pi p)+1}{5\cdot 7}\cos 6\omega_1 t + \cdots \right) +$$
$$+ \frac{2}{\pi}\left(\frac{2\sin(2\pi p)\cdot\cos(4\pi p)-\cos(2\pi p)\cdot\sin(4\pi p)}{1\cdot 3}\sin 2\omega_1 t + \right.$$
$$+ \frac{4\sin(2\pi p)\cdot\cos(8\pi p)-\cos(2\pi p)\cdot\sin(8\pi p)}{3\cdot 5}\sin 4\omega_1 t +$$
$$\left. + \frac{6\sin(2\pi p)\cdot\cos(12\pi p)-\cos(2\pi p)\cdot\sin(12\pi p)}{5\cdot 7}\sin 6\omega_1 t + \cdots \right)$$
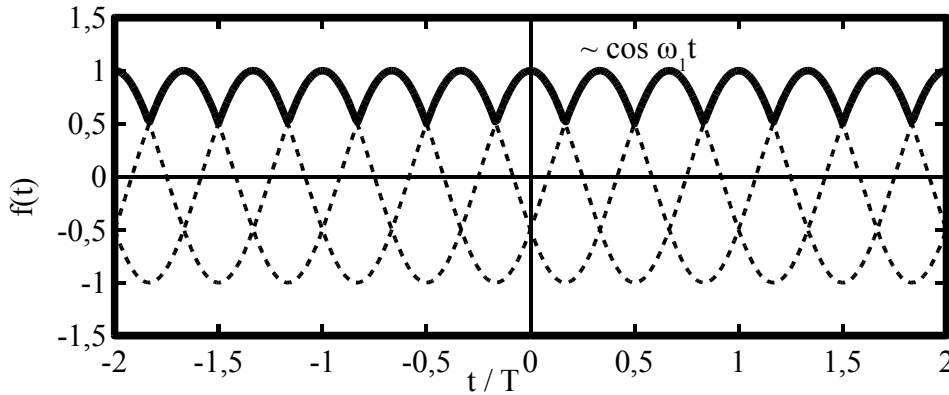
8.1

$\sim \sin \omega_1 t$

$$\overline{|f(t)|} = 1 - 4p + 2\frac{1 - \cos 2\pi p}{\pi \sin 2\pi p}$$

$$f_{\text{eff}} = \left(1 - 4p + \frac{4\pi p - \sin 4\pi p}{2\pi \sin^2 2\pi p}\right)^{\frac{1}{2}}$$

$$f(t) = \frac{2}{\pi \sin 2\pi p}\left[\left(2\pi p + \frac{\sin 4\pi p}{1\cdot 2}\right)\sin \omega_1 t + \left(\frac{\sin 4\pi p}{2\cdot 3} + \frac{\sin 8\pi p}{3\cdot 4}\right)\sin 3\omega_1 t + \left(\frac{\sin 8\pi p}{4\cdot 5} + \frac{\sin 12\pi p}{5\cdot 6}\right)\sin 5\omega_1 t + \cdots\right]$$
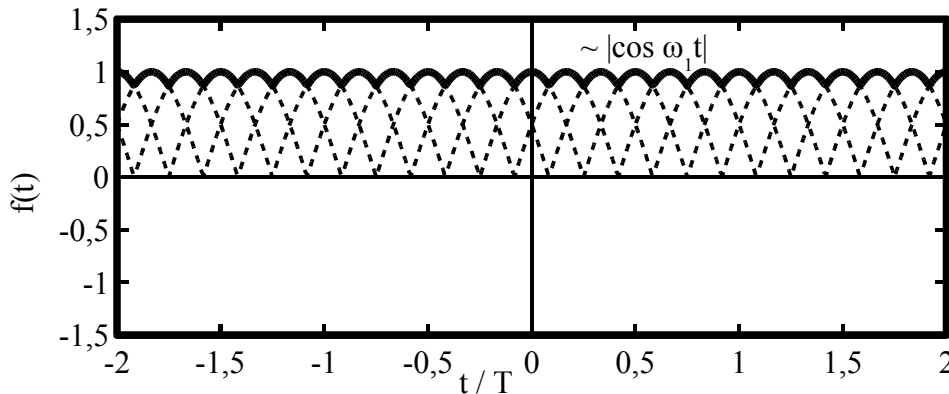
8.2

$\sim \cos \omega_1 t$

$$\overline{|f(t)|} = \frac{3\sqrt{3}}{2\pi}$$

$$f_{\text{eff}} = \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{8\pi}}$$

$$f(t) = \frac{3\sqrt{3}}{2\pi} + \frac{3\sqrt{3}}{\pi}\left(\frac{\cos 3\omega_1 t}{2\cdot 4} - \frac{\cos 6\omega_1 t}{5\cdot 7} + \frac{\cos 9\omega_1 t}{8\cdot 10} - \cdots\right)$$

8.3

$\sim |\cos \omega_1 t|$

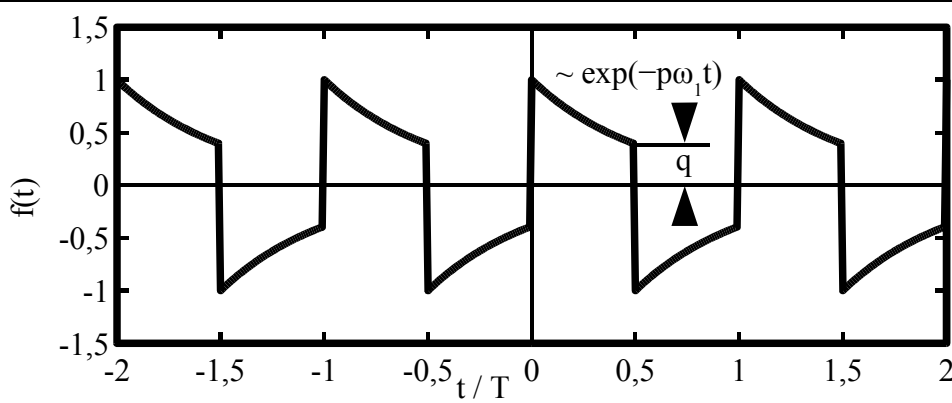$$\overline{|f(t)|} = \frac{3}{\pi}$$

$$f_{\text{eff}} = \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}}$$

$$f(t) = \frac{3}{\pi} + \frac{6}{\pi}\left(\frac{\cos 6\omega_1 t}{5\cdot 7} - \frac{\cos 12\omega_1 t}{11\cdot 13} + \frac{\cos 18\omega_1 t}{17\cdot 19} - \cdots\right)$$
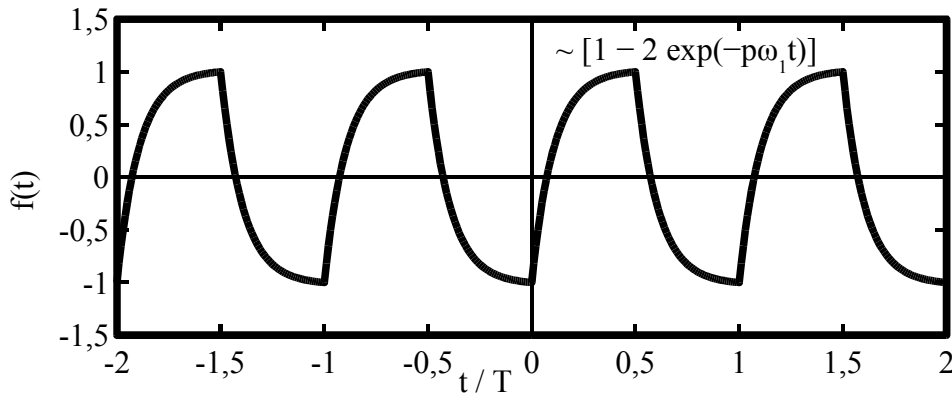
9.1

$$q = e^{-\pi p}$$

$$\overline{|f(t)|} = \frac{1-q}{\pi p}$$

$$f_{\text{eff}} = \sqrt{\frac{1-q^2}{2\pi p}}$$

$$f(t) = \frac{2(1+q)p}{\pi}\left(\frac{\cos\omega_1 t}{1^2+p^2} + \frac{\cos 3\omega_1 t}{3^2+p^2} + \frac{\cos 5\omega_1 t}{5^2+p^2} + \cdots\right) +$$
$$+ \frac{2(1+q)}{\pi}\left(\frac{\sin\omega_1 t}{1^2+p^2} + \frac{3\sin 3\omega_1 t}{3^2+p^2} + \frac{5\sin 5\omega_1 t}{5^2+p^2} + \cdots\right)$$

9.2

$$q = e^{-\pi p}$$

$$\overline{|f(t)|} = \frac{1+q}{1-q}\left(1 - \frac{2}{\pi p}\ln\frac{2}{1+q}\right)$$

$$f_{\text{eff}} = \sqrt{\frac{q}{\pi p(1-q)}\left[(\pi p - 2)\frac{1}{q} + 2\pi p + (\pi p + 2)q\right]}$$

$$f(t) = -\frac{4p}{\pi}\cdot\frac{1+q^2}{1-q}\left(\frac{\cos\omega_1 t}{1^2+p^2} + \frac{\cos 3\omega_1 t}{3^2+p^2} + \frac{\cos 5\omega_1 t}{5^2+p^2} + \cdots\right) +$$
$$+ \frac{4p^2}{\pi}\cdot\frac{1+q^2}{1-q}\left(\frac{\sin\omega_1 t}{1(1^2+p^2)} + \frac{\sin 3\omega_1 t}{3(3^2+p^2)} + \frac{\sin 5\omega_1 t}{5(5^2+p^2)} + \cdots\right)$$