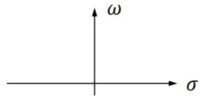
Elementare LTI-Systeme: P-Glied (Proportionalglied)

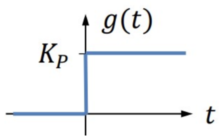
- Differentialgleichung $y = K_P \cdot x$
- Übertragungsfunktion $\underline{H}(s)$ $\underline{H}(s) = K_P$
- PN-Diagramm



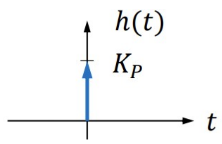
keine Nullstellen,
keine Pole

- Sprungantwort $g(t)$ und Impulsantwort $h(t)$

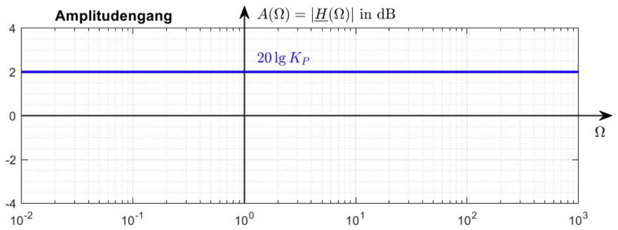
$g(t) = K_P \cdot \varepsilon(t)$



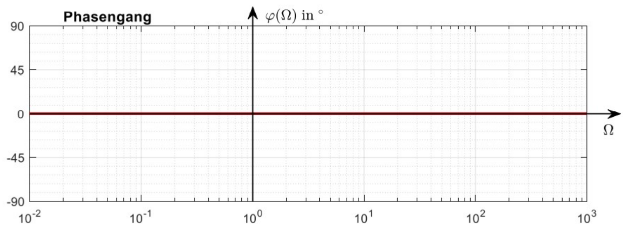
$h(t) = K_P \cdot \delta(t)$



- Frequenzgang (Bode-Diagramm)



$H(\omega) = K_P$



$\varphi(\omega) = 0$

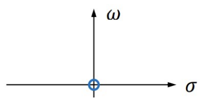
$\Omega = \frac{\omega}{\omega_B}$

ω_B: beliebig

- Eigenschaften: kausal, stabil, speicherfrei

Elementare LTI-Systeme: D-Glied (Differenzierer)

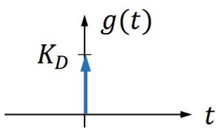
- Differentialgleichung $y = K_D \cdot \dot{x}$
- Übertragungsfunktion $\underline{H}(s)$ $\underline{H}(s) = K_D \cdot s$
- PN-Diagramm



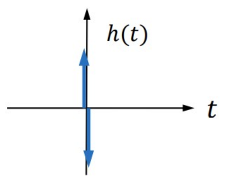
eine Nullstelle bei $s_o=0$ (Ursprung)
keine Pole

- Sprungantwort $g(t)$ und Impulsantwort $h(t)$

$g(t) = K_D \cdot \delta(t)$



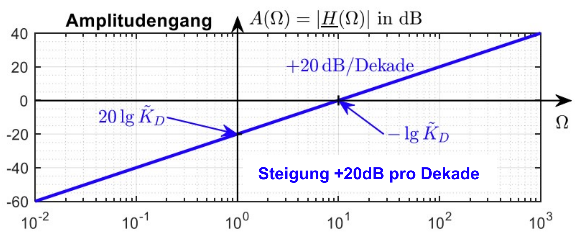
$h(t) = K_D \cdot \dot{\delta}(t)$



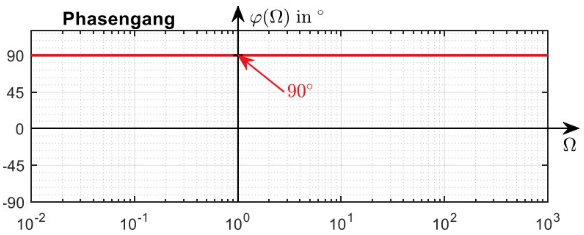
$\dot{\delta}(t)$: „Doublette“

$\int_{-\infty}^{\infty} x(t)\,\dot{\delta}(t)\,dt = -\dot{x}(t)$

- Frequenzgang (Bode-Diagramm)



$H(\omega) = K_D \cdot \omega$



$\varphi(\omega) = 90°$

$\Omega = \frac{\omega}{\omega_B}$

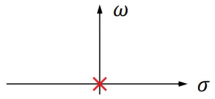
ω_B: beliebig

$\tilde{K}_D = \omega_B K_D$

- Eigenschaften: akausal, instabil, speicherbehaftet

Elementare LTI-Systeme: I-Glied (Integrierer)

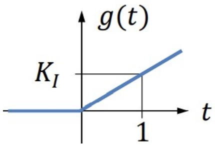
- Differentialgleichung $\dot{y} = K_I \cdot x$ $\qquad y = K_I \cdot \int_{-\infty}^{t} x(\tau)\, d\tau$
- Übertragungsfunktion $\underline{H}(s)$ $\qquad \underline{H}(s) = \frac{K_I}{s}$
- PN-Diagramm

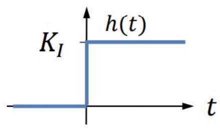


keine Nullstellen
eine Polstelle bei s_x=0 (Ursprung)

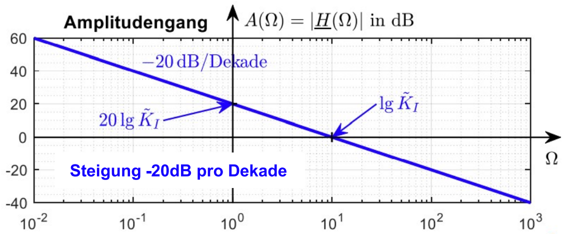
- Sprungantwort $g(t)$ und Impulsantwort $h(t)$

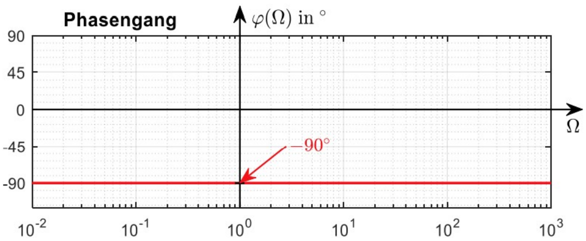
$g(t) = K_I \cdot t \cdot \varepsilon(t)$ $\qquad h(t) = K_I \cdot \varepsilon\,(t)$





- Frequenzgang (Bode-Diagramm)





$$\Omega = \frac{\omega}{\omega_B}$$

ω_B: beliebig

$$\tilde{K}_I = \frac{K_I}{\omega_B}$$

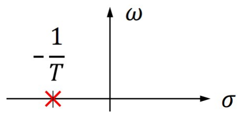
$$H(\omega) = \frac{K_I}{\omega}$$

$$\varphi(\omega) = -90°$$

- Eigenschaften: kausal, instabil, speicherbehaftet

Elementare LTI-Systeme: PT_1-Glied (Tiefpass 1. Ordnung)

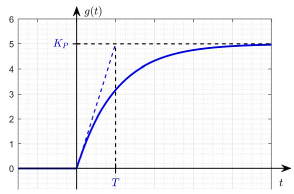
- Differentialgleichung $T \cdot \dot{y} + y = K_P \cdot x$ $\qquad T$: Zeitkonstante
- Übertragungsfunktion $\underline{H}(s)$ $\qquad \underline{H}(s) = \frac{K_P}{T \cdot s + 1} = \frac{K_P \cdot \omega_E}{s + \omega_E}$ $\qquad \omega_E = \frac{1}{T}$: Eckfrequenz
- PN-Diagramm

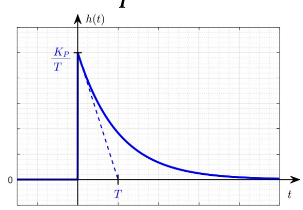


keine Nullstellen
eine Polstelle bei s_x= $-1/T$

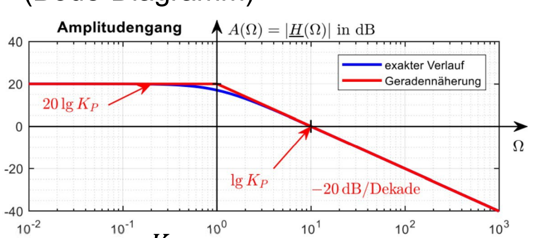
- Sprungantwort $g(t)$ und Impulsantwort $h(t)$

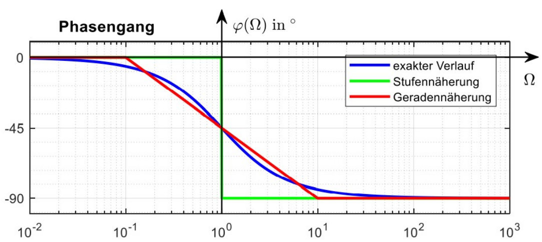
$g(t) = K_P \cdot \left(1 - e^{-\frac{t}{T}}\right) \cdot \varepsilon(t)$ $\qquad h(t) = \frac{K_P}{T} \cdot e^{-\frac{t}{T}} \cdot \varepsilon(t)$





- Frequenzgang (Bode-Diagramm)





$$\Omega = \frac{\omega}{\omega_E}$$

üblich: $\omega_B = \omega_E$

$$H(\Omega) = \frac{K_P}{\sqrt{1+\Omega^2}} \qquad H(\omega) = \frac{K_P}{\sqrt{1+\omega^2 T^2}} = \frac{K_P \cdot \omega_E}{\sqrt{\omega_E^2 + \omega^2}}$$

$$\varphi(\Omega) = -\arctan \Omega \qquad \varphi(\omega) = -\arctan \omega T$$

- Eigenschaften: kausal, stabil, speicherbehaftet

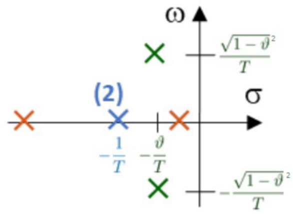
Elementare LTI-Systeme: PT$_2$-Glied (Tiefpass 2. Ordnung)

T: Kennzeit (Zeitkonstante)

$\omega_0 = \frac{1}{T}$: Kennkreisfreq., Polfreq.

$\vartheta = \frac{\delta}{\omega_0}$: Dämpfungsgrad, -maß $(\vartheta = D)$

- Differentialgleichung $\quad T^2 \cdot \ddot{y} + 2\vartheta T \cdot \dot{y} + y = K_P \cdot x$
- Übertragungsfunktion $\underline{H}(s)$ $\quad \underline{H}(s) = \frac{K_P}{T^2 s^2 + 2\vartheta T s + 1} = \frac{K_P \cdot {\omega_0}^2}{s^2 + 2\vartheta\omega_0 s + {\omega_0}^2}$
- PN-Diagramm **Pole:** $s_{x1,2} = \frac{1}{T}\left(-\vartheta \pm \sqrt{\vartheta^2 - 1}\right)$



$0 < \vartheta < 1$ Schwingung: konjugiert komplexes Polpaar

$$\underline{H}(s) = \frac{K_P}{T^2} \frac{1}{(s - s_{x1})(s - s_{x2})}$$

zu Schwingfall: $s_{x1,2} = -\vartheta\omega_0 \pm j\omega_d$

mit Eigen-, Ausschwing-frequenz ω_d

$$\omega_d = \omega_0\sqrt{1 - \vartheta^2}$$

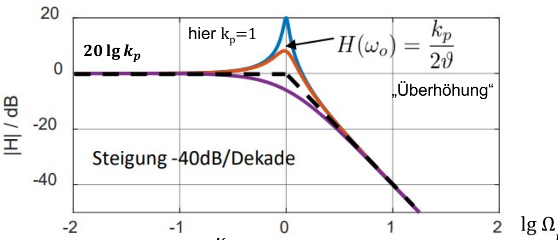
$\vartheta = 1$ aperiod. Grenzfall: doppelter reeller Pol bei $-\omega_0$

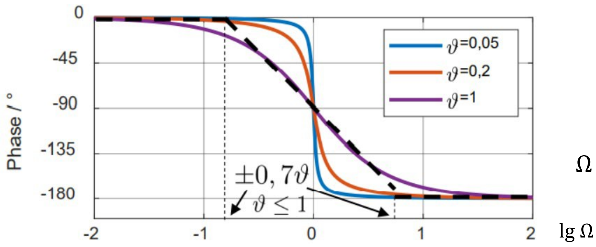
$$\underline{H}(s) = K_P \frac{1}{(1 + sT)^2}$$

$\vartheta > 1$ aperiod. Dämpfung: zwei versch. reelle Pole $s_{x1,2} = \omega_0(-\vartheta \pm \sqrt{\vartheta^2 - 1})$

$$\underline{H}(s) = K_P \frac{1}{1 + sT_1} \frac{1}{1 + sT_2} \quad \text{mit: } T_{1,2} = -\frac{1}{s_{x1,2}} = \frac{T}{\vartheta \mp \sqrt{\vartheta^2 - 1}} = T(\vartheta \pm \sqrt{\vartheta^2 - 1})$$

- Frequenzgang (Bode-Diagramm) $\quad \underline{H}(\omega) = \frac{K_P}{1 - \omega^2 T^2 + j\omega 2\vartheta T}$





$$\Omega = \frac{\omega}{\omega_0}$$

$$H(\omega) = \frac{K_P}{\sqrt{(1 - \omega^2 T^2)^2 + 4\vartheta^2\omega^2 T^2}} \qquad H(\Omega) = \frac{K_P}{\sqrt{(1 - \Omega^2)^2 + 4\vartheta^2\Omega^2}}$$

$$\varphi(\omega) = -\arctan\frac{2\vartheta\omega T}{1 - \omega^2 T^2} \qquad \varphi(\Omega) = -\arctan\frac{2\Omega\vartheta}{1 - \Omega^2}$$

- Eigenschaften: kausal, stabil, speicherbehaftet

Elementare LTI-Systeme: PT$_2$-Glied (Tiefpass 2. Ordnung)

- Sprungantwort g(t) und Impulsantwort h(t)

$0 < \vartheta < 1$ Schwingung

$$g(t) = \left(1 - \frac{1}{\sqrt{1 - \vartheta^2}} \cdot e^{-\vartheta\omega_0 t} \cdot \cos(\omega_d t - \Theta)\right) K_P \cdot \varepsilon(t) \qquad h(t) = \frac{K_P \omega_0}{\sqrt{1 - \vartheta^2}} \cdot e^{-\vartheta\omega_0 t} \cdot \sin(\omega_d t)\, \varepsilon(t)$$

mit $\Theta = \operatorname{asin}(\vartheta)$

$\vartheta = 1$ aperiod. Grenzfall

$$g(t) = \left(1 - \left(1 + \frac{t}{T}\right) \cdot e^{-\frac{t}{T}}\right) K_P \cdot \varepsilon(t) \qquad h(t) = K_P \cdot \frac{t}{T^2} e^{-\frac{t}{T}} \cdot \varepsilon(t)$$

$\vartheta > 1$ aperiod. Dämpfung

$$g(t) = \left(1 - \frac{T_1}{T_1 - T_2} \cdot e^{-\frac{t}{T_1}} + \frac{T_2}{T_1 - T_2} \cdot e^{-\frac{t}{T_2}}\right) K_P \cdot \varepsilon(t) \qquad h(t) = \left(\frac{1}{T_1 - T_2} \cdot \left(e^{-\frac{t}{T_1}} - e^{-\frac{t}{T_2}}\right)\right) K_P \cdot \varepsilon(t)$$