5. Multiplication of Vectors

In Data Science

there are totally 3 types of (Mul of Vectors)

which we do

3 Types

① Dot Product (Inner Product)

② Element wise Multiplication

③ Scalar Multiplication

① Dot Product

Dot product of 2 vectors result in a scalar and its corresponding components → is calculated as the sum of

Eg

$$A = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$A \cdot B = (2 \cdot 4) + (3 \cdot 5)$ → Product of 2 vectors

$= 8 + 15$ (results in Scalar)

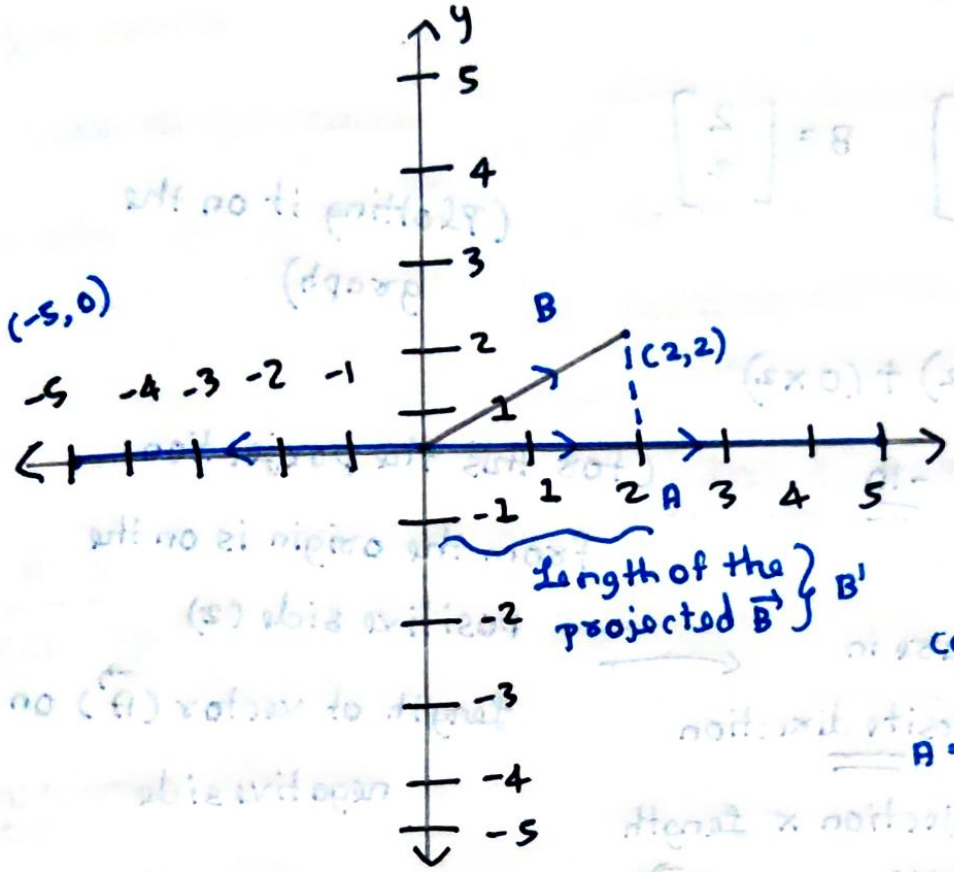
usually $= 23$ (Sum of corresponding comp)

$A \cdot B^T$

(This is how it will be calculative) → $\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 \end{bmatrix}$ → $(2 \times 4) + (3 \times 5)$

→ $8 + 15 = 23$

Visualizing them in the co-ordinate system



considering

$A = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Now, our aim is to take the $\vec{A}$ (do a dot product with B)

$\vec{A} \cdot \vec{B} = (5 \times 2) + (0 \times 2)$

$= \boxed{10}$ (Ans)

(wrt scalar system)

Based on the co-ord sys how we will find the ans

$\vec{A} \cdot \vec{B}$ = (Length of projected $\vec{B}$ (B')) · (Length of vector $\vec{A}$)

$= (2) \cdot (5)$

$\boxed{\vec{A} \cdot \vec{B} = 10}$ (Dot Product is a positive value)

The projection (B') and the $\vec{A}$ are in the same direction

Taking Another Scenario **Case 2**

where

$$A = \begin{bmatrix} -5 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

(Plotting it on the graph)

Sol

$(-5 \times 2) + (0 \times 2)$

$\Rightarrow -10$

(for this the projection from the origin is on the positive side (2)

Both are in the opposite direction ←

Length of vector ($\vec{A}$) on the negative side

$\Rightarrow$ Projection $\times$ Length

(B') ($\vec{A}$)

$\Rightarrow 2 \times -5$

$\Rightarrow \boxed{-10}$ (final ans)

Case 3

Suppose say I have a point $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ → on the y axis

$\vec{A} \cdot \vec{B} = 0$ (how)

This happens when we project the vector to the origin

Projection (B') $\times$ Length (A)

0×5

$0 \rightarrow$ (Ans)

Applications of Dot Product in Data Science → Gen AI Apps
↳ RAG

① Cosine Similarity

Measure used to determine how similar 2 vectors are

(calculates the cosine of the angle b/w 2 vectors providing a similarity score that ranges b/w -1 (dissimilar) to 1 (Similar))

formula

$$\cos\theta = \frac{A \cdot B}{\|A\| \|B\|}$$

Recommendation Systems → Netflix → watch an action movie
↓
Recommends you similar movies based on Action

Say I have movies → Say this is represented in a 5D

Avengers → [1, 2, 0, 3, 1]
Action, Comedy, Drama, Romance

B → [2, 0, 1, 1, 1]

Now we calc Cosine Similarity
↓
we find the Dot Product

Then we find the magnitude by Euclidian Norm
→ (Step 2)

Step 2

$\vec{A} \cdot \vec{B} = (1 \cdot 2) + (2 \cdot 0) + (0 \cdot 1) + (3 \cdot 1) + (1 \cdot 1)$

$= \boxed{6}$ (Ans)

Step 2 we find magnitude

$||A||$ $||B||$

$||A|| \rightarrow \sqrt{(1)^2 + (2)^2 + (0)^2 + (3)^2 + (1)^2} \rightarrow \sqrt{15} = \approx 3.872$

$||B|| \rightarrow \sqrt{(2)^2 + (0)^2 + (1)^2 + (1)^2 + (1)^2}$

$\rightarrow \sqrt{7} = 2.646$

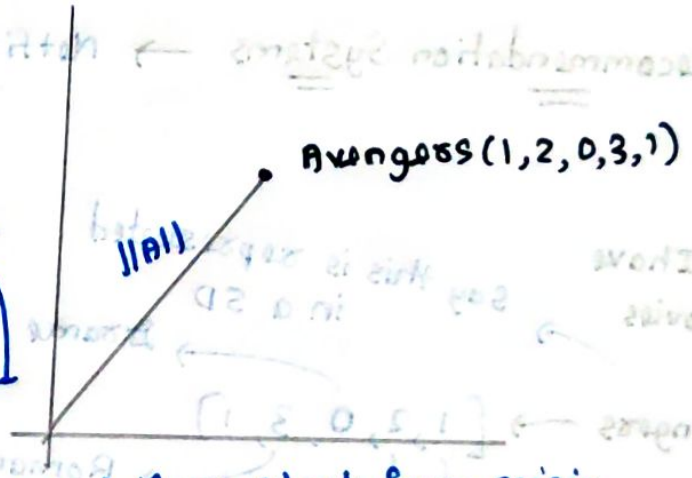
(how are we doing this)

once after computing everything we finally compute

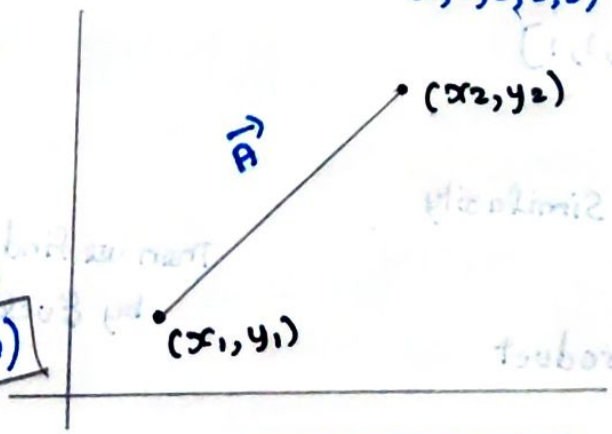
$$\cos\theta = \frac{6}{3.872 * 2.646} = \approx 0.586$$

nearly close to 1

56.6% +vely similar



(what if we start from origin (0,0,0,0,0) $(1-0)^2 + (2-0)^2 + \cdots$ (gaussian)



(In general we find the distance b/w 2 points using Euclidean distance)

calculated as $\rightarrow ||A|| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

So, if a person is watching Avengers there is a chance of 56.6% movie B can pop up