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2 authors:

Andre Borgato Morelli
University of São Paulo

André Luiz Cunha
University of São Paulo

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Assessing vulnerabilities in transport networks: a graph-theoretic approach

Verificação de vulnerabilidades em redes de transporte: uma abordagem pela teoria dos grafos

André Borgato Morelli[1], **André Luiz Cunha**[2]

[1]University of São Paulo, São Paulo – Brazil, andre.morelli@usp.br
[2]University of São Paulo, São Paulo – Brazil, alcunha@usp.br

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ABSTRACT
The design and maintenance of sustainable and resilient transport systems depend on the identification of possible vulnerabilities before crises occur so that infrastructure and strategies of action are effectively developed for times of crisis. However, given the complexity of transport systems, the proposed methods for assessing vulnerabilities are difficult to implement and require data inaccessible to most Brazilian municipalities. Given this scenario, and intending to simplify the preliminary analysis of a system in the search for vulnerabilities, the objective of this paper is to present the centrality measure from graph theory that best represents the local vulnerability of inland transport networks in Brazilian cities. The method proposed in the study was the systematic degradation of the network measuring the decay in continuity on the system, defined as the proportion of valid paths that remain in the network after the removal of a certain number of roads. The results pointed out the betweenness centrality is the metric that best reflects vulnerability since the attack strategy that progressively removes the roads with greater betweenness centrality presents a faster decay of continuity. With this result, we expect to facilitate the detection of vulnerabilities in transport systems and to guide the creation of more resilient transport systems.

RESUMO
O projeto e a manutenção de sistemas de transporte sustentáveis e resilientes dependem da identificação de possíveis vulnerabilidades antes que as crises ocorram, para que a infraestrutura e as estratégias de ação sejam efetivamente desenvolvidas em tempos de crise. No entanto, dada a complexidade dos sistemas de transporte, os métodos propostos para avaliação de vulnerabilidades são de difícil implementação e requerem dados inacessíveis para a maioria dos municípios brasileiros. Diante desse cenário, e com o intuito de simplificar a análise preliminar de um sistema em busca de vulnerabilidades, o objetivo deste trabalho é apresentar a medida de centralidade da teoria dos grafos que melhor representa a vulnerabilidade local das redes de transporte terrestre nas cidades brasileiras. O método proposto no estudo foi a degradação sistemática da rede medindo o decaimento de continuidade no sistema, definida como a proporção de caminhos válidos que permanecem na rede após a remoção de um determinado número de vias. Os resultados apontaram que a centralidade de intermediação é a métrica que melhor reflete a vulnerabilidade, uma vez que a estratégia de ataque que remove progressivamente as estradas com maior centralidade de intermediação apresenta um declínio mais rápido da continuidade. Com este resultado, esperamos facilitar a detecção de vulnerabilidades nos sistemas de transporte e orientar a criação de sistemas de transporte mais resilientes.

1. INTRODUCTION

Resilience studies have recently gained attention due to their importance in conceiving and maintaining sustainable transport systems. The mass movements of goods and people are fundamental to the contemporary economy and lifestyle. The systems that make these movements possible are frequently subject to congestion, overcrowding, and widespread delays. Besides, natural phenomena such as floods and hurricanes can also impact land transport networks, damaging or obstructing infrastructure, and limiting the traffic of both vehicles and pedestrians. Given this scenario, studying potential causes for vulnerabilities gain importance since the detection of weaknesses in systems allows for the adoption of strategies during crises that can render the system more resilient. Moreover, with the current development of climate change and its increasing potential to exacerbate already catastrophic events, resilience takes on a central role in urban planning to make cities more adaptable and able to resist impacts at the lowest possible social, technical, and economic cost.

Resilience can be defined as the inherent ability of a system to adapt to adverse situations avoiding potential losses (Westrum, 2006). In this context, accurately measuring resilience depends on assumptions about the type of impact and the elements of the system that will be reached. One of the pillars of resilience studies is the detection of vulnerabilities in transport systems, which lends itself to understanding the regions where the most significant impacts are expected and, based on this information, guide resilient design (Mattsson and Jenelius, 2015). Vulnerability detection generally falls into two categories: specific or generalized. The firstanalyzes the impact of a particular event or shocks such as floods or natural disasters. The latter is interested in topological characteristics of the network and how they affect the system's ability to resist shocks in general, even when the phenomenon causing the damage is not previously known (Folke *et al.*, 2010). This paper falls into the second category, focusing on detecting possible structural vulnerabilities in networks regardless of the phenomenon that may cause them. Thus, the main objective of this paper is to determine the centrality measure that more closely reflects structural vulnerabilities in road network systems. We focus our analysis on topological characteristics of the network obtained from graph theory, to assess the centrality measures (local means of complex networks) that favor or hinder the continuity of the transport system. More specifically, three measures of centrality were evaluated: (1) closeness centrality, which represents how close one edge is, on average, to other edges, so that roads with high closeness centrality occupy the central portion of the network; (2) betweenness centrality, which expresses what extent a road is needed as an intermediary to connect two points, so that bridges, viaducts, tunnels and other sources of network bottlenecks generally have high betweenness centrality; and (3) degree centrality, which indicates how many roads are directly connected to an edge, through at least one intersection.

This paper is divided into five parts: (1) this introduction; (2) a brief review of previous work on the topics of resilience and vulnerability in transport networks; (3) a description of the proposed method for the paper; (4) a presentation of results and discussion; and (5) a conclusion.

2. PREVIOUS WORK

Within the scope of transport vulnerabilities, we look at papers that consider weaknesses to specific events as well as those that consider the structural vulnerability of the network. Specific analyses consider recurrent phenomena such as congestion and disturbances generated by accidents in regions where these are more likely (Cox, Prager and Rose, 2011; Wang *et al.*, 2015). Also, there are studies which consider natural disasters such as storms and floods (Litman, 2005; Lu, Peng and Zhang, 2014; Morelli and Cunha, 2019) and others that investigated possible oil supply crises and their impacts on the transport system (Newman, Beatley and Boyer, 2009;

Martins, Rodrigues da Silva and Pinto, 2019). These specific analyses generally depend on complex and difficult-to-obtain data, which bring difficulties to applications in most Brazilian cities in which even origin-destination (OD) surveys are rare. Regarding the generalized view of vulnerability as a structural property, most studies focus on the network's properties such as connectivity and redundancy of infrastructure, taking into account the morphology of the road system to identify possible vulnerabilities (Leu, Abbass and Curtis, 2010; Ip and Wang, 2011; Zhang, Miller-Hooks and Denny, 2015). There is a wide variety of studies proposing metrics that can identify vulnerabilities in transport networks (Appert and Chapelon, 2007; Berche *et al.*, 2009; Rodríguez-Núñez and García-Palomares, 2014). Although some studies seem easily applicable to several urban networks, none of them compares a significant number of cities, with the majority presenting a comparison of two or three networks, making it impossible to determine which measures influence network resilience.

In this regard, Appert and Chapelon (2007) developed vulnerability metrics and applied them in the city of Montpellier, France. The metrics were based on the impact on the length of the minimum paths in the network when removing an edge or node. The logic is that, if the loss of a connection causes a significant increase in the average distance in a city, that connection must have greater importance in the network. However, there are problems with this approach, the most immediate being the fact that the index cannot be applied when some destinations become unreachable, e.g. if an edge is the only connection between two regions of a city, such as a single bridge between two sides of a river. Another problem is the computational load required to calculate vulnerability in large networks. With this method, for each edge or node removed, all paths in the new network must be recalculated, which makes it impossible to use computational shortcuts generally used in the calculation of centralities.

In a slightly more comprehensive work, Berche *et al.* (2009) studied the resilience of 14 transit systems from the perspective of deactivating connections from the system, to which the authors give the name “attacks” on the system. The authors raise the discussion that there are several types of attack strategies for these systems, and links can be removed according to several criteria, some of which are more harmful than others to the system. To find the most effective attack strategies, the authors used graph theory and found that removing nodes from a network in decreasing order of betweenness centrality degrade the network more quickly, significantly reducing the size of the largest connected block with a small proportion of nodes removed. Rodríguez-Núñez and García-Palomares (2014) conducted a similar analysis on rail transit in the city of Madrid measuring the delay that a failure of a subway segment would cause to users. The authors also analyzed the number of subway journeys that are rendered impossible due to the removal of segments from the system.

In this paper, we build upon the basis of these reported works, with methodological expansions for application in urban road networks and analyze the 309 largest Brazilian cities to find the metrics of graph theory that can be used to assess local vulnerability in cities.

3. PROPOSED METHOD

We work with a database of Brazilian cities with a population of over 100 thousand inhabitants, which constitute a total of 309 urban centers as estimated by the Brazilian Institute of Geography and Statistics (*IBGE* in Portuguese) in 2018 (*IBGE*, 2018). We proposed a method with four steps to assess the centrality measure that best reflects the local vulnerability in a transport network:

- Extraction of a representative graph of the network;
- Calculation of centrality measures;
- Generation of impact scenarios (attacks on the system);
- Determination of the strategy that best reflects the vulnerability of the system.

3.1. Graph extraction

In the first stage of the work, we extracted representative graphs of the networks from the collaborative mapping platform OpenStreetMap (OpenStreetMap, 2019). In it, the municipal limits or centers are queried through the Nominatim system (OpenStreetMap Nominatim, 2019). Most of this information come out from IBGE dataset, including the central points, which is often marked by a structure of importance to the municipality or monument. To extract the networks was used the package OSMnx (Boeing, 2017) – a Python library for geospatial analysis.

The graphs obtained for this paper do not represent the entire region contained in the municipal boundaries since most cities have a small urbanized area in proportion to the municipal area. In this case, as this analysis is more focused on urbanized regions, we adopted only the innermost 100 km^2 of the city (the square with 10 km side centralized in the point of greater economic activity, not necessarily the geometric center). The area enclosed by the square is sufficient to encompass most urbanized regions entirely and, even in cases where this does not happen, the square contains the most economically active part of the city. As an example of extracted graphs, Figure 1 includes the graphs of the Brazilian cities of São Carlos-SP and Florianópolis-SC.

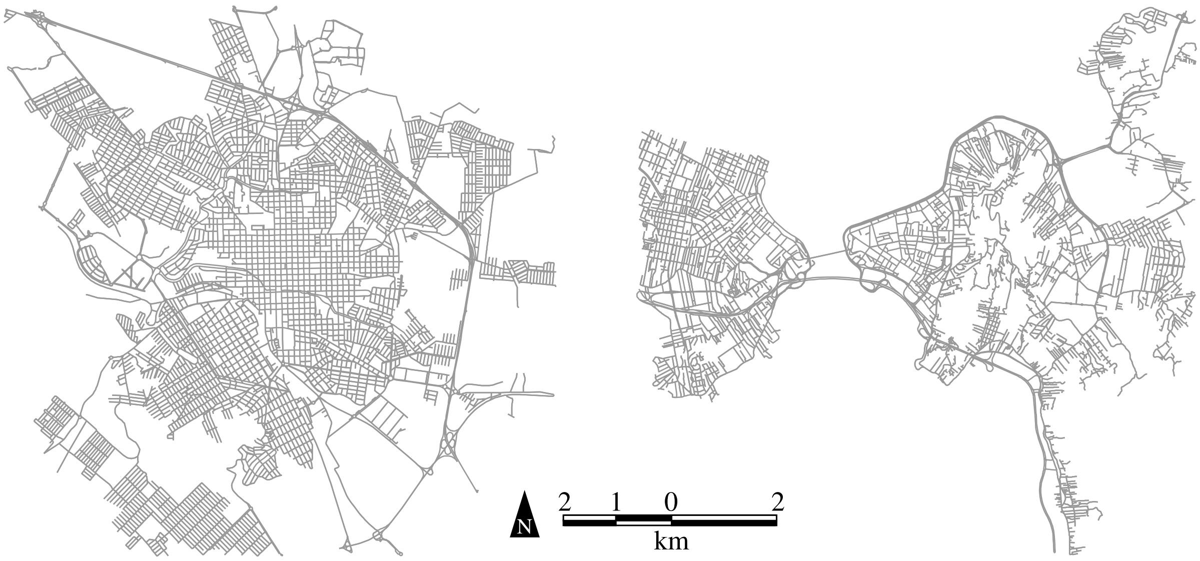

Figure 1. Graphs of the cities of São Carlos-SP (left) and Florianópolis-SC (right)

3.2. Centrality metrics

Centralities are metrics from graph theory that define how an element of the network (node or edge) relates to all other elements. These metrics depend only on the characteristics of the graph and can be easily calculated through NetworkX (Hagberg, Schult and Swart, 2008), a library for graph computations in Python. Three types of centrality were calculated for the extracted graphs:

Closeness Centrality: expresses the proximity of one element to all others. Centermost elements in the network have shorter average distances to all other elements and therefore have greater **closeness** in the graph. This centrality is defined as the inverse of the average distance from one node to all others in the graph, normalized by the number of nodes in a graph:

$$\mathbf{C_c(v)} = \frac{\mathbf{N}}{\sum_{t \in V} \mathbf{d(v,t)}} \tag{1}$$

where N: number of nodes in the graph;
v: node of reference;
t: node belonging to the graph;
d(v,t): distance from **v** to **t**

Since we evaluate vulnerability of road links (edges) in this paper, we assume the closeness centrality of an edge as the average centrality of its nodes.

Betweenness centrality: expresses how important a given edge is as a link within a network. The greater the number of minimum paths passing through an edge, the greater its betweenness centrality and, consequently, the greater its importance for travel in an urban network. The equation (2) defines the edge betweenness centrality .

$$\mathbf{C_b(e)} = \sum_{s \neq t \in V} \frac{\sigma_{st}(e)}{(N-1)(N-2) \cdot \sigma_{st}} \tag{2}$$

where **e**: edge of reference;
s,t: nodes belonging to the graph;
$\sigma_{st}(e)$: number of shortest paths departing from ***s*** and passing through ***e***;
σ_{st}: number of shortest paths departing from ***s***

Degree Centrality: The degree of a node refers to the number of edges directly connected to that node. In a road network, the degree of an intersection indicates how many roads intersect at that point. Thus, the degree centrality expresses the local connectivity of a node. The greater the number of edges connected to a given node, the greater its degree centrality, so that the degree centrality can be defined, in its normalized form, as:

$$\mathbf{C_g(v)} = \frac{\mathbf{g(v)}}{\mathbf{N}} \tag{3}$$

where g(v): degree of node v

The degree of an edge is defined as the average degrees of its nodes.

3.3. Scenario Generation

The proposed scenarios simulate the systematic removal of edges in the network in different ways. The removal of edges is analogous to blocking traffic through the road segment represented by the edge and an **attack strategy** to the system is defined by the order in which the edges of the system are removed (Berche *et al.*, 2009). In this paper, we assume that the strategy that harms the system the most with the lowest proportion of edges removed is the strategy that imposes the greatest vulnerability to the system. If a strategy is defined from a deterministic metric, as a measure of the centrality, it is possible to infer that this characteristic of the edge is an indicator of vulnerability in the system. The most straightforward attack strategy is to remove edges at random in a graph. However, more complex rules can be used to maximize or reduce the impact of an attack, such as attacks using the centrality of the graph elements.

We defined random edge removal as a control group and the deterministic heuristics evaluated were defined by removing edges ordered by each of the centrality metrics discussed previously (degree, closeness, and betweenness). Two ordering schemes ware evaluated: from the largest to the smallest (descending), and the smallest to the largest (ascending).

3.4. Evaluation Metric

In addition to attack strategies, a comparative metric is needed to define what is meant by "degradation" in a road system. In this work, we measure the general cohesion of the system after an attack as a proxy for vulnerability. When any phenomenon blocks part of a network's infrastructure, isolation of sectors of the city can occur, especially if the affected infrastructure is an essential link between a neighborhood or region to the rest of the network. Isolated sectors are inaccessible to users and emergency vehicles, which can be particularly harmful in periods of crisis, such as natural disasters.

We defined the **continuity** metric to measure the cohesion of the network after an impact. **Continuity** measures the proportion of valid routes in a system after an impact. To illustrate the concept of a valid route, Figure 2 contains two paths in an urban network with two blocks separated by a river. In the event of a problem with the single bridge that connects the two regions, the two blocks are disconnected, rendering Route A invalid, which would not happen if there was redundancy between the two sides of the river which would make it possible to deviate the route. In the illustrated case, not only Route A, but all routes connecting one side of the river to the other are not feasible in this event, reducing by half the number of valid routes in the network. Thus, the measure of continuity depends directly on the number of valid routes that remain after an impact on the network.

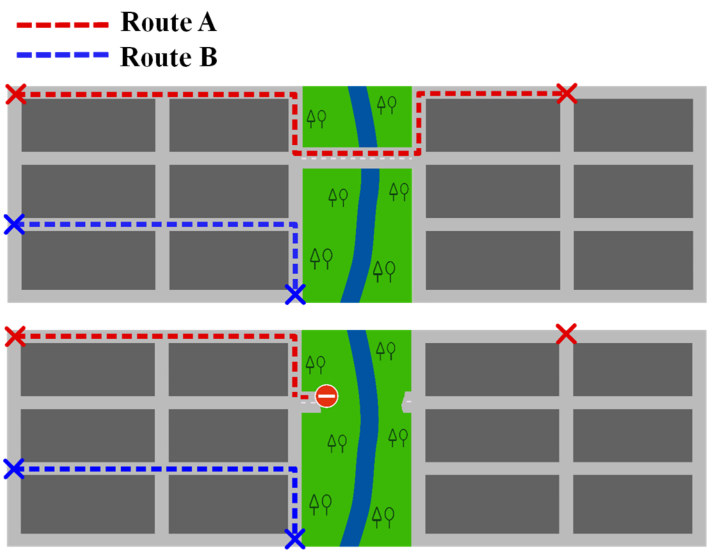

Figure 2. Exemplification of an event that reduces the number of valid routes in a transport network

We propose that a pair of nodes have a **valid path** if they have at least one route connecting one node to the other. Hence, to measure the number of pairs that have a valid path bateween then, we consider the network as an undirected graph and search for the connected components of it. Connected components of a graph are blocks of nodes connected by at least one path

(Newman, 2010). When all the nodes of a graph are interconnected, the graph has only one component, but as the connections are damaged, a network splits into blocks, as in the case of Figure 2, where the deterioration of a bridge separated the network into two blocks, one unreachable from the other. As a node has valid paths for all others in the same block and no valid path to any node outside of that block, each node in the block with N vertices will have N-1 valid paths, resulting in N (N-1) valid paths in the component or half of that number considering the path and its reverse as one. Therefore, for the entire network, the number of valid paths is the sum of the number of minimum valid paths contained in each connected. This method was chosen for computational efficiency since algorithms for finding connected blocks in undirected graphs have less computational complexity than minimum path algorithms. Thus:

$$\mathbf{VP} = \sum_{\mathbf{G'} \in \mathbf{G}} \mathbf{N}_{\mathbf{G'}}(\mathbf{N}_{\mathbf{G'}} - \mathbf{1}) \cdot \frac{\mathbf{1}}{\mathbf{2}} \tag{4}$$

where VP: Number of valid paths in the system;

G: evaluated Graph;

G': Connected component of G;

$N_{G'}$: Number of nodes in the component G'.

From Equation 4, we based our analysis on the evolution of the proportion of valid non-duplicated paths with the gradual removal of edges. Therefore, continuity after an impact $\boldsymbol{i}$ on the network is defined as the proportion of paths that remain valid in the system.

$$\boldsymbol{C}(\boldsymbol{i}) = \frac{\boldsymbol{VP}(\boldsymbol{i})}{\boldsymbol{VP_0}} \tag{5}$$

where $\boldsymbol{C}(\boldsymbol{i})$: Continuity after an impact $\boldsymbol{i}$ to the network;

$\boldsymbol{VP}(\boldsymbol{i})$: Valid paths after the impact;

$\boldsymbol{VP_0}$: Valid paths before the impact;

3.4. Centrality that best reflects vulnerability in the system.

The behavior of the system's continuity was analyzed from the systematic removal of roads according to strategies based on the centrality metrics of the system. An attack strategy that results in a more abrupt drop in continuity is more damaging to the system, which means that the measure that serves as the basis for the strategy has greater significance in measuring the vulnerability of a system.

For each strategy created, edges were disabled in steps of 1% until all edges of the network were removed. For each step, the total number of valid paths on the network was computed and divided by the initial total before the attack. After, we conducted a random edge removal strategy with 10 different random seeds, and the plotted curve refers to the average of these cases to capture the behavior of a process with stochastic nature. On the other hand, the other six strategies evaluated are deterministic, i.e. they do not require repetition of the process as they depend on a measure of network centrality.

4. RESULTS

As examples of results from the database, Figure 3 shows the continuity decay curves for the cities of São Carlos-SP and Florianópolis-SC. It is noticeable that in these cases, the random strategy is the one that least hurts the system initially, while the strategies of removing centrality from descending betweenness (bigger centralities first) and ascending degree (lowest degree first) degrade the system faster. The strategy of descending betweenness proved to be more robust for impacts that affect a large number of edges in the network (more than 50% of the roads in these cities).

Notice that Florianópolis network begins with a sharp decrease on descending betweenness and descending degree strategies. Both results occurs due to high dependence on the bridges connecting island to continent, integrating the municipality. When the bridges are removed from the system, the two regions are isolated from each other, causing loss of continuity.

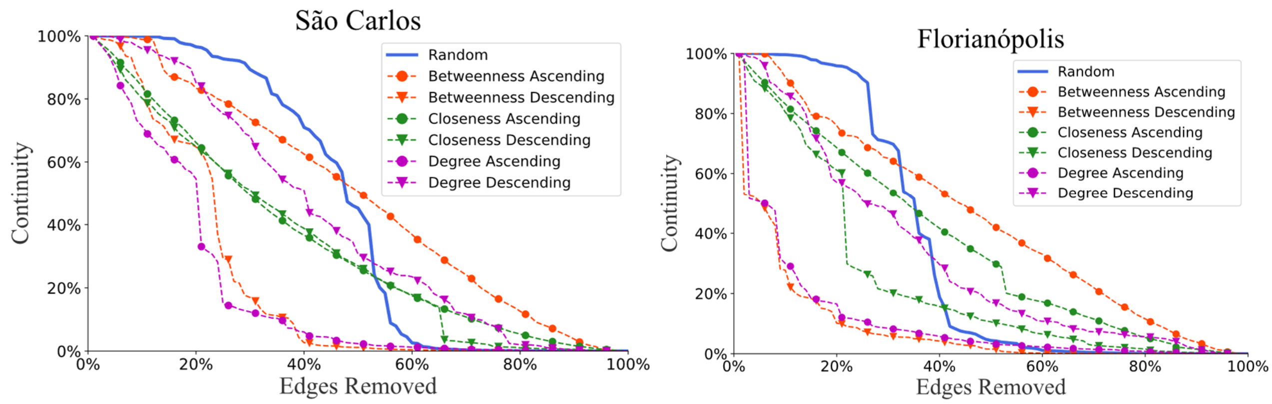

Figure 3. Curves of continuity decay in the networks of São Carlos-SP (left) and Florianópolis SC (right)

Figure 4 contains the curves of the average behavior of the strategies in Brazilian cities. The behavior observed in the previous cities is somewhat reflected in this result, the random strategy being less harmful in minor impacts (less than 40% of the edges removed) and the strategies of descending betweenness and ascending degree being the most damaging as a whole. The closeness centrality is a middle ground, not reflecting well the vulnerability of the system. This is because the strategy of removing connections in order of closeness centrality corrodes the network from its center, not necessarily dividing the network, since the connections of the periphery remain intact (or, in the ascending case, from outside to inside leaving the center intact).

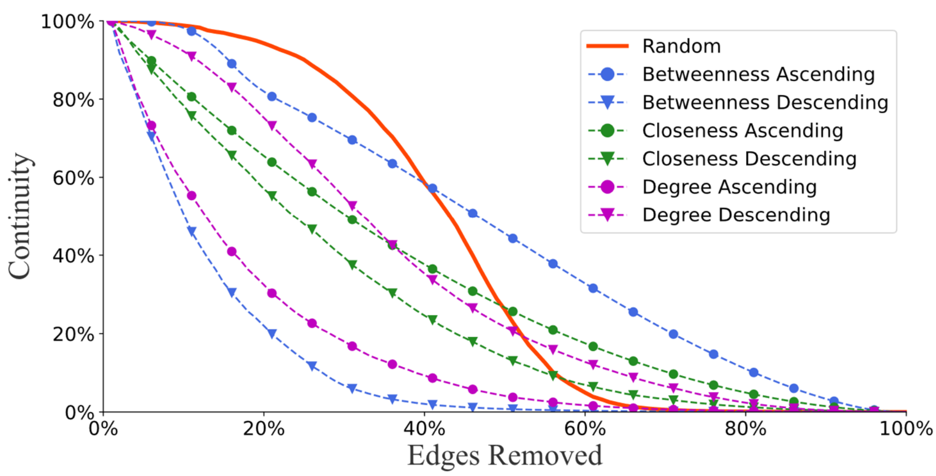

Figure 4. Average behavior in the 309 largest Brazilian cities

Figure 5 shows the strategy histograms for 10%, 20%, and 50% of the edges removed. With 10% of the edges removed, most strategies do not show significant disturbances. Descending betweenness and ascending degree are exceptions in this aspect since the histograms presented were slightly shifted to the left (low continuity values) and relatively high dispersion, with some cities showing resistance to removing the edges and maintaining high continuity while others lose a significant portion of continuity. At the 20% mark, on the other hand, the strategy of descending betweenness shifts the distribution to lower values of continuity while the ascending degree still maintains its uniform behavior, and the other strategies keep their highest concentrations above 50% of continuity. With 50% of the edges removed, all strategies are on the lower spectrum of continuity and the descending betweenness strategy leads all cities to a negligible continuity. Furthermore, the strategy of descending betweenness tends to make the system more vulnerable than the other strategies, with the centrality of degree following.

Regarding the strategies that cause less vulnerability, the betweenness centrality in ascending order showed relative consistency in not causing impacts, with the curve moving slowly to the left with small dispersions while random and descending degree strategies have more erratic behavior. Therefore, we concluded that the betweenness centrality is more related to the vulnerability of the system: edges with higher betweenness centrality present more vulnerability in the system, while those of lower betweenness show less vulnerability.

Figure 6 depict the dispersion of the average behavior in the database with an envelope representing 25th and 75th percentiles of the samples for the more promising strategies. There is a large region of intersection between the scenarios that make the system more vulnerable (descending betweenness and ascending degree). However, the betweenness strategy is still clearly more indicative of the system's vulnerability. On the other hand, removing edges by betweenness in ascending order is a very consistent strategy, with a high average curve and reduced deviation at all points.

For a numerical assessment of these results, we analyze the areas below each curve in the previous graphs. The area under a continuity curve demonstrates average behavior in the range of 0% to 100% of the edges removed. Thus, a small area under the curve expresses that the strategy causes the system to lose continuity faster, so we can associate an area close to zero with a very effective strategy in making the system vulnerable, In contrast, area value closer to 1.0 tells us a strategy that does not affect the system. Table 1 has the average and quartile values of the area under the curve of each strategy for the cities in the database. We conclude that the betweenness centrality on the descending strategy has the lowest result (greatest vulnerability) presenting an area under the curve significantly smaller than in the random case.

Table 1 – Area under the average and quartile curves

Estratégia	25 percentile	Median	75 percentile	Average
Random	0.37	0.41	0.45	0.41
Ascending Betweenness	0.45	0.46	0.48	0.46
Descending Betweenness	0.08	0.11	0.15	0.12
Ascending Closeness	0.33	0.34	0.34	0.34
Descending Closeness	0.24	0.27	0.30	0.26
Ascending Degree	0.13	0.16	0.20	0.16
Descending Degree	0.31	0.35	0.39	0.34

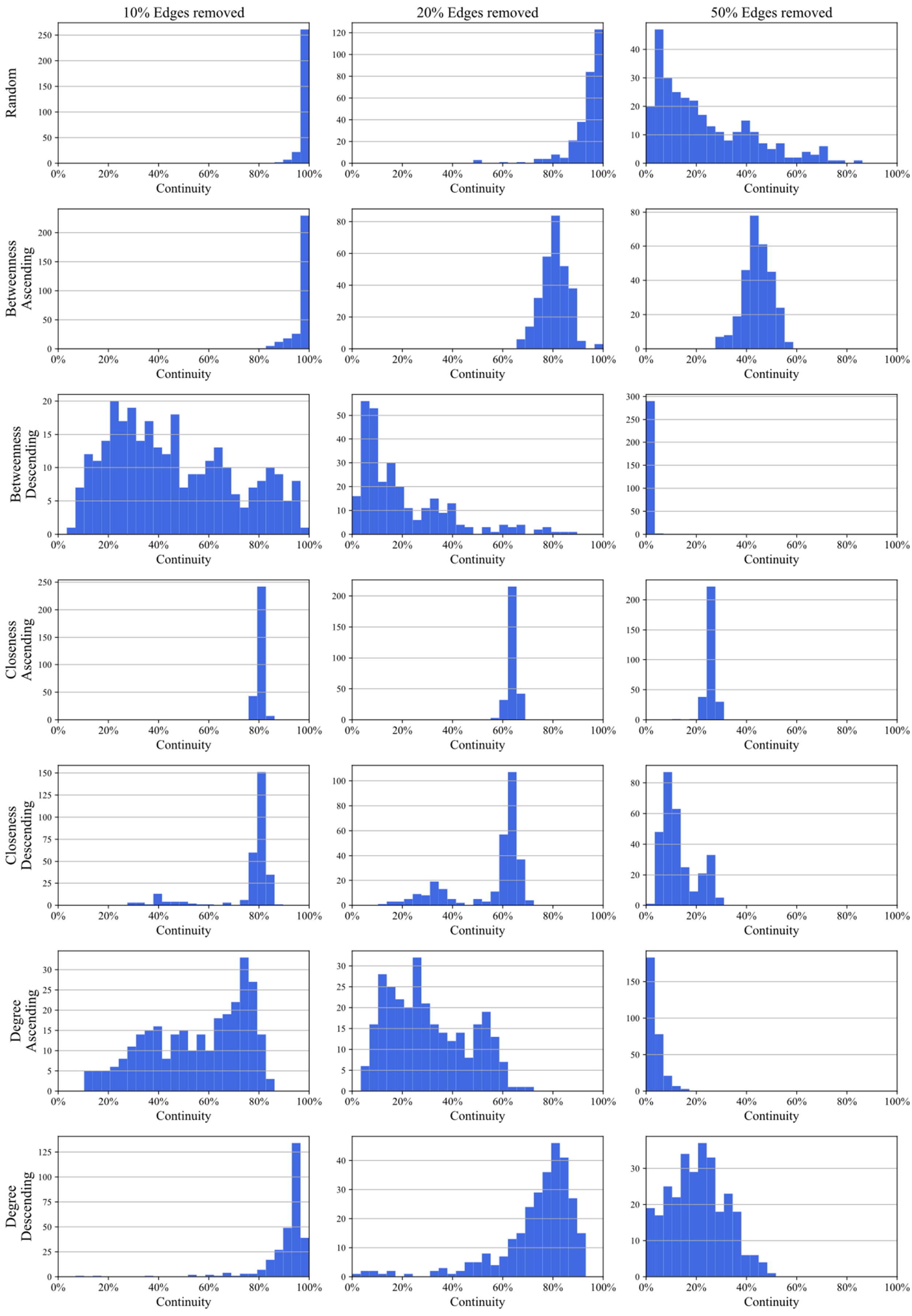

Figure 5. Continuity frequencies for each strategy with 10%, 20%, and 50% of the edges removed

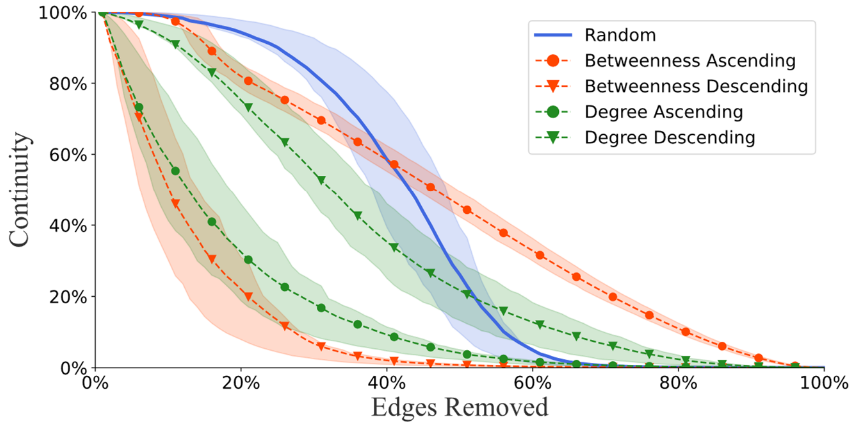

Figure 6. Average behavior of the curves, with envelopes representing the region between the 25th and the 75th percentiles of the distribution.

5. CONCLUSION

This paper had the goal of determining the centrality measure that best represents the vulnerability of land transport networks in Brazilian cities. We propose a systematic degradation of the road system to measure how fast the continuity (proportion of valid paths that remain after an attack) of a network decays. The significant contribution of this work to the scientific literature is a new way of analyzing vulnerability from a structural point of view, which underpins an effort to understand more comprehensively how the morphology of a network can influence its vulnerability.

We found that the betweenness centrality is the measure that best reflects the vulnerability of the system, with the systematic removal of edges in decreasing order of betweenness centrality causing the most significant impacts, whereas the removal in ascending order tends to cause the least impact. Removal at random had mild effect on the system compared to other strategies, especially when it comes to smaller-scale effects (less than 40% of the network removed). Such results indicated that the deterministic strategies impact continuity more profoundly than a random one, possibly because the measures evaluated tend to have smooth variation in the network, with elements of high centrality being generally close to other elements of high centrality, which means that portions of relatively close edges are removed in deterministic methods, creating a tendency to disconnect the system, while the random method tends to distribute the impact across the network. However, deterministic removal methods tend to represent the impacts on the network more accurately. Natural phenomena, for example, tend to reach a concentrated region in space and not random sections of the city.

In this paper, we considered only the continuity of the system without analyzing the effective distance that the valid paths have. In future works, interesting conclusions can be drawn from the evolution of the average length of the minimum paths according to the strategy of attack on the network.

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