Ans.

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19BCE0215

CAT-II

Applied Linear Algebra

Q1)

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 7 \\ -6 & 1 \end{bmatrix}_{3\times 2} ; \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}_{3\times 1}$$

augmented matrix :

$$\left[\begin{array}{cc:c} 2 & 3 & b_1 \\ 3 & 7 & b_2 \\ -6 & 1 & b_3 \end{array}\right] \quad R_1 \to R_1/2$$

$$\left[\begin{array}{cc:c} 1 & 3/2 & b_1/2 \\ 3 & 7 & b_2 \\ -6 & 1 & b_3 \end{array}\right] \quad R_2 \to R_2 + (-3)R_1$$

$$\left[\begin{array}{cc:c} 1 & 3/2 & b_1/2 \\ 0 & 5/2 & b_2 - 3b_1/2 \\ -6 & 1 & b_3 \end{array}\right] \quad R_3 \to R_3 + 6R_1$$

$$\left[\begin{array}{cc:c} 1 & 3/2 & b_1/2 \\ 0 & 5/2 & b_2 - 3b_1/2 \\ 0 & 10 & b_3 + 3b_1 \end{array}\right] \quad R_2 \to R_2 \times \frac{2}{5}$$

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$$\Rightarrow \left[\begin{array}{cc|c} 1 & 3/2 & b_1/2 \\ 0 & 1 & (b_2 - 3b_1/2) \times 2/5 \\ 0 & 10 & b_3 + 3b_1 \end{array}\right] \quad R_3 \to R_3 - 10R_2$$

$$\left[\begin{array}{cc|c} 1 & 3/2 & b_1/2 \\ 0 & 1 & (b_2 - 3b_1/2) \times 2/5 \\ 0 & 0 & b_3 + 3b_1 - 4(b_2 - 3b_1/2) \end{array}\right] \quad R_1 \to R_1 + \frac{-3R_2}{2}$$

Reduced row echlon form :

$$\left[\begin{array}{cc|c} 1 & 0 & b_1/2 - (b_2 - 3b_1/2) \times 3/5 \\ 0 & 1 & (b_2 - 3b_1/2) \times 2/5 \\ 0 & 0 & b_3 + 3b_1 - 4(b_2 - 3b_1/2) \end{array}\right]$$

~~↳ The system has unique solution when:~~

~~$b_3 + 3b_1 - 4(b_2 - 3b_1/2) \neq 0$~~

~~$b_3 + 3b_1 - 4b_2 + 6b_1 \neq 0$~~

~~$\boxed{b_3 - 4b_2 + 9b_1 \neq 0}$~~ //

~~↳ it will have infinitely many solutions if~~

↳ The system has infinite no. of sol^n when

$b_3 + 3b_1 - 4(b_2 - 3b_1/2) = 0$

$\Rightarrow \boxed{b_3 - 4b_2 + 9b_1 = 0}$ //

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Q2)

$$A = \begin{bmatrix} 0 & 1 & -1 & -2 & 1 \\ 1 & 1 & -1 & 3 & 1 \\ 2 & 1 & -1 & 8 & 3 \\ 3 & 5 & -5 & 5 & 10 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

Row space:

Reduce to reduced row echlon form:

$$\begin{bmatrix} 1 & 1 & -1 & 3 & 1 \\ 0 & 1 & -1 & -2 & 1 \\ 2 & 1 & -1 & 8 & 3 \\ 3 & 5 & -5 & 5 & 10 \end{bmatrix} \quad R_3 \to R_3 - 2R_1$$

$$\begin{bmatrix} 1 & 1 & -1 & 3 & 1 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & -1 & 1 & 2 & 1 \\ 3 & 5 & -5 & 5 & 10 \end{bmatrix} \quad R_4 \to R_4 - 3R_1$$

$$\begin{bmatrix} 1 & 1 & -1 & 3 & 1 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & -1 & 1 & 2 & 1 \\ 0 & 2 & -2 & -4 & 7 \end{bmatrix} \quad R_3 \to R_3 + R_2$$

$$\begin{bmatrix} 1 & 1 & -1 & 3 & 1 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & -2 & -4 & 7 \end{bmatrix} \quad R_4 \to R_4 - 2R_2$$

$$\begin{bmatrix} 1 & 1 & -1 & 3 & 1 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \quad R_3 \to R_3 \times 1/2$$

$$\begin{bmatrix} 1 & 1 & -1 & 3 & 1 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \quad R_4 \to R_4 - 5R_3$$

$$\begin{bmatrix} 1 & 1 & -1 & 3 & 1 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2 \to R_2 - R_3$$

$$\begin{bmatrix} 1 & 1 & -1 & 3 & 1 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 \to R_1 - R_3$$

$$\begin{bmatrix} 1 & 1 & -1 & 3 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 \to R_1 - R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & 5 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

basis for → Row space: [non zero rows]

$$= \left\{ [1, 0, 0, 5, 0], [0, 1, -1, -2, 0], [0, 0, 0, 0, 1] \right\}$$

$$\boxed{\dim R(A) = 3}$$

* Column space:

To obtain basis for row space we first need to look at the leading 1's in $Ux = 0$.

$$= \left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \\ 10 \end{bmatrix} \right\}$$

* Null space

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 5 & 0 & 0 \\ 0 & 1 & -1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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convert the matrix equation back to equation :

$$x_1 + 5x_4 = 0$$
$$x_2 - x_3 - 2x_4 = 0$$
$$x_5 = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ x_3 \\ x_3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -5x_4 \\ 2x_4 \\ 0 \\ x_4 \\ 0 \end{bmatrix}$$

$$= x_3 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -5 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

basis for null space :

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right.$$

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Q4)

$T(x,y,z) = (2x - 7y + 4z;\ 3x + y - 4z,$
$6x - 8y + z)$

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x - 7y + 4z \\ 3x + y - 4z \\ 6x - 8y + z \end{bmatrix}$$

Q3) The given line $= \frac{x}{\sqrt{3}} = y$

slope $m = 1/\sqrt{3}$

$\theta = \pi/6$

Since R_θ

$$= \begin{bmatrix} \cos\pi/6 & -\sin\pi/6 \\ \sin\pi/6 & \cos\pi/6 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$R_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

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Q4

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$$R_{\pi/6} = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$$

The matrix of reflection of line about $y = \frac{x}{\sqrt{3}}$ is $R_{\pi/6}$ ~~ooo~~ $R_{-\pi/6}$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix}$$

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Q(4) cont.

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = x\begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix} + y\begin{bmatrix} -7 \\ 1 \\ -8 \end{bmatrix} + z\begin{bmatrix} 6 \\ -8 \\ 1 \end{bmatrix}$$

$$[T]_{\alpha} = \begin{bmatrix} 2 & -7 & 6 \\ 3 & 1 & -8 \\ 6 & -8 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 3 & 5 \end{bmatrix}$$