FALL SEMESTER 2020-21

MAT3004

APPLIED LINEAR ALGEBRA

DIGITAL ASSIGNMENT-2

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II)

Q1) **Find bases and dimensions for the Row space, Column space, Null space of the matrix:**

$$A=\begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ -2 & 5 & 7 & 0 & -6 \\ -2 & 3 & -2 & 1 & -3 \\ -3 & 8 & -9 & 1 & -9 \end{bmatrix}$$

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A1.

$$A=\begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ -2 & 5 & -7 & 0 & -6 \\ -1 & 3 & -2 & 1 & -3 \\ -3 & 8 & -9 & 1 & -9 \end{bmatrix}$$

$R_2 = R_2 + 2R_1$ —①.

first, lets find the reduced row echlon form of matrix A.

$$\begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ -1 & 3 & -2 & 1 & -3 \\ -3 & 8 & -9 & 1 & -9 \end{bmatrix} \quad R_3 = R_3 + R_1 \text{ —②}$$

$$\begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 \\ -3 & 8 & -9 & 1 & -9 \end{bmatrix} \quad R_4 = R_4 + 3R_1 \text{ —③}$$

$$\begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 \\ 0 & 2 & 6 & 1 & 0 \end{bmatrix} \quad R_3 = R_3 - R_2 \text{ —④}$$

$$\begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 6 & 1 & 0 \end{bmatrix} \quad R_4 = R_4 - 2R_2 \text{ —⑤}$$

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$$\begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad R_4 = R_4 - R_3 \text{ — (6)}$$

$$\begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 = R_1 + 2R_2$$

$$U = \begin{bmatrix} 1 & 0 & 11 & 0 & 3 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

∴ The number of non-zero rows in U will be the bases for row space of A.

* → Basis for row space (A) :

$\boxed{\dim R(A) = 3}$

$$\boxed{\left\{ \begin{array}{l} [1, 0, 11, 0, 3], \\ [0, 1, 3, 0, 0], \\ [0, 0, 0, 1, 0] \end{array} \right\}}$$

⇒ For basis for column space for A, we need to find the columns of A corresponding to the columns with leading 1s in U.

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* ⇒ Basis for column space of A :

$\boxed{\dim C(A) = 3}$ $\boxed{\left\{ \begin{bmatrix} 1 \\ -2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 3 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}}$

⇒ To find the basis for null space of A, we need to represent U matrix in the $Ux = 0$ manner :

$$Ux = 0 \Rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 11 & 0 & 3 & 0 \\ 0 & 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

converting to equation form :

$$\begin{aligned} x_1 + 11x_3 + 3x_5 &= 0 \\ x_2 + 3x_3 &= 0 \\ x_4 &= 0 \\ &= 0 \end{aligned}$$

$$\therefore \begin{aligned} x_1 &= -11x_3 - 3x_5 \\ x_2 &= -3x_3 \\ x_3 &= x_3 \\ x_4 &= 0 \\ x_5 &= x_5. \end{aligned}$$

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$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} -11 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

* ⇒ Basis for null space of A :

$$\boxed{\left\{ \begin{bmatrix} -11 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}}$$

$\boxed{\dim N(A) = 2}$

Q2) **Let T: $M^3 \wedge M^3$ be the mapping given by T(x, y, z) = (2x + y + z, x + 2 y + z, x + y + 2 z).**

a) Prove that T is linear

b) Find Ker(T)

c) Is T invertible? Find T^{-1} if T is invertible.

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A2.

a) To prove: $T(x,y,z) = (2x+y+z, x+2y+z, x+y+2z)$ is linear.

(i) To show $T(x+y+z) = T(x) + T(y) + T(z) \in R^3$

Take any $(x_1,y_1,z_1), (x_2,y_2,z_2), (x_3,y_3,z_3) \in R^3$

LHS:

$= T((x_1,y_1,z_1) + (x_2,y_2,z_2) + (x_3,y_3,z_3))$

$= T((x_1+x_2+x_3), (y_1+y_2+y_3), (z_1+z_2+z_3))$

$$= \begin{pmatrix} 2(x_1+x_2+x_3) + (y_1+y_2+y_3) + (z_1+z_2+z_3), \\ (x_1+x_2+x_3) + 2(y_1+y_2+y_3) + (z_1+z_2+z_3), \\ (x_1+x_2+x_3) + (y_1+y_2+y_3) + 2(z_1+z_2+z_3) \end{pmatrix}$$

$$= \begin{pmatrix} (2x_1+y_1+z_1) + (2x_2+y_2+z_2) + (2x_3+y_3+z_3), \\ (x_1+2y_1+z_1) + (x_2+2y_2+z_2) + (x_3+2y_3+z_3), \\ (x_1+y_1+2z_1) + (x_2+y_2+2z_2) + (x_3+y_3+2z_3) \end{pmatrix}$$

$= T(x_1,y_1,z_1) + T(x_2,y_2,z_2) + T(x_3,y_3,z_3)$
$= RHS$

$\therefore$ $\boxed{LHS = RHS}$ Hence proved!

(ii) $T(cx) = cT(x)$

LHS

$T(c(x_1,y_1,z_1)) = T(cx_1, cy_1, cz_1)$

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$$= T\big(2(cx_1) + (cy_1) + (cz_1),\ (cx_1) + 2(cy_1) + (cz_1),\ (cx_1) + (cy_1) + 2(cz_1)\big)$$

$$= \big(c(2x_1 + y_1 + z_1),\ c(x_1 + 2y_1 + z_1),\ c(x_1 + y_1 + 2z_1)\big)$$

$$= c\,(2x_1 + y_1 + z_1,\ x_1 + 2y_1 + z_1,\ x_1 + y_1 + 2z_1)$$
$$= RHS$$

$\therefore \boxed{LHS = RHS}$ Hence proved!

Since both condition / properties are true

$\therefore$ $\boxed{T \text{ is linear}}$ //.

b) Find Ker(T)

$$T(x, y, z) = (2x + y + z,\ x + 2y + z,\ x + y + 2z)$$

Kernel of T is a set of all vectors such that :

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0_{R^3}$$

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equations:

$$2x+y+z=0$$
$$x+2y+z=0$$
$$x+y+2z=0.$$

matrix:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad R_1 = R_1/2 \quad -①$$

find reduced row echlon form:

$$\begin{bmatrix} 1 & 1/2 & 1/2 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad R_2 = R_2 - R_1 \quad -②$$

$$\begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & 3/2 & 1/2 \\ 1 & 1 & 2 \end{bmatrix} \quad R_3 = R_3 - R_1 \quad -③$$

$$\begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & 3/2 & 1/2 \\ 0 & 1/2 & 3/2 \end{bmatrix} \quad R_2 = R_2 \times (2/3) \quad -④$$

$$\begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & 1 & 1/3 \\ 0 & 1/2 & 3/2 \end{bmatrix} \quad R_3 = R_3 - \frac{R_2}{2} \quad -⑤$$

$$\begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & 1 & 1/3 \\ 0 & 0 & 4/3 \end{bmatrix} \quad R_4 = \frac{3}{4} R_4 \quad -⑥$$

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$$\begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & 1 & 1/3 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2 = R_2 - \frac{R_3}{3} \quad - (7)$$

$$\begin{bmatrix} 1 & 1/2 & 1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_1 = R_1 - \frac{R_2}{2} \quad - (8)$$

$$R_1 = R_1 - \frac{R_3}{2} \quad - (9)$$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \; x + 0.y + 0.z = 0$$
$$0.x + y + 0.z = 0$$
$$0.x + 0.y + z = 0.$$

$$\rightarrow \quad x = 0$$
$$y = 0$$
$$z = 0.$$

$\therefore$ $\boxed{\text{Ker(T) has only zero vector } [0 \;\; 0 \;\; 0]}$

c)

↳ T is invertible since T is linear.

To find T^{-1}:

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$$T = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$T = \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \quad R_1 = R_1/2 \text{ - (1)}$$

repeat all row operations (1) to (9)

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \quad R_2 = R_2 - R_1 \text{ - (2)}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 3/2 & 1/2 & -1/2 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \quad R_3 = R_3 - R_1 \text{ - (3)}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 3/2 & 1/2 & -1/2 & 1 & 0 \\ 0 & 1/2 & 3/2 & -1/2 & 0 & 1 \end{array}\right] \quad R_2 = R_2 \times \frac{2}{3} \text{ - (4)}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 1/3 & -1/3 & 2/3 & 0 \\ 0 & 1/2 & 3/2 & -1/2 & 0 & 1 \end{array}\right] \quad R_3 = R_3 - \frac{R_2}{2} \text{ - (5)}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 1/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & 4/3 & -1/3 & -1/3 & 1 \end{array}\right] \quad R_4 = \frac{3}{4} R_4 \text{ - (6)}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 1/3 & -1/3 & 2/3 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & 3/4 \end{array}\right] \quad R_2 = R_2 - \frac{R_3}{3} \text{ - (7)}$$

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$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & -1/4 & 3/4 & -1/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & 3/4 \end{array}\right] \quad \text{— (8)}$$

$R_1 = R_1 - \frac{R_2}{2}$

$R_1 = R_1 - \frac{R_3}{2}$

(9)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/4 & -1/4 & -1/4 \\ 0 & 1 & 0 & -1/4 & 3/4 & -1/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & 3/4 \end{array}\right]$$

$$= \left[\begin{array}{c|ccc} I_3 & 3/4 & -1/4 & -1/4 \\ & -1/4 & 3/4 & -1/4 \\ & -1/4 & -1/4 & 3/4 \end{array}\right] \to [I_3 \mid T^{-1}]$$

$$\Rightarrow T^{-1}(v) = \begin{bmatrix} 3/4 & -1/4 & -1/4 \\ -1/4 & 3/4 & -1/4 \\ -1/4 & -1/4 & 3/4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{3x/4} & \cancel{-y/4} & \cancel{-z/4} \\ \cancel{-x/4} & \cancel{3y/4} & \cancel{-x/4} \\ \cancel{-x/4} & \cancel{-y/4} & \end{bmatrix}$$

$$= \begin{bmatrix} 3x/4 & -y/4 & -z/4 \\ -x/4 & 3y/4 & -z/4 \\ -x/4 & -y/4 & 3z/4 \end{bmatrix}$$

$$\therefore \boxed{T^{-1}(x,y,z) = \left(\frac{3x}{4} - \frac{y}{4} - \frac{z}{4},\ -\frac{x}{4} + \frac{3y}{4} - \frac{z}{4},\ -\frac{x}{4} - \frac{y}{4} + \frac{3z}{4}\right)}$$

ans.