

Concept question

Icy wind shield:

We will get an icy windshield when the windshield is facing away from the house because at night when temperature drops below 0°C the water vapour in the atmosphere will condense and freeze on the windshield. When the windshield faces the house, it is exposed to thermal radiation which prevents the water vapour from freezing and because the temperature of the windshield is higher than 0°C .

Cooking a roast:

- A)** The hollow metal tube is good at conducting heat, so the exposed parts of the metal tube will conduct heat and make the wet wick/the water inside evaporate, this will ensure that the heat is evenly and quickly distributed throughout the tube. The water vapour will then transfer heat in the middle of the tube and then the tube to the meat.
- B)** The shiny side should be on the inside (in contact with the meat) as it can reflect the heat back to the meat, the dull side on the outside won't reflect heat radiation as much as the shiny side and therefore is best to have on the outside.
- C)** When the lid is not on, we lose the water vapour as it rises and flies away and cooler air sinks in, this means that this air will be heated up by the boiling water due to the big temperature difference which means that the water would need longer time to boil. When the lid is on the water vapour stays there minimising the temperature gradient, this means the water does not lose heat energy as fast and therefore requiring less time to boil.

Otto Diesel engine

D) Does the entropy of the gas change? (in a single cycle)

Since the question detail states that we may assume it behaves as a reversible quastatic engine going through the idealised cycle with an ideal gas, we know that the net entropy change of the gas has to be 0.

E)

$$\boxed{\Delta S = C \ln \frac{T_2}{T_1}}$$

heat capacity has two components:
 C_p = heat capacity at constant pressure
 C_v = heat capacity at constant volume

Cycle 2 \rightarrow 3: ~ constant pressure:

$$\Delta S_{23} = C_p \ln \frac{T_3}{T_2}$$

Cycle 4 \rightarrow 1: constant volume

$$\Delta S_{41} = C_v \ln \frac{T_1}{T_4}$$

As it is a reversible cycle we see that $\Delta S_{23} = -\Delta S_{41}$

$$\Rightarrow C_p \ln \left(\frac{T_3}{T_2} \right) = -C_v \ln \left(\frac{T_1}{T_4} \right)$$

$$\Rightarrow -\frac{C_p}{C_v} \ln \frac{T_3}{T_2} = \ln \frac{T_1}{T_4} \quad \left| \quad \frac{C_p}{C_v} = \gamma \right.$$

$$\Rightarrow -\gamma \ln \frac{T_3}{T_2} = \ln \frac{T_1}{T_4}$$

$$\Rightarrow \ln \frac{T_3^{-\gamma}}{T_2^{-\gamma}} = \ln \frac{T_1}{T_4}$$

$$\Rightarrow \ln \frac{T_2^{\gamma}}{T_3^{\gamma}} = \ln \frac{T_1}{T_4} \quad \left| \quad \text{take both sides to the power of 'e'}
$$\Rightarrow \frac{T_2^{\gamma}}{T_3^{\gamma}} = \frac{T_1}{T_4}$$

$$\Rightarrow T_2^{\gamma} T_4 = T_3^{\gamma} T_1$$$$

F) adiabatic compression process

using Charles law: $\frac{V_i}{T_i} = \frac{V_f}{T_f} \Rightarrow \frac{T_i}{V_i} = \frac{T_f}{V_f}$

we see that the adiabatic constant is γ

the compression ratio is given by $V_i = \frac{V}{V^{\gamma}} = V V^{\gamma-1}$

$$\frac{T_1}{V_1} = \frac{T_2}{V_2} \Rightarrow T_1 \frac{V_1^{\gamma}}{V_1} = T_2 \frac{V_2^{\gamma}}{V_2} \Rightarrow T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\frac{T_3}{V_3} = \frac{T_4}{V_4} \Rightarrow T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1}$$

in the cycle $4 \rightarrow 1$ we see that $V_1 = V_4$
therefore: $T_3 V_3^{\gamma-1} = T_4 V_1^{\gamma-1}$

G)

got formulas needed for question G from:

https://www.ohio.edu/mechanical/thermo/Intro/Chapt.1_6/Chapter3c.html

G1) Q_{41} and Q_{23}

$$Q_{23} = -Q_{41}$$

$$Q_{23} = Q_{in} = \Delta h = C_p \Delta T = C_p (T_3 - T_2)$$

$$Q_{41} = Q_{out} = -\Delta U = -C_v \Delta T = -C_v (T_1 - T_4) = C_v (T_4 - T_1)$$

H)

H) from the book: efficiency = $e = 1 - \frac{Q_c}{Q_h}$

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{-C_v(T_1 - T_4)}{C_p(T_3 - T_2)} = 1 + \frac{1}{\gamma} \frac{T_1 - T_4}{T_3 - T_2}$$

ideal gas law: $PV = nRT \Rightarrow T = \frac{PV}{nR} = \frac{1}{nR} PV$

$$\Rightarrow 1 + \frac{\frac{1}{nR} (P_1 V_1 - P_4 V_4)}{\frac{1}{nR} (P_3 V_3 - P_2 V_2)} = 1 + \frac{P_1 V_1 - P_4 V_4}{P_3 V_3 - P_2 V_2}$$

we know that $V_1 = V_4 \Rightarrow 1 + \frac{P_1 V_1 - P_4 V_1}{P_3 V_3 - P_2 V_2}$

$$P_2 = P_3 \Rightarrow 1 + \frac{V_1 (P_1 - P_4)}{P_2 (V_3 - V_2)}$$

$$= 1 + \frac{V_1}{V_3 - V_2} \frac{P_1 - P_4}{P_2} = 1 + \left(\frac{V_3}{V_1} - \frac{V_2}{V_1} \right)^{-1} \left(\frac{P_1 - P_4}{P_2} \right)$$

$$= 1 + \left(r_1^{-1} - r_2^{-1} \right)^{-1} \left(\frac{P_1 - P_4}{P_2} \right) \Rightarrow \text{we can use the ideal gas relation: } PV^\gamma = \text{constant}$$

$$\frac{P_1}{P_3} = \left(\frac{V_3}{V_1} \right)^\gamma \text{ and } \frac{P_4}{P_2} = \left(\frac{V_2}{V_1} \right)^\gamma \Rightarrow \frac{P_1}{P_3} = (r_1^{-1})^\gamma, \frac{P_4}{P_2} = (r_2^{-1})^\gamma$$

$$\Rightarrow e = 1 + \frac{r_1^{-\gamma} - r_2^{-\gamma}}{r_1^{-1} - r_2^{-1}}$$

I)

$$I) P_1 = 100 \text{ kPa} \quad V_1 = 2 \text{ m}^3 \quad \gamma = \frac{7}{5} = 1.4 \quad r_2 = r_1 = 8$$

$$\text{Combined gas law: } \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

from 1 to 2: assuming room temperature $\approx 300 \text{ K} (= 27^\circ\text{C})$
and engine temp. at $327^\circ\text{C} = 600 \text{ K}$
max

$$\text{finding } P_2: P_2 = \frac{V_1 T_2 P_1}{V_2 T_1} = 8 \cdot 2 \cdot 100 = 1600 \text{ kPa}$$

$$W_{1 \rightarrow 2} = C_V (T_1 - T_2) = -300 \text{ kJ/kg}$$

$R_{\text{air}} = 1$ for simplicity

$$W_{2 \rightarrow 3} = \text{no work, heat in} = C_P (T_3 - T_2)$$

J) To reach the maximum possible efficiency is not really possible but, we could nearly reach this value by reducing friction and thermally isolating the system to minimise loss to the 'universe'

Vacancies in crystals

K)

$$k) \text{ multiplicity: } \Omega(N, n) = \binom{N}{n} = \frac{N!}{n! (N-n)!}$$

L)

$$L) S = k_B \ln \Omega \frac{V_F}{V_0} = k_B \ln \left(\frac{N!}{n! (N-n)!} \right) \frac{V_F}{V_0}$$

m)

using Stirling's approximation: $\ln(N!) \approx N \ln N - N$

$$\ln \left(\frac{N!}{n! (N-n)!} \right) = \ln N! - \ln n! - \ln (N-n)!$$

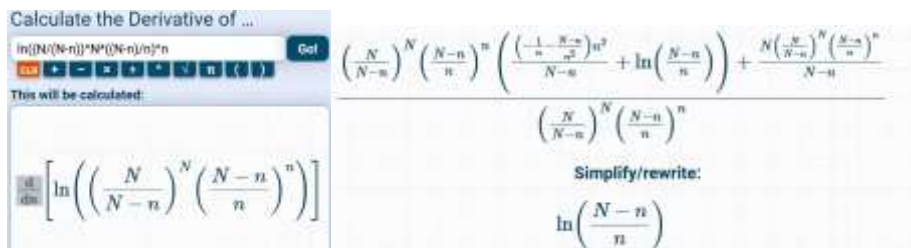
$$\approx N \ln N - N - n \ln n + n - (N-n) \ln (N-n) + N - n$$

$$= N \ln N - n \ln n - (N-n) \ln (N-n)$$

$$= \ln \left(\frac{N^N}{n^n} \right) + \ln \left(\frac{1}{N-n} \right)^{N-n} = \ln \frac{N^N}{n^n} \left(\frac{1}{N-n} \right)^{N-n} (N-n)^n$$

$$= \ln \left(\left(\frac{N}{N-n} \right)^N \left(\frac{N-n}{n} \right)^n \right)$$

N) Used derivative calculator



Calculate the Derivative of ...
 $\ln \left(\left(\frac{N}{N-n} \right)^N \left(\frac{N-n}{n} \right)^n \right)$
 This will be calculated:
 $\frac{d}{dn} \left[\ln \left(\left(\frac{N}{N-n} \right)^N \left(\frac{N-n}{n} \right)^n \right) \right]$
 Simplify/rewrite:
 $\ln \left(\frac{N-n}{n} \right)$

$$n) \quad U = n \Delta E, \quad \frac{1}{T} = \frac{1}{\Delta E} \frac{\partial S}{\partial n}$$

$$\frac{\partial S}{\partial n} = \frac{\partial}{\partial n} k_B \ln \left(\left(\frac{N}{N-n} \right)^N \left(\frac{N-n}{n} \right)^n \right) = k_B \ln \left(\frac{N-n}{n} \right)$$

$$\frac{1}{T} = \frac{1}{\Delta E} k_B \ln \left(\frac{N-n}{n} \right) \Rightarrow T = \frac{\Delta E}{k_B} \ln \left(\frac{N-n}{n} \right)^{-1}$$

$$O) \quad T = \frac{\Delta E}{k_B} \ln \left(\frac{N-n}{n} \right)^{-1} \Rightarrow \frac{T k_B}{\Delta E} = \ln \left(\frac{N-n}{n} \right)^{-1}$$

$$\frac{\Delta E}{T k_B} = \ln \left(\frac{N-n}{n} \right)$$

$$e^{\frac{\Delta E}{T k_B}} = \frac{N-n}{n}$$

$$\Rightarrow n e^{\Delta E / T k_B} = N - n$$

$$n e^{\Delta E / T k_B} + n = N$$

$$n (e^{\Delta E / T k_B} + 1) = N$$

$$n = \frac{N}{e^{\Delta E / T k_B} + 1}$$

P)

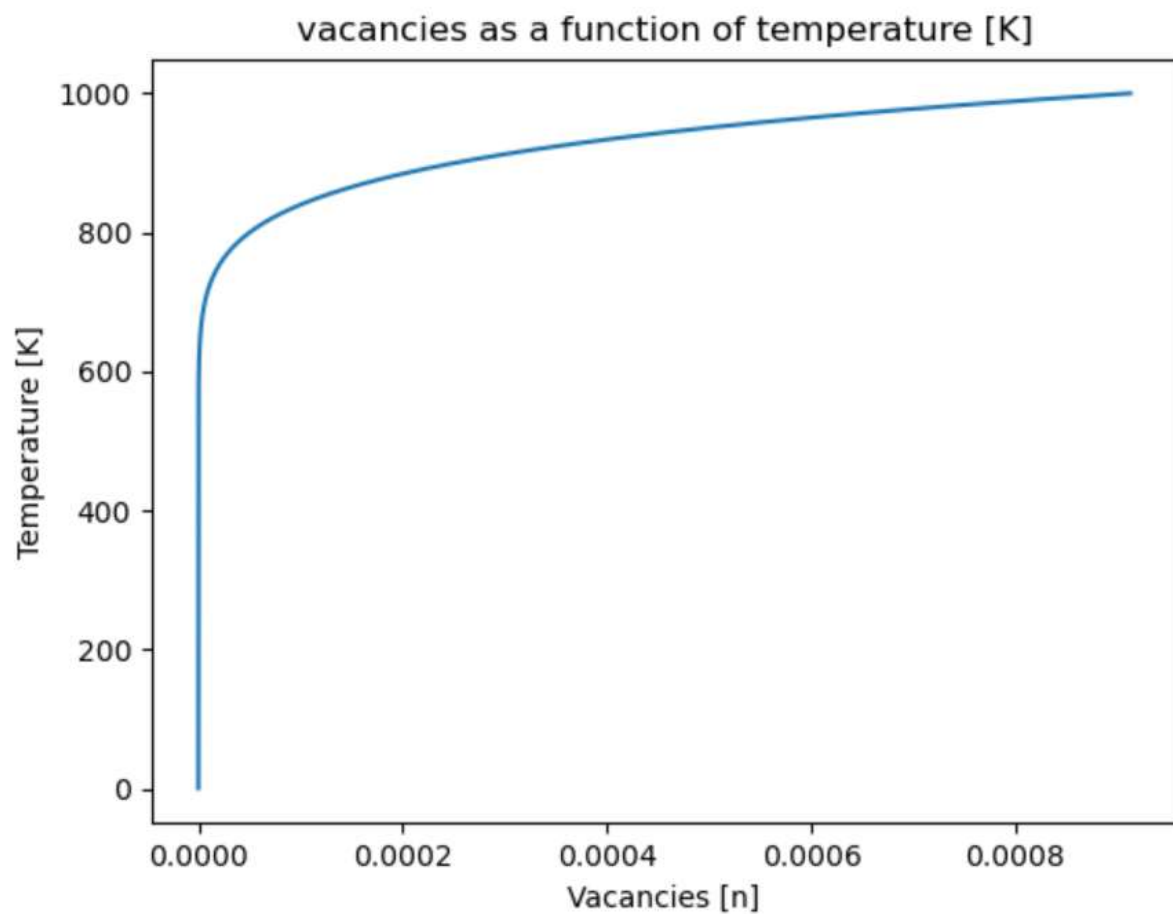
P) Using what we got in 'O'

$$n = \frac{N}{e^{\Delta E / T k_B} + 1} \quad \text{When } T \rightarrow 0 : e^{\Delta E / T k_B} \rightarrow \infty$$

$$\text{therefore when } T \rightarrow 0 : n = \frac{N}{\infty} = 0$$

Q)

```
1  import matplotlib.pyplot as plt
2  import numpy as np
3  from numpy import exp
4
5
6  T = np.linspace(1, 1000, 1000) #10,000 data points
7  delta_e = 1.602176565e-19 #[1eV]
8  Kb = 1.38064852e-23 #boltzmann constant
9  N = 100
10 n = N/(exp(delta_e/(T*Kb))+1) #vacancies
11
12 plt.title('vacancies as a function of temperature [K]')
13 plt.ylabel('Temperature [K]')
14 plt.xlabel('Vacancies [n]')
15 plt.plot(n,T)
16 plt.show()
```



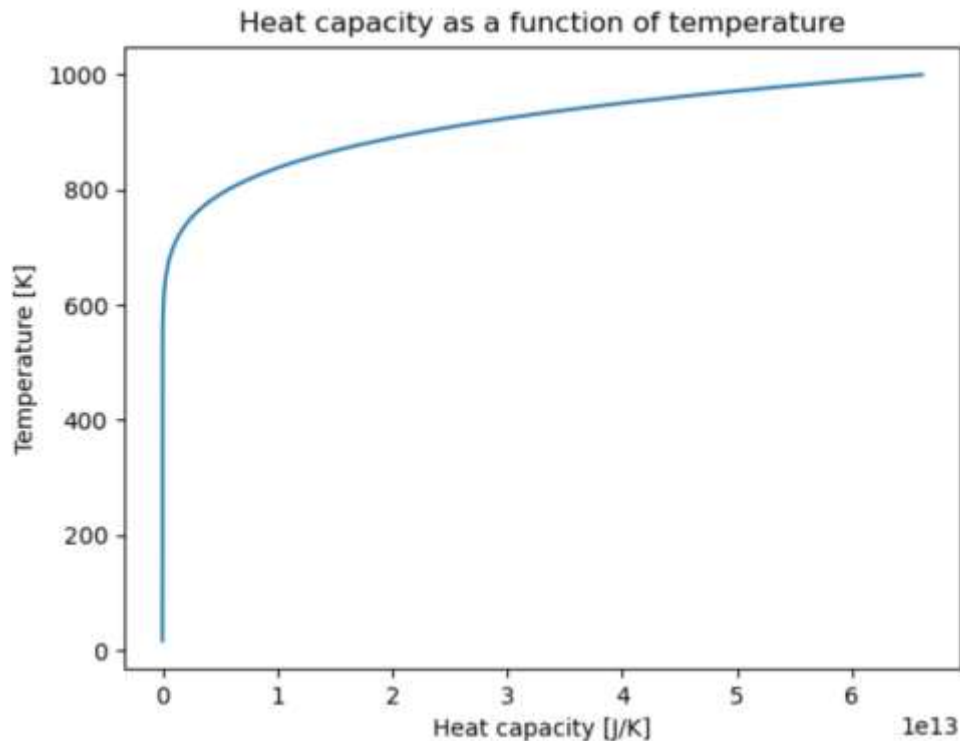
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R)

$$\begin{aligned}
 R) \quad C_v &= T \left(\frac{\partial S}{\partial T} \right)_{v, N}, \quad T = \frac{\Delta \epsilon}{k_B} \ln \left(\frac{N-n}{n} \right)^{-1} \\
 C_v &= T \left(\frac{\partial S}{\partial n} \frac{\partial n}{\partial T} \right) \quad S = k_B \ln \left(\left(\frac{N}{N-n} \right)^N \left(\frac{N-n}{n} \right)^n \right) \\
 &\quad \uparrow = \frac{1}{T \Delta \epsilon} \quad n = \frac{N}{e^{\Delta \epsilon / T k_B} + 1} \\
 C_v &= T \times \frac{1}{T \Delta \epsilon} \left(\frac{\partial n}{\partial T} \right) = \frac{\partial n}{\partial T} \frac{1}{\Delta \epsilon} \\
 C_v &= \text{Using derivative calculator} \Rightarrow \frac{N \Delta \epsilon e^{\Delta \epsilon / T k_B}}{k_B T^2 (e^{\Delta \epsilon / T k_B} + 1)^2} \Delta \epsilon
 \end{aligned}$$

```

Cv = Nexp(delta_e/(T*Kb))/(((Kb*T**2)*(exp(delta_e/(T*Kb))+1)**2))#heat capaci
ty [J/K]
plt.title('Heat capacity as a function of temperature')
plt.xlabel('Heat capacity [J/K]')
plt.ylabel('Temperature [K]')
plt.plot(Cv, T)
plt.show()

```



We see that at higher temperatures the heat capacity starts to increase exponentially, this means that there are smaller changes in temperature for the same amount of heat applied at higher temperatures.

S) Interstitial atoms, or impurities, can lower the heat capacity of the solid as it affects the way the structure exists. It could act as a vacancy depending on how it is bonded to the other molecules. If the molecule is between the regular lattice space it could also increase the heat capacity as it would absorb some of the heat energy itself. The base vibrations of the crystal also store some of the heat energy, making the heat capacity even higher. The crystalline structure could end up using the heat supplied to rotate, bend, and vibrate.