BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

First Semester, 2020-21
MATH F111 : Mathematics-I
Tutorial sheet-IV

Q.1. Find the derivative of the function $h(x, y, z) = \cos(xy) + e^{yz} + \ln(zx)$ at the point $P_0(1, 0, 1/2)$ in the direction of $\mathbf{u} = \mathbf{i} + \mathbf{2j} + \mathbf{2k}$.

Q.2. Let $f(x, y) = \frac{(x-y)}{(x+y)}$. Find the direction $\mathbf{u}$ for which the value of $D_u f(-\frac{1}{2}, \frac{3}{2})$ is largest and also find the largest value of $D_u f(-\frac{1}{2}, \frac{3}{2})$.

Q.3. If $f(x, y, z)$ has directional derivative 1, -1 and 3 in the direction of vectors $\mathbf{i}$, $\mathbf{i} + \mathbf{j}$ and $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ respectively at P_0. Find directional derivative of f in the direction of $\mathbf{i} + \mathbf{j} - \mathbf{k}$ at P_0.

Q.4. Find equations for the tangent plane and normal line on the surface $x^2 + y^2 - 2xy - x + 3y - z = -4$ at the point $P_0(2, -3, 18)$.

Q.5. Suppose that the Celsius temperature at the point (x, y) in the xy-plane is $T(x, y) = x \sin 2y$ and that distance in the xy-plane is measured in meters. A particle is moving clockwise around the circle of radius $1m$ centered at the origin at the constant rate of $2m/sec$.

(a) How fast is the temperature experienced by the particle changing in degree Celsius per meter at the point $P(1/2, \sqrt{3}/2)$?

(b) How fast is the temperature experienced by the particle changing in degree Celsius per second at P?

Q.6. Find the linearization $L(x, y)$ of the function function $f(x, y) = xy^2 + y\cos(x - 1)$ at $p_0(1, 2)$. Also find an upper bound for the magnitude $|E|$ of the error in the approximation $f(x, y) = L(x, y)$ over the rectangle $R : |x - 1| \leq 0.1, |y - 2| \leq 0.1$.

Q.7. Discuss the maxima and minima of $x^3y^2(1 - x - y)$.

Q.8. Find the absolute maxima and minima of the function $D(x, y) = x^2 - xy + y^2 + 1$ on the closed triangular plate in the first quadrant bounded by the lines $x = 0, y = 4, y = x$.

Q.9. Find the maximum and minimum distance of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 1$.

Q.10. If A, B and C are the angles of triangle, locate and classify the critical points of the function $f(A, B, C) = \cos A \cos B \cos C$ without using the method of Lagrange multipliers.

Q.11. Prove that he equation $\frac{l^2a^4}{1-a^2u} + \frac{m^2b^4}{1-b^2u} + \frac{n^2c^4}{1-c^2u} = 0$ is satisfied by the extreme values of $u = \frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}$ subject to the constraints $lx + my + nz = 0$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Here a, b, c, l, m, n are real constants.