

## Errors

For  $N$  measurements of a quantity  $Q$ :

$$\bar{Q} = \frac{1}{N} \sum Q_i$$

$$\text{Deviation } d = \sqrt{\frac{1}{N} \sum (Q_i - \bar{Q})^2}$$

$$\text{Resultant measurement } Q = \bar{Q} \pm d$$

$$\text{Error } \Delta Q = \text{Standard error} = d$$

For a quantity  $Q = Q(x, y, z)$ , the general error is given as:

$$(\Delta Q)^2 = (\Delta Q_x)^2 + (\Delta Q_y)^2 + (\Delta Q_z)^2$$

Where  $\Delta Q_x = \left(\frac{\partial Q}{\partial x}\right) \Delta x$  and so on for  $Q_y$  and  $Q_z$

Combination of errors:

$$Q = x \pm y \quad \Delta Q = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$Q = xy \text{ or } \frac{x}{y} \quad \left(\frac{\Delta Q}{Q}\right) = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

$$Q = x^n \quad \left(\frac{\Delta Q}{Q}\right) = n \frac{\Delta x}{x}$$

$$Q = \ln x \quad \frac{\Delta Q}{Q} = \frac{\Delta x}{x}$$

$$Q = e^x \quad \frac{\Delta Q}{Q} = \Delta x$$

## Planck's constant

Boltzmann constant ( $k$ ) =  $1.38 \cdot 10^{-23} \text{ J/K}$

Planck's constant =  $h$

$U(\nu)d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left(e^{\frac{h\nu}{kT}} - 1\right)} d\nu$  (now, all relevant approximations can be made)

At a particular frequency  $\nu$ , the photocurrent  $I_{\text{ph}}$  is approximated to be:

$$I_{\text{ph}} = \frac{8\pi h\nu^3}{c^3} e^{-\frac{h\nu}{kT}}$$

If we take the *natural logarithm* on both sides, the following is obtained:

$$\ln I_{\text{ph}} = -\frac{h\nu}{kT} + \text{constant}$$

Temperature dependence of resistance of tungsten filament:

$$R = R_0(1 + \alpha T + \beta T^2)$$

$\alpha, \beta$  being empirical constants used for calibration

## EMI

$$\varepsilon = -\frac{d\phi}{dt}$$

Where  $\varepsilon$  = EMF and  $\phi$  = flux through the coil

$$\omega_{\text{max}} = 2\sqrt{\frac{Mgl}{I}} \sin \frac{\theta_0}{2}$$

Here,  $\sqrt{\frac{Mgl}{I}}$  is the *frequency* of oscillations, which can also be used to find the time period  $T$

Considering  $R$  to be the radius of the arc (and  $v_{\text{max}}$  to be the maximum velocity of the arc), we get:

$$v_{\text{max}} = R\omega_{\text{max}} = \frac{4\pi R}{T} \sin \frac{\theta_0}{2}$$

After a bunch of approximations, it is concluded that  $\varepsilon_{\text{max}} \propto v_{\text{max}}$  which is given a bit more precisely as:

$$\varepsilon_{\text{max}} = -\frac{1}{R} \frac{d\phi}{d\theta} \bigg|_{\theta_{\text{max}}} v_{\text{max}}$$

Consider the pulse width of one oscillation to be  $\tau$  and the time constant of the circuit to be  $RC$ . If:

- $RC < \tau \Rightarrow$  Capacitor fully charged in one oscillation
- $RC > \tau \Rightarrow$  Capacitor charges in multiple oscillations

Total charge delivered to capacitor after each swing =  $q = \frac{\Delta\phi}{R}$

For a damped oscillation, the angular displacement is given as:

$$\theta_A(t) = \theta_{A0} e^{-\frac{\omega_0 t}{2Q}}$$

$Q$  being the **Quality factor** or strength of damping

$$Q \propto \frac{1}{\text{Amount of damping}}$$

In a plot of  $\ln \theta_{An}$  vs.  $n$ , the equation is found to be:

$$\ln \theta_{An} = \ln \theta_{A0} - \frac{\pi}{Q} n$$

$n$  being the number of oscillations

## Newton's rings

Optical path difference for interfering waves =  $2\mu t$   
Conditions for:

1. Constructive interference:  $2t = m\lambda$
2. Destructive interference:  $2t = m\left(m + \frac{1}{2}\right)$

Where  $m \in \mathbb{Z}^+ \cup 0$ .

Denoting the radius of the  $m^{\text{th}}$  order of the ring by  $r_m$ , the following can be proved after a bit of work:

$$r_m = \sqrt{m\lambda R}$$

$$r_m = \sqrt{\left(m + \frac{1}{2}\right) \lambda R}$$

for the **dark** and **bright** rings respectively. In the above results,  $R$  is the radius of the spherical surface and  $t$  is the thickness of the film

$$\text{Diameter of } m^{\text{th}} \text{ order ring } D_m = \sqrt{4m\lambda R}$$

## Diffraction grating

$$a = (n-1)\Delta$$

**Single slit diffraction:**

If the rays make an angle  $\theta$  with the normal, the corresponding phase difference  $\phi$  is given as  $\phi = \frac{2\pi}{\lambda} \Delta \sin \theta$ ,  $\Delta$  being the distance b/w consecutive points

Intensity distribution:  $I = I_0 \frac{\sin^2 \beta}{\beta^2}$  where  $I_0$  is the intensity at  $\theta = 0$  and  $\beta = \frac{\pi a \sin \theta}{\lambda}$

$$\text{Minima: } a \sin \theta = m\lambda (m \neq 0)$$

$$\text{Maxima: } \tan \beta = \beta$$

**Double slit diffraction:**

$$\text{Similar as above, } \phi_1 = \frac{2\pi}{\lambda} d \sin \theta$$

Intensity distribution:  $I = 4I_0 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma$  where  $\gamma = \frac{\phi_1}{2} = \frac{\pi}{\lambda} d \sin \theta$

$$\text{Minima: } \gamma = (2n+1)\frac{\pi}{2} \Rightarrow a \sin \theta = m\lambda \text{ where } m = (1, 2, 3, \dots)$$

$$\text{Maxima: } \gamma = n\pi \Rightarrow d \sin \theta = n\lambda \text{ where } n = (0, 1, 2, \dots)$$

In all of the equations above,  $d$  represents the **distance** between the two point sources and  $a$  denotes the **width** of two parallel slits