LISTA GA 7

1. a) $A=(5,4,1)$ $B=(-2,3,2)$

$x = A + \lambda \vec{u}$

$\vec{u} = \vec{AB} = B - A = (-2,3,2)-(5,4,1) = (-7,-1,1)$

$x = (5,4,1) + \lambda(-7,-1,-1)$

$x=(x,y,z)$

$$\begin{cases} x = 5 - \lambda 7 \\ y = 4 - \lambda \quad , \lambda \in \mathbb{R} \\ z = 1 - \lambda \end{cases}$$

simétrica:

$$s: \frac{-x+5}{7} = \frac{-y+4}{1} = \frac{-z+1}{1}$$

b) $A=(0,-1,0)$ $B=(1,0,0)$

$r: x = A + \lambda \vec{u}$

$\vec{u} = (1,0,0)-(0,-1,0) = (1,1,0)$

$r: x = (0,-1,0) + \lambda(1,1,0)$

$x=(x,y,z)$

$$r: \begin{cases} x = \lambda \\ y = -1 + \lambda \\ z = 0 \end{cases}$$

não é possível ter uma equação na forma simétrica pois $u_z = 0$

c) $A=(0,1,-1)$ $B=(0,0,0)$

$r: x = A + \lambda \vec{u}$

$\vec{u} = (0,0,0)-(0,1,-1) = (0,-1,1)$

$r: x = (0,1,-1) + \lambda(0,-1,1)$

$x=(x,y,z)$

$$r: \begin{cases} x = 0 \\ y = 1 - \lambda \\ z = -1 + \lambda \end{cases}$$

como $u_1 = 0$ não é possível ter uma equação na forma simétrica.

d) $A=(3,2,1)$ $B=(6,1,-4)$

$r: x = A + \lambda \vec{u}$

$\vec{u} = (6,1,-4)-(3,2,1) = (3,-1,-5)$

$r: x = (3,2,1) + \lambda(3,-1,-5)$

$x=(x,y,z)$

$$\begin{cases} x = 3 + \lambda 3 \\ y = 2 - \lambda \\ z = 1 - 5\lambda \end{cases}$$

simétrica:

$$r: \frac{x-3}{3} = \frac{-y+2}{1} = \frac{-z+1}{5}$$

2. a) Como $\vec{u}$ já é um vetor diretor $\vec{v} = \lambda \vec{u}$ também é, logo:

$\vec{u} = (-1,1,2)$

$\vec{v} = 2\vec{u}$

$\vec{v} = (-2,2,4)$

$\vec{w} = -7\vec{u}$

$\vec{w} = (7,-7,-14)$

• Ponto x_1

$\lambda = -3$

$$\begin{cases} x = 1-(-3) \\ y = -3 \\ z = 4-6 \end{cases} \rightarrow x_1 = (4,-3,-2)$$

• Ponto x_2

$\lambda = 2$

$$\begin{cases} x = 1-2 \\ y = 2 \\ z = 4+2\cdot 2 \end{cases} \rightarrow x_2 = (-1,2,8)$$

b) $P \in r$:

$$\begin{cases} 1 = 1 - \lambda \\ 3 = \lambda \\ -3 = 4 + 2\lambda \end{cases} \rightarrow \begin{matrix} \lambda = 0 \\ \lambda = 3 \\ \lambda = -\frac{7}{2} \end{matrix} \quad \therefore P \notin r$$

$Q \in r$:

$$\begin{cases} -3 = 1 - \lambda \\ 4 = \lambda \\ 12 = 4 + 2\lambda \end{cases} \rightarrow \begin{matrix} \lambda = 4 \\ \lambda = 4 \\ \lambda = 4 \end{matrix} \quad \therefore Q \in r$$

c)

$\vec{v}_r = (-1, 1, 2)$ $\vec{v}_s = (u_1, u_2, u_3)$ $P = (1, 4, -7)$

Para ser paralela $\vec{v}_r = \lambda \vec{v}_s$, logo L.D

Como o λ não foi determinado $\lambda = 1$, logo

$\vec{v}_s = \vec{v}_r = (-1, 1, 2)$

$$\begin{cases} x = 1 - \lambda \\ y = 4 + \lambda \\ z = -7 + 2\lambda \end{cases} \qquad s: x = (1, 4, -7) + \lambda(-1, 1, 2)$$

3. a)

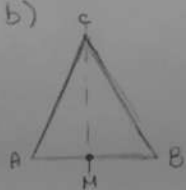
$A = (3, 6, -7)$, $B = (-5, 2, 3)$ $C = (4, -7, -6)$

Para esses 3 pontos serem vertices de um triangulo precisam ser L.I

$$\det x = \begin{vmatrix} 3 & 6 & -7 \\ -5 & 2 & 3 \\ 4 & -7 & -6 \end{vmatrix} = -36 + 72 - 245 - (-56 - 63 + 180) = -270 \neq 0$$

$\therefore \{A, B, C\}$ é L.I

b)



$$M = \left(\frac{3 + (-5)}{2}, \frac{6 + 2}{2}, \frac{-7 + 3}{2}\right)$$

$M = (-1, 4, -2)$

$\vec{v}_m = M - C = (-1, 4, -2) - (4, -7, -6)$

$= (-5, 11, 4)$

$M: x = (4, -7, -6) + \lambda(-5, 11, 4)$, $\lambda \in \mathbb{R}$

4. a) Para que as condições se realizem o produto escalar do triangulo precisa ser 0

$\vec{AB} = B - A = (-3, 0, 9) - (0, 1, 8) = (-3, -1, 1)$

Para achar as coordenadas de C:

$X = (1, 2, 0) + \lambda(1, 1, -3)$

$C = (1 + \lambda, 2 + \lambda, -3\lambda)$

Assim:

$\vec{AC} = C - A = (1 + \lambda, 2 + \lambda, -3\lambda) - (0, 1, 8) = (\lambda + 1, 1 + \lambda, -8 - 3\lambda$

$\vec{BC} = C - B = (1 + \lambda, 2 + \lambda, -3\lambda) - (-3, 0, 9) = (4 + \lambda, 2 + \lambda, -9 - 3\lambda)$

Como $\vec{AB}$ e $\vec{AC}$ testamos sua ortogonalidade

$AB \perp AC$

$\vec{AB} \cdot \vec{AC} = (-3, -1, 1)(1 + \lambda, 1 + \lambda, -8 - 3\lambda)$

$= (-3 - 3\lambda - 1 - \lambda - 8 - 3\lambda) = -12 - 7\lambda = 0$

$\lambda = -\dfrac{12}{7}$

Assim calculamos o ponto C:

$C = (1 + \lambda, 2 + \lambda, -3\lambda)$

$= \left(1 - \frac{12}{7}, 2 - \frac{12}{7}, -3\left(-\frac{12}{7}\right)\right)$

$= \left(-\frac{5}{7}, \frac{2}{7}, \frac{36}{7}\right)$

b)

$r: x = (1,0,0) + \lambda(1,1,1)$

$x = (1+\lambda, \lambda, \lambda)$

$A: (1,1,1) \quad B(0,0,1)$

Para que A e B sejam equidistantes de x

$\|x - A\| = \|x - B\|$

$x - A = (1+\lambda, \lambda, \lambda) - (1,1,1) = (\lambda, \lambda-1, \lambda-1)$

$x - B = (1+\lambda, \lambda, \lambda) - (0,0,1) = (1+\lambda, \lambda, \lambda-1)$

$\sqrt{\lambda^2 + (\lambda-1)^2 + (\lambda-1)^2} = \sqrt{(1+\lambda)^2 + \lambda^2 + (\lambda-1)^2}$

$\sqrt{\lambda^2 + \lambda^2 - 2\lambda + 1 + \lambda^2 - 2\lambda + 1} = \sqrt{\lambda^2 + 2\lambda + 1 + \lambda^2 + \lambda^2 - 2\lambda + 1}$

$\left(\sqrt{3\lambda^2 - 4\lambda + 2}\right)^2 = \left(\sqrt{3\lambda^2 + 2}\right)^2$

$3\lambda^2 - 4\lambda + 2 = 3\lambda^2 + 2$

$-4\lambda = 0$

$\lambda = 0 \rightarrow x = (1+0, 0, 0) = (1,0,0)$

$\therefore$ O único ponto equidistante a A e B é $x = (1,0,0)$ //

5. a) $A = (1,2,0) \quad \vec{u} = (1,1,0) \quad \vec{v} = (2,3,-1)$

$x = A + \alpha\vec{u} + \beta\vec{v}$

$x = (1,2,0) + \alpha(1,1,0) + \beta(2,3,-1)$

• Equação Paramétrica

$$\begin{cases} x = 1 + \alpha + 2\beta \\ y = 2 + \alpha + 3\beta \\ z = -\beta \end{cases}$$

b) $A = (1,1,0) \quad B = (1,-1,-1) \quad \vec{v} = (2,1,0)$

$x = A + \alpha\vec{u} + \beta\vec{v}$

$\vec{u} = \overrightarrow{AB} = B - A = (1,-1,-1) - (1,1,0) = (0,-2,-1)$

$\pi: x = (1,1,0) + \alpha(0,-2,-1) + \beta(2,1,0)$

• Equação paramétrica

$$\pi: \begin{cases} x = 1 + 2\beta \\ y = 1 - 2\alpha + \beta \\ z = -\alpha \end{cases}$$ //

c) $A = (1,0,1) \quad B = (2,1,-2) \quad C = (1,-1,0)$

$x = A + \alpha\vec{u} + \beta\vec{v} \quad \vec{u} = \overrightarrow{AB} = (2,1,-2) - (1,0,1) = (1,0,-3)$

$\vec{v} = \overrightarrow{AC} = (1,-1,0) - (1,0,1) = (0,-1,-1)$

$\pi: x = (1,0,1) + \alpha(1,0,-3) + \beta(0,-1,-1)$

$$\pi: \begin{cases} x = 1 + \alpha \\ y = -\beta \\ z = 1 - 3\alpha - \beta \end{cases}$$ //

6. a) $A = (9,-1,0) \quad \vec{u} = (0,1,0) \quad \vec{v} = (1,1,1)$

$x \in \mathbb{R} \leftrightarrow \overrightarrow{AX} \cdot (\vec{u} \times \vec{v}) = 0$

$\overrightarrow{AX} = (x,y,z) - (9,-1,0) = (x-9, y+1, z)$

$$\overrightarrow{AX} \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} x-9 & y+1 & z \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (x-9) + 0 + 0 - (z + 0 + 0) = 0$$

$= x - 9 - z = 0$

$\pi: x - z - 9 = 0$ //

b) $A = (1,0,1) \quad B = (-1,0,1) \quad C = (2,1,2)$

$\vec{u} = \overrightarrow{AB} = (-1,0,1) - (1,0,1) = (-2,0,0)$

$\vec{v} = \overrightarrow{AC} = (2,1,2) - (1,0,1) = (1,1,1)$

$\overrightarrow{AX} = (x,y,z) - (1,0,1) = (x-1, y, z-1)$

$$\overrightarrow{AX} \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} x-1 & y & z-1 \\ -2 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 0 + 0 - 2z + 2 - (0 + 0 - 2y) = 0$$

$= 2y - 2z + 2 = 0 \rightarrow y - z + 1 = 0$

$\pi: y - z + 1 = 0$ //

c) $A=(1,1,0)$ $B=(1,-1,-1)$ $\vec{u}=(2,1,0)$

$\vec{v}=\vec{AB}=(1,-1,-1)-(1,1,0)=(0,-2,-1)$

$\vec{AX}=(x,y,z)-(1,1,0)=(x-1,y-1,z)$

$$\vec{AX}\cdot(\vec{u}\times\vec{v})=\begin{vmatrix} x-1 & y-1 & z \\ 2 & 1 & 0 \\ 0 & -2 & -1 \end{vmatrix} = -x+1+0-4z-(0+0-2y+2) = -x+2y-4z-1=0$$

$\pi: x-2y+4z-1=0$

d) $P=(1,-1,1)$ $X=(0,2,2)+\lambda(1,1,-1)$

$A=(0,2,2)$ $\vec{u}=(1,1,-1)$ $P=(1,-1,1)$

$\vec{v}=\vec{AP}=(1,-1,1)-(0,2,2)=(1,-3,-1)$

$\vec{AX}=(x,y,z)-(0,2,2)=(x,y-2,z-2)$

$$\vec{AX}\cdot(\vec{u}\times\vec{v})=\begin{vmatrix} x & y-2 & z-2 \\ 1 & -3 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 3x-y+2+z-2-(-3z+6-x-y+2)=0$$

$= 4x+4z-8=0 \rightarrow x+z-2=0$

$\pi: x+z-2=0$

7. a) $4x+2y-z+5=0$

$x=\alpha$ $y=\beta$

$4\alpha+2\beta-z+5=0$

$z=5+4\alpha+2\beta$

$$\begin{cases} x=\alpha \\ y=\beta \\ z=5+4\alpha+2\beta \end{cases}$$

b) $5x-y-1=0$

$x=\alpha$

$z=\beta$

$5\alpha-y-1=0$

$y=-1+5\alpha$

$$\begin{cases} x=\alpha \\ y=-1+5\alpha \\ z=\beta \end{cases}$$

c) $z-3=0$

$$\begin{cases} x=\alpha \\ y=\beta \\ z=3 \end{cases}$$

d) $y-z-2=0$

$x=\alpha$

$y=\beta$

$\beta-z-2=0$

$z=-2+\beta$

$$\begin{cases} x=\alpha \\ y=\beta \\ z=-2+\beta \end{cases}$$

8. a) $\begin{cases} x=1+\lambda-\mu \\ y=2\lambda+\mu \\ z=3-\mu \end{cases}$

$A=(1,0,3)$ $\vec{u}=(1,2,0)$ $\vec{v}=(-1,1,-1)$

$AX=(x-1,y,z-3)$

$$AX\cdot(\vec{u}\times\vec{v})=\begin{vmatrix} x-1 & y & z-3 \\ 1 & 2 & 0 \\ -1 & 1 & -1 \end{vmatrix} = -2x+2+0+z-3-(-2z+6+0-y) = -2x+y+3z-7=0$$

$\pi: 2x-y-3z+7=0$

b) $\begin{cases} x=1+\lambda \\ y=2 \\ z=3-\lambda+\mu \end{cases}$

$AX=(x-1,y-2,z-3)$

$\vec{u}=(1,0,-1)$ $\vec{v}=(0,0,1)$

$$\vec{AX}\cdot(\vec{u}\times\vec{v})=\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$=0+0+0-(0+0+y-2)$

$=-y+2=0$

$\pi: y-2=0$

c) $\begin{cases} x=-2+\lambda-\mu \\ y=2\lambda+2\mu \\ z=\lambda+\mu \end{cases}$

$AX=(x+2,y,z)$

$\vec{u}=(1,2,1)$ $\vec{v}=(-1,2,1)$

$$AX\cdot(\vec{u}\times\vec{v})=\begin{vmatrix} x+2 & y & z \\ 1 & 2 & 1 \\ -1 & 2 & 1 \end{vmatrix} = 2x+4-y+2z-(-2z+2x+4+y)=0$$

$=-2y+4z=0$

$\pi: y-2z=0$

LISTA GA 7

9. a) $r: \begin{cases} x = 1 + 2\lambda \\ y = \lambda \\ z = 1 + 3\lambda \end{cases}$ $\quad s: \begin{cases} x = -1 + 4\mu \\ y = -1 + 2\mu \\ z = -2 + 6\mu \end{cases}$

$\vec{u} = (2, 1, 3)$ $\quad \vec{v} = (4, 2, 6)$

$\{\vec{u}, \vec{v}\}$ são L.D

$A = (1, 0, 1)$ $\quad B = (-1, -1, -2)$

$\vec{AB} = (-1, -1, -2) - (1, 0, 1) = (-2, -1, -3)$

$\{\vec{AB}, \vec{u}\}$ é L.D, logo são retas coincidentes//

b) $r: X = (1, 1, 0) + \lambda(1, 2, 3)$

$s: X = (2, 3, 3) + \mu(3, 2, 1)$

$\vec{u} = (1, 2, 3)$ $\quad \vec{v} = (3, 2, 1)$

$\{\vec{u}, \vec{v}\}$ são L.I

$A = (1, 1, 0)$ $\quad B = (2, 3, 3)$

$\vec{AB} = (2, 3, 3) - (1, 1, 0) = (1, 2, 3)$

$P = r \cap s \Rightarrow x = (2, 3, 3) + 0(3, 2, 1)$

$\boxed{P = (2, 3, 3)}$//

$AB \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 2 + 18 + 6 - (18 + 6 + 2) = 0$

$\{AB, \vec{u}, \vec{v}\}$ são L.D, logo são concorrentes

• Ponto de intersecção:

$1 + \lambda = 2 + 3\mu$ $\qquad \lambda = \dfrac{3+\mu}{3}$

$1 + 2\lambda = 3 + 2\mu$

$3\lambda = 3 + \mu$ $\qquad 1 + \dfrac{3+\mu}{3} = 2 + 3\mu$

$3 + 3 + \mu = 6 + 9\mu \rightarrow 8\mu = 0 \quad \mu = 0$//

$\lambda = 1$//

$PX \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} x-2 & y-3 & z-3 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 2x - 4 + 9y - 27 + 2z - 6 - (6z - 18 + 6x - 12 + y - 3)$

$= -4x + 8y - 4z - 4 = 0$

$\pi: x - 2y + z + 1 = 0$//

c) $r: \begin{cases} x = 2 - 4\lambda \\ y = 4 + 5\lambda \\ z = 11 \end{cases}$ $\quad s: \dfrac{x}{2} = \dfrac{y-1}{-2} = z \rightarrow \begin{cases} \dfrac{x}{2} = z \\ \dfrac{y-1}{-2} = z \\ z = \mu \end{cases} \rightarrow \begin{cases} x = 2\mu \\ y = -2\mu + 1 \\ z = \mu \end{cases}$

$\vec{u} = (-4, 5, 0)$ $\quad \vec{v} = (2, -2, 1)$

$\{\vec{u}, \vec{v}\}$ são L.I

$A = (2, 4, 11)$ $\quad B = (0, 1, 0)$

$\vec{AB} = (-2, -3 - 11)$

$AB \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} -2 & -3 & -11 \\ -4 & 5 & 0 \\ 2 & -2 & 1 \end{vmatrix} =$

$= -10 + 0 - 88 - (-110 + 0 + 12) = 0$//

$\{AB, \vec{u}, \vec{v}\}$ são L.D, logo são concorrentes

• Ponto de intersecção (P):

$2 - 4\lambda = 2\mu$ $\qquad 2 - 4\lambda = 2 \cdot 11$

$4 + 5\lambda = -2\mu + 1 \rightarrow$ $\qquad \lambda = -5$//

$11 = \mu$ $\qquad \mu = 11$//

$P = (-10, 11, -5)$//

$PX \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} x+10 & y-11 & z+5 \\ -4 & 5 & 0 \\ 2 & -2 & 1 \end{vmatrix} = 5x + 50 + 0 + 8z + 40 - (10z + 50 + 0 - 4y + 44)$

$= 5x + 4y - 2z - 4 = 0$

$\pi: 5x + 4y - 2z - 4 = 0$//

d) $r: \frac{x-2}{3} = \frac{y+2}{4} = z$ $\quad s: \frac{x}{4} = \frac{y}{2} = \frac{z-3}{2}$

$A:(2,-2,0)$ $\quad B:(0,0,3)$ $\quad AB=(-2,2,3)$

$$\begin{cases} \frac{x-2}{3} = z \\ \frac{y+2}{4} = z \\ z = \alpha \end{cases} \rightarrow \begin{cases} x = 3\alpha + 2 \\ y = 4\alpha - 2 \\ z = \alpha \end{cases} \qquad \begin{cases} x = 4\lambda \\ y = 2\lambda \\ z = 2\lambda + 3 \end{cases}$$

$\vec{u} = (3,4,1)$ $\quad \vec{v} = (4,2,2)$

$\{\vec{u}, \vec{v}\}$ são L.I

$$AB \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} -2 & 2 & 3 \\ 3 & 4 & 1 \\ 4 & 2 & 2 \end{vmatrix} = -16 + 8 + 18 - (48 - 4 + 12) = -46$$

$\{AB, \vec{u}, \vec{v}\}$ são L.I, logo são reversas.

10.

a) $r: \begin{cases} x + 2y + 3z - 1 = 0 \ (1) \\ x - y + 2z = 0 \ (2) \end{cases}$

$x = y - 2z$ (3)

(3) → (1):

$y - 2z + 2y + 3z - 1 = 0$

$3y + z - 1 = 0$

$y = \frac{1-z}{3}$

$x = \frac{1-z}{3} - 2z$

$3x = 1 - z - 6z$

$3x = 1 - 7z$

$x = \frac{1-7z}{3}$

$$\begin{cases} x = \frac{1-7z}{3} \\ y = \frac{1-z}{3} \\ z = \alpha \end{cases} \rightarrow \begin{cases} x = \frac{1-7\alpha}{3} \\ y = \frac{1-\alpha}{3} \\ z = \alpha \end{cases}$$

Equação vetorial:

$r: X = \left(\frac{1}{3}, \frac{1}{3}, 0\right) + \alpha\left(-\frac{7}{3}, -\frac{1}{3}, 1\right)$

b) $r: \begin{cases} x + y + z - 1 = 0 \ (1) \\ x + y - z = 0 \ (2) \end{cases}$

(2) − (1): $-2z + 1 = 0$

$z = \frac{1}{2}$

(1) + (2): $2x + 2y - 1 = 0$

$x = \frac{1-2y}{2}$

$$\begin{cases} x = \frac{1-2y}{2} \\ y = \alpha \\ z = \frac{1}{2} \end{cases} \rightarrow \begin{cases} x = \frac{1-2\alpha}{2} \\ y = \alpha \\ z = \frac{1}{2} \end{cases}$$

Equação vetorial:

$r: X = \left(\frac{1}{2}, 0, \frac{1}{2}\right) + \alpha(-1, 1, 0)$

c) $r: \begin{cases} x=3 \\ 2x-z+1=0 \end{cases}$ $\begin{cases} x=3 \\ y=\alpha \\ z=7 \end{cases}$ $r: x=(3,0,7)+\alpha(0,1,0)$

d) $r: \begin{cases} y=2 \\ z=0 \end{cases} \to \begin{cases} x=\alpha \\ y=2 \\ z=0 \end{cases}$ Equação vetorial:
$r: x=(0,2,0)+\alpha(1,0,0)$

11. a)

$r: x=(1,-1,1)+\lambda(-2,1,-1)$ $A=(1,-1,1)$

$s: \begin{cases} y+z=3 \\ x+y-z=6 \end{cases}$ $n_1=(0,1,1)$ $n_2=(1,1,-1)$

$\begin{cases} -1+1=3 \\ 1-1-1=6 \end{cases} \to \begin{matrix} 0=3 \\ -1=6 \end{matrix}$ (inconsistente)

$\vec{v}_s = n_1 \times n_2 = \begin{vmatrix} j & k & i \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -j+k+0-(i+j+0) = -2j-i+k = (-2,1,-1)$

$\vec{v}_r=(-2,1,-1)$

As retas r e s são paralelas distintas.

b) $r: \frac{x+1}{2}=\frac{y}{3}=\frac{z+1}{2}$

$s: x=(0,0,0)+\lambda(1,2,0)$

$r: \begin{cases} x=-1+2\alpha \\ y=3\alpha \\ z=-1+2\alpha \end{cases} \to \vec{v}_r=(2,3,2)$ $\vec{v}_s=(1,2,0)$

$\{\vec{v}_r, \vec{v}_s\}$ são L.I

$A=(-1,0,-1)$ $B=(0,0,0)$ $\vec{AB}=(1,0,1)$

$AB\cdot(\vec{v}_r \times \vec{v}_s) = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 0 \end{vmatrix} = 0+0+4-(3+4+0) = 3 \neq 0$

$\{AB, \vec{v}_r, \vec{v}_s\}$ são L.I, logo são retas reversas

c) $r: x=(8,1,9)+\lambda(2,-1,3)$ $A=(8,1,9)$ $B=(3,-4,4)$

$s: x=(3,-4,4)+\mu(1,-2,2)$ $AB=(-5,-5,-5)$

$\vec{u}=(2,-1,3)$ $\vec{v}=(1,-2,2)$ $AB\cdot(\vec{u}\times\vec{v}) = \begin{vmatrix} -5 & -5 & -5 \\ 2 & -1 & 3 \\ 1 & -2 & 2 \end{vmatrix} =$

$\{\vec{u}, \vec{v}\}$ são L.I

$= 10-15+20-(5+30-20) = 0$

$\{AB, \vec{u}, \vec{v}\}$ são L.D, logo são retas concorrentes

d) $r: \frac{x+1}{2}=y=z$ $r: \begin{cases} x=-1+2\lambda \\ y=\lambda \\ z=\lambda \end{cases}$

$s: \begin{cases} x+y-3z=1 \; (1) \\ 2x-y-2z=0 \; (2) \end{cases}$

$s: n_1=(1,1,-3)$
$n_2=(2,-1,-2)$
$n_1 \times n_2 = \vec{v}$

$\begin{vmatrix} i & j & k \\ 1 & 1 & -3 \\ 2 & -1 & -2 \end{vmatrix} = -2i-6j-k-(2k+3i-2j) = -5i-4j-3k$

$(-5,-4,-3)$

$v=(-5,-4,-3)$ $u=(2,1,1)$

$\{v, u\}$ são L.I

(1) + (2):
$3x-5z=1$
$x=\frac{1+5z}{3}$

2·(1) − (2)
$3y-4z=2$
$y=\frac{2+4z}{3}$

$\begin{cases} x=\frac{1+5\alpha}{3} \\ y=\frac{2+4\alpha}{3} \\ z=\alpha \end{cases}$

$A=(-1,0,0)$ $B=\left(\frac{1}{3},\frac{2}{3},1\right)$

$AB=\left(-\frac{2}{3},\frac{2}{3},1\right)$

$AB\cdot(u\times v) = \begin{vmatrix} -\frac{2}{3} & \frac{2}{3} & 1 \\ -5 & -4 & -3 \\ 2 & 1 & 1 \end{vmatrix} = \frac{8}{3}-\frac{12}{3}-5-\left(-8+\frac{6}{3}-\frac{10}{3}\right) = 3 \neq 0$

$\{AB, u, v\}$ são L.I, logo são retas reversas

12.

a) $r: X=(1,1,0)+\lambda(0,1,1)$

$\tilde{\pi}: x-y-z=2$

$\vec{w}=(0,1,1) \quad \vec{n}=u\times v=(1,-1,-1)$

$\vec{w}\cdot\vec{n}=(0,1,1)\cdot(1,-1,-1)=(0-1-1)=-2\neq 0$

$\{n, w\}$ são L.I, logo são transversais

$$\begin{cases} x=1 \\ y=1+\lambda \\ z=\lambda \end{cases}$$

Substituindo

$\tilde{\pi}: x-y-z=2$

$= 1-(1+\lambda)-\lambda=2$

$= 1-1-\lambda-\lambda=2$

$= -2\lambda=2$

$\lambda=-1$

Intersecção (P):

$$\begin{cases} x=1 \\ y=1-1 \\ z=-1 \end{cases} \quad P=(1,0,-1)$$

b) $r: \frac{x-1}{2}=y=z$

$\tilde{\pi}: X=(3,0,1)+\lambda(1,0,1)+\mu(2,2,0)$

$$r: \begin{cases} x=1+2\alpha \\ y=\alpha \\ z=\alpha \end{cases}$$

$\vec{w}=(2,1,1)$

$\vec{u}=(1,0,1)$

$\vec{v}=(2,2,0)$

$$\vec{w}\cdot(\vec{u}\times\vec{v})=\begin{vmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 2 & 0 \end{vmatrix}=0+2+2-(0+4+0)=0$$

$A=(1,0,0)$ vendo se $A\in r$

$(1,0,0)=(3,0,1)+\lambda(1,0,1)+\mu(2,2,0)$

$$\begin{cases} 1=3+\lambda+2\mu \\ 0=2\mu \\ 0=\lambda \end{cases} \rightarrow \text{sistema inconsistente}$$

$\therefore A\notin\tilde{\pi}$, então r e $\tilde{\pi}$ são paralelas (disjuntas).

c) $r: \begin{cases} x-y=1 \\ x-2y=0 \end{cases}$ $\quad n_1=(1,-1,0)$ $\quad n_2=(1,-2,0)$

$\tilde{\pi}: x+y=2 \quad n_3=(1,1,0)$

$$v_r=\begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & -2 & 0 \end{vmatrix}=0+0-2k-(-k+0+0)=-k+0+0=(0,0,-1)$$

$\vec{v_r}\cdot\vec{n_3}=(0,0,-1)\cdot(1,1,0)=0$

$$r: \begin{cases} x=1+y \\ y=\frac{x}{2} \\ z=\alpha \end{cases} \rightarrow \begin{cases} x=1+\frac{x}{2} \\ y=\frac{x}{2} \\ z=\alpha \end{cases} \rightarrow \begin{cases} x=2 \\ y=1 \\ z=\alpha \end{cases}$$

$A=(2,1,0)$ vendo se $A\in\tilde{\pi}$

$\tilde{\pi}=x+y=2$

$2+1=2$

$3=2$ (inconsistente)

$\therefore A\notin\tilde{\pi}$, então r e $\tilde{\pi}$ sao paralelos (disjuntos)

LISTA GA 7

d) $r: x-2y = 3-2z+y = 2x-z$

$\pi: X = (1,4,0) + \lambda(1,1,1) + \mu(2,1,0)$

$r: \begin{cases} x-2y = 3-2z+y \\ 3-2z+y = 2x-z \end{cases} \rightarrow \begin{cases} x = 3+3y-2z \\ y = 2x+z-3 \\ z = \alpha \end{cases}$

$y = 2(3+3y-2z)+z-3$

$y = 6+6y-4z+z-3$

$y = 3+6y-3z$

$y = -\frac{3}{5} + \frac{3z}{5}$

$x = 3+3\left(-\frac{3}{5}+\frac{3z}{5}\right)-2z$

$x = 3 - \frac{9}{5} + \frac{9z}{5} - 2z$

$x = \frac{6}{5} - \frac{z}{5}$

$\begin{cases} x = \frac{6}{5} - \frac{\alpha}{5} \\ y = \frac{-3+3\alpha}{5} \\ z = \alpha \end{cases}$

$\vec{u} = (1,1,1)$ $\vec{v} = (2,1,0)$

$\vec{w} = \left(-\frac{1}{5}, \frac{3}{5}, 1\right)$

$\vec{w}\cdot(\vec{u}\times\vec{v}) = \begin{vmatrix} -\frac{1}{5} & \frac{3}{5} & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix} = 0 + \frac{6}{5} + 1 - \left(2 - \frac{1}{5} + 0\right) = \frac{7}{5} - 1 = \frac{2}{5} \neq 0$

$\{\vec{w}, \vec{u}, \vec{v}\}$ são L.I, logo são transversais

Substituindo

$\begin{cases} \frac{6-\alpha}{5} = 1+\lambda+2\mu \\ \frac{-3+3\alpha}{5} = 4+\lambda+\mu \\ \alpha = \lambda \end{cases} \rightarrow$

$\frac{6-\lambda}{5} = 1+\lambda+2\mu$

$6-\lambda = 5+5\lambda+10\mu$

$6\lambda = 1-10\mu$

$\mu = \frac{6\lambda-1}{-10}$

$\frac{-3+3\lambda}{5} = 4+\lambda+\mu$

$-3+3\lambda = 20+5\lambda+5\mu$

$3\lambda = 23+5\lambda+5\mu$

$\mu = \frac{-2\lambda-23}{5}$

$\frac{-6\lambda+1}{10} = \frac{-2\lambda-23}{5}$

$-6\lambda+1 = -4\lambda-46$

$-2\lambda = -47$

$\lambda = \frac{47}{2}$

$x = \frac{6}{5} - \frac{\frac{47}{2}}{5} = -\frac{\frac{35}{2}}{5} = -\frac{7}{2}$

$y = \frac{-3+3\left(\frac{47}{2}\right)}{5} = \frac{-3+\frac{141}{2}}{5} = \frac{\frac{135}{2}}{5} = \frac{27}{2}$

$z = \frac{47}{2}$

Ponto de intersecção (P):

$P = \left(-\frac{7}{2}, \frac{27}{2}, \frac{47}{2}\right)$

13. a)

$r: X = (1,1,1) + \lambda(2,m,1)$

$\pi: X = (0,0,0) + \alpha(1,2,0) + \beta(1,0,1)$

$\vec{w} = (2,m,1)$

$\vec{u} = (1,2,0)$ $\vec{v} = (1,0,1)$

$\vec{w}\cdot(\vec{u}\times\vec{v}) = \begin{vmatrix} 2 & m & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 4+0+0-(2+0+m) = 0$

$-m+2 = 0$

$m = 2$

$A = (1,1,1)$

$r \notin \pi$:

$\begin{cases} 1 = \alpha+\beta \\ 1 = 2\alpha \\ 1 = \beta \end{cases}$

$\beta = 1$, $\alpha = \frac{1}{2}$

$1 = \frac{1}{2} + 1$

$1 = \frac{3}{2}$ (falso)

$\therefore r \notin \pi$ e é paralelo para $m = 2$

LISTA GA 7

d) $r: x-2y = 3-2z+y = 2x-z$

$\pi: X = (1,4,0) + \lambda(1,1,1) + \mu(2,1,0)$

$r: \begin{cases} x-2y = 3-2z+y \\ 3-2z+y = 2x-z \end{cases} \rightarrow \begin{cases} x = 3+3y-2z \\ y = 2x+z-3 \\ z = \alpha \end{cases}$

$y = 2(3+3y-2z)+z-3$

$y = 6+6y-4z+z-3$

$y = 3+6y-3z$

$y = -\frac{3}{5} + \frac{3z}{5}$

$x = 3+3\left(-\frac{3}{5}+\frac{3z}{5}\right)-2z$

$x = 3 - \frac{9}{5} + \frac{9z}{5} - 2z$

$x = \frac{6}{5} - \frac{z}{5}$

$\begin{cases} x = \frac{6}{5} - \frac{\alpha}{5} \\ y = \frac{-3+3\alpha}{5} \\ z = \alpha \end{cases}$

$\vec{u} = (1,1,1)$ $\vec{v} = (2,1,0)$

$\vec{w} = \left(-\frac{1}{5}, \frac{3}{5}, 1\right)$

$\vec{w} \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} -\frac{1}{5} & \frac{3}{5} & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix} = 0 + \frac{6}{5} + 1 - \left(2 - \frac{1}{5} + 0\right) = \frac{7}{5} - 1 = \frac{2}{5} \neq 0$

$\{\vec{w}, \vec{u}, \vec{v}\}$ são L.I, logo são transversais

Substituindo

$\begin{cases} \frac{6-\alpha}{5} = 1+\lambda+2\mu \\ \frac{-3+3\alpha}{5} = 4+\lambda+\mu \\ \alpha = \lambda \end{cases} \rightarrow$

$\frac{6-\lambda}{5} = 1+\lambda+2\mu$

$6-\lambda = 5+5\lambda+10\mu$

$6\lambda = 1-10\mu$

$\mu = \frac{6\lambda-1}{-10}$

$\frac{-3+3\lambda}{5} = 4+\lambda+\mu$

$-3+3\lambda = 20+5\lambda+5\mu$

$3\lambda = 23+5\lambda+5\mu$

$\mu = \frac{-2\lambda-23}{5}$

$\frac{-6\lambda+1}{10} = \frac{-2\lambda-23}{5}$

$-6\lambda+1 = -4\lambda-46$

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$r: X = (1,1,1) + \lambda(2,m,1)$

$\pi: X = (0,0,0) + \alpha(1,2,0) + \beta(1,0,1)$

$\vec{w} = (2,m,1)$

$\vec{u} = (1,2,0)$ $\vec{v} = (1,0,1)$

$\vec{w} \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} 2 & m & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 4+0+0-(2+0+m) = 0$

$-m+2 = 0$

$m = 2$

$A = (1,1,1)$

$r \notin \pi$:

$\begin{cases} 1 = \alpha+\beta \\ 1 = 2\alpha \\ 1 = \beta \end{cases}$

$\beta = 1$, $\alpha = \frac{1}{2}$

$1 = \frac{1}{2} + 1$

$1 = \frac{3}{2}$ (falso)

$\therefore r \notin \pi$ e é paralelo para $m = 2$

b) $r: X=(n,2,0)+\lambda(2,m,m)$

$\pi: x-3y+z=1$

$\vec{u}=(2,m,m)$

$n=(1,-3,1)$

$\vec{u}\cdot n=(2,m,m)\cdot(1,-3,1)=(2-3m+m)=0$

$2-2m=0 \quad m=1$

$r: \begin{cases} x=n \\ y=2 \\ z=0 \end{cases}$

Substituindo:

$\pi: n-3\cdot 2+0=1$

$n-6=1$

$n=7$

c) $r: \frac{x-1}{m}=\frac{y}{2}=\frac{z}{m}$

$\pi: x+my+z=0$

$r: \begin{cases} \frac{x-1}{m}=\frac{y}{2} \\ \frac{y}{2}=\frac{z}{m} \end{cases} \to \begin{cases} x=\frac{ym}{2}+1 \\ z=\frac{ym}{2} \\ y=\alpha \end{cases} \to \begin{cases} x=\frac{\alpha m}{2}+1 \\ y=\alpha \\ z=\frac{\alpha m}{2} \end{cases}$

$n=(1,m,1)$

$\vec{u}=(\frac{m}{2},1,\frac{m}{2})$

$\vec{u}\cdot n=(\frac{m}{2},1,\frac{m}{2})(1,m,1)=(\frac{m}{2}+m+\frac{m}{2})$

$2m\neq 0 \quad m\neq 0$

14.

a) $\pi_1: X=(4,2,4)+\lambda(1,1,2)+\mu(3,3,1)$

forma geral π_1:

$$\begin{vmatrix} x-4 & y-2 & z-4 \\ 1 & 1 & 2 \\ 3 & 3 & 1 \end{vmatrix} = x-4+6y-12+3z-12-(3z-12+6x-24+y-2)$$

$= -5x+5y+10=0$

$= x-y-2=0$

$\pi_2: X=(3,0,0)+\lambda(1,1,0)+\mu(0,1,4)$

forma geral π_2:

$$\begin{vmatrix} x-3 & y & z \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 0 \to 4x-12+0+z-(0+0+4y)=0$$

$4x-4y+z-12=0$

$\vec{n_1}=(1,-1,0) \quad \vec{n_1}\neq\lambda\vec{n_2}$

$\vec{n_2}=(4,-4,1) \quad \{\vec{n_1},\vec{n_2}\}$ são L.I, logo são transversais.

• forma planar

$r: \begin{cases} x-y-2=0 \\ 4x-4y+z-12=0 \end{cases}$

$x-y=2$

$8+z=12$

$z=4$

$r: \begin{cases} x=2+y \\ y=\alpha \\ z=4 \end{cases} \to \begin{cases} x=2+\alpha \\ y=\alpha \\ z=4 \end{cases}$

• forma vetorial

$r: (2,0,4)+\alpha(1,1,0)$

b) $\pi_1: x-y+2z-2=0$

$\pi_2: X=(0,0,1)+\lambda(1,0,3)+\mu(-1,1,1)$

forma geral π_2:

$$\begin{vmatrix} x & y & z-1 \\ 1 & 0 & 3 \\ -1 & 1 & 1 \end{vmatrix} = 0$$

$=0-3y+z-1-(0+3x+y)=0$

$-3x-4y+z-1=0$

$\vec{n_1}=(1,-1,2) \quad \vec{n_2}=(-3,-4,1)$

$\vec{n_1}\neq\lambda\vec{n_2}$

$\{\vec{n_1},\vec{n_2}\}$ são L.I, logo são transversais.

• forma planar de r:

$r: \begin{cases} x-y+2z-2=0 & (1) \\ -3x-4y+z-1=0 & (2) \end{cases}$

$x=2+y-2z$

(1) - (2)

$4x+3y+z-1=0 \to z=1-4x-3y$

$z=1-4(2+y-2z)-3y$

$z=1-8-4y+8z-3y$

$z=\frac{-7y-7}{-7}=y+1$

$x=2+y-2(y+1)$

$x=2+y-2y-2=-y$

$\begin{cases} x=-y \\ y=\alpha \\ z=y+1 \end{cases} \to \begin{cases} x=-\alpha \\ y=\alpha \\ z=\alpha+1 \end{cases}$

• forma vetorial de r:

$r: X=(0,0,1)+\alpha(-1,1,1)$

c) $\tilde{\pi}_1: 2x - y + z - 1 = 0$

$\tilde{\pi}_2: 4x - 2y + 2z - 3 = 0$

$\vec{n}_1 = (2, -1, 1)$

$\vec{n}_2 = (4, -2, 2)$

$\vec{n}_2 = 2\vec{n}_1 \quad \{\vec{n}_1, \vec{n}_2\}$ são L.D

Como:

$d_2 = 3d_1$

∴ São paralelos coincidentes.

15.

a) $\tilde{\pi}_1: \begin{cases} x = -\lambda_1 + 2\mu_1 \\ y = m\lambda_1 \\ z = \lambda_1 + \mu_1 \end{cases} \quad \tilde{\pi}_2: \begin{cases} x = 1 + m\lambda_2 + \mu_2 \\ y = 2 + \lambda_2 \\ z = 3 + m\mu_2 \end{cases}$

$$\begin{vmatrix} x & y & z \\ -1 & m & 1 \\ 2 & 0 & 1 \end{vmatrix} = 0 \qquad \begin{vmatrix} x-1 & y-2 & z-3 \\ m & 1 & 0 \\ 1 & 0 & m \end{vmatrix} = 0$$

$mx + 2y + 0 - (2mz + 0 - y) = 0 \qquad xm - m + 0 + 0 - (z - 3 + 0 + m^2y - 3m^2) = 0$

$mx + 3y - 2mz = 0 \qquad xm - m^2y - z + 3m^2 - m + 3 = 0$

$\vec{n}_1 = (m, 3, -2m) \qquad \vec{n}_2 = (m, -m^2, -1)$

$\{\vec{n}_1, \vec{n}_2\}$ são L.I, pois $\vec{n}_1 \neq \lambda\vec{n}_2$, logo são transversais:

$\frac{m}{m} = \frac{3}{-m^2} = \frac{-1}{-2m} \quad (m \neq 0)$

$1 = -\frac{3}{m^2} = \frac{1}{2}$ (inconsistente)

$m = 0$:

$\vec{n}_1 = (0, 3, 0) \quad \vec{n}_2 = (0, 0, -1)$

$\vec{n}_1 \neq \lambda\vec{n}_2$

d) $\tilde{\pi}_1: A = (0, 1, 6) \quad B = (5, 0, 1) \quad C = (4, 0, 0)$

$\vec{u} = \vec{AB} = (5, -1, -5) \quad \vec{v} = \vec{AC} = (4, -1, -6)$

$\tilde{\pi}_1: X = (0, 1, 6) + \lambda(5, -1, -5) + \beta(4, -1, -6)$

$$\begin{vmatrix} x & y-1 & z-6 \\ 5 & -1 & -5 \\ 4 & -1 & -6 \end{vmatrix} = 0$$

$6x - 20y + 20 - 5z + 30 - (-4z + 24 + 5x - 30y + 30) = 0$

$\tilde{\pi}_1: x + 10y - z - 4 = 0 \quad \vec{n}_1 = (1, 10, -1) \quad \vec{n}_2 = 4\vec{n}_1$

$\tilde{\pi}_2: 4x + 40y - 4z - 16 = 0 \quad \vec{n}_2 = (4, 40, -4)$

$\{\vec{n}_1, \vec{n}_2\}$ são L.D

$d_2 = 4d_1$ ∴ São paralelos coincidentes.

b) $\tilde{\pi}_1: X = (1, 1, 0) + \lambda(m, 1, 1) + \mu(1, 1, m)$

$$\begin{vmatrix} x-1 & y-1 & z \\ m & 1 & 1 \\ 1 & 1 & m \end{vmatrix} = 0$$

$mx - m + y - \cancel{1} + mz - (z + x - \cancel{1} + m^2y - m^2)$

$\tilde{\pi}_1: (m-1)x + (-m^2+1)y + (m-1)z - m + m^2 = 0 \quad \vec{n}_1 = (m-1, -m^2+1, m-1)$

$\vec{n}_1 = (m-1, -(m-1)(m+1), m-1) = (1, -(m+1), 1)$

$\tilde{\pi}_2: 2x + 3y + 2z + n = 0 \quad \vec{n}_2 = (2, 3, 2)$

$K\vec{n}_2 = \vec{n}_1$

$1 = 2K$

$-m - 1 = 3K$

$1 = 2k$

$k = \frac{1}{2}$

$-m = \frac{3}{2} + 1$

$m = -\frac{5}{2}$

$A = (1, 1, 0)$

Substituindo $\tilde{\pi}_2$

$2 \cdot 1 + 3 \cdot 1 + 2 \cdot 0 + n = 0$

$n = -5$

$n \neq -5$

∴ Para ser paralelos distintos $m = -\frac{5}{2}$ e $n \neq -5$.