tal que $AB = BA = I_n$, onde $I_n = (\delta_{ij})_{n\times n}$ é a identidade de ordem n. B é denominada matriz inversa de A

• Notação: $B = A^{-1}$

LISTA 2 (GA) - DETERMINANTES E MATRIZ

1.a)

$$\det(A) = \begin{vmatrix} 1 & 2 \\ -4 & 3 \end{vmatrix} \quad \det(A) = a_{11}\cdot\tilde{a}_{11} + a_{12}\cdot\tilde{a}_{12}$$
$$= 1\cdot(-1)^{1+1}\cdot\det|3| + 2\cdot(-1)^{1+2}\cdot\det|-4|$$
$$= 1\cdot 1\cdot 3 + 2(-1)(-4) = 3 + 8 = 11$$

b)

$$\det(B) = \begin{vmatrix} \sqrt{2} & 3\sqrt{6} \\ 2 & \sqrt{3} \end{vmatrix} \quad \det(B) = b_{11}\cdot\tilde{b}_{11} + b_{12}\cdot\tilde{b}_{12}$$
$$= \sqrt{2}\cdot(-1)^{1+1}\cdot\sqrt{3} + 3\sqrt{6}\cdot(-1)^{1+2}\cdot 2$$
$$= \sqrt{6} - 6\sqrt{6} = -5\sqrt{6}$$

c)

$$\det(C) = \begin{vmatrix} \pi & 0 & 2 \\ 5 & -1 & 1 \\ 1 & 0 & 0 \end{vmatrix} \quad \det(C) = c_{31}\cdot\tilde{c}_{31} + c_{32}\cdot\tilde{c}_{32} + c_{33}\cdot\tilde{c}_{33}$$
$$= 1\cdot(-1)^{3+1}\cdot\det\begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} + 0\cdot(-1)^{3+2}\cdot\det\begin{vmatrix} \pi & 2 \\ 5 & 1 \end{vmatrix} + 0\cdot(-1)^{3+3}\cdot\det\begin{vmatrix} \pi & 0 \\ 5 & -1 \end{vmatrix}$$
$$= 1\cdot 1\cdot(c'_{11}\cdot\tilde{c}'_{11} + c'_{12}\cdot\tilde{c}'_{12})$$
$$= 1\cdot 1(0\cdot(-1)^{1+1}\cdot\det|1| + 2\cdot(-1)^{1+2}\cdot\det|-1|)$$
$$= 1\cdot 1(2) = 2$$

d)

$$\det(D) = \begin{vmatrix} -2 & 1 & -1 \\ 1 & 5 & 4 \\ -3 & 4 & 2 \end{vmatrix} \quad \det(D) = d_{11}\cdot\tilde{d}_{11} + d_{12}\cdot\tilde{d}_{12} + d_{13}\cdot\tilde{d}_{13}$$
$$= -2\cdot(-1)^{1+1}\cdot\det\begin{vmatrix} 5 & 4 \\ 4 & 2 \end{vmatrix} + 1\cdot(-1)^{1+2}\cdot\det\begin{vmatrix} 1 & 4 \\ -3 & 2 \end{vmatrix} + (-1)\cdot(-1)^{1+3}\cdot\det\begin{vmatrix} 1 & 5 \\ -3 & 4 \end{vmatrix}$$
$$= -2\cdot 1\cdot(d'_{11}\cdot\tilde{d}'_{11} + d'_{12}\cdot\tilde{d}'_{12}) + 1\cdot(-1)\cdot(d'_{11}\cdot\tilde{d}'_{11} + d'_{12}\cdot\tilde{d}'_{12}) + (-1)\cdot 1\cdot(d'_{11}\cdot\tilde{d}'_{11} + d'_{12}\cdot\tilde{d}'_{12})$$
$$= -2(5\cdot(-1)^{1+1}\cdot\det|2| + 4\cdot(-1)^{1+2}\cdot\det|4|) + (-1)(1\cdot(-1)^{1+1}\cdot\det|2| + 4\cdot(-1)^{1+2}\cdot\det|-3|) + (-1)\cdot(1\cdot(-1)^{1+1}\cdot\det|4| + 5\cdot(-1)^{1+2}\cdot\det|-3|)$$
$$= -2(5\cdot 1\cdot 2 + 4\cdot(-1)\cdot 4) + (-1)(1\cdot 1\cdot 2 + 4\cdot(-1)\cdot(-3)) + (-1)(1\cdot 1\cdot 4 + 5\cdot(-1)\cdot(-3))$$
$$= -2(10 - 16) + (-1)(2 + 12) + (-1)(4 + 15) = 12 - 14 - 19 = -21$$

e)

$$\det(E) = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & 5 \\ 2 & -1 & 2 \end{vmatrix} \quad \det(E) = e_{11}\cdot\tilde{e}_{11} + e_{12}\cdot\tilde{e}_{12} + e_{13}\cdot\tilde{e}_{13}$$
$$= 0\cdot(-1)^{1+1}\cdot\det\begin{vmatrix} 3 & 5 \\ -1 & 2 \end{vmatrix} + 2\cdot(-1)^{1+2}\cdot\det\begin{vmatrix} 1 & 5 \\ 2 & 2 \end{vmatrix} + 0\cdot(-1)^{1+3}\cdot\det\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}$$
$$= 2\cdot(-1)\cdot(e'_{11}\cdot\tilde{e}'_{11} + e'_{12}\cdot\tilde{e}'_{12}) = -2\cdot(1\cdot(-1)^{1+1}\cdot\det|2| + 5\cdot(-1)^{1+2}\cdot\det|2|)$$
$$= -2(2 + 10(-1)) = -2\cdot(-8) = 16$$

f)

$$\det(F) = \begin{vmatrix} 3 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} \quad \det(F) = f_{11}\cdot\tilde{f}_{11} + f_{21}\cdot\tilde{f}_{21} + f_{31}\cdot\tilde{f}_{31} + f_{41}\cdot\tilde{f}_{41}$$
$$= 3\cdot(-1)^{1+1}\cdot\det\begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} + 0\cdot(-1)^{2+1}\cdot\det\begin{vmatrix} -1 & 1 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} + 0\cdot(-1)^{3+1}\cdot\det\begin{vmatrix} -1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} + 0\cdot(-1)^{4+1}\cdot\det\begin{vmatrix} -1 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$
$$= 3\cdot 1\cdot(f'_{11}\cdot\tilde{f}'_{11} + f'_{12}\cdot\tilde{f}'_{12} + f'_{13}\cdot\tilde{f}'_{13}) = 3\cdot 1(1\cdot(-1)^{1+1}\cdot\det\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} + 0\cdot(-1)^{1+2}\cdot\det\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} + 1\cdot(-1)^{1+3}\cdot\det\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix}$$
$$= 3(1\cdot(-1-0) + 0 + 1(-1-0)) = 3(-1+0-1) = 3(-2) = -6$$

g)

$$\det(G) = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 2 & 5 & 3 \\ 7 & 2 & \sqrt{5} & 0 & 0 \\ 10 & -3 & 6 & 1 & 0 \\ -2 & -3 & 0 & 0 & 0 \end{vmatrix} \quad \det(G) = g_{11}\cdot\tilde{g}_{11} + \cancel{g_{12}\cdot\tilde{g}_{12}} + \cancel{g_{13}\cdot\tilde{g}_{13}} + \cancel{g_{14}\cdot\tilde{g}_{14}} + \cancel{g_{15}\cdot\tilde{g}_{15}}$$

0 0 0 0

$$= 1\cdot(-1)^{1+1}\cdot\det\begin{vmatrix} 1 & 2 & 5 & 3 \\ 2 & \sqrt{5} & 0 & 0 \\ -3 & 6 & 1 & 0 \\ -3 & 0 & 0 & 0 \end{vmatrix}$$
$$= 1\cdot(-3\cdot(-1)^{4+1}\cdot\det\begin{vmatrix} 2 & 5 & 3 \\ \sqrt{5} & 0 & 0 \\ 6 & 1 & 0 \end{vmatrix})$$
$$= 3(\sqrt{5}\cdot(-1)^{2+1}\cdot\det\begin{vmatrix} 5 & 3 \\ 1 & 0 \end{vmatrix}) = 3\cdot(\sqrt{5}\cdot(-1)\cdot(0-3)) = 9\sqrt{5}$$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

eorema — $A = (a_{ij})_{n \times n}$ possui matriz inversa, essa inversa é única [demo]. Sejam B e C inversas de A. AB = BA = I

h)

$$\det(H) = \begin{vmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{vmatrix}$$

$$\det(H) = h_{11} \cdot \tilde{h}_{11} + \underbrace{h_{12} \cdot \tilde{h}_{12}}_{0} + \underbrace{h_{13} \cdot \tilde{h}_{13}}_{0} + \underbrace{h_{14} \cdot \tilde{h}_{14}}_{0} + \underbrace{h_{15} \cdot \tilde{h}_{15}}_{0}$$

$$= 3 \cdot (-1)^{1+1} \cdot \det \begin{vmatrix} 0 & 0 & 0 & -2 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 \end{vmatrix} = 3 \cdot (\underbrace{h'_{11} \cdot \tilde{h}'_{11}}_{0} + \underbrace{h'_{12} \cdot \tilde{h}'_{12}}_{0} + \underbrace{h'_{13} \cdot \tilde{h}'_{13}}_{0} + h'_{14} \cdot \tilde{h}'_{14}$$

$$= 3 \cdot (-2 \cdot (-1)^{1+4} \cdot \det \begin{vmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ -2 & 0 & 0 \end{vmatrix}) = 3 \cdot 2 \cdot (\underbrace{h''_{11} \cdot \tilde{h}''_{11}}_{0} + h''_{12} \cdot \tilde{h}''_{12} + \underbrace{h''_{13} \cdot \tilde{h}''_{13}}_{0})$$

$$= 6(2 \cdot (-1)^{1+2} \cdot \det \begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix} = -12 \cdot (0 + 2) = -24 \,//$$

2.a) $A+B=\begin{bmatrix}3&-5&7\\4&2&8\\1&-9&6\end{bmatrix}+\begin{bmatrix}4&3&7\\-1&0&2\\3&1&-4\end{bmatrix}=\begin{bmatrix}7&-2&14\\3&2&10\\4&-8&2\end{bmatrix}$

$\det(A+B)=\begin{vmatrix}7&-2&14\\3&2&10\\4&-8&2\end{vmatrix}\begin{matrix}7&-2\\3&2\\4&-8\end{matrix}$

$-112 \quad +560 \quad 12 \quad 28 \quad -80 \quad -336$

$\det(A+B)=28-80-336-112+560+12$

$=72$

b) $\det(AB)=\det A\cdot\det B$

$\det(A)=\begin{vmatrix}3&-5&7\\4&2&8\\1&-9&6\end{vmatrix}\begin{matrix}3&-5\\4&2\\1&-9\end{matrix}$

$-14 \quad 216 \quad 120 \quad 36 \quad -40 \quad -252$

$\det(B)=\begin{vmatrix}4&3&7\\-1&0&2\\3&1&-4\end{vmatrix}\begin{matrix}4&3\\-1&0\\3&1\end{matrix}$

$0 \quad -8 \quad -12 \quad 0 \quad 18 \quad -7$

$\det(A)\cdot\det(B)=66\cdot(-9)=-594$

$\det(A)=-14+216+120+36-40-252=66$

$\det(B)=18-7-8-12=-9$

c) $\det(A^tB^t)=\det(A^t)\cdot\det(B^t)=\det(A)\cdot\det(B)$

$\therefore$ Como calculado na questão anterior, então $\det(A^tB^t)=-594$

d) $\det(3A-2C+B)$

$\det\left(3\begin{bmatrix}3&-5&7\\4&2&8\\1&-9&6\end{bmatrix}-2\begin{bmatrix}2&3&-1\\6&9&-2\\8&12&-3\end{bmatrix}+\begin{bmatrix}4&3&7\\-1&0&2\\3&1&-4\end{bmatrix}\right)=\det\left(\begin{bmatrix}9&-15&21\\12&6&24\\3&-27&18\end{bmatrix}-\begin{bmatrix}4&6&-2\\12&18&-4\\16&24&-6\end{bmatrix}+\begin{bmatrix}4&3&7\\-1&0&2\\3&1&-4\end{bmatrix}\right)=\det\begin{vmatrix}9&-18&30\\-1&-12&30\\-10&-50&20\end{vmatrix}\begin{matrix}9&-18\\-1&-12\\-10&-50\end{matrix}$

$3600 \quad 13.500 \quad -360 \quad -2160 \quad 5400 \quad 1500$

$\det(3A-2C+B)=+3600+13.500-360-2160+5400+1500=14.280$

e) $\det(AC^t) = \det(A)\cdot\det(C^t) = \det(A)\cdot\det(C)$

$\det(A) = \begin{vmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{vmatrix}\begin{matrix} 3 & -5 \\ 4 & 2 \\ 1 & -9 \end{matrix}$ $\qquad$ $\det(C) = \begin{vmatrix} 2 & 3 & -1 \\ 6 & 9 & -2 \\ 8 & 12 & -3 \end{vmatrix}\begin{matrix} 2 & 3 \\ 6 & 9 \\ 8 & 12 \end{matrix}$ $\qquad$ $\det(A)\cdot\det(C) = 66\cdot 0 = 0$

$-14 \quad 216 \quad 120 \quad 36 \quad -40 \quad -252$ $\qquad$ $72 \quad 48 \quad 54 \quad -54 \quad -48 \quad -72$

$\det(A) = -252 - 14 - 40 + 36 + 120 + 216 = 66$ $\qquad$ $\det(C) = 72 + 48 + 54 - 54 - 48 - 72 = 0$

3.a) $\det(A^t) = \det(A) = -2$ $\qquad$ b) $\det(6A) = 6^4\cdot\det(A) = 6^4\cdot(-2) = -2592$

c) $\det(A^7) = (\det(A))^7 = (-2)^7 = -128$, $\quad$ d) $\det(A^{-1}) = (\det(A))^{-1} = (-2)^{-1} = -\frac{1}{2}$

4. a) $\begin{vmatrix} a & b & c \\ d & e & f \\ 4g & 4h & 4i \end{vmatrix} = 4\cdot\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4\cdot(-3) = -12$

b) $\begin{vmatrix} a & b & -2c \\ 3d & 3e & -6f \\ g & h & -2i \end{vmatrix} = \begin{vmatrix} a & b & -2+1\cdot 2 \\ 3d & 3e & -6+3\cdot 2 \\ g & h & -2+1\cdot 2 \end{vmatrix} = \begin{vmatrix} a & b & 0 \\ 3d & 3e & 0 \\ g & h & 0 \end{vmatrix} = 0$

c) $\begin{vmatrix} -a & -b & -c \\ g & h & i \\ -d & -e & -f \end{vmatrix} = (-1)\cdot(-1)\cdot\begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} = \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} \xrightarrow{\text{permutação}} -\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -(-3) = 3$

d) $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix} \xrightarrow{\ell_1 \leftrightarrow \ell_2} \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} \xrightarrow{\ell_2 \leftrightarrow \ell_3} \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ ∴ Como foram feitas permutações pares então o sinal permanece o mesmo, então: $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix} = -3$

e) $\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{vmatrix} + \begin{vmatrix} a & b & c \\ a & b & c \\ g & h & i \end{vmatrix} \rightarrow 2\cdot\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + 0 \rightarrow 2\cdot(-3)+0 \rightarrow -6$

f) $\begin{vmatrix} ka+a & kb+b & ck+c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} ka & kb & ck \\ d & e & f \\ g & h & i \end{vmatrix} \rightarrow -3 + k(-3) \rightarrow -3(1+k)$

5. $A = \begin{vmatrix} 10 & 8 & 40 & -2 \\ 4 & 6 & 20 & -4 \\ -5 & -7 & -30 & 1 \\ 3 & -6 & -30 & 12 \end{vmatrix} \rightarrow 2\cdot 10 \begin{vmatrix} 5 & 4 & 2 & -1 \\ 4 & 6 & 2 & -4 \\ -5 & -7 & -3 & 1 \\ 3 & -6 & 3 & 12 \end{vmatrix} \rightarrow 3\cdot 2\cdot 10 \begin{vmatrix} 5 & 4 & 2 & -1 \\ 4 & 6 & 2 & -4 \\ -5 & -7 & -3 & 1 \\ 1 & -2 & 1 & 4 \end{vmatrix}\ 60\cdot 2 \begin{vmatrix} 5 & 4 & 2 & -1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 1 \\ 1 & -2 & 1 & 4 \end{vmatrix} \xrightarrow{\ell_1 \rightarrow \ell_1+\ell_3} \begin{vmatrix} 0 & -3 & -1 & 0 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 1 \\ 1 & -2 & 1 & 4 \end{vmatrix}$

$\det(A) = a_{11}\cdot(-1)^{1+1}\cdot\det|a_{11}| + a_{12}\cdot(-1)^{1+2}\cdot\det|a_{12}| + a_{13}\cdot(-1)^{1+3}\cdot\det|a_{13}| + a_{14}\cdot(-1)^{1+4}\cdot\det|a_{14}|$

$= 0\cdot 1\cdot\det\begin{vmatrix} 3 & 1 & -2 \\ -7 & -3 & 1 \\ -2 & 1 & 4 \end{vmatrix} + (-3)\cdot(-1)\cdot\det\begin{vmatrix} 2 & 1 & -2 \\ -5 & -3 & 1 \\ 1 & 1 & 4 \end{vmatrix} + (-1)\cdot 1\cdot\det\begin{vmatrix} 2 & 3 & -2 \\ -5 & -7 & 1 \\ 1 & -2 & 4 \end{vmatrix} + 0\cdot(-1)\cdot\det\begin{vmatrix} 2 & 3 & 1 \\ -5 & -7 & -3 \\ 1 & -2 & 1 \end{vmatrix}$

$= 3\cdot\begin{vmatrix} 2 & 1 & -2 \\ -5 & -3 & 1 \\ 1 & 1 & 4 \end{vmatrix}\begin{matrix} 2 & 1 \\ -5 & -3 \\ 1 & 1 \end{matrix} + (-1)\begin{vmatrix} 2 & 3 & -2 \\ -5 & -7 & 1 \\ 1 & -2 & 4 \end{vmatrix}\begin{matrix} 2 & 3 \\ -5 & -7 \\ 1 & -2 \end{matrix} = 3(41-32)+(-1)(67-82) = 3(9)+(-1)(-15) = 27+15 = 42 \cdot 120 = 5040$

$-6 \quad -2 \quad 20 \quad -24 \quad 1 \quad 20 \quad -14 \quad 4 \quad 60 \quad -48 \quad 3 \quad -20$

6.a) $\begin{vmatrix} 4 & 6 & x \\ 7 & 4 & 2x \\ 5 & 2 & -x \end{vmatrix} = -128 \rightarrow x \begin{vmatrix} 4 & 6 & 1 \\ 7 & 4 & 2 \\ 5 & 2 & -1 \end{vmatrix} \rightarrow 2x \begin{vmatrix} 4 & 3 & 1 \\ 7 & 2 & 2 \\ 5 & 1 & -1 \end{vmatrix} = -128 \rightarrow 2x \cdot 32 = -128 \rightarrow 64x = -128 \rightarrow x = -2$

$\begin{vmatrix} 4 & 3 & 1 \\ 7 & 2 & 2 \\ 5 & 1 & -1 \end{vmatrix}\begin{matrix} 4 & 3 \\ 7 & 2 \\ 5 & 1 \end{matrix} \rightarrow 32$

$-10 \quad -8 \quad 21 \quad -8 \quad 80 \quad 7$

b) $\begin{vmatrix} 3 & 5 & 7 \\ 2x & x & 3x \\ 4 & 6 & 7 \end{vmatrix} = 39 \rightarrow x \begin{vmatrix} 3 & 5 & 7 \\ 2 & 1 & 3 \\ 4 & 6 & 7 \end{vmatrix} = 39 \rightarrow x \cdot 13 = 39 \rightarrow x = 3$

$\begin{vmatrix} 3 & 5 & 7 \\ 2 & 1 & 3 \\ 4 & 6 & 7 \end{vmatrix}\begin{matrix} 3 & 5 \\ 2 & 1 \\ 4 & 6 \end{matrix} \rightarrow 84 - 71 = 13$

$-28 \quad -54 \quad -90 \quad 21 \quad 60 \quad 84$

67
+35
102

75
+28
103

c) $\begin{vmatrix} x+3 & x+1 & x+4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix} = -7 \rightarrow x \cdot \begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix} + \begin{vmatrix} 3 & 1 & 4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix} = -7 \rightarrow x \cdot \begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix}\begin{matrix} 1 & 1 \\ 4 & 5 \\ 9 & 10 \end{matrix} + \begin{vmatrix} 3 & 1 & 4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix}\begin{matrix} 3 & 1 \\ 4 & 5 \\ 9 & 10 \end{matrix} \rightarrow x \cdot (-1) + (-6) = -7$

$-45 \quad -30 \quad -28 \quad 35 \quad 27 \quad 40 \qquad -180 \quad -90 \quad -28 \quad 105 \quad 27 \quad 160$

$x = 1$

d) $\begin{vmatrix} x & x+2 \\ 1 & x \end{vmatrix} = 0 \rightarrow x^2 - x - 2 = 0 \rightarrow S = 1 \quad P = -2 \quad x_1 = -1 \quad x_2 = 2$

e) $\begin{pmatrix} x-4 & 0 & 3 \\ 2 & 0 & x-9 \\ 0 & 3 & 0 \end{pmatrix} \neq 0 \qquad \det(E) = \begin{vmatrix} x-4 & 0 & 3 \\ 2 & 0 & x-9 \\ 0 & 3 & 0 \end{vmatrix}\begin{matrix} x-4 & 0 \\ 2 & 0 \\ 0 & 3 \end{matrix}$

$0 - (x-4)(x-9)(3) \quad 0 \quad 0 \quad 0 \quad 18$

$-3(-(x-4)(x-9)+18 = 0 \qquad S = 13 \quad P = 30$

$-3(x-4)(x-9)+18 = 0 \qquad x_1 = 3 \quad x_2 = 10$

$-3(x^2 - 13x + 36) + 18 = 0$

$-3x^2 + 39x - 108 + 18 = 0$

$-3x^2 + 39x - 90 = 0$

$x^2 - 13x + 30 = 0$

∴ Logo, como é inversível $\det(E) \neq 0$, assim $x_1 \neq 3$ e $x_2 \neq 10$.

7. a) $A^{-1} \cdot A = A \cdot A^{-1} = I_n$

$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A) \qquad \operatorname{adj}(A) = [\operatorname{cof}(A)]^t$

$\tilde{a}_{11} = (-1)^{1+1} \cdot \det|d| \qquad \tilde{a}_{12} = (-1)^{1+2} \cdot \det|c| \qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\tilde{a}_{11} = 1 \cdot d = d \qquad \tilde{a}_{12} = (-1) \cdot c = -c$

$A^{-1} = \frac{1}{ad - cb} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$\tilde{a}_{21} = (-1)^{2+1} \cdot \det|b| \qquad \tilde{a}_{22} = (-1)^{2+2} \cdot \det|a|$

$\tilde{a}_{21} = (-1) \cdot b = -b \qquad \tilde{a}_{22} = 1 \cdot a = a$

$\det(A) \neq 0$

$\operatorname{cof}(A) = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \qquad \operatorname{adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

7.b) $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 7 \\ 1 & 2 \end{pmatrix}$

AB		4	7
		1	2
3	1	13	23
5	2	22	39

$AB = \begin{pmatrix} 13 & 23 \\ 22 & 39 \end{pmatrix}$

$A^{-1} = \dfrac{1}{3\cdot 2 - 1\cdot 5} \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$ $B^{-1} = \dfrac{1}{4\cdot 2 - 7\cdot 1} \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix}$ $AB^{-1} = \dfrac{1}{13\cdot 39 - 23\cdot 22} \begin{pmatrix} 39 & -23 \\ -22 & 13 \end{pmatrix}$

$A^{-1} = \dfrac{1}{1} \cdot \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$ $B^{-1} = \dfrac{1}{1} \cdot \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix}$ $AB^{-1} = \dfrac{1}{1} \begin{pmatrix} 39 & -23 \\ -22 & 13 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$ // $B^{-1} = \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix}$ // $AB^{-1} = \begin{pmatrix} 39 & -23 \\ -22 & 13 \end{pmatrix}$ //

8. a) $A=\begin{pmatrix} 2 & -2 \\ 3 & 1 \end{pmatrix}$ $\tilde{a}_{11}=(-1)^{1+1}\cdot\det|1|$ $\tilde{a}_{12}=(-1)^{1+2}\cdot\det|3|$ $a_{21}=(-1)^{2+1}\cdot\det|-2|$ $\tilde{a}_{22}=(-1)^{2+2}\cdot\det|2|$ $\operatorname{cof} A=\begin{pmatrix} 1 & -3 \\ 2 & 2 \end{pmatrix}$

$\tilde{a}_{11}=1\cdot 1=1$ $\tilde{a}_{12}=(-1)\cdot 3=-3$ $a_{21}=(-1)(-2)=2$ $\tilde{a}_{22}=1\cdot 2=2$

b) $B=\begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ $\tilde{b}_{11}=(-1)^{1+1}\cdot\det\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$ $\tilde{b}_{12}=(-1)^{1+2}\cdot\det\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix}$ $\tilde{b}_{13}=(-1)^{1+3}\cdot\det\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$ $\tilde{b}_{21}=(-1)^{2+1}\cdot\det\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix}$

$\tilde{b}_{11}=1\cdot(-2-1)$ $\tilde{b}_{12}=(-1)(-1-0)$ $\tilde{b}_{13}=1\cdot(1-0)$ $\tilde{b}_{21}=(-1)(2-0)$

$\tilde{b}_{11}=-3$ $\tilde{b}_{12}=1$ $\tilde{b}_{13}=1$ $\tilde{b}_{21}=-2$

$\tilde{b}_{22}=(-1)^{2+2}\cdot\det\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix}$ $\tilde{b}_{23}=(-1)^{2+3}\cdot\det\begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix}$ $\tilde{b}_{31}=(-1)^{3+1}\cdot\det\begin{vmatrix} -2 & 0 \\ 2 & 1 \end{vmatrix}$ $\tilde{b}_{32}=(-1)^{3+2}\cdot\det\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$ $\tilde{b}_{33}=(-1)^{3+3}\cdot\det\begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix}$

$\tilde{b}_{22}=1(-2-0)$ $\tilde{b}_{23}=(-1)(2-0)$ $\tilde{b}_{31}=1\cdot(-2-0)$ $\tilde{b}_{32}=(-1)(2-0)$ $\tilde{b}_{33}=1\cdot(4-(-2))$

$\tilde{b}_{22}=-2$ $\tilde{b}_{23}=-2$ $\tilde{b}_{31}=-2$ $\tilde{b}_{32}=-2$ $\tilde{b}_{33}=1(4+2)=6$

$$\tilde{B}=\begin{pmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{pmatrix}$$

9. a) $A^{-1}=\frac{1}{\det(A)}\cdot\operatorname{adj}(A)$

$\operatorname{adj}(A)=(\tilde{A})^t$ $\tilde{A}=\begin{pmatrix} 1 & -3 \\ 2 & 2 \end{pmatrix}$

$A_{11}=(-1)^{1+1}\cdot\det|1|$ $A_{12}=(-1)^{1+2}\cdot\det|3|$ $A_{21}=(-1)^{2+1}\cdot\det|-2|$ $A_{22}=(-1)^{2+2}\cdot\det|2|$

$A_{11}=1$ $A_{12}=-3$ $A_{21}=2$ $A_{22}=2$

$\operatorname{adj}(A)=\begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix}$ $\det(A)=\begin{vmatrix} 2 & -2 \\ 3 & 1 \end{vmatrix}=2\cdot 1-(-2\cdot 3)=2+6=8$ $A^{-1}=\frac{1}{8}\begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix}$ $A^{-1}=\begin{pmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{3}{8} & \frac{1}{4} \end{pmatrix}$

b) $\begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ $\operatorname{adj}(B)=(\tilde{B})^t$

$B_{11}=(-1)^{1+1}\cdot\det\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$ $B_{12}=(-1)^{1+2}\cdot\det\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix}$ $B_{13}=(-1)^{1+3}\cdot\det\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$ $B_{21}=(-1)^{2+1}\cdot\det\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix}$

$B_{11}=1\cdot(-2)-1$ $B_{12}=(-1)\cdot(-1)$ $B_{13}=1\cdot(1-0)$ $B_{21}=(-1)\cdot(2-0)$

$B_{11}=1\cdot(-3)=-3$ $B_{12}=1$ $B_{13}=1$ $B_{21}=-2$

$B_{22}=(-1)^{2+2}\cdot\det\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix}$ $B_{23}=(-1)^{2+3}\cdot\det\begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix}$ $B_{31}=(-1)^{3+1}\cdot\det\begin{vmatrix} -2 & 0 \\ 2 & 1 \end{vmatrix}$ $B_{32}=(-1)^{3+2}\cdot\det\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$

$B_{22}=1\cdot(-2-0)$ $B_{23}=(-1)\cdot(2-0)$ $B_{31}=1\cdot(-2-0)$ $B_{32}=(-1)\cdot(2-0)$

spiral $B_{22}=-2$ $B_{23}=-2$ $B_{31}=-2$ $B_{32}=-2$

$B_{33} = (-1)^{3+3} \cdot \det \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix}$

$B_{33} = 1 \cdot (4-2)$

$B_{33} = 2$

$\tilde{B} = \begin{pmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 2 \end{pmatrix}$ $\quad \text{adj } B = (\tilde{B})^t \quad \text{adj } B = \begin{pmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & -2 \end{pmatrix}$ $\quad B^{-1} = \frac{1}{\det B} \cdot \text{adj } B$

$\det(B) = \begin{vmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{vmatrix} \begin{matrix} 2 & -2 \\ 1 & 2 \\ 0 & 1 \end{matrix}$

$0 \quad -2 \quad -2 \qquad -4 \quad 0 \quad 0$

$\det(B) = -8$

$B^{-1} = \frac{1}{-8} \cdot \begin{pmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & -2 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{pmatrix}$

c) $\begin{pmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$

$C_{11} = (-1)^{1+1} \cdot \det \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$ $\quad C_{12} = (-1)^{1+2} \cdot \det \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix}$ $\quad C_{13} = (-1)^{1+3} \cdot \det \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}$ $\quad C_{21} = (-1)^{2+1} \cdot \det \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix}$

$C_{11} = 1 \cdot (0+1)$ $\quad C_{12} = (-1) \cdot (0+1)$ $\quad C_{13} = 1 \cdot (2-0)$ $\quad C_{21} = (-1) \cdot (-1)$

$C_{11} = 1$ $\quad C_{12} = -1$ $\quad C_{13} = 2$ $\quad C_{21} = 1$

$C_{22} = (-1)^{2+2} \cdot \det \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$ $\quad C_{23} = (-1)^{2+3} \cdot \det \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}$ $\quad C_{31} = (-1)^{3+1} \cdot \det \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix}$ $\quad C_{32} = (-1)^{3+2} \cdot \det \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix}$ $\quad C_{33} = (-1)^{3+3} \cdot \det \begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix}$

$C_{22} = 1 \cdot (0-1)$ $\quad C_{23} = (-1) \cdot (0+1)$ $\quad C_{31} = 1 \cdot (1-0)$ $\quad C_{32} = (-1) \cdot (0-2)$ $\quad C_{33} = 1 \cdot (0+2)$

$C_{22} = -1$ $\quad C_{23} = -1$ $\quad C_{31} = 1$ $\quad C_{32} = 2$ $\quad C_{33} = 2$

$\tilde{C} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -1 & -1 \\ 1 & 2 & 2 \end{pmatrix}$ $\quad \text{adj } C = (\tilde{C})^t \quad \text{adj } C = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{pmatrix}$

$\det C = 0 + ((-1)\cdot(-1)\cdot 1) + (1 \cdot 2 \cdot 1) - ((1 \cdot 0 \cdot 1) + (0 \cdot (-1) \cdot 1) + ((-1) \cdot 2 \cdot 0))$

$\det C = 0 + 1 + 2 - 0 - 0 - 0 = 3$

$C^{-1} = \frac{1}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{pmatrix} \rightarrow C^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$

d) $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 3 \\ 0 & 0 & -1 & 0 \end{pmatrix}$

$D_{11} = (-1)^{1+1} \cdot \det \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix}$ $\quad D_{12} = (-1)^{1+2} \cdot \det \begin{vmatrix} 0 & 0 & 0 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix}$ $\quad D_{13} = (-1)^{1+3} \cdot \det \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix}$ $\quad D_{14} = (-1)^{1+4} \cdot \det \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix}$

$D_{11} = 1 \cdot (3)$ $\quad D_{12} = (-1) \cdot 0$ $\quad D_{13} = 1 \cdot 0$ $\quad D_{14} = (-1) \cdot 2$

$D_{11} = 3$ $\quad D_{12} = 0$ $\quad D_{13} = 0$ $\quad D_{14} = -2$

$D_{21} = (-1)^{2+1} \cdot \det \begin{vmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix}$ $\quad D_{22} = (-1)^{2+2} \cdot \det \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix}$ $\quad D_{23} = (-1)^{2+3} \cdot \det \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix}$ $\quad D_{24} = (-1)^{2+4} \cdot \det \begin{vmatrix} 1 & 0 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix}$

$D_{21} = (-1) \cdot 0$ $\quad D_{22} = 1 \cdot (-2+3)$ $\quad D_{23} = (-1) \cdot 0$ $\quad D_{24} = 1 \cdot 0$

$D_{21} = 0$ $\quad D_{22} = 1(1) = 1$ $\quad D_{23} = 0$ $\quad D_{24} = 0$

$$D_{31} = (-1)^{3+1} \cdot \det \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \qquad D_{31} = 1 \cdot (-1) \qquad D_{31} = -1$$

$$D_{32} = (-1)^{3+2} \cdot \det \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \qquad D_{32} = (-1) \cdot 0 \qquad D_{32} = 0$$

$$D_{33} = (-1)^{3+3} \cdot \det \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \qquad D_{33} = 1 \cdot 0 \qquad D_{33} = 0$$

$$D_{34} = (-1)^{3+4} \cdot \det \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \qquad D_{34} = (-1)(-1) \qquad D_{34} = 1$$

$$D_{41} = (-1)^{4+1} \cdot \det \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & 3 \end{vmatrix} \qquad D_{41} = (-1) \cdot (2) \qquad D_{41} = -2$$

$$D_{42} = (-1)^{4+2} \cdot \det \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 2 & 3 \end{vmatrix} \qquad D_{42} = 1 \cdot 0 \qquad D_{42} = 0$$

$$D_{43} = (-1)^{4+3} \cdot \det \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{vmatrix} \qquad D_{43} = (-1)(3-2) \qquad D_{43} = (-1) \cdot 1 = -1$$

$$D_{44} = (-1)^{4+4} \cdot \det \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{vmatrix} \qquad D_{44} = 1 \cdot 2 \qquad D_{44} = 2$$

$$\operatorname{adj}(D) = [\operatorname{cof}(D)]^t \qquad \operatorname{cof}(D) = \begin{pmatrix} 3 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -2 & 0 & -1 & 2 \end{pmatrix} \qquad \operatorname{adj}(D) = \begin{pmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & 2 \end{pmatrix}$$

$$\operatorname{Det}(D) = \underbrace{d_{41} \cdot \tilde{d}_{41}}_{0} + \underbrace{d_{42} \cdot \tilde{d}_{42}}_{0} + d_{43} \cdot \tilde{d}_{43} + \underbrace{d_{44} \cdot \tilde{d}_{44}}_{0} = (-1)(-1)^{4+3} \cdot \det \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 1 \cdot (3-2) = 1$$

$$D^{-1} = \frac{1}{\det D} \cdot \operatorname{adj}(D) \qquad D^{-1} = \frac{1}{1} \begin{pmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & 2 \end{pmatrix} \qquad D^{-1} =$$

10. a) $2A^t = C - XB$

$2A^t - C = \bar{0} - XB$

$B^{-1}\cdot(2A^t - C) = -XB\cdot B^{-1}$

$B^{-1}\cdot(2A^t - C) = -X(B\cdot B^{-1})$

$B^{-1}\cdot(2A^t - C) = -XI_n$

$B^{-1}\cdot(2A^t - C) = -X$

$X = (C - 2A^t)B^{-1}$

b) Como mostrado na questão anterior $X = (C - 2A^t)B^{-1}$

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & -1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 3 & -2 & 6 \\ 2 & -1 & 5 \\ 1 & 0 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 4 & 4 & 0 \\ 0 & 8 & -2 \\ -2 & 0 & 6 \end{pmatrix}$$

$$\det(B) = \left|\begin{matrix} 3 & -2 & 6 \\ 2 & -1 & 5 \\ 1 & 0 & 3 \end{matrix}\right|\begin{matrix} 3 & -2 \\ 2 & -1 \\ 1 & 0 \end{matrix} = 6 + 12 - 9 - 10 = -1$$

$6 \quad 0 \quad 12 \qquad -9 \quad -10 \quad 0$

$\mathrm{adj}(B) = [\mathrm{cof}(B)]^t \qquad B^{-1} = \dfrac{1}{\det B}\cdot \mathrm{adj}(B)$

$\tilde{b}_{11} = (-1)^{1+1}\cdot\det\begin{vmatrix} -1 & 5 \\ 0 & 3 \end{vmatrix}$, $\tilde{b}_{11} = 1(-3-0)$, $\tilde{b}_{11} = -3$

$\tilde{b}_{12} = (-1)^{1+2}\cdot\det\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix}$, $\tilde{b}_{12} = (-1)(6-5)$, $\tilde{b}_{12} = (-1)(1) = -1$

$\tilde{b}_{13} = (-1)^{1+3}\cdot\det\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix}$, $\tilde{b}_{13} = 1\cdot(0-(-1))$, $\tilde{b}_{13} = -1$

$\tilde{b}_{21} = (-1)^{2+1}\cdot\det\begin{vmatrix} -2 & 6 \\ 0 & 3 \end{vmatrix}$, $\tilde{b}_{21} = 1\cdot(-6-0)$, $\tilde{b}_{21} = -6$

$\tilde{b}_{22} = (-1)^{2+2}\cdot\det\begin{vmatrix} 3 & 6 \\ 1 & 3 \end{vmatrix}$, $\tilde{b}_{22} = 1\cdot(9-6)$, $\tilde{b}_{22} = 1\cdot 3 = 3$

$\tilde{b}_{23} = (-1)^{2+3}\cdot\det\begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix}$, $\tilde{b}_{23} = (-1)(0-(-2))$, $\tilde{b}_{23} = (-1)(-2) = 2$

$\tilde{b}_{31} = (-1)^{3+1}\cdot\det\begin{vmatrix} -2 & 6 \\ -1 & 5 \end{vmatrix}$, $\tilde{b}_{31} = 1\cdot(-10-(-6))$, $\tilde{b}_{31} = 1(-4) = -4$

$\tilde{b}_{32} = (-1)^{3+2}\cdot\det\begin{vmatrix} 3 & 6 \\ 2 & 5 \end{vmatrix}$, $\tilde{b}_{32} = (-1)(15-12)$, $\tilde{b}_{32} = (-1)\cdot 3 = -3$

$\tilde{b}_{33} = (-1)^{3+3}\cdot\det\begin{vmatrix} 3 & -2 \\ 2 & -1 \end{vmatrix}$, $\tilde{b}_{33} = 1\cdot(-3-(-4))$, $\tilde{b}_{33} = 1(-3+4) = 1\cdot(1) = 1$

$$\mathrm{cof}(B) = \begin{pmatrix} -3 & -1 & -1 \\ -6 & 3 & 2 \\ -4 & -3 & 1 \end{pmatrix} \quad \mathrm{adj}(B) = \begin{pmatrix} -3 & -6 & -4 \\ -1 & 3 & -3 \\ -1 & 2 & 1 \end{pmatrix} \quad B^{-1} = \frac{1}{-1}\begin{pmatrix} -3 & -6 & -4 \\ -1 & 3 & -3 \\ -1 & 2 & 1 \end{pmatrix} \quad B^{-1} = \begin{pmatrix} 3 & 6 & 4 \\ 1 & -3 & 3 \\ 1 & -2 & -1 \end{pmatrix}$$

$$X = \left(\begin{pmatrix} 4 & 4 & 0 \\ 0 & 8 & -2 \\ -2 & 0 & 6 \end{pmatrix} - 2\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ -1 & 0 & 2 \end{pmatrix}\right)\cdot B^{-1} \rightarrow X = \left(\begin{pmatrix} 4 & 4 & 0 \\ 0 & 8 & -2 \\ -2 & 0 & 6 \end{pmatrix} - \begin{pmatrix} 2 & 4 & 0 \\ 0 & 6 & -2 \\ -2 & 0 & 4 \end{pmatrix}\right)B^{-1} \rightarrow X = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}\cdot B^{-1}$$

$$\begin{array}{c|c} & \begin{pmatrix} 3 & 6 & 4 \\ 1 & -3 & 3 \\ 1 & -2 & -1 \end{pmatrix} \\ X & \\ \hline \begin{matrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{matrix} & \begin{pmatrix} 6 & 12 & 8 \\ 2 & -6 & 6 \\ 2 & -4 & -2 \end{pmatrix} \end{array}$$

$$\therefore X = \begin{pmatrix} 6 & 12 & 8 \\ 2 & -6 & 6 \\ 2 & -4 & -2 \end{pmatrix}$$