LISTA 4 – GA

1) a. $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix}$

$\tilde{A}X = \tilde{B} \leftrightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\therefore$ Logo, $x = 2$ e $y = -1$.

b. $\begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

$\tilde{B}X = \tilde{C} \leftrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ h \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$

$\therefore$ Logo, solução $(x, y, z, h) = (4, 3, 2, 1)$

c. $\begin{bmatrix} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$

$\tilde{C}X = \tilde{B} \leftrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ h \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$

$\begin{cases} x = 6 \\ y = 3 \\ z = 2 - h \end{cases}$

$\therefore$ Logo, as variáveis x e y estão definidas, porem as incognitas z e h estão dependentes da outra.

d. $\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & 2 \end{bmatrix}$

$\tilde{D}X = \tilde{E}$

$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\begin{cases} x + 3z = 1 \\ y - z = 2 \end{cases} \rightarrow \begin{cases} x = 1 - 3z \\ y = 2 + z \\ z = y - 2 \end{cases}$

$\therefore$ Logo x, y, z são dependentes uns dos outros.

e. $\begin{bmatrix} 1 & 0 & 0 & -7 & 8 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 1 & -5 \end{bmatrix}$

$\tilde{E}X = \tilde{B} \leftrightarrow \begin{bmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ h \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ -5 \end{bmatrix}$

$\begin{cases} x - 7h = 8 \\ y + 3h = 2 \\ z + h = -5 \end{cases} \begin{cases} x = 7h + 8 \\ y = -3h + 2 \\ z = -h - 5 \\ h \in \mathbb{R} \end{cases}$

f. $\begin{bmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{cases} x - 6y + 3j = -2 \\ z + 4j = 7 \\ h + 5j = 8 \end{cases} \begin{cases} x = -2 + 6y - 3j \\ y \in \mathbb{R} \\ z = 7 - 4j \\ h = 8 - 5j \\ j \in \mathbb{R} \end{cases}$

2) a. $\begin{cases} 3x - 4y = 1 \\ x + 3y = 9 \end{cases}$

$A = \begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 9 \end{bmatrix}$ $\rightarrow \left[\begin{array}{cc|c} 3 & -4 & 1 \\ 1 & 3 & 9 \end{array}\right] \rightarrow \begin{bmatrix} 3 & -4 & 1 \\ 1 & 3 & 9 \end{bmatrix} \xrightarrow{\text{permutação}} \begin{bmatrix} 1 & 3 & 9 \\ 3 & -4 & 1 \end{bmatrix} \rightarrow \ell_2 \leftarrow \ell_2 - 3\ell_1 \rightarrow \begin{bmatrix} 1 & 3 & 9 \\ 0 & -13 & -26 \end{bmatrix} \ell_2 \leftarrow \ell_2/(-13) \rightarrow$

$\begin{bmatrix} 1 & 3 & 9 \\ 0 & 1 & 2 \end{bmatrix} \ell_1 \leftarrow \ell_1 - 3\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

Sistema equivalente:

$\begin{cases} x = 3 \\ y = 2 \end{cases}$

$\therefore$ Solução $(x, y) = (3, 2)$

b. $\begin{cases} 5x + 8y = 34 \\ 10x + 16y = 50 \end{cases}$

$A = \begin{bmatrix} 5 & 8 \\ 10 & 16 \end{bmatrix}$; $B = \begin{bmatrix} 34 \\ 50 \end{bmatrix}$ $\rightarrow \left[\begin{array}{cc|c} 5 & 8 & 34 \\ 10 & 16 & 50 \end{array}\right] \rightarrow \begin{bmatrix} 5 & 8 & 34 \\ 10 & 16 & 50 \end{bmatrix} \ell_2 \leftarrow \ell_2 - 2\ell_1 \rightarrow \begin{bmatrix} 5 & 8 & 34 \\ 0 & 0 & -18 \end{bmatrix} \ell_1 \leftarrow \ell_1 + \ell_2 \rightarrow \begin{bmatrix} 5 & 8 & 16 \\ 0 & 0 & -18 \end{bmatrix}$

$\begin{bmatrix} 5/8 & 1 & 2 \\ 0 & 0 & -18 \end{bmatrix}$

Sistema equivalente:

$\begin{cases} \frac{5}{8}x + y = 2 \\ 0x + 0y = -18 \end{cases} \rightarrow 0 = -18$

$\therefore$ Portanto, esse sistema não tem solução (SI)

c. $\begin{cases} x+2y=5 \\ 2x-3y=-4 \end{cases}$

$A=\begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix};\ B=\begin{bmatrix} 5 \\ -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 5 \\ 2 & -3 & -4 \end{bmatrix} \ell_2 \leftarrow \ell_2 - 2\ell_1 \rightarrow \begin{bmatrix} 1 & 2 & 5 \\ 0 & -7 & -14 \end{bmatrix} \ell_2 \leftarrow \ell_2/_{-7} \rightarrow \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \end{bmatrix} \ell_1 \leftarrow \ell_1 - 2\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

sistema equivalente:

$\begin{cases} x=1 \\ y=2 \end{cases}$ $\therefore$ Solução: $(x,y)=(1,2)$

d. $\begin{cases} 3x+2y-5z=8 \\ 2x-4y-2z=-4 \\ x-2y-3z=-4 \end{cases}$

$A=\begin{bmatrix} 3 & 2 & -5 \\ 2 & -4 & -2 \\ 1 & -2 & -3 \end{bmatrix};\ B=\begin{bmatrix} 8 \\ -4 \\ -4 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & -5 & 8 \\ 2 & -4 & -2 & -4 \\ 1 & -2 & -3 & -4 \end{bmatrix} \ell_2 \leftarrow \ell_2/_{-2} \rightarrow \begin{bmatrix} 3 & 2 & -5 & 8 \\ -1 & 2 & 1 & 2 \\ 1 & -2 & -3 & -4 \end{bmatrix} \ell_2 \leftarrow \ell_2 + \ell_3 \rightarrow \begin{bmatrix} 3 & 2 & -5 & 8 \\ 0 & 0 & -2 & -2 \\ 1 & -2 & -3 & -4 \end{bmatrix} \ell_1 \leftarrow \ell_3 + \ell_1 \rightarrow \begin{bmatrix} 4 & 0 & -8 & 4 \\ 0 & 0 & -2 & -2 \\ 1 & -2 & -3 & -4 \end{bmatrix}$

$\begin{matrix} \ell_1 \leftarrow \ell_1/4 \\ \ell_2 \leftarrow \ell_2/2 \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 0 & -1 & -1 \\ 1 & -2 & -3 & -4 \end{bmatrix} \ell_3 \leftarrow \ell_1 - \ell_3 \rightarrow \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & -2 & 1 & 5 \end{bmatrix} \ell_2 \leftarrow \ell_2 \cdot (-1) \rightarrow \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -2 & 1 & 5 \end{bmatrix} \ell_2 \leftarrow \ell_2 - \ell_3 \rightarrow \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 2 & 0 & -4 \\ 0 & -2 & 1 & 5 \end{bmatrix} \ell_3 \leftarrow \ell_2 + \ell_3$

$\begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 2 & 0 & -4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \ell_2 \leftarrow \ell_2/2 \rightarrow \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \ell_1 \leftarrow \ell_1 + 2\ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

sistema equivalente:

$\begin{cases} x=3 \\ y=2 \\ z=1 \end{cases}$ $\therefore$ Solução: $(x,y,z)=(3,2,1)$

e. $\begin{cases} 2x-6y=-4 \\ x+3y=1 \\ 4+12y=2 \end{cases}$ $A=\begin{bmatrix} 2 & -6 \\ 1 & 3 \\ 4 & 12 \end{bmatrix};\ B=\begin{bmatrix} -4 \\ 1 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 2 & -6 & -4 \\ 1 & 3 & 1 \\ 4 & 12 & 2 \end{bmatrix} \ell_1 \leftrightarrow \ell_2 \rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 2 & -6 & -4 \\ 4 & 12 & 2 \end{bmatrix} \ell_2 \leftarrow \ell_2 - 2\ell_1 \rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & -12 & -6 \\ 4 & 12 & 2 \end{bmatrix} \ell_3 \leftarrow \ell_3 - 4\ell_1 \rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & -12 & -6 \\ 0 & 0 & -2 \end{bmatrix} \ell_2 \leftarrow \ell_2/(-6) \rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{bmatrix}$

$\begin{matrix} \ell_2 \leftarrow \ell_2/2 \\ \ell_3 \leftarrow \ell_3/(-2) \end{matrix} \rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 1/2 \\ 0 & 0 & -1 \end{bmatrix} \ell_1 \leftarrow \ell_1 - 3\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 1/2 \\ 0 & 0 & -1 \end{bmatrix}$

sistema equivalente:

$\begin{cases} x=-1/2 \\ y=\frac{1}{2} \\ 0=-1 \end{cases}$ $\therefore$ Esse sistema é impossível (SI)

f. $\begin{cases} x+2y-z=2 \\ 2x-y+3z=9 \\ 3x+3y-2z=3 \end{cases}$ $A=\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 3 & -2 \end{bmatrix};\ B=\begin{bmatrix} 2 \\ 9 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & -1 & 3 & 9 \\ 3 & 3 & -2 & 3 \end{bmatrix} \ell_2 \leftarrow \ell_2 - 2\ell_1 \rightarrow \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & -5 & 5 & 5 \\ 3 & 3 & -2 & 3 \end{bmatrix} \ell_3 \leftarrow \ell_3 - 3\ell_1 \rightarrow \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & -5 & 5 & 5 \\ 0 & -3 & 1 & 3 \end{bmatrix} \ell_2 \leftarrow \ell_2/_{-5} \rightarrow \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & -3 & 1 & 3 \end{bmatrix} \ell_3 \leftarrow \ell_3 + 2\ell_2 \rightarrow$

$\begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & -6 \end{bmatrix} \ell_3 \leftarrow \ell_3/_{-2} \rightarrow \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix} \ell_1 \leftarrow \ell_1 + \ell_3 \rightarrow \begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix} \ell_2 \leftarrow \ell_2 + \ell_3 \rightarrow \begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \ell_1 \leftarrow \ell_1 - 2\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

Sistema equivalente:

$$\begin{cases} x = 1 \\ y = 2 \\ z = 3 \end{cases} \quad \text{Solução } (x, y, z) = (1, 2, 3)$$

g) $$\begin{cases} x + 3z = -8 \\ 2x - 4y = -4 \\ 3x - 2y - 5z = 26 \end{cases} \quad A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -4 & 0 \\ 3 & -2 & -5 \end{bmatrix} ; \; B = \begin{bmatrix} -8 \\ -4 \\ 26 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 & -8 \\ 2 & -4 & 0 & -4 \\ 3 & -2 & -5 & 26 \end{bmatrix} \ell_2 \leftarrow \ell_2 - 2\ell_1 \rightarrow \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & -4 & -6 & 0 \\ 3 & -2 & -5 & 26 \end{bmatrix} \ell_3 \leftarrow \ell_3 - 3\ell_1 \rightarrow \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & -4 & -6 & 0 \\ 0 & -2 & -11 & 2 \end{bmatrix} \ell_2 \leftarrow \ell_2/(-2) \rightarrow \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & 2 & 3 & 0 \\ 0 & -2 & -11 & 2 \end{bmatrix} \ell_2 \leftarrow \ell_2 + \ell_3 \rightarrow$$

$$\begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & 0 & -8 & 2 \\ 0 & -2 & -11 & 2 \end{bmatrix} \ell_2 \leftrightarrow \ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & -2 & -11 & 2 \\ 0 & 0 & -8 & 2 \end{bmatrix} \ell_2 \leftarrow \ell_2 - \ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & -2 & -3 & 0 \\ 0 & 0 & -8 & 2 \end{bmatrix} \ell_3 \leftarrow \ell_3/2 \rightarrow \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & -2 & -3 & 0 \\ 0 & 0 & -4 & 0 \end{bmatrix} \ell_3 \leftarrow \ell_3/(-4) \rightarrow \begin{bmatrix} 1 & 0 & 3 & -8 \\ 0 & -2 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\ell_1 \leftarrow \ell_1 - 3\ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & -2 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \ell_2 \leftarrow \ell_2 + 3\ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \ell_2 \leftarrow \ell_2/(-2) \rightarrow \begin{bmatrix} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Sistema equivalente:

$$\begin{cases} x = -8 \\ y = 0 \\ z = 0 \end{cases} \quad \therefore \text{Solução: } (x, y, z) = (-8, 0, 0)$$

h. $$\begin{cases} x + 2y + 3z = 10 \\ 3x + 4y + 6z = 23 \\ 2x + 2y + 3z = 13 \end{cases} \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \\ 2 & 2 & 3 \end{bmatrix} ; \; B = \begin{bmatrix} 10 \\ 23 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 10 \\ 3 & 4 & 6 & 23 \\ 2 & 2 & 3 & 13 \end{bmatrix} \ell_2 \leftarrow \ell_2 - 3\ell_1 \rightarrow \begin{bmatrix} 1 & 2 & 3 & 10 \\ 0 & -2 & -3 & -7 \\ 2 & 2 & 3 & 13 \end{bmatrix} \ell_3 \leftarrow \ell_3 - 2\ell_1 \rightarrow \begin{bmatrix} 1 & 2 & 3 & 10 \\ 0 & -2 & -3 & -7 \\ 0 & -2 & -3 & -7 \end{bmatrix} \ell_3 \leftarrow \ell_3 + \ell_2 \rightarrow \begin{bmatrix} 1 & 2 & 3 & 10 \\ 0 & -2 & -3 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \ell_1 \leftarrow \ell_1 + \ell_2 \rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & -2 & -3 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \ell_2 \leftarrow \ell_2 \cdot \left(-\frac{1}{2}\right) \rightarrow \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & \frac{3}{2} & \frac{7}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Sistema equivalente:

$$\begin{cases} x = 3 \\ y + \frac{3}{2}z = \frac{7}{2} \end{cases} \therefore \begin{cases} x = 3 \\ y = \frac{7}{2} - \frac{3}{2}z \\ z \in \mathbb{R} \end{cases} \quad \text{Solução: } (x, y, z) = \left(3, \frac{7}{2} - \frac{3}{2}z, z\right)$$

i. $$\begin{cases} x - 3y + 4z - w = 2 \\ 2x - y + 3z - 2w = 19 \end{cases} \quad A = \begin{bmatrix} 1 & -3 & 4 & -1 \\ 2 & -1 & 3 & -2 \end{bmatrix} ; \; B = \begin{bmatrix} 2 \\ 19 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 4 & -1 & 2 \\ 2 & -1 & 3 & -2 & 19 \end{bmatrix} \ell_2 \leftarrow \ell_2 - 2\ell_1 \rightarrow \begin{bmatrix} 1 & -3 & 4 & -1 & 2 \\ 0 & 5 & -5 & 0 & 15 \end{bmatrix} \ell_2 \leftarrow \ell_2/5 \rightarrow \begin{bmatrix} 1 & -3 & 4 & -1 & 2 \\ 0 & 1 & -1 & 0 & 3 \end{bmatrix} \ell_1 \leftarrow \ell_1 + 3\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 1 & -1 & 11 \\ 0 & 1 & -1 & 0 & 3 \end{bmatrix}$$

Sistema equivalente:

$$\begin{cases} x + z - w = 11 \\ y - z = 3 \end{cases} \quad \begin{cases} x = 11 - z + w \\ y = 3 + z \\ z \in \mathbb{R} \\ w \in \mathbb{R} \end{cases} \quad \text{Solução: } (x, y, z, w) = (11 - z + w, 3 + z, z, w)$$

3) a. $\begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{bmatrix} \ell_2 \leftarrow \ell_2 - \ell_1 \rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 1 & 0 & 2 & 1 & 3 \end{bmatrix} \ell_3 \leftarrow \ell_3 - \ell_1 \rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{bmatrix} \ell_3 \leftarrow \ell_3 + 2\ell_2 \begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -7 & 0 & -7 \end{bmatrix}$

$\ell_3 \leftarrow \ell_3/_{-7} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \ell_1 \leftarrow \ell_1 - \ell_3 \cdot 3 \rightarrow \begin{bmatrix} 1 & 2 & 0 & 1 & 5 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \ell_2 \leftarrow \ell_2 + 3\ell_3 \rightarrow \begin{bmatrix} 1 & 2 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \ell_1 \leftarrow \ell_1 - 2\ell_2 \rightarrow$

$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

Sistema equivalente:

$\begin{cases} x + w = 1 \\ y = 2 \\ z = 1 \end{cases} \quad \begin{cases} x = 1 - w \\ y = 2 \\ z = 1 \\ w \in \mathbb{R} \end{cases}$

$\therefore$ Solução: $(x, y, z, w) = (1-w, 2, 1, w)$

b. $\begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{bmatrix} \ell_3 \leftarrow \ell_3 - \ell_1 \rightarrow \begin{bmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{bmatrix} \ell_1 \leftarrow \ell_1 + \ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{bmatrix} \ell_2 \leftarrow \ell_2 + \ell_3 \begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & -1 & -1 & 2 & -1 \end{bmatrix} \ell_3 \leftarrow \ell_3 + \ell_2 \rightarrow$

$\begin{bmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 3 & 1 & 1 \end{bmatrix} \ell_1 \leftarrow \ell_1 - \ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 3 & -2 & -2 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \ell_1 \leftarrow \ell_1 + \ell_3 \cdot 3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \ell_3 \leftarrow \ell_3 \cdot (-1) \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix}$

Sistema equivalente:

$\begin{cases} x + w = 1 \\ y - w = 2 \\ z - w = -1 \end{cases} \quad \begin{cases} x = 1 - w \\ y = 2 + w \\ z = -1 + w \\ w \in \mathbb{R} \end{cases}$

Solução $(x, y, z, w) = (1-w, 2+w, -1+w, w)$

c. $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{bmatrix} \ell_2 \leftarrow \ell_2 - \ell_4 \rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -2 & -2 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{bmatrix} \ell_1 \leftarrow \ell_1 + \ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -2 & -2 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{bmatrix} \ell_2 \leftarrow \ell_2/_{-2} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{bmatrix} \ell_3 \leftarrow \ell_3 - \ell_4 \rightarrow$

$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & -1 & 0 \\ 1 & 3 & 3 & 0 \end{bmatrix} \ell_3 \leftarrow \ell_3 + 2\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 3 & 3 & 0 \end{bmatrix} \ell_4 \leftarrow \ell_4 - 3\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \ell_1 \leftarrow \ell_1 - \ell_3 \\ \ell_2 \leftarrow \ell_2 - \ell_3 \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \ell_4 \rightarrow \ell_4 - \ell_1 \rightarrow$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Sistema equivalente:

$\begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$

$\therefore$ Sistema: $(x, y, z) = (0, 0, 0)$

4. $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & -5 & 1 \\ 3 & -7 & 2 \end{bmatrix}$; $B_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$; $B_2 = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ 2 & -5 & 1 & -2 & -1 \\ 3 & -7 & 2 & -1 & 2 \end{bmatrix} \ell_2 \leftarrow \ell_2 - 2\ell_1 \rightarrow \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ 0 & -1 & -1 & -4 & -5 \\ 3 & -7 & 2 & -1 & 2 \end{bmatrix} \ell_3 \leftarrow \ell_3 - 3\ell_1 \rightarrow \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ 0 & -1 & -1 & -4 & -5 \\ 0 & -1 & -1 & -4 & -4 \end{bmatrix} \ell_1 \leftarrow \ell_1 - 2\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 3 & 9 & 10 \\ 0 & -1 & -1 & -4 & -5 \\ 0 & -1 & -1 & -4 & -4 \end{bmatrix}$$

$$\ell_3 \leftarrow \ell_3 - \ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 3 & 9 & 10 \\ 0 & -1 & -1 & -4 & -5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \ell_2 \leftarrow \ell_2 \cdot (-1) \rightarrow \begin{bmatrix} 1 & 0 & 3 & 9 & 10 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$; $B_1 = \begin{bmatrix} 9 \\ 4 \\ 0 \end{bmatrix}$; $B_2 = \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix}$

Sistema equivalente:

$\begin{cases} x + 3z = 9 \\ y + z = 4 \end{cases}$ $\begin{cases} x = 9 - 3z \\ y = 4 - z \\ z \in \mathbb{R} \end{cases}$ $B_2 : \begin{cases} x + 3z = 10 \\ y + z = 5 \\ 0 = 1 \end{cases}$ Sistema impossível $(0 = 1)$

Solução $B_1 : (x, y, z) = (9 - 3z, 4 - z, z)$.

5. a) $(A+4I_3)X=0$

$$A=\begin{bmatrix}1&0&5\\1&1&1\\0&1&-4\end{bmatrix} \quad (A+4I_3)=\begin{bmatrix}1+4&0&5\\1&1+4&1\\0&1&-4+4\end{bmatrix}=\begin{bmatrix}5&0&5\\1&5&1\\0&1&0\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{cases}5x_1+5x_3=0\\x_1+5x_2+x_3=0\\x_2=0\end{cases} \qquad x_1+x_3=0 \qquad \begin{cases}x_1=-x_3\\x_2=0\\x_3\in\mathbb{R}\end{cases} \qquad X=\begin{bmatrix}x_1\\0\\-x_1\end{bmatrix}$$ = Solução geral

b) $AX=2X$

$AX-2X=0$

$X(A-2I_n)=0$

$$A=\begin{bmatrix}1&0&5\\1&1&1\\0&1&-4\end{bmatrix} \quad (A-2I_n)=\begin{bmatrix}1-2&0&5\\1&1-2&1\\0&1&-4-2\end{bmatrix}=\begin{bmatrix}-1&0&5\\1&-1&1\\0&1&-6\end{bmatrix} \qquad \begin{matrix}-1&0&5\\1&-1&1\\0&1&-6\end{matrix}$$

$$\begin{bmatrix}-1&0&5&0\\1&-1&1&0\\0&1&-6&0\end{bmatrix} \xrightarrow{\ell_1\leftarrow\ell_1+\ell_2} \begin{bmatrix}0&-1&6&0\\1&-1&1&0\\0&1&-6&0\end{bmatrix} \xrightarrow{\ell_1\leftarrow\ell_1+\ell_3} \begin{bmatrix}0&0&0&0\\1&-1&1&0\\0&1&-6&0\end{bmatrix} \xrightarrow{\ell_2\leftarrow\ell_2+\ell_3} \begin{bmatrix}0&0&0&0\\1&0&-5&0\\0&1&-6&0\end{bmatrix}$$

$$\begin{cases}x_1-5x_3=0\\x_2-6x_3=0\end{cases} \rightarrow \begin{cases}x_1=5x_3\\x_2=6x_3\\x_3\end{cases} \qquad X=\begin{bmatrix}5x_3\\6x_3\\x_3\end{bmatrix}$$ = Solução geral

6. a)

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & (a^2-1) \end{bmatrix}$; $B = \begin{bmatrix} 2 \\ 5 \\ a+1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & (a^2-1) & (a+1) \end{bmatrix} \xrightarrow{\ell_2 \leftarrow \ell_2 - 2\ell_1} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 2 & 3 & (a^2-1) & (a+1) \end{bmatrix} \xrightarrow{\ell_3 \leftarrow \ell_3 - 2\ell_1} \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & (a^2-3) & (a-3) \end{bmatrix} \xrightarrow{\ell_3 \leftarrow \ell_3 - \ell_2}$

$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & (a^2-3) & (a-4) \end{bmatrix}$

Solução:

- Sistema não tem solução: $a = \pm\sqrt{3}$
- Sistema com solução única: $a \neq \pm\sqrt{3}$
- Sem solução = Não ocorre para nenhum a.

b)

$A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 5 \\ 4 & 1 & (a^2-14) \end{bmatrix}$; $B = \begin{bmatrix} 4 \\ 2 \\ a+2 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & (a^2-14) & (a+2) \end{bmatrix} \xrightarrow{\ell_2 \leftarrow \ell_2 - 3\ell_1} \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 4 & 1 & (a^2-14) & (a+2) \end{bmatrix} \xrightarrow{\ell_3 \leftarrow \ell_3 - 4\ell_1} \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & (a^2-2) & (a-14) \end{bmatrix} \xrightarrow{\ell_2 \leftarrow \ell_2/_{-7}} \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & 10/_7 \\ 0 & -7 & (a^2-2) & (a-14) \end{bmatrix}$

$$\ell_1 \leftarrow \ell_1 - 2\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 1 & 8/7 \\ 0 & 1 & -2 & 10/7 \\ 0 & -7 & (a^2-2) & (a-14) \end{bmatrix} \quad \ell_3 \leftarrow \ell_3 + 7\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 1 & 8/7 \\ 0 & 1 & -2 & 10/7 \\ 0 & 0 & (a^2-16) & (a-4) \end{bmatrix}$$

Solução:

- Sistema não tem solução: $a = -4$
- Sistema com solução única: $a \neq \pm 4$
- Sistema com soluções infinitas: $a = 4$

$t = 140$ $s = 20$

7. a) $\left[\begin{array}{cc|cc} 2 & -2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right] \; \ell_1 \leftarrow \ell_1/2 \rightarrow \begin{bmatrix} 1 & -1 & \frac{1}{2} & 0 \\ 3 & 1 & 0 & 1 \end{bmatrix} \; \ell_2 \leftarrow \ell_2 - 3\ell_1 \; \begin{bmatrix} 1 & -1 & \frac{1}{2} & 0 \\ 0 & 4 & -\frac{3}{2} & 1 \end{bmatrix} \; \ell_2 \leftarrow \ell_2/4 \rightarrow \begin{bmatrix} 1 & -1 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{3}{8} & \frac{1}{4} \end{bmatrix}$

$\ell_1 \leftarrow \ell_1 + \ell_2 \rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{8} & \frac{1}{4} \\ 0 & 1 & -\frac{3}{8} & \frac{1}{4} \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{3}{8} & \frac{1}{4} \end{bmatrix}$

b) $\left[\begin{array}{ccc|ccc} 2 & -2 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \; \ell_1 \leftarrow \ell_1 - \ell_2 \rightarrow \begin{bmatrix} 1 & -4 & -1 & 1 & -1 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \; \ell_2 \leftarrow \ell_2 - \ell_1 \rightarrow \begin{bmatrix} 1 & -4 & -1 & 1 & -1 & 0 \\ 0 & 6 & 2 & -1 & 2 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \; \ell_2 \leftarrow \ell_2 - 6\ell_3 \longrightarrow$

$\begin{bmatrix} 1 & -4 & -1 & 1 & -1 & 0 \\ 0 & 0 & 8 & -1 & 2 & -6 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \; \ell_2 \leftrightarrow \ell_3 \rightarrow \begin{bmatrix} 1 & -4 & -1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 8 & -1 & 2 & -6 \end{bmatrix} \; \ell_1 \leftarrow \ell_1 + 4\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & -5 & 1 & -1 & 4 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 8 & -1 & 2 & -6 \end{bmatrix} \; \ell_3 \leftarrow \ell_3/8 \rightarrow$

$\begin{bmatrix} 1 & 0 & -5 & 1 & -1 & 4 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{bmatrix} \; \ell_1 \leftarrow \ell_1 + 5\ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{bmatrix} \; \ell_2 \leftarrow \ell_2 + \ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$

$-\frac{5}{4}$ $\frac{5}{8}$ 2 -1

$\begin{bmatrix} \cdots & 4 \\ 1 & 5 & 2 \\ 1 & (a^2-14) & (a+2) \end{bmatrix} \; \ell_2 \leftarrow \ell_2 - 3\ell_1 \rightarrow \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 4 & 1 & (a^2-14) & (a+2) \end{bmatrix} \; \ell_3 \leftarrow \ell_3 - 4\ell_1 \rightarrow \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ \cdots \end{bmatrix} \; \ell_2 \leftarrow \ell_2/\ldots \rightarrow \begin{bmatrix} 1 & 2 & -3 & 4 \\ \cdots \end{bmatrix}$

$$B^{-1} = \begin{bmatrix} 3/8 & 1/4 & 1/4 \\ -1/8 & 1/4 & 1/4 \\ -1/8 & 1/4 & -3/4 \end{bmatrix}$$

c) $\left[\begin{array}{cc|cc} 3 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right] \ell_1 \leftarrow \ell_1 - 3\ell_2 \rightarrow \begin{bmatrix} 0 & -1 & 1 & -3 \\ 1 & 2 & 0 & 1 \end{bmatrix} \ell_1 \leftrightarrow \ell_2 \rightarrow \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -1 & 1 & -3 \end{bmatrix} \ell_1 \leftarrow \ell_1 + 2\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & -1 & 1 & -3 \end{bmatrix}$

$\ell_2 \leftarrow \ell_2 \cdot (-1) \rightarrow \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & 3 \end{bmatrix}$ $\quad C^{-1} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

d) $\left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \ell_2 \leftarrow \ell_2 - 2\ell_3 \rightarrow \begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 0 & 1 & -2 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \ell_2 \leftarrow \ell_2 - 2\ell_1 \rightarrow \begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -3 & -2 & 1 & -2 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \ell_1 \leftrightarrow \ell_3$

$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -3 & -2 & 1 & -2 \\ 0 & -1 & 1 & 1 & 0 & 0 \end{bmatrix} \ell_2 \leftrightarrow \ell_3 \rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -3 & -2 & 1 & -2 \end{bmatrix} \ell_1 \leftarrow \ell_1 + \ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -3 & -2 & 1 & -2 \end{bmatrix} \begin{matrix} \ell_2 \leftarrow \ell_2 \cdot (-1) \longrightarrow \\ \ell_3 \leftarrow \ell_3 / -3 \end{matrix}$

$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2/3 & -1/3 & 2/3 \end{bmatrix} \ell_1 \leftarrow \ell_1 - \ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1/3 & 4/3 & 1/3 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 2/3 & -1/3 & 2/3 \end{bmatrix} \ell_2 \leftarrow \ell_2 + \ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1/3 & 4/3 & 1/3 \\ 0 & 1 & 0 & -1/3 & -1/3 & 2/3 \\ 0 & 0 & 1 & 2/3 & -1/3 & 2/3 \end{bmatrix}$

$$D^{-1} = \begin{bmatrix} 1/3 & 4/3 & 1/3 \\ -1/3 & -1/3 & 2/3 \\ 2/3 & -1/3 & 2/3 \end{bmatrix}$$

e) $\left[\begin{array}{cccc|cccc} 2 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \ell_3 \leftrightarrow \ell_2 \begin{bmatrix} 2 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \ell_1 \leftarrow \ell_1 - \ell_2 \rightarrow \begin{bmatrix} 1 & -1 & -2 & -2 & 1 & 0 & -1 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \ell_1 \leftarrow \ell_1 - \ell_2 \rightarrow$

$\begin{bmatrix} 0 & -1 & -4 & -5 & 1 & 0 & -2 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \ell_1 \leftrightarrow \ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 1 & 0 & -2 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \ell_3 \leftarrow \ell_3 + 2\ell_2 \rightarrow \begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 1 & 0 & -2 & 0 \\ 0 & 0 & -4 & -6 & 2 & 1 & -4 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} \ell_2 \leftarrow \ell_2(-1) \\ \rightarrow \\ \ell_3 \leftarrow \ell_3 - 2\ell_4 \end{matrix}$

$\frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$ $\qquad$ $\frac{1}{4} - \frac{81}{4} = -\frac{80}{4} = -20$

$$\begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 5 & -1 & 0 & 2 & 0 \\ 0 & 0 & 0 & -6 & 2 & 1 & -4 & -2 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \ell_3 \leftrightarrow \ell_4 \\ \ell_4 \leftarrow \ell_{4/(-2)} \end{array} \rightarrow \begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 5 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & -6 & 2 & 1 & -4 & -2 \end{bmatrix} \ell_1 \leftarrow \ell_1 - 2\ell_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 3 & 0 & 0 & 1 & 1 \\ 0 & 1 & 4 & 5 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & -6 & 2 & 1 & -4 & -2 \end{bmatrix} \ell_4 \leftarrow \ell_{4}/-6 \rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 0 & 0 & 1 & 1 \\ 0 & 1 & 4 & 5 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & 1 & -1/3 & -1/6 & 2/3 & 1/3 \end{bmatrix} \ell_1 \leftarrow \ell_1 - 3\ell_4 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1/3 & -1 & 2/3 \\ 0 & 1 & 4 & 5 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & 1 & -1/3 & -1/6 & 2/3 & 1/3 \end{bmatrix} \ell_2 \leftarrow \ell_2 - 4\ell_3 \longrightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1/3 & -1 & 2/3 \\ 0 & 1 & 0 & 5 & -1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & 1 & -1/3 & -1/6 & 2/3 & 1/3 \end{bmatrix} \ell_2 \leftarrow \ell_2 - 5\ell_4 \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1/3 & -1 & 2/3 \\ 0 & 1 & 0 & 0 & 2/3 & 5/6 & -4/3 & 1/3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1/2 \\ 0 & 0 & 0 & 1 & -1/3 & -1/6 & 2/3 & 1/3 \end{bmatrix}$$

$-1 + \frac{5}{3}$ $\quad$ $\frac{2}{3}$ $\quad$ $2 - \frac{10}{3}$ $\quad$ $-\frac{4}{3}$ $\quad$ $2 - \frac{5}{3}$ $\quad$ $1 - \frac{5}{3}$ $\quad$ $1 - 2$

$$E^{-1} = \begin{bmatrix} 1 & 1/3 & -1 & 2/3 \\ 2/3 & 5/6 & -4/3 & 1/3 \\ 0 & 0 & 0 & -1/2 \\ -1/3 & -1/3 & 2/3 & 1/3 \end{bmatrix}$$

8. $\begin{cases} c + 2s + 3b = 26 & (1) \\ 2c + 5s + 6b = 60 & (2) \\ 2c + 3s + 4b = 40 & (3) \end{cases}$

(2) − 2(1)

$\begin{cases} 2c + 5s + 6b = 60 \\ 2c + 4s + 6b = 52 \end{cases} -$

$s = 8$

(2) − (3)

$\begin{cases} 2c + 5s + 6b = 60 \\ 2c + 3s + 4b = 40 \end{cases} -$

$2s + 2b = 20 \rightarrow$

$2s + 2b = 20$

$s + b = 10 \rightarrow 8 + b = 10$

$b = 2$

$c + 2(8) + 3(2) = 26$

$c + 16 + 6 = 26$

$c = 4$

∴ Os preços unitários da calça, blusa e shorts são, respectivamente, 4, 2 e 8 reais.

8. $\begin{cases} 5s + 2c + 6b = 2.200 \\ c = 3b \\ c = b + s \end{cases}$

$3b = b + s$

$2b = s$

$5(2b) + 2(3b) + 6b = 2200$

$10b + 6b + 6b = 2200$

$22b = 2200$

$b = 100$

$s = 2b$

$s = 200$

$c = 300$

10.

$\begin{cases} 40t + 30s + 10p = 7000 \\ 20t + 40s + 30p = 6000 \\ 10t + 20s + 40p = 5000 \end{cases} \xrightarrow{\div 10} \begin{cases} 4t + 3s + p = 700 \\ 2t + 4s + 3p = 600 \\ t + 2s + 4p = 500 \end{cases}$

$\begin{cases} 2t + 4s + 3p = 600 \\ 2t + 4s + 8p = 1000 \end{cases} -$

$5p = 400$

$p = 80$

$\begin{cases} 4t + 3s + 80 = 700 \\ 4t + 8s + 16(80) = 2000 \end{cases} -$

$\begin{cases} 4t + 3s = 620 \\ 4t + 8s = 720 \end{cases} -$

$-5s = -100$

$s = 20$

$t + 40 + 320 = 500$

$t = 140$

11. $\begin{cases} 2A + 3B + 1C = 8420 \\ A + 2B + 2C = 7940 \\ 4A + 3B = 8110 \end{cases}$

$3B = 8110 - 4A$

$2A + 8110 - 4A + C = 8420$

$-2A + C = 310$

$C = 310 + 2A$

$A + 2\left(\frac{8110 - 4A}{3}\right) + 2(310 + 2A) = 7940$

$A + 620 + 4A + \left(\frac{16220 - 8A}{3}\right) = 7940$

$3A + 1860 + 12A + 16\,220 - 8A = 23.820$

$7A = 5740$

$A = 820$ // $C = 1950$ //

$4(820) + 3B = 8110$

$3280 + 3B = 8110$

$3B = 4830$

$B = 1610$ //

$1950 - 820 = 1130$

∴ O comprimento da maior excede a menor em 1130 m.