LISTA GA 8

1.

a) $r: X = (-5, \frac{2}{3}, 0) + \lambda(\frac{1}{2}, 1, 1)$

$s: z = 3x = 2y - 16$

$s: \begin{cases} z = 3x \\ 3x = 2y - 16 \end{cases} \rightarrow \begin{cases} z = 3x \\ y = \frac{3x + 16}{2} \end{cases} \quad \begin{cases} x = \alpha \\ y = \frac{3\alpha}{2} + 8 \\ z = 3\alpha \end{cases}$

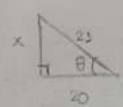
$\vec{u} = (\frac{1}{2}, 1, 1) \quad \vec{v} = (1, \frac{3}{2}, 3)$

$\|\vec{u}\| = \sqrt{(\frac{1}{2})^2 + 1^2 + 1^2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

$\|\vec{v}\| = \sqrt{1^2 + (\frac{3}{2})^2 + 3^2} = \sqrt{\frac{49}{4}} = \frac{7}{2}$

$\cos\theta = \frac{|\vec{u} \cdot \vec{v}|}{\|\vec{u}\| \cdot \|\vec{v}\|} = \frac{|(\frac{1}{2}, 1, 1) \cdot (1, \frac{3}{2}, 3)|}{\frac{3}{2} \cdot \frac{7}{2}}$

$= \frac{|(\frac{1}{2} + \frac{3}{2} + 3)|}{\frac{21}{2}} = \frac{5}{\frac{21}{4}} = \frac{20}{21}$



$x^2 + 20^2 = 21^2$

$x^2 = 41 \quad x = \sqrt{41}$

$\text{sen}\theta = \frac{\sqrt{41}}{21}$

b) $r: X = (1, 1, 0) + \lambda(0, -1, 1)$

$s: x - y + 3 = z = 4$

$s: \begin{cases} x - y + 3 = z \\ z = 4 \end{cases} \rightarrow \begin{cases} x = 1 + \alpha \\ y = \alpha \\ z = 4 \end{cases}$

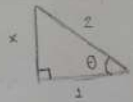
$\vec{u} = (0, -1, 1) \quad \vec{v} = (1, 1, 0)$

$\|\vec{u}\| = \sqrt{0 + (-1)^2 + 1^2} = \sqrt{2}$

$\|\vec{v}\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$

$\cos\theta = \frac{|\vec{u} \cdot \vec{v}|}{\|\vec{u}\| \cdot \|\vec{v}\|} = \frac{|(0, -1, 1) \cdot (1, 1, 0)|}{\sqrt{2} \cdot \sqrt{2}}$

$= \frac{|(0 - 1 + 0)|}{2} = \frac{1}{2}$



$x^2 + 1 = 4$

$x = \sqrt{3}$

$\text{sen}\theta = \frac{\sqrt{3}}{2}$

c) $r: \begin{cases} x + 3z = 7 \\ y = 0 \end{cases} \rightarrow \begin{cases} x = 7 - 3\alpha \\ y = 0 \\ z = \alpha \end{cases}$

$s: \begin{cases} x - 4y - 2z = 5 \\ y = 0 \end{cases} \rightarrow \begin{cases} x = 5 + 2\beta \\ y = 0 \\ z = \beta \end{cases}$

$\vec{u} = (-3, 0, 1) \quad \vec{v} = (2, 0, 1)$

$\|\vec{u}\| = \sqrt{-3^2 + 0^2 + 1^2} = \sqrt{10}$

$\|\vec{v}\| = \sqrt{2^2 + 0^2 + 1^2} = \sqrt{5}$

$\cos\theta = \frac{|\vec{u} \cdot \vec{v}|}{\|\vec{u}\| \|\vec{v}\|} =$

$= \frac{|(-3, 0, 1) \cdot (2, 0, 1)|}{\sqrt{5} \cdot \sqrt{10}}$

$= \frac{|(-6 + 0 + 1)|}{\sqrt{50}} = \frac{5}{5\sqrt{2}} = \frac{\sqrt{2}}{2}$

d)

$s: \begin{cases} 3x - y - 5z = 0 \\ x - 2y + 3z + 1 = 0 \end{cases} \rightarrow \begin{bmatrix} 3 & 1 & -5 & 0 \\ 1 & -2 & 3 & -1 \end{bmatrix} \ell_1 \leftarrow \ell_1 - 3\ell_2 \rightarrow \begin{bmatrix} 0 & 7 & -14 & 3 \\ 1 & -2 & 3 & -1 \end{bmatrix} \ell_1 \leftarrow \ell_1/7 \rightarrow$

$\begin{bmatrix} 0 & 1 & -2 & 3/7 \\ 1 & -2 & 3 & -1 \end{bmatrix} \ell_2 \leftarrow \ell_2 + 2\ell_1 \rightarrow \begin{bmatrix} 0 & 1 & -2 & 3/7 \\ 1 & 0 & -1 & -1/7 \end{bmatrix} \ell_1 \leftrightarrow \ell_2 \rightarrow \begin{bmatrix} 1 & 0 & -1 & -1/7 \\ 0 & 1 & -2 & 3/7 \end{bmatrix} \rightarrow$

$\begin{cases} x - z = -\frac{1}{7} \\ y - 2z = \frac{3}{7} \end{cases} \rightarrow \begin{cases} x = -\frac{1}{7} + \alpha \\ y = \frac{3}{7} + 2\alpha \\ z = \alpha \end{cases}$

$\|\vec{u}\| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14} \quad \vec{u} = (1, -2, 3)$

$\|\vec{v}\| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6} \quad \vec{v} = (1, 2, 1)$

$\cos\theta = \frac{|\vec{u} \cdot \vec{v}|}{\|\vec{u}\| \cdot \|\vec{v}\|} = \frac{|(1 - 4 + 3)|}{\sqrt{14} \cdot \sqrt{6}} = \frac{0}{\sqrt{84}} = 0$

$r: x = \frac{1 - y}{2} = \frac{z}{3}$

2. $r: x = (0,2,0) + \lambda(0,1,0)$ $\quad s: x = (1,2,0) + \mu(0,0,1)$

$\vec{u} = (0,1,0)$ $\quad \vec{v} = (0,0,1)$

$r: P = (0, 2+\lambda, 0)$ $\quad s: Q = (1,2,\mu)$

$\vec{PQ} = Q - P = (1,2,\mu) - (0,2+\lambda,0) = (1,-\lambda,\mu)$

$\|\vec{PQ}\| = \sqrt{1^2 + (-\lambda)^2 + \mu^2} = \sqrt{1+\lambda^2+\mu^2}$

$\|\vec{u}\| = \sqrt{1} = 1$ $\quad \|\vec{v}\| = \sqrt{1} = 1$

$$\cos 45^\circ = \frac{|\vec{PQ}\cdot\vec{u}|}{\|\vec{PQ}\|\cdot\|\vec{u}\|} = \frac{|(1,-\lambda,\mu)\cdot(0,1,0)|}{\sqrt{1+\lambda^2+\mu^2}\cdot 1} = \frac{\lambda}{\sqrt{1+\lambda^2+\mu^2}} \to \left(\frac{\sqrt{2}}{2}\right)^2 = \left(\frac{\lambda}{\sqrt{1+\lambda^2+\mu^2}}\right)^2 = \frac{1}{2} = \frac{\lambda^2}{1+\lambda^2+\mu^2} =$$

$$\Rightarrow 2\lambda^2 = 1+\lambda^2+\mu^2 \to \lambda^2 = 1+\mu^2 \to \lambda^2 - \mu^2 = 1 \quad ①$$

$$\cos 60^\circ = \frac{|\vec{PQ}\cdot\vec{v}|}{\|\vec{PQ}\|\|\vec{v}\|} = \frac{|(1,-\lambda,\mu)\cdot(0,0,1)|}{\sqrt{1+\lambda^2+\mu^2}\cdot 1} = \frac{\mu}{\sqrt{1+\lambda^2+\mu^2}} \to \left(\frac{1}{2}\right)^2 = \left(\frac{\mu}{\sqrt{1+\lambda^2+\mu^2}}\right)^2 \to \frac{1}{4} = \frac{\mu^2}{1+\lambda^2+\mu^2}$$

$$\to 4\mu^2 = 1+\lambda^2+\mu^2 \to 3\mu^2 - \lambda^2 = 1 \quad ②$$

$$\begin{cases} \lambda^2 - \mu^2 = 1 \\ -\lambda^2 + 3\mu^2 = 1 \end{cases}$$

$-\lambda^2 = -\mu^2 - 1$

$-\mu^2 - 1 + 3\mu^2 = 1$

$2\mu^2 = 2$

$\mu^2 = 1 \to \mu = \pm 1$

$-\lambda^2 + 3\cdot 1 = 1$

$-\lambda^2 = -2$

$\lambda = \pm\sqrt{2}$

$\therefore$ Logo, $P = (0, 2\pm\sqrt{2}, 0)$ $\quad Q = (1, 2, \pm 1)$

3.

a) $r: x = y - z = 0$

$$r: \begin{cases} x = y - z \\ y - z = 0 \end{cases} \to \begin{cases} x = y - z \\ y = z \\ z = \alpha \end{cases} \to \begin{cases} x = 0 \\ y = \alpha \\ z = \alpha \end{cases}$$

$\pi: z = 0$

$\vec{n} = (0,0,1)$ $\quad \vec{u} = (0,1,1)$

$$\operatorname{sen}\theta = \frac{|\vec{u}\cdot\vec{n}|}{\|\vec{u}\|\cdot\|\vec{n}\|} = \frac{|(0,1,1)\cdot(0,0,1)|}{\sqrt{2}\cdot 1} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$\therefore \theta = 45^\circ = \frac{\pi}{4}$

b) $r: -x = y = \frac{z-1}{2}$

$$\begin{cases} -x = y \\ y = \frac{z-1}{2} \end{cases} \to \begin{cases} x = -y \\ y = \alpha \\ z = 2y + 1 \end{cases} \to \begin{cases} x = -\alpha \\ y = \alpha \\ z = 2\alpha + 1 \end{cases}$$

$\pi: 2x - y = 0$

$\vec{n} = (2,-1,0)$ $\quad \vec{u} = (-1,1,2)$

$$\operatorname{sen}\theta = \frac{|\vec{u}\cdot\vec{n}|}{\|\vec{u}\|\cdot\|\vec{n}\|} = \frac{|(-2-1+0)|}{\sqrt{6}\cdot\sqrt{5}} = \frac{3}{\sqrt{30}} = \frac{\sqrt{30}}{10}$$

$\therefore \theta = \operatorname{arcsen}\left(\frac{\sqrt{30}}{10}\right)$ rad

c) $r: x = (1,0,0) + \lambda(1,1,-2)$

$\pi: x + y - z - 1 = 0$

$\vec{u} = (1,1,-2)$ $\quad \vec{n} = (1,1,-1)$

$\|\vec{u}\| = \sqrt{6}$

$\|\vec{n}\| = \sqrt{3}$

$$\operatorname{sen}\theta = \frac{|\vec{u}\cdot\vec{n}|}{\|\vec{u}\|\cdot\|\vec{n}\|} = \frac{|(1+1+2)|}{\sqrt{6}\cdot\sqrt{3}} = \frac{4}{\sqrt{18}} = \frac{4}{3\sqrt{2}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

$\theta = \operatorname{arcsen}\left(\frac{2\sqrt{2}}{3}\right)$ rad

4. $\pi_1: x+y+z=0 \quad \pi_2: x-y=0$

$\vec{u}=(a,b,c)$

Paralelo:

$(a,b,c)(1,1,1)=0$

$a+b+c=0 \; (1)$

$\|\vec{u}\|=1$

$\|\vec{n_2}\|=\sqrt{2}$

$\vec{n_2}=(1,-1,0)$

$$\operatorname{sen} 45° = \frac{|\vec{u}\cdot\vec{n_2}|}{\|\vec{u}\|\cdot\|\vec{n_2}\|} = \frac{|(a,b,c)\cdot(1,-1,0)|}{\sqrt{2}\cdot 1}$$

$$\Rightarrow \frac{|a-b|}{\sqrt{2}} = \frac{\sqrt{2}}{2} \to |a-b| = 1 \; (2)$$

Como u é um vetor unitário:

$\sqrt{a^2+b^2+c^2}=1$

$a^2+b^2+c^2=1 \; (3)$

$$\begin{cases} a+b+c=0 \\ |a-b|=1 \\ a^2+b^2+c^2=1 \end{cases}$$

$c=-a-b$

$a^2+b^2+(-a^2-b^2)=1 \to a^2+b^2+a^2+2ab+b^2=1$

$\to 2a^2+2ab+2b^2=1 \to a^2+ab+b^2=\frac{1}{2}$

caso 1:

$a-b=1$

$b=a-1$

$a^2+a(a-1)+(a-1)^2=\frac{1}{2}$

$a^2+a^2-a+a^2-2a+1=\frac{1}{2}$

$3a^2-3a+1=\frac{1}{2}$

$6a^2-6a+2=1$

$6a^2-6a+1=0$

$\Delta=(-6)^2-4\cdot 6\cdot 1=12$

$x=\frac{-(-6)\pm\sqrt{12}}{12}=\frac{6\pm 2\sqrt{3}}{12}=\frac{3\pm\sqrt{3}}{6}$

$a=\frac{3\pm\sqrt{3}}{6}$

$\to a=\frac{3+\sqrt{3}}{6},\; b=\frac{-3+\sqrt{3}}{6},\; c=-\frac{\sqrt{3}}{3}$

$\to a=\frac{3-\sqrt{3}}{6},\; b=\frac{-3-\sqrt{3}}{6},\; c=\frac{\sqrt{3}}{3}$

caso 2:

$a-b=-1$

$b=a+1$

$a^2+(a+1)^2+a(a+1)=\frac{1}{2}$

$\to a^2+a^2+2a+1+a^2+a=\frac{1}{2}$

$3a^2+3a+1=\frac{1}{2}$

$6a^2+6a+1=0$

$\Delta=12$

$a=\frac{-3+\sqrt{3}}{6},\; b=\frac{3+\sqrt{3}}{6},\; c=-\frac{\sqrt{3}}{3}$

$a=\frac{-3-\sqrt{3}}{6},\; b=\frac{3-\sqrt{3}}{6},\; c=\frac{\sqrt{3}}{3}$

$\therefore$ Os 4 vetores diretores são válidos, então uma opção é $\vec{u}=\left(\frac{-3+\sqrt{3}}{6}, \frac{3+\sqrt{3}}{6}, -\frac{\sqrt{3}}{3}\right)$

5.

a) $\pi_1: 2x+y-z-1=0$

$\pi_2: x-y+3z-10=0$

$\vec{n_1}=(2,1,-1) \quad \vec{n_2}=(1,-1,3)$

$$\cos\theta = \frac{|\vec{n_1}\cdot\vec{n_2}|}{\|\vec{n_1}\|\|\vec{n_2}\|} = \frac{|(2,1,-1)\cdot(1,-1,3)|}{\sqrt{6}\cdot\sqrt{11}}$$

$$= \frac{|2-1-3|}{\sqrt{66}} = \frac{2}{\sqrt{66}} = \frac{2\sqrt{66}}{66} = \frac{\sqrt{66}}{33}$$

$\theta = \arccos\left(\frac{\sqrt{66}}{33}\right)$

b) $\pi_2: x+y+z=0$

$\pi_1: x=(1,0,0)+\lambda(1,0,1)+\mu(-1,0,0)$

$$\vec{n_1}=\begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ -1 & 0 & 0 \end{vmatrix} = 0-j+0-(0+0+0) = -j \; (0,-1,0)$$

$\vec{n_2}=(1,1,1)$

$$\cos\theta = \frac{|\vec{n_1}\cdot\vec{n_2}|}{\|\vec{n_1}\|\|\vec{n_2}\|} = \frac{|(0,-1,0)(1,1,1)|}{1\cdot\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$\theta = \arccos\left(\frac{\sqrt{3}}{3}\right)$

c) $\pi_1: x=(0,0,0)+\lambda(1,0,0)+\mu(1,1,1)$

$\pi_2: x=(1,0,0)+\lambda(-1,2,0)+\mu(0,1,0)$

$$\vec{n_1}=\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 0+0+k-(0+0+j) = k-j \to (0,-1,1)$$

$$\vec{n_2}=\begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0+0-k-(0+0+0) = -k \to (0,0,-1)$$

$$\cos\theta = \frac{|\vec{n_1}\cdot\vec{n_2}|}{\|\vec{n_1}\|\cdot\|\vec{n_2}\|} = \frac{|(0,0,-1)(0,-1,1)|}{\sqrt{2}\cdot 1}$$

$$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$\theta = 45°$

6.

Ângulo entre os planos é o mesmo que o ângulo entre os vetores normais:

$\tilde{\pi}_1 : 2x - y + z = 0$

$\vec{n}_1 = (2, -1, 1)$

Como o plano $\tilde{\pi}_2$ é perpendicular ao $\vec{v} = \vec{i} - 2\vec{j} + \vec{k}$ então esse é o vetor normal de $\tilde{\pi}_2$.

$\vec{n}_2 = (1, -2, 1)$

$$\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\| \|\vec{n}_2\|} = \frac{|(2,-1,1)(1,-2,1)|}{\sqrt{6} \cdot \sqrt{6}} = \frac{|2+2+1|}{6} = \frac{5}{6}$$

$$\theta = \arccos\left(\frac{5}{6}\right)$$

LISTA 8 GA

7.

a) $r: x-1 = 2y = z$

$A=(1,1,0)$ $B=(0,1,1)$

$r: \frac{x-1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{1} \rightarrow r: \frac{x-1}{2} = \frac{y}{1} = \frac{z}{2}$

$r: (1,0,0) + \lambda(2,1,2)$

Ponto generico r

$P:(1+2\lambda, \lambda, 2\lambda)$

$\|\vec{AX}\| = \|\vec{BX}\| \leftrightarrow \|\vec{AX}\|^2 = \|BX\|^2$

$AX = (2\lambda + \lambda - 1 + 2\lambda)$

$BX = (1+2\lambda + \lambda - 1 + 2\lambda - 1)$

$(2\lambda)^2 + (\lambda-1)^2 + (2\lambda)^2 = (1+2\lambda)^2 + (\lambda-1)^2 + (2\lambda-1)^2$

$1 + 4\lambda + 1 - 4\lambda = 0$

$2 = 0$ (inconsistente)

b) $r: X = (0,0,4) + \lambda(4,2,-3)$ $A=(2,2,5)$ $B=(0,0,1)$

Ponto genérico r:

$P=(4\lambda, 2\lambda, 4-3\lambda)$

$AX = (4\lambda-2, 2\lambda-2, -3\lambda-1)$

$BX = (4\lambda, 2\lambda, 3-3\lambda)$

pontos equidistantes:

$(4\lambda-2)^2 + (2\lambda-2)^2 + (-3\lambda-1)^2 = (4\lambda)^2 + (2\lambda)^2 + (3-3\lambda)^2$

$\cancel{16\lambda^2} - 16\lambda + 4 + \cancel{4\lambda^2} - 8\lambda + 4 + \cancel{9\lambda^2} + 6\lambda + 1 = 16\lambda^2 + 4\lambda^2 + 9\lambda^2 - 18\lambda + 9$

$29\lambda^2 - 18\lambda + 9 = 29\lambda^2 - 18\lambda + 9$

$0=0$

$\therefore$ Todos os pontos de r equidistam A e B //

c) $r: X = (2,3,-3) + \lambda(1,1,1)$

$A=(1,1,0)$ $B=(2,2,4)$

- ponto genérico r:

$P=(\lambda+2, \lambda+3, \lambda-3)$

$AX = (\lambda+1, \lambda+2, \lambda-3)$

$BX = (\lambda, \lambda+1, \lambda-7)$

pontos equidistantes:

$(\lambda+1)^2 + (\lambda+2)^2 + (\lambda-3)^2 = (\lambda)^2 + (\lambda+1)^2 + (\lambda-7)^2$

$\lambda^2 + 2\lambda + 1 + \lambda^2 + 4\lambda + 4 + \lambda^2 - 6\lambda + 9 = \lambda^2 + \lambda^2 + 2\lambda + 1 + \lambda^2 + 14\lambda + 49$

$\cancel{3\lambda^2} + 14 = \cancel{3\lambda^2} + 12\lambda + 50$

$12\lambda = 36$

$\lambda = 3$

$P=(5,6,0)$ //

8.

a) $P=(-2,0,1)$ $\quad r: X=(1,-2,0)+\lambda(3,2,1)$

$$d(AP,r)=\frac{\|\vec{AP}\times\vec{u}\|}{\|\vec{u}\|}$$

$A=(1,-2,0)$

$P-A=(-2,0,1)-(1,-2,0)$
$=(-3,2,1)$

$\vec{u}=(3,2,1)$

$$\vec{AP}\times\vec{u}=\begin{vmatrix} i & j & k \\ -3 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix}=2i+3j-6k-(6k+2i-3j)$$
$$=6j-12k$$

$$\|\vec{AP}\times\vec{u}\|=\sqrt{0^2+6^2+12^2}=\sqrt{180}=6\sqrt{5}$$

$$d(AP,r)=\frac{6\sqrt{5}}{\sqrt{14}}=\frac{6\sqrt{70}}{14}=\frac{3\sqrt{70}}{7}$$

b) $P=(1,-1,4)$ $\quad r: \frac{x-2}{4}=\frac{y}{-3}=\frac{1-z}{2}$

$$\begin{cases} \frac{x-2}{4}=\lambda \\ \frac{y}{-3}=\lambda \\ \frac{1-z}{2}=\lambda \end{cases} \rightarrow \begin{cases} x=4\lambda+2 \\ y=-3\lambda \\ z=-2\lambda+1 \end{cases}$$

$A=(2,0,1)$

$P-A=(1,-1,4)-(2,0,1)=(-1,-1,3)$

$\vec{u}=(4,-3,-2)$

$$\vec{AP}\times\vec{u}=\begin{vmatrix} i & j & k \\ -1 & -1 & 3 \\ 4 & -3 & -2 \end{vmatrix}=2i+12j+3k-(-4k-9i+2j)$$
$$=11i+10j+7k \quad (11,10,7)$$

$$\|\vec{AP}\times\vec{u}\|=\sqrt{11^2+10^2+7^2}=\sqrt{270}=3\sqrt{30}$$

$$d(AP,r)=\frac{3\sqrt{30}}{\sqrt{29}}=\frac{3\sqrt{870}}{29}$$

c) $P=(0,-1,0)$ $\quad r: x=2y-3=2z-1$

$$r:\begin{cases} x=2y-3 \\ 2y-3=2z-1 \end{cases} \rightarrow \begin{cases} x=2y-3 \\ z=\frac{2y-2}{2} \\ y=\lambda \end{cases} \rightarrow \begin{cases} x=2\lambda-3 \\ y=\lambda \\ z=\lambda-1 \end{cases}$$

$A=(-3,0,-1)$

$\vec{u}=(2,1,1)$

$P-A=(0,-1,0)-(-3,0,-1)=(3,-1,1)$

$$\vec{AP}\times\vec{u}=\begin{vmatrix} i & j & k \\ 3 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}=-i+2j+3k-(-2k+i+3j)$$
$$=-2i-j+5k \quad (-2,-1,5)$$

$$\|\vec{AP}\times\vec{u}\|=\sqrt{(-2)^2+(-1)^2+5^2}=\sqrt{30}$$

$$d(\vec{AP},\vec{u})=\frac{\sqrt{30}}{\sqrt{6}}=\frac{\sqrt{180}}{6}=\frac{6\sqrt{5}}{6}=\sqrt{5}$$

LISTA GA 8

9. $\tilde{\pi}_1: x+y=2 \quad \tilde{\pi}_2: x=y+z$

$r: \begin{cases} x+y=2 \\ x=y+z \end{cases}$ forma planar $\rightarrow$ $x=2-y$, $2-y=y+z$, $2=2y+z$ $\begin{cases} x=2-y \\ y=y \\ z=2-2y \end{cases}$ $r: x=(2,0,2)+y(-1,1,-2)$

$P=(2-y, y, 2-2y)$

$A=(0,0,-1)$

$P-A=(2-y, y, 3-2y)$

$s: \begin{cases} x=y \\ y=z+1 \end{cases} \rightarrow \begin{cases} x=\alpha \\ y=\alpha \\ z=\alpha-1 \end{cases} \rightarrow s: x=(0,0,-1)+\alpha(1,1,1)$

$d = \dfrac{\|\vec{AP}\times\vec{u}\|}{\|\vec{u}\|} =$

$AP\times\vec{u} = \begin{vmatrix} i & j & k \\ 2-y & y & 3-2y \\ 1 & 1 & 1 \end{vmatrix} = yi+3j-2yj+2k-yk-(ky+3i-2yi+2j-yj)$

$= 3yi-3i-yj+j-2yk+2k$

$(3y-3, -y+1, -2y+2)$

$\|\vec{AP}\times\vec{u}\| = \sqrt{(3y-3)^2+(-y+1)^2+(-2y+2)^2} = \sqrt{(9y^2-18y+9)+(y^2-2y+1)+(4y^2-8y+4)}$

$= \sqrt{14y^2-28y+14} = \sqrt{14(y^2-2y+1)} = \sqrt{14(y-1)^2} = \sqrt{14}|y-1|$

$\|\vec{u}\| = \sqrt{3}$

$= \dfrac{\sqrt{14}|y-1|}{\sqrt{3}} = \dfrac{\sqrt{14}}{\sqrt{3}}$

$|y-1|=1$

$y-1=1 \quad y-1=-1$

$y=2 \quad y=0$

- Pontos de interseccão:

$P_1=(0,2,-2)$

$P_2=(2,0,2)$

10.

a) $P(1,3,4)$ $\tilde{\pi}: x=(1,0,0)+\lambda(1,0,0)+\mu(-1,0,3)$

$A=(1,0,0)$

$\vec{AP} = P-A = (1,3,4)-(1,0,0) = (0,3,4)$

$\vec{n} = \vec{u}\times\vec{v}$

$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 0+0+0-(0+0+3j)$

$= -3j + (0,-1,0)$

$d(P,\tilde{\pi}) = \dfrac{|\vec{AP}\cdot\vec{n}|}{\|\vec{n}\|} = \dfrac{|(0,3,4)\cdot(0,-1,0)|}{1}$

$= \dfrac{|0-3+0|}{1} = 3$

b) $P(0,0,-6)$ $\tilde{\pi}: x-2y-2z-6=0$

$\vec{n}=(1,-2,-2)$

$\begin{cases} x=2y+2z+6 \\ y=0 \\ z=0 \end{cases} \rightarrow \begin{cases} x=6 \\ y=0 \\ z=0 \end{cases}$ $A=(6,0,0)$

$\vec{AP} = P-A = (-6,0,-6)$

$d(P,\tilde{\pi}) = \dfrac{|(-6,0,-6)\cdot(1,-2,-2)|}{\sqrt{9}}$

$= \dfrac{|-6+0+12|}{3} = \dfrac{6}{3} = 2$

c) $P(1,1,1)$ $\tilde{\pi}: 2x-y+2z-3=0$

$\vec{n}=(2,-1,2)$

$\begin{cases} x=0 \\ y=2x+2z-3 \\ z=0 \end{cases} \rightarrow \begin{cases} x=0 \\ y=-3 \\ z=0 \end{cases}$ $A=(0,-3,0)$

$\vec{AP} = P-A = (1,1,1)-(0,-3,0) = (1,4,1)$

$d(P,\tilde{\pi}) = \dfrac{|(1,4,1)\cdot(2,-1,2)|}{\sqrt{9}} = \dfrac{|2-4+2|}{3} = \dfrac{0}{3} = 0$

11. $r: x = 2 - y = y + z$

$\pi: x - 2y - z = 1$

$$\begin{cases} x = 2 - y \\ 2 - y = y + z \end{cases} \rightarrow \begin{cases} x = 2 - y \\ y = \alpha \\ z = 2 - 2y \end{cases} \rightarrow \begin{cases} x = 2 - \alpha \\ y = \alpha \\ z = 2 - 2\alpha \end{cases}$$

$P = (2 - \alpha, \alpha, 2 - 2\alpha)$

$n = (1, -2, -1)$

$$\pi: \begin{cases} x = 1 + 2y + z \\ y = 0 \\ z = 0 \end{cases} \rightarrow \begin{cases} x = 1 \\ y = 0 \\ z = 0 \end{cases}$$

$A = (1, 0, 0)$

$P - A = (1 - \alpha, \alpha, 2 - 2\alpha)$

$$d = \frac{|\vec{AP} \cdot \vec{n}|}{\|\vec{n}\|} = \frac{|(1 - \alpha, \alpha, 2 - 2\alpha) \cdot (1, -2, -1)|}{\sqrt{6}}$$

$$= \frac{|1 - \alpha - 2\alpha - 2 + 2\alpha|}{\sqrt{6}} = \frac{|-\alpha - 1|}{\sqrt{6}}$$

$$\rightarrow \frac{|-\alpha - 1|}{\sqrt{6}} = \sqrt{6} \rightarrow |-\alpha - 1| = 6$$

$-\alpha - 1 = 6 \rightarrow \alpha = -7$

$-\alpha - 1 = -6 \rightarrow \alpha = 5$

$\therefore P_1 = (9, -7, 16)$

$P_2 = (-3, 5, -8)$

12.

a) $r: X = (2, 1, 0) + \lambda(1, -1, 1)$

$s: x + y + z = 2x - y - 1 = 0$

$$s: \begin{cases} x + y + z = 0 & \vec{n_1} = (1, 1, 1) \\ 2x - y - 1 = 0 & \vec{n_2} = (2, -1, 0) \end{cases}$$

$$s: \begin{cases} x + y + z = 0 \\ 2x - y - 1 = 0 \\ z = 0 \end{cases} \rightarrow \begin{cases} x + y = 0 \\ 2x - y = 1 \\ z = 0 \end{cases} \rightarrow \begin{cases} x = -y \\ -2y - y = 1 \\ z = 0 \end{cases} \rightarrow \begin{cases} x = -y \\ y = -\frac{1}{3} \\ z = 0 \end{cases} \rightarrow \begin{cases} x = \frac{1}{3} \\ y = -\frac{1}{3} \\ z = 0 \end{cases}$$

$$\vec{u} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = 0 + 2j - k - (2k - i + 0) = i + 2j - 3k \rightarrow (1, 2, -3)$$

$\vec{v} = (1, -1, 1)$

$A = \left(\frac{1}{3}, -\frac{1}{3}, 0\right) \quad B = (2, 1, 0)$

$\vec{AB} = B - A = (2, 1, 0) - \left(\frac{1}{3}, -\frac{1}{3}, 0\right) = \left(\frac{5}{3}, \frac{4}{3}, 0\right)$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 1 & -1 & 1 \end{vmatrix} = 2i - 3j - k - (2k + 3i + j) = -i - 4j - 3k \rightarrow (-1, -4, -3)$$

$\|\vec{u} \times \vec{v}\| = \sqrt{(-1)^2 + (-4)^2 + (-3)^2} = \sqrt{26}$

$$d(r, s) = \frac{|\vec{AB} \cdot (\vec{u} \times \vec{v})|}{\|\vec{u} \cdot \vec{v}\|}$$

$$= \frac{\left|\left(\frac{5}{3}, \frac{4}{3}, 0\right) \cdot (-1, -4, -3)\right|}{\sqrt{26}}$$

$$= \frac{\left|\left(-\frac{5}{3} - \frac{16}{3}\right)\right|}{\sqrt{26}} = \frac{\left|-\frac{21}{3}\right|}{\sqrt{26}}$$

$$= \frac{7}{\sqrt{26}} = \frac{7\sqrt{26}}{26}$$

b) $r: \frac{x + 4}{3} = \frac{y}{4} = \frac{z + 5}{-2} \quad s: X = (21, -5, 2) + \lambda(6, -4, -1)$

$$r: \begin{cases} x = -4 + 3\alpha \\ y = 4\alpha \\ z = -5 - 2\alpha \end{cases} \rightarrow r: X = (-4, 0, -5) + \alpha(3, 4, -2)$$

$\vec{u} = (6, -4, -1) \quad \vec{v} = (3, 4, -2)$

$A = (-4, 0, -5) \quad B = (21, -5, 2)$

$\vec{AB} = B - A = (25, -5, 7)$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 6 & -4 & -1 \\ 3 & 4 & -2 \end{vmatrix} = 8i - 3j + 24k - (-12k - 4i - 12j) = 12i + 9j + 36k$$

$(12, 9, 36)$

$\|\vec{u} \times \vec{v}\| = \sqrt{(12)^2 + 9^2 + 36^2}$

$= \sqrt{144 + 81 + 1296} = \sqrt{1521} = 39$

$$d(r, s) = \frac{|(25, -5, 7) \cdot (12, 9, 36)|}{39} = \frac{|(300 - 45 + 252)|}{39}$$

$$= \frac{507}{39} = 13$$

c) $r: \frac{x-1}{-2} = 2y = z$ $s: X = (0,0,2) + \lambda(-4,1,2)$

$r: \begin{cases} x = 1 - 2\alpha \\ y = \frac{1}{2}\alpha \\ z = \alpha \end{cases}$ $\vec{u} = (-2, \frac{1}{2}, 1)$ $\vec{v} = (-4, 1, 2)$

$A = (1,0,0)$ $B = (0,0,2)$

$\vec{AB} = (-1,0,2)$

$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ -2 & \frac{1}{2} & 1 \\ -4 & 1 & 2 \end{vmatrix} = i - 4j - 2k - (-2k + i - 4j) = 0 \rightarrow (0,0,0)$

$\|\vec{u} \times \vec{v}\| = 0$

∴ Como $\|\vec{u} \times \vec{v}\|$ faz parte da divisão do cálculo da distância $d(r,s) = 0$

13.

a) $r: X = (1,9,4) + \lambda(3,3,3)$

$\pi: X = (5,7,9) + \lambda(1,0,0) + \mu(0,1,0)$

$\vec{u} = (3,3,3)$ $\vec{v} = (1,0,0)$ $\vec{w} = (0,1,0)$

$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} 3 & 3 & 3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0 + 0 + 3 - (0 + 0 + 0) = 3 \neq 0$

Como a reta não é paralela ao plano e sim transversal $d(r, \pi) = 0$

b) $r: x - y + z = 0 = 2x + y - z - 3$

$\pi: y - z = 4$

$r: \begin{cases} x - y + z = 0 \\ 2x + y - z - 3 = 0 \end{cases} \rightarrow \begin{cases} x = y - \alpha \\ y = 3 + \alpha - 2x \\ z = \alpha \end{cases} \rightarrow \begin{matrix} y = 3 + \alpha - 2y + 2\alpha \\ 3y = 3\alpha + 3 \\ y = \alpha + 1 \end{matrix}$

$x = \alpha + 1 - \alpha = x = 1$

$\rightarrow \begin{cases} x = 1 \\ y = \alpha + 1 \\ z = \alpha \end{cases}$ $\vec{n} = (0,1,-1)$ $\vec{u} = (0,1,1)$

$\vec{u} \cdot \vec{n} = (0,1,-1) \cdot (0,1,1) = (0 + 1 - 1) = 0$

$A = (1,1,0)$

$\pi: x = 0 = 1$
$y = 1 = 1$ (inconsistente)
$z = -1 = 0$

Como r é paralelo ao plano π:

$X = (1,1,0)$ $\pi: y - z = 4$

$\vec{n} = (0,1,-1)$

$A = (0,4,0)$

$\begin{cases} y = 4 + z \\ x = \alpha \\ z = \beta \end{cases} \rightarrow \begin{cases} x = \alpha \\ y = 4 + \beta \\ z = \beta \end{cases}$

$AX = (1,-3,0)$

$d(A, \pi) = \frac{|(1,-3,0) \cdot (0,1,-1)|}{\sqrt{2}} = \frac{|(0 - 3 + 0)|}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

c) $r: x = y - 1 = z + 3$

$\pi: 2x + y - 3z - 10 = 0$

$r: \begin{cases} x = y - 1 \\ y - 1 = z + 3 \end{cases} \rightarrow \begin{cases} x = \alpha - 1 \\ y = \alpha \\ z = \alpha - 4 \end{cases}$ $\pi: \begin{cases} y = -2x + 3z + 10 \\ x = \alpha \\ z = \beta \end{cases} \rightarrow \begin{cases} x = \alpha \\ y = -2\alpha + 3\beta + 10 \\ z = \beta \end{cases}$

$\vec{n} = (2,1,-3)$ $X = (-1,0,-4)$

$\vec{u} = (1,1,1)$ $A = (0,10,0)$

$\vec{u} \cdot \vec{n} = (1,1,1) \cdot (2,1,-3) = (2 + 1 - 3) = 0$

$AX = (-1,-10,-4)$

$d(r, \pi) = \frac{|(-1,-10,-4) \cdot (2,1,-3)|}{\sqrt{14}} = \frac{|(-2 - 10 + 12)|}{\sqrt{14}} = 0$

14.

a) $\tilde{\pi}_1: 2x - y + 2z + 0 = 0 \quad \tilde{\pi}_2: 4x - 2y + 4z - 21 = 0$

$\vec{n}_1 = (2, -1, 2) \quad \vec{n}_2 = (4, -2, 4)$

$\{\vec{n}_1, \vec{n}_2\}$ são L.D

$$\tilde{\pi}_1: \begin{cases} x = \alpha \\ y = 2x + 2z \\ z = \beta \end{cases} \rightarrow \begin{cases} x = \alpha \\ y = 2\alpha + 2\beta \\ z = \beta \end{cases}$$

$\{\tilde{\pi}_1, \tilde{\pi}_2\}$ são paralelos distintos

$P_1 = (0, 0, 0)$

$\tilde{\pi}_2: 4 \cdot 0 - 2 \cdot 0 + 4 \cdot 0 - 21 = 0$

$= -21 = 0$ (inconsistente)

$$\tilde{\pi}_2: \begin{cases} x = \alpha \\ y = \dfrac{-21 + 4\alpha + 4\beta}{2} \\ z = \beta \end{cases}$$

$A = (0, -\frac{21}{2}, 0)$

$\vec{AP} = (0, \frac{21}{2}, 0)$

$\vec{n} = (4, -2, 4)$

$$d = \frac{|(0, \frac{21}{2}, 0) \cdot (4, -2, 4)|}{\sqrt{36}}$$

$$= \frac{|-21|}{6} = \frac{21}{6} = \frac{7}{2}$$

b) $\tilde{\pi}_1: 2x + 2y + 2z = 5 \quad \tilde{\pi}_2: x = (2, 1, 2) + \lambda(-1, 0, 3) + \mu(1, 1, 0)$

$\vec{n}_1 = (2, 2, 2)$

$$\vec{n}_2 = \vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ -1 & 0 & 3 \\ 1 & 1 & 0 \end{vmatrix}$$

$= 0 + 3j - k - (0 + 3i + 0)$

$= -3i + 3j - k \; (-3, 3, -1)$

$\{\vec{n}_1, \vec{n}_2\}$ são L.I, logo não são paralelos então:

$d(\tilde{\pi}_1, \tilde{\pi}_2) = 0$

c) $\tilde{\pi}_1: x + y + z = 0 \quad \tilde{\pi}_2: 2x + y + z + 2 = 0$

$\vec{n}_1 = (1, 1, 1)$

$\vec{n}_2 = (2, 1, 1)$

$d(\tilde{\pi}_1, \tilde{\pi}_2) = 0$

$\{\vec{n}_1, \vec{n}_2\}$ não são paralelos logo, sua distância é 0.

15.

$r: x + z = 5 = y + 4$

$s: x = (4, 1, 1) + \lambda(4, 2, -3)$

$$\begin{cases} x + z = 5 \\ y + 4 = 5 \end{cases} \rightarrow \begin{cases} x = 5 - z \\ y = 1 \\ z = \alpha \end{cases} \rightarrow \begin{cases} x = 5 - \alpha \\ y = 1 \\ z = \alpha \end{cases}$$

$r: x = (5, 1, 0) + \alpha(-1, 0, 1)$

$s: x = (4, 1, 1) + \lambda(4, 2, -3)$

$$\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ -1 & 0 & 1 \\ 4 & 2 & -3 \end{vmatrix} = 0 + 4j - 2k - (0 + 2i + 3j)$$

$= j - 2i - 2k \rightarrow (-2, 1, -2)$

$$d = \frac{|d_2 - d_1|}{\|\vec{n}_1\|} = 2$$

$$\frac{|d_2 + 9|}{3} = 2$$

$\tilde{\pi}: 2x - y + 2z - 9 = 0 \quad d_1 = -9$

$2(x - 4) - 1 \cdot (y - 1) + 2 \cdot (z - 1) = 2x - 8 - y + 1 + 2z - 2$

$= 2x - y + 2z - 9 = 0$

$|d_2 + 9| = 6$

$d_2 + 9 = 6$

$d_2 = -3$

$d_2 + 9 = -6$

$d_2 = -15$

$\tilde{\pi}_1: 2x - y + 2z - 3 = 0$

$\tilde{\pi}_2: 2x - y + 2z - 15 = 0$