sistema possível e determinado (SPD):

→ Redundância: $x+y=2$

LISTA 3 - GA

1. $AX=B$

$X=B\cdot A^{-1}$

$$A:\begin{bmatrix}2\cdot1-1 & 1+2 & 1+3\\ 1-2 & 2\cdot2-2 & 2+3\\ 1-3 & 2-3 & 2\cdot3-3\end{bmatrix}=\begin{bmatrix}1&3&4\\1&2&5\\2&1&3\end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc}1&3&4&1&0&0\\1&2&5&0&1&0\\2&1&3&0&0&1\end{array}\right] \ell_2\leftarrow\ell_2-\ell_1 \rightarrow \left[\begin{array}{ccc|ccc}1&3&4&1&0&0\\0&-1&1&-1&1&0\\2&1&3&0&0&1\end{array}\right] \ell_3\leftarrow\ell_3-2\ell_1\rightarrow \left[\begin{array}{ccc|ccc}1&3&4&1&0&0\\0&-1&1&-1&1&0\\0&-5&-5&-2&0&1\end{array}\right] \ell_2\leftarrow\ell_2\cdot(-1)\rightarrow \left[\begin{array}{ccc|ccc}1&3&4&1&0&0\\0&1&-1&1&-1&0\\0&-5&-5&-2&0&1\end{array}\right]$$

$$\ell_1\leftarrow\ell_1-3\ell_2\rightarrow\left[\begin{array}{ccc|ccc}1&0&7&-2&3&0\\0&1&-1&1&-1&0\\0&-5&-5&-2&0&1\end{array}\right] \ell_3\leftarrow\ell_3+5\ell_2\rightarrow\left[\begin{array}{ccc|ccc}1&0&7&-2&3&0\\0&1&-1&1&-1&0\\0&0&-10&3&-5&1\end{array}\right] \ell_3\leftarrow\ell_3/_{-10}\rightarrow\left[\begin{array}{ccc|ccc}1&0&7&-2&3&0\\0&1&-1&1&-1&0\\0&0&1&-3/10&1/2&-1/10\end{array}\right] \ell_2\leftarrow\ell_2+\ell_3\rightarrow$$

$$\left[\begin{array}{ccc|ccc}1&0&7&-2&3&0\\0&1&0&7/10&-1/2&-1/10\\0&0&1&-3/10&1/2&-1/10\end{array}\right] \ell_1\leftarrow\ell_1-7\ell_3\rightarrow\left[\begin{array}{ccc|ccc}1&0&0&1/10&-1/2&7/10\\0&1&0&7/10&-1/2&-1/10\\0&0&1&-3/10&1/2&-1/10\end{array}\right]$$

$$A^{-1}=\begin{bmatrix}1/10&-1/2&7/10\\7/10&-1/2&-1/10\\-3/10&1/2&-1/10\end{bmatrix}$$

$A^{-1}\cdot B$

$$\begin{bmatrix}1\\2\\-3\end{bmatrix}$$

$$\begin{bmatrix}0,1&-0,5&0,7\\0,7&-0,5&-0,1\\-0,3&0,5&-0,1\end{bmatrix}\quad\begin{matrix}x\\y\\z\end{matrix}$$

$\rightarrow$ $0,1-1-2,1=x$

$0,7-1+0,3=y$

$-0,3+1+0,3=z$

$$\begin{cases}x=-3\\y=0\\z=1\end{cases}$$

2. a) $A=\begin{bmatrix}1&4\\2&3\end{bmatrix}$; $B=\begin{bmatrix}2\\-1\end{bmatrix}$ $\quad AX=B\rightarrow X=B\cdot A^{-1}$

$$A^{-1}=\begin{bmatrix}3&-4\\-2&1\end{bmatrix}\cdot\frac{1}{1\cdot3-2\cdot4}\rightarrow\begin{bmatrix}3&-4\\-2&1\end{bmatrix}\cdot-\frac{1}{5}\rightarrow\begin{bmatrix}-\frac{3}{5}&\frac{4}{5}\\\frac{2}{5}&-\frac{1}{5}\end{bmatrix}$$

$A^{-1}\cdot B$ $\quad\begin{bmatrix}2\\-1\end{bmatrix}$

$$\begin{bmatrix}-\frac{3}{5}&\frac{4}{5}\\\frac{2}{5}&-\frac{1}{5}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$-\frac{6}{5}-\frac{4}{5}=x \rightarrow x=-\frac{10}{5}=-2$

$y=\frac{4}{5}+\frac{1}{5}=\frac{5}{5}=1$

$$X=\begin{bmatrix}-2\\1\end{bmatrix}$$

b) $A=\begin{bmatrix}2&3\\5&5\end{bmatrix}$; $B=\begin{bmatrix}1&2\\3&5\end{bmatrix}$; $C=\begin{bmatrix}1&3\\2&7\end{bmatrix}$

$A+BY=C$

$BY=C-A$

$Y=(C-A)B^{-1}$

$$B^{-1}=\begin{bmatrix}5&-2\\-3&1\end{bmatrix}\cdot\frac{1}{5\cdot1-(-3)\cdot(-2)}=\begin{bmatrix}5&-2\\-3&1\end{bmatrix}\cdot-1=\begin{bmatrix}-5&2\\3&-1\end{bmatrix}$$

$$Y=\left(\begin{bmatrix}1&3\\2&7\end{bmatrix}-\begin{bmatrix}2&3\\5&5\end{bmatrix}\right)\begin{bmatrix}-5&2\\3&-1\end{bmatrix}\rightarrow\begin{bmatrix}-1&0\\-3&2\end{bmatrix}\cdot\begin{bmatrix}-5&2\\3&-1\end{bmatrix}$$

Y $\quad\begin{bmatrix}-1&0\\-3&2\end{bmatrix}$

$$\begin{bmatrix}-5&2\\3&-1\end{bmatrix}\begin{bmatrix}Y_{11}&Y_{12}\\Y_{21}&Y_{22}\end{bmatrix}$$

$y_{11}=(-5)(-1)+2(-3)=-1$

$y_{12}=0+4=4$

$y_{21}=-3+3=0$

$y_{22}=0-2=-2$

$$Y=\begin{bmatrix}-1&4\\0&-2\end{bmatrix}$$

c. $A=\begin{bmatrix}1&0&0\\2&-1&0\\2&3&1\end{bmatrix}$; $B=\begin{bmatrix}5\\7\\2\end{bmatrix}$ $\quad A\cdot W=B$

$W=B\cdot A^{-1}$

$$\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\2&-1&0&0&1&0\\2&3&1&0&0&1\end{array}\right]\ell_2\leftarrow\ell_2-2\ell_1\rightarrow\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&-1&0&-2&1&0\\2&3&1&0&0&1\end{array}\right]\ell_3\leftarrow\ell_3-2\ell_1\rightarrow\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&-1&0&-2&1&0\\0&3&1&0&0&1\end{array}\right]\ell_3\leftarrow\ell_3+3\ell_2\rightarrow\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&-1&0&-2&1&0\\0&0&1&-6&3&1\end{array}\right]$$

$x=\frac{1}{\;}\ |\tilde{a}_{12}\ \tilde{a}_{22}\ \dots\ \tilde{a}_{n2}|\ |b_2|$

Sistema possível e determinado (SPD):

→ Retas ou planos concorrentes (uma única solução)

→ Redundância: $x+y=2$

Seja $x=\lambda$, $\lambda \in \mathbb{R}$ (um parâmetro)

$\lambda + y = 2 \leftrightarrow y = 2-\lambda$

b) $\begin{cases} x+y=2 \;\cdots\; ① \\ \ldots \end{cases} \quad ②^{(+\cdot 3)} \longleftrightarrow \begin{cases} x+y=2 \\ x+u=2 \end{cases}$

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$$\ell_2 \leftarrow \ell_2 \cdot (-1) \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & -6 & 3 & 1 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -6 & 3 & 1 \end{bmatrix}$$

$$\underset{w}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -6 & 3 & 1 \end{bmatrix}} \begin{bmatrix} 5 \\ 7 \\ 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x = 5$

$y = 10 - 7 = 3$

$z = -30 + 21 + 2 = -7$

$$\begin{cases} x = 5 \\ y = 3 \\ z = -7 \end{cases}$$

3. a) $AXB = C$

$X = C \cdot A^{-1} \cdot B^{-1}$

b) $A(B+X) = A$

$B + X = \underbrace{A \cdot A^{-1}}_{I_n}$

$X = I_n - B$

$X = -B$ //

c) $ACXB = C$

$CXB = C \cdot A^{-1}$

$CX = C \cdot A^{-1} \cdot B^{-1}$

$X = C \cdot A^{-1} \cdot B^{-1} \cdot C^{-1}$

d) $(AB)^{-1}(AX) = \underbrace{C \cdot C^{-1}}_{I_n}$

$(AB)^{-1}(AX) = I_n$

$A^{-1} \cdot B^{-1} \cdot A \cdot X = I_n$

$\underbrace{A^{-1} \cdot A}_{I_n} \cdot B^{-1} \cdot X = I_n$

$B^{-1} \cdot X = I_n$

$\underbrace{B \cdot B^{-1}}_{I_n} \cdot X = I_n \cdot B$

$X = B$ //

e) $AB^t X B^{-1} = A^t$

$\underbrace{(B^t)^{-1} \cdot B^t}_{I_n} \cdot \underbrace{A^{-1} \cdot A}_{I_n} \cdot X \cdot B^{-1} = A^t \cdot (B^t)^{-1} \cdot A^{-1}$

$X \cdot B^{-1} = A^t (B^t)^{-1} \cdot A^{-1}$

$\underbrace{B \cdot B^{-1}}_{I_n} \cdot X = A^t \cdot B \cdot (B^t)^{-1} \cdot A^{-1}$

$X = A^t B (B^t)^{-1} \cdot A^{-1}$ //

f) $2AX - X = 3B$

$(2A - I_n) X = 3B$

$\underbrace{(2A - I_n)^{-1} \cdot (2A - I_n)}_{I_n} X = 3B$

$X = 3B (2A - I_n)^{-1}$ //

b) $\{ x + y + z = 0$

c) $\{ x + y + 2z = 0$

$(AB^t)^{-1} \cdot AB^t X = A^t \cdot B (AB^t)^{-1}$ In

√ $X = A^t \cdot B (AB^t)^{-1}$

4. a) $\begin{cases} 3x - 4y = 1 \\ 2x + 6y = 18 \end{cases}$

$$\underbrace{\begin{bmatrix} 3 & -4 \\ 2 & 6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 \\ 18 \end{bmatrix}}_{B}$$

• $\det(A) = \begin{vmatrix} 3 & -4 \\ 2 & 6 \end{vmatrix} = 3 \cdot 6 - 2 \cdot (-4) = 26$

• $\det(A_x) = \begin{vmatrix} 1 & -4 \\ 18 & 6 \end{vmatrix} = 1 \cdot 6 - (-4) \cdot 18 = 6 + 72 = 78$

• $\det(A_y) = \begin{vmatrix} 3 & 1 \\ 2 & 18 \end{vmatrix} = 3 \cdot 18 - 1 \cdot 2 = 54 - 2 = 52$

$x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{78}{26} = 3$

$y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{52}{26} = 2$

$\therefore$ Logo $(x, y) = (3, 2)$

b) $\begin{cases} 5x + 8y = 34 \\ 10x + 16y = 50 \end{cases}$

$$\underbrace{\begin{bmatrix} 5 & 8 \\ 10 & 16 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 34 \\ 50 \end{bmatrix}}_{B}$$

• $\det(A) = \begin{vmatrix} 5 & 8 \\ 10 & 16 \end{vmatrix} = 5 \cdot 16 - 8 \cdot 10 = 80 - 80 = 0$

• $\det(A_x) = \begin{vmatrix} 34 & 8 \\ 50 & 16 \end{vmatrix} = 34 \cdot 16 - (8 \cdot 50) = 544 - 400 = 144$

• $\det(A_y) = \begin{vmatrix} 5 & 34 \\ 10 & 50 \end{vmatrix} = 5 \cdot 50 - (34 \cdot 10) = 250 - 340 = -90$

$\therefore$ Como os $\det(A_x)$ e $\det(A_y)$ tiveram um valor diferente de zero esse é um sistema sem solução

c) $\begin{cases} x + 2y = 5 \\ 2x - 3y = -4 \end{cases}$

$$\underbrace{\begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 5 \\ -4 \end{bmatrix}}_{B}$$

$x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{-7}{-7} = 1$

$y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{-14}{-7} = 2$

$\therefore$ Logo, $(x, y) = (1, 2)$

• $\det(A) = \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = 1 \cdot (-3) - 2 \cdot 2 = -3 - 4 = -7$

• $\det(A_x) = \begin{vmatrix} 5 & 2 \\ -4 & -3 \end{vmatrix} = 5 \cdot (-3) - (2 \cdot (-4)) = -15 + 8 = -7$

• $\det(A_y) = \begin{vmatrix} 1 & 5 \\ 2 & -4 \end{vmatrix} = 1 \cdot (-4) - 5 \cdot 2 = -4 - 10 = -14$

d) $\begin{bmatrix} 3 & 2 & -5 \\ 2 & -4 & -2 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ -4 \end{bmatrix}$

• $\det(D) = \left|\begin{array}{ccc|cc} 3 & 2 & -5 & 3 & 2 \\ 2 & -4 & -2 & 2 & -4 \\ 1 & -2 & -3 & 1 & -2 \end{array}\right| = -20 + 20 - 12 + 12 + 36 - 4 = 32$

$-20 \quad -12 \quad 12 \qquad 36 \quad -4 \quad 20$

• $\det(D_x) = \left|\begin{array}{ccc|cc} 8 & 2 & -5 & 8 & 2 \\ -4 & -4 & -2 & -4 & -4 \\ -4 & -2 & -3 & -4 & -2 \end{array}\right| = 80 + 96 + 16 - 40 - 32 - 24 = 192 - 96 = 96$

$80 \quad -32 \quad -24 \qquad 96 \quad 16 \quad -40$

$x = \dfrac{\det(D_x)}{\det(D)} = \dfrac{96}{32} = 3$

• $\det(D_y) = \left|\begin{array}{ccc|cc} 3 & 8 & -5 & 3 & 8 \\ 2 & -4 & -2 & 2 & -4 \\ 1 & -4 & -3 & 1 & -4 \end{array}\right| = -20 - 24 - 16 + 40 + 36 + 48 = 64$

$y = \dfrac{\det(D_y)}{\det(D)} = \dfrac{64}{32} = 2$

$-20 \quad -24 \quad 48 \qquad 36 \quad -16 \quad 40$

$z = \dfrac{\det(D_z)}{\det(D)} = \dfrac{32}{32} = 1$

• $\det(D_z) = \left|\begin{array}{ccc|cc} 3 & 2 & 8 & 3 & 2 \\ 2 & -4 & -4 & 2 & -4 \\ 1 & -2 & -4 & 1 & -2 \end{array}\right| = -24 - 8 - 32 + 32 + 16 + 48 = 32$

$\therefore$ logo, $(x, y, z) = (3, 2, 1)$

$32 \quad -24 \quad 16 \qquad 48 \quad -8 \quad -32$

e) $\begin{cases} x+2y-z=2 \\ 2x-y+3z=9 \\ 3x+3y-2z=3 \end{cases}$ $\quad \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \\ 3 \end{bmatrix}$

• $\det(E) = \left|\begin{array}{ccc|cc} 1 & 2 & -1 & 1 & 2 \\ 2 & -1 & 3 & 2 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{array}\right| = 2+18+8-6-9-3 = 10$

$-3 \quad -9 \quad 8 \qquad 2 \quad 18 \quad -6$

• $\det(E_x) = \left|\begin{array}{ccc|cc} 2 & 2 & -1 & 2 & 2 \\ 9 & -1 & 3 & 9 & -1 \\ 3 & 3 & -2 & 3 & 3 \end{array}\right| = 36+18+4-27-18-3 = 10$

$-3 \quad -18 \quad 36 \qquad 4 \quad 18 \quad -27$

• $\det(E_y) = \left|\begin{array}{ccc|cc} 1 & 2 & -1 & 1 & 2 \\ 2 & 9 & 3 & 2 & 9 \\ 3 & 3 & -2 & 3 & 3 \end{array}\right| = -18-9-6+27+8+18 = 20$

$27 \quad -9 \quad 8 \qquad -18 \quad 18 \quad -6$

• $\det(E_z) = \left|\begin{array}{ccc|cc} 1 & 2 & 2 & 1 & 2 \\ 2 & -1 & 9 & 2 & -1 \\ 3 & 3 & 3 & 3 & 3 \end{array}\right| = -27-12-3+12+54+6 = 30$

$6 \quad -27 \quad -12 \qquad -3 \quad 54 \quad 12$

$x = \dfrac{\det(E_x)}{\det(E)} = \dfrac{10}{10} = 1 \qquad y = \dfrac{\det(E_y)}{\det(E)} = \dfrac{20}{10} = 2 \qquad z = \dfrac{\det(E_z)}{\det(E)} = \dfrac{30}{10} = 3$

•∴ Logo, $(x, y, z) = (1, 2, 3)$

f) $\begin{cases} x+3z+6=-2 \\ 2x-4y=-4 \\ 3x-2y-5z=26 \end{cases}$ $\quad \begin{bmatrix} 1 & 0 & 3 \\ 2 & -4 & 0 \\ 3 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ -4 \\ 26 \end{bmatrix}$

• $\det(F) = \left|\begin{array}{ccc|cc} 1 & 0 & 3 & 1 & 0 \\ 2 & -4 & 0 & 2 & -4 \\ 3 & -2 & -5 & 3 & -2 \end{array}\right| = -12+20+36 = 44$

$36 \quad 0 \quad 0 \qquad 20 \quad 0 \quad -12$

• $\det(F_x) = \left|\begin{array}{ccc|cc} -8 & 0 & 3 & -8 & 0 \\ -4 & -4 & 0 & -4 & -4 \\ 26 & -2 & -5 & 26 & -2 \end{array}\right| = 312+24-160 = 176$

$312 \quad 0 \quad 0 \qquad -160 \quad 0 \quad 24$

• $\det(F_y) = \left|\begin{array}{ccc|cc} 1 & -8 & 3 & 1 & -8 \\ 2 & -4 & 0 & 2 & -4 \\ 3 & 26 & -5 & 3 & 26 \end{array}\right| = 156+20+36-80 = 132$

$36 \quad 0 \quad -80 \qquad 20 \quad 0 \quad 156$

• $\det(F_z) = \left|\begin{array}{ccc|cc} 1 & 0 & -8 & 1 & 0 \\ 2 & -4 & -4 & 2 & -4 \\ 3 & -2 & 26 & 3 & -2 \end{array}\right| = -96-8-104+32 = -176$

$-96 \quad -8 \quad 0 \qquad -104 \quad 0 \quad 32$

$x = \dfrac{\det(F_x)}{\det(F)} = \dfrac{176}{44} = 4 \qquad y = \dfrac{\det(F_y)}{\det(F)} = \dfrac{132}{44} = 3 \qquad z = \dfrac{\det(F_z)}{\det(F)} = \dfrac{-176}{44} = -4$

•∴ Logo, $(x, y, z) = (4, 3, -4)$

g) $\begin{cases} x + 2y + 3z = 10 \\ 3x + 4y + 6z = 23 \\ 3x + 2y + 3z = 10 \end{cases}$ $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \\ 3 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 23 \\ 10 \end{bmatrix}$

$\det(G) = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \\ 3 & 2 & 3 \end{vmatrix}\begin{matrix} 1 & 2 \\ 3 & 4 \\ 3 & 2 \end{matrix} = 0$

$12 \quad 36 \quad 18$

$-36 \quad -12 \quad -18$

• $\det(G_x) = \begin{vmatrix} 10 & 2 & 3 \\ 23 & 4 & 6 \\ 10 & 2 & 3 \end{vmatrix}\begin{matrix} 10 & 2 \\ 23 & 4 \\ 10 & 2 \end{matrix} = 0$

$120 \quad 120 \quad 138$

$-120 \quad -120 \quad -138$

• $\det(G_y) = \begin{vmatrix} 1 & 10 & 3 \\ 3 & 23 & 6 \\ 3 & 10 & 3 \end{vmatrix}\begin{matrix} 1 & 10 \\ 3 & 23 \\ 3 & 10 \end{matrix} = -18$

$69 \quad 180 \quad 90$

$-207 \quad -60 \quad -90$

$\det(G_z) = \begin{vmatrix} 1 & 2 & 10 \\ 3 & 4 & 23 \\ 3 & 2 & 10 \end{vmatrix}\begin{matrix} 1 & 2 \\ 3 & 4 \\ 3 & 2 \end{matrix} = 60 + 138 + 40 - 120 - 46 - 60 = 12$

$40 \quad 138 \quad 60$

$-120 \quad -46 \quad -60$

∴ Logo, esse sistema é sem solução.

$x + y + 3 \cdot 0 = 0$ ∴ Assim esse sistema é SPI

I_n

$X = A^t B (B^t)^{-1} \cdot A^{-1}$

5. a) $\begin{cases} 3x_1 - 4x_2 = 0 \\ -6x_1 + 8x_2 = 0 \end{cases}$

$\begin{bmatrix} 3 & -4 \\ -6 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

∴ Há infinitas soluções, logo é, (SPI)

b) $\begin{cases} x + y + z = 0 \\ 2x + 2y + 4z = 0 \\ x + y + 3z = 0 \end{cases}$

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

∴ Há infinitas soluções, então é (SPI)

c) $\begin{cases} x + y + 2z = 0 \\ x - y - 3z = 0 \\ x + 4y = 0 \end{cases}$

$\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & -3 \\ 1 & 4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

∴ Há infinitas soluções, logo (SPI)

6. a) $\begin{cases} 3x + my = 2 \\ x - y = 1 \end{cases}$ $\quad \underbrace{\begin{bmatrix} 3 & m \\ 1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{B}$

• $\det(A) = \begin{vmatrix} 3 & m \\ 1 & -1 \end{vmatrix} = 3\cdot(-1) - (m\cdot 1) = -3 - m$

$-3 - m \neq 0 \quad m \neq 3$

b) $\begin{cases} 3x + 2(m-1)y = 1 \\ mx - 4y = 0 \end{cases}$ $\quad \underbrace{\begin{bmatrix} 3 & 2(m-1) \\ m & -4 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{C}$

$\det(B) = 3\cdot(-4) - m(2(m-1)) \neq 0$

$= -12 - m(2m - 2) = -12 - 2m^2 + 2m \neq 0$

$-2m^2 + 2m - 12 \neq 0 \qquad m^2 - m + 6 \neq 0$

$\Delta = b^2 - 4ac = (-1)^2 - 4\cdot 1\cdot 6 = 1 - 24 = -23$

$\Delta = -23$

∴ Como o $\Delta = -23$ para todo $\det(B) \in \mathbb{R}$ há soluções possíveis e determinados.

c) $\begin{cases} x - y = 2 \\ x + my = -z \\ -x + y - z = 4 \end{cases}$ $\quad \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 1 & m & 1 \\ -1 & 1 & -1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}}_{B}$

$\det(C) = \left|\begin{matrix} 1 & -1 & 0 \\ 1 & m & 1 \\ -1 & 1 & -1 \end{matrix}\right| \begin{matrix} 1 & -1 \\ 1 & m \\ -1 & 1 \end{matrix} = -1 - 1 - m + 1 = -1 - m$

$0 \quad -1 \quad -1 \qquad -m \quad 1 \quad 0$

$-1 - m \neq 0$

$m \neq -1$

d) $\begin{cases} mx + y - z = 4 \\ x + my + z = 0 \\ x - y = 2 \end{cases}$ $\quad \underbrace{\begin{bmatrix} m & 1 & -1 \\ 1 & m & 1 \\ 1 & -1 & 0 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}}_{B}$

$\det(D) = \left|\begin{matrix} m & 1 & -1 \\ 1 & m & 1 \\ 1 & -1 & 0 \end{matrix}\right| \begin{matrix} m & 1 \\ 1 & m \\ 1 & -1 \end{matrix} = 2m + 2 \neq 0$

$m \quad m \quad 0 \qquad 0 \quad 1 \quad 1$

$m + 1 \neq 0$

$m \neq -1$

7.) x → peça s/ defeito   y → peças c/ defeito

$$\begin{cases} 6x - 2y = 750 \quad (1) \\ x + y = 225 \quad (2) \end{cases}$$

(1) ÷2 substitui com (2)

$3x - y = 375$

$x = 225 - y$

$3(225 - y) - y = 375$

$675 - 3y - y = 375$

$-4y = -300$

$y = 75 \qquad x = 225 - 75 = 150$

8.) $$\begin{cases} 60m + 20c = 30000 \\ m + c = 540 \end{cases} \xrightarrow{\div 20} \begin{cases} 3m + c = 1500 \\ m + c = 540 \end{cases}$$

$m = 540 - c$

$3(540 - c) + c = 1500$

$1620 - 3c + c = 1.500$

$-2c = -12 \qquad c = 60$

$m = 480$

9) x = 2 reais y = 10 reais z = 5 reais

$$\begin{cases} x = y \quad (1) \\ 2x + 10y + 5z = 500 \quad (2) \\ x + y + z = 92 \quad (3) \end{cases}$$

(1) → (3)

$y + y + z = 92$

$2y + z = 92$

(1) → (2)

$2y + 10y + 5z = 500$

$12y + 5z = 500$

$-6 \cdot (2y + z = 92) = -12y - 6z = -552$

$$\begin{cases} 12y + 5z = 500 \\ -12y - 6z = -552 \end{cases} \rightarrow 0y - z = -52$$

$z = 52$

$2y + z = 92$

$2y + 52 = 92$

$2y = 40$

$y = 20 \qquad x = 20$

10)

$$\begin{cases} k + A = 109 \quad (1) \\ k + T = 142 \quad (2) \\ T + A = 97 \quad (3) \end{cases}$$

$A = 109 - k$ $\quad (1) \rightarrow (3)$

$T + (109 - k) = 97$

$T - k = 97 - 109$

$T - k = -12$

(2)

$$\begin{cases} T - k = -12 \\ T + k = 142 \end{cases}$$

$2T = 130$

$T = 65 kg$ $\quad k = 77 kg$

$k + A = 109$ $\quad -77 + 109 = A$

$A = 32 kg$