LISTA 2 (GA) - DETERMINANTES E MATRIZ

Lista 1 - Matrizes

1.

a) $A + 2B =$

$$\begin{pmatrix}1&0\\2&7\end{pmatrix} + 2\begin{pmatrix}0&5\\3&-2\end{pmatrix} =$$

$$= \begin{pmatrix}1&0\\2&7\end{pmatrix} + \begin{pmatrix}0&10\\6&-4\end{pmatrix} =$$

$$= \begin{pmatrix}1&10\\8&3\end{pmatrix} //$$

b) $AB - BA$

$$\begin{array}{c|c} AB & \begin{pmatrix}0&5\\3&-2\end{pmatrix} \\ \hline \begin{pmatrix}1&0\\2&7\end{pmatrix} & \begin{pmatrix}0&5\\21&-4\end{pmatrix} \end{array} \qquad \begin{array}{c|c} BA & \begin{pmatrix}1&0\\2&7\end{pmatrix} \\ \hline \begin{pmatrix}0&5\\3&-2\end{pmatrix} & \begin{pmatrix}10&35\\-1&-14\end{pmatrix} \end{array}$$

$$AB = \begin{pmatrix}0&5\\21&-4\end{pmatrix} \qquad BA = \begin{pmatrix}10&35\\-1&-14\end{pmatrix}$$

$$AB - BA = \begin{pmatrix}0&5\\21&-4\end{pmatrix} - \begin{pmatrix}10&35\\-1&-14\end{pmatrix} = \begin{pmatrix}-10&-30\\22&10\end{pmatrix} //$$

c) $2C - D =$

$$2\begin{pmatrix}-2&3&-7\\7&-3&-2\end{pmatrix} - \begin{pmatrix}-3&2&0\\1&1&4\\-2&0&2\end{pmatrix}$$

Não é possível subtrair essas matrizes pois elas são de diferentes ordens.

d) $2D^t - 3E^t =$

$$2\begin{pmatrix}-3&1&-2\\2&1&0\\0&4&2\end{pmatrix} - 3\begin{pmatrix}2&-1&-6\\4&0&0\\-3&-4&-1\end{pmatrix} =$$

$$= \begin{pmatrix}-6&2&-4\\4&2&0\\0&8&4\end{pmatrix} - \begin{pmatrix}6&-3&-18\\12&0&0\\-9&-12&-3\end{pmatrix} =$$

$$= \begin{pmatrix}-12&5&14\\-8&2&0\\9&20&7\end{pmatrix} //$$

e) $D^2 + DE =$

$$\begin{array}{c|c} D^2 & \begin{pmatrix}-3&2&0\\1&1&4\\-2&0&2\end{pmatrix} \\ \hline \begin{pmatrix}-3&2&0\\1&1&4\\-2&0&2\end{pmatrix} & \begin{matrix}11&-4&8\\-10&3&12\\2&-4&4\end{matrix} \end{array} \qquad \begin{array}{c|c} DE & \begin{pmatrix}2&4&-3\\-1&0&-4\\-6&0&-1\end{pmatrix} \\ \hline \begin{pmatrix}-3&2&0\\1&1&4\\-2&0&2\end{pmatrix} & \begin{matrix}-8&-12&1\\-23&4&-11\\-16&-8&4\end{matrix} \end{array}$$

$$= \begin{pmatrix}11&-4&8\\-10&3&12\\2&-4&4\end{pmatrix} + \begin{pmatrix}-8&-12&1\\-23&4&-11\\-16&-8&4\end{pmatrix} = \begin{pmatrix}3&-16&9\\-23&7&1\\-14&-12&8\end{pmatrix} //$$

f) $C^tA =$

$$\begin{array}{c|c} C^tA & \begin{pmatrix}1&0\\2&7\end{pmatrix} \\ \hline \begin{pmatrix}-2&7\\3&-3\\-7&-2\end{pmatrix} & \begin{matrix}12&49\\-3&-21\\-11&-14\end{matrix} \end{array}$$

$$= \begin{pmatrix}12&49\\-3&-21\\-11&-14\end{pmatrix} //$$

g) $E - AC$

$$\begin{array}{c|c} AC & \begin{pmatrix}-2&3&-7\\7&-3&-2\end{pmatrix} \\ \hline \begin{pmatrix}1&0\\2&7\end{pmatrix} & \begin{pmatrix}-2&3&-7\\45&-15&-28\end{pmatrix} \end{array}$$

$$= \begin{pmatrix}2&4&-3\\-1&0&-4\\-6&0&-1\end{pmatrix} - \begin{pmatrix}-2&3&-7\\45&-15&-28\end{pmatrix}$$

Não é possível realizar essa subtração pois as matrizes são de ordens diferentes.

h) $F^tE =$

$$\begin{array}{c|c} F^tE & \begin{pmatrix}2&4&-3\\-1&0&-4\\-6&0&-1\end{pmatrix} \\ \hline \begin{pmatrix}1&-2&0\end{pmatrix} & \begin{pmatrix}4&4&5\end{pmatrix} \end{array}$$

$$= \begin{pmatrix}4&4&5\end{pmatrix} //$$

i) $BCF =$

$$\begin{array}{c|c} CF & \begin{pmatrix}1\\-2\\0\end{pmatrix} \\ \hline \begin{pmatrix}-2&3&-7\\7&-3&-2\end{pmatrix} & \begin{pmatrix}-8\\13\end{pmatrix} \end{array} \qquad \begin{array}{c|c} BCF & \begin{pmatrix}-8\\13\end{pmatrix} \\ \hline \begin{pmatrix}0&5\\3&-2\end{pmatrix} & \begin{matrix}65\\-50\end{matrix} \end{array}$$

$$BCF = \begin{pmatrix}65\\-50\end{pmatrix} //$$

2.

a)

$$\begin{array}{c|c} A_{2\times3}\,B_{3\times4} & \begin{pmatrix}b_{11}&b_{12}&b_{13}&b_{14}\\b_{21}&b_{22}&b_{23}&b_{24}\\b_{31}&b_{32}&b_{33}&b_{34}\end{pmatrix} \\ \hline \begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\end{pmatrix} & \begin{pmatrix}c_{11}&c_{12}&c_{13}&c_{14}\\c_{21}&c_{22}&c_{23}&c_{24}\end{pmatrix} \end{array}$$

$\therefore C_{2\times4}$, mas o produto BA não é possível.

b)

$$\begin{array}{c|c} A_{4\times1}\,B_{1\times2} & \begin{pmatrix}b_{11}&b_{12}\end{pmatrix} \\ \hline \begin{pmatrix}a_{11}\\a_{21}\\a_{31}\\a_{41}\end{pmatrix} & \begin{pmatrix}c_{11}&c_{12}\\c_{21}&c_{22}\\c_{31}&c_{32}\\c_{41}&c_{42}\end{pmatrix} \end{array}$$

$\therefore C_{4\times2}$, mas nesse caso o produto BA não é possível

c) O produto AB nesse caso não é possível devido a ordem das matrizes, porém BA é possível:

$$\begin{array}{c|c} B_{3\times1}\,A_{1\times2} & \begin{pmatrix}a_{11}&a_{12}\end{pmatrix} \\ \hline \begin{pmatrix}b_{11}\\b_{21}\\b_{31}\end{pmatrix} & \begin{matrix}d_{11}&d_{12}\\d_{21}&d_{22}\\d_{31}&d_{32}\end{matrix} \end{array} \qquad B_{3\times1} \cdot A_{1\times2} = D_{3\times2}$$

d) $C_{5\times3}$, mas BA não é possível

$$\begin{array}{c|c} A_{5\times2}\,B_{2\times3} & \begin{pmatrix}b_{11}&b_{12}&b_{13}\\b_{21}&b_{22}&b_{23}\end{pmatrix} \\ \hline \begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\\a_{31}&a_{32}\\a_{41}&a_{42}\\a_{51}&a_{52}\end{pmatrix} & \begin{matrix}c_{11}&c_{12}&c_{13}\\c_{21}&c_{22}&c_{23}\\c_{31}&c_{32}&c_{33}\\c_{41}&c_{42}&c_{43}\\c_{51}&c_{52}&c_{53}\end{matrix} \end{array}$$

e) Nesse caso tanto AB quanto BA não é possível realizar essa multiplicação.

f) $A_{4\times2}B_{2\times4}$

$$\begin{array}{c|c} & \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \end{pmatrix} \\ \hline \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix} & \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{pmatrix} \end{array}$$

$B_{2\times4}A_{4\times2}$

$$\begin{array}{c|c} & \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix} \\ \hline \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \end{pmatrix} & \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \end{array}$$

∴ Nesse caso, tanto AB quanto BA geram produtos sendo

$A_{4\times2}B_{2\times4} = C_{4\times4}$ e $B_{2\times4}A_{4\times2} = D_{2\times2}$.

g) $A_{2\times1}B_{1\times3}$

$$\begin{array}{c|c} & \begin{pmatrix} b_{11} & b_{12} & b_{13} \end{pmatrix} \\ \hline \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} & \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{pmatrix} \end{array}$$

∴ O produto BA não é possível

mas o $A_{2\times1} \cdot B_{1\times3} = C_{2\times3}$.

h) $A_{2\times2}B_{2\times2}$

$$\begin{array}{c|c} & \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \\ \hline \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} & \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \end{array}$$

$B_{2\times2}A_{2\times2}$

$$\begin{array}{c|c} & \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \\ \hline \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} & \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix} \end{array}$$

∴ É possível realizar a multiplicação em ambos os casos, sendo $A_{2\times2} \cdot B_{2\times2} = C_{2\times2}$ e $B_{2\times2} \cdot A_{2\times2} = D_{2\times2}$.

3.

a) $a_{ij} = 3i - 2j$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \end{pmatrix}$$

$a_{11} = 3\cdot1-2\cdot1 = 1$ $\quad a_{21} = 3\cdot2-2\cdot1 = 4$

$a_{12} = 3\cdot1-2\cdot2 = -1$ $\quad a_{22} = 3\cdot2-2\cdot2 = 2$

$a_{13} = 3\cdot1-2\cdot3 = -3$ $\quad a_{23} = 3\cdot2-2\cdot3 = 0$

b) $b_{ij} = \begin{cases} 3i+j, & i=j \\ i^2-j, & i\neq j \end{cases}$

$$B = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 4 & -1 & -2 \\ 3 & 8 & 1 \\ 8 & 7 & 12 \end{pmatrix}$$

$a_{11} = 3\cdot1+1 = 4$ $\quad a_{21} = 2^2-1 = 3$ $\quad a_{31} = 3^2-1 = 8$

$a_{12} = 1^2-2 = -1$ $\quad a_{22} = 3\cdot2+2 = 8$ $\quad a_{32} = 3^2-2 = 7$

$a_{13} = 1^2-3 = -2$ $\quad a_{23} = 2^2-3 = 1$ $\quad a_{33} = 3\cdot3+3 = 12$

c) $c_{ij} = j^i$

$C = (a_{11}\ a_{12}\ a_{13}\ a_{14}) = (1\ 2\ 3\ 4)$

$c_{11} = 1^1$ $\quad c_{13} = 3^1$

$c_{12} = 2^1$ $\quad c_{14} = 4^1$

d) $d_{ij} = \begin{cases} i^2+j^2, & i=j \\ 2ij, & i\neq j \end{cases}$

$$D = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} = \begin{pmatrix} 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 6 & 12 & 18 & 24 \\ 8 & 16 & 24 & 32 \end{pmatrix}$$

$a_{11} = 1^2+1^2 = 2$ $\quad a_{21} = 2\cdot2\cdot1 = 4$ $\quad a_{31} = 2\cdot3\cdot1 = 6$ $\quad a_{41} = 2\cdot4\cdot1 = 8$

$a_{12} = 2\cdot1\cdot2 = 4$ $\quad a_{22} = 2^2+2^2 = 8$ $\quad a_{32} = 2\cdot3\cdot2 = 12$ $\quad a_{42} = 2\cdot4\cdot2 = 16$

$a_{13} = 2\cdot1\cdot3 = 6$ $\quad a_{23} = 2\cdot2\cdot3 = 12$ $\quad a_{33} = 3^2+3^2 = 18$ $\quad a_{43} = 2\cdot4\cdot3 = 24$

$a_{14} = 2\cdot1\cdot4 = 8$ $\quad a_{24} = 2\cdot2\cdot4 = 16$ $\quad a_{34} = 2\cdot3\cdot4 = 24$ $\quad a_{44} = 4^2+4^2 = 32$

4. Sabendo que:

$$\begin{array}{c|c} AB & \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -7 \end{pmatrix} \\ \hline \begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix} & \begin{pmatrix} 2 & -3 & 4 \\ -14 & 1 & -22 \\ -6 & -9 & -16 \end{pmatrix} \end{array}$$

$$\begin{array}{c|c} BA & \begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix} \\ \hline \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -7 \end{pmatrix} & \begin{pmatrix} 4 & 14 & 16 \\ 8 & 23 & 20 \\ -8 & -31 & -40 \end{pmatrix} \end{array}$$

$$\begin{array}{c|c} B^2 & \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -7 \end{pmatrix} \\ \hline \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -7 \end{pmatrix} & \begin{pmatrix} -8 & -3 & -18 \\ -12 & -3 & -20 \\ 16 & 8 & 36 \end{pmatrix} \end{array}$$

$$B^t = \begin{pmatrix} 1 & 2 & -3 \\ 0 & -1 & -1 \\ 3 & 4 & -7 \end{pmatrix}$$

a) $[BA]_{23} = 20$

b) $[AB]_{23} = -32$

c) $[B^2]_{31} = 16$

d) $tr(A) = a_{11} + a_{22} + a_{33}$

$= 1 + (-3) + 5$

$= 3$

e) $tr(B^t) = [B^t]_{11} + [B^t]_{22} + [B^t]_{33}$

$= 1 + (-1) + (-7)$

$= -7$

f) $tr(A-B)$:

$$= \begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -7 \end{pmatrix} = \begin{pmatrix} 0 & 2 & -2 \\ -4 & -2 & -2 \\ 4 & 5 & 12 \end{pmatrix}$$

$= tr(A-B) = 0 + (-2) + 12 = 10$

g) $tr(AB) = [AB]_{11} + [AB]_{22} + [AB]_{33}$

$= 2 + 1 + (-16)$

$= -13$

5.

a) $2X + A = 3B + C$

$$2X = \begin{pmatrix} 6 & 3 \\ 12 & 9 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}$$

$$2X = \begin{pmatrix} 7 & -2 \\ 11 & 3 \end{pmatrix}$$

$$X = \begin{pmatrix} \frac{7}{2} & -1 \\ \frac{11}{2} & \frac{3}{2} \end{pmatrix}$$

b) $Y + A = \frac{1}{2}(B - C)^t$

$$Y = \frac{1}{2}\left(\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}\right) - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}$$

$$Y = \frac{1}{2}\begin{pmatrix} 2 & 3 \\ -1 & 3 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}$$

$$Y = \begin{pmatrix} 1 & \frac{3}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}$$

$$Y = \begin{pmatrix} 2 & -\frac{11}{2} \\ -\frac{5}{2} & -\frac{9}{2} \end{pmatrix}$$

c) $3X + A = B - X$

$4X = B - A$

$$4X = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}$$

$$4X = \begin{pmatrix} 3 & -6 \\ 2 & -3 \end{pmatrix}$$

$$X = \begin{pmatrix} \frac{3}{4} & -\frac{3}{2} \\ \frac{1}{2} & -\frac{3}{4} \end{pmatrix}$$

d) $\begin{cases} X + Y = 3A \quad \text{I} \\ X - Y = 2B + C \quad \text{II} \end{cases}$

I $\quad X + Y = \begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix} \rightarrow X = \begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix} - Y$

II $\quad X - Y = \begin{pmatrix} 4 & 2 \\ 8 & 6 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$

$$X - Y = \begin{pmatrix} 4 & 4 \\ 9 & 6 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix} - Y - Y = \begin{pmatrix} 4 & 4 \\ 9 & 6 \end{pmatrix}$$

$$-2Y = \begin{pmatrix} 4 & 4 \\ 9 & 6 \end{pmatrix} - \begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix}$$

$$-2Y = \begin{pmatrix} 7 & -17 \\ 3 & -12 \end{pmatrix}$$

$$Y = \begin{pmatrix} -\frac{7}{2} & \frac{17}{2} \\ -\frac{3}{2} & 6 \end{pmatrix}$$

$$X = \begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix} - \begin{pmatrix} -\frac{7}{2} & \frac{17}{2} \\ -\frac{3}{2} & 6 \end{pmatrix}$$

$$X = \begin{pmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{pmatrix}$$

6. $A^2 = 2A$

$$A^2 = \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} + \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{pmatrix}$$

∴ E quando se multiplica $A \cdot A$ o valor é

A^2	$\begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix}$
$\begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{pmatrix}$

Logo, tanto A^2 quanto $2A$ geram a mesma matriz: $\begin{pmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{pmatrix}$

A expressão para A^n seria:

$$A^n = \begin{pmatrix} a_1 \cdot q^{n-1} & \frac{a_1 \cdot q^{n-1}}{x} \\ (a_1 \cdot q^{n-1})x & a_1 \cdot q^{n-1} \end{pmatrix}$$

Sendo $a_1 = 1$ devido a matriz A e $q = 2$ por ser uma PG que cresce multiplicando por 2.

7. Sabendo que $X = AB$ e $Y = AC$

a) $A(B + C) =$

$= AB + AC$

$= X + Y$

b) $B^t A^t$

como $B^t A^t = (AB)^t$ então:

$B^t A^t = (AB)^t$

$= X^t$

c) $C^t A^t = (AC)^t$

$= Y^t$

d) $(ABA)C =$

$(AB)(AC) =$

$= XY$

8.

a)

$$A = \begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$$

$$A^t = \begin{pmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{pmatrix}$$

$x + 2 = 2x - 3$

$x = 5$

$$A = \begin{pmatrix} 4 & 7 \\ 7 & 6 \end{pmatrix}$$

b) $B^t = -B$

$$\begin{pmatrix} 0 & x & y \\ -4 & 0 & 2z \\ 2 & 1-z & 0 \end{pmatrix} = -\begin{pmatrix} 0 & -4 & 2 \\ x & 0 & 1-z \\ y & 2z & 0 \end{pmatrix}$$

$y = -2$

$x = 4$

$-1 + z = 2z$

$z = -1$

9.

$$3\begin{pmatrix}x & y\\ z & t\end{pmatrix}=\begin{pmatrix}x & 6\\ -1 & 2t\end{pmatrix}+\begin{pmatrix}4 & x+y\\ z+t & 3\end{pmatrix}$$

$3x = x+4$ $\qquad$ $3y = 6+x+y$ $\qquad$ $3t = 2t+3$ $\qquad$ $3z = -1+z+t$

$x = 2$ $\qquad$ $2y = 8$ $\qquad$ $t = 3$ $\qquad$ $2z = -1+3$

$y = 4$ $\qquad$ $z = 1$

10.

a)

$$\begin{array}{c|c} R(\theta)\cdot R(\theta)^t & \begin{matrix}\cos\theta & -\text{sen}\,\theta\\ \text{sen}\,\theta & \cos\theta\end{matrix} \\ \hline \begin{matrix}\cos\theta & \text{sen}\,\theta\\ -\text{sen}\,\theta & \cos\theta\end{matrix} & \begin{pmatrix}\cos\theta^2+\text{sen}\,\theta^2 & -\text{sen}\,\theta\cdot\cos\theta+\text{sen}\,\theta\cdot\cos\theta\\ -\text{sen}\,\theta\cdot\cos\theta+\text{sen}\,\theta\cdot\cos\theta & \text{sen}^2\theta+\cos^2\theta\end{pmatrix}\end{array}$$

Sabendo que $\text{sen}^2x+\cos^2x=1$ simplificamos a matriz formada que resulta na seguinte matriz:

$$\begin{pmatrix}\cos^2\theta+\text{sen}^2\theta & -\text{sen}\,\theta\cdot\cos\theta+\text{sen}\,\theta\cdot\cos\theta\\ -\text{sen}\,\theta\cdot\cos\theta+\text{sen}\,\theta\cdot\cos\theta & \text{sen}^2\theta+\cos^2\theta\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=I$$

Logo, a matriz $R(\theta)$ é ortogonal.

b)

$$\begin{array}{c|c} A\cdot A^t & \begin{pmatrix}1 & 0 & 0\\ 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2}\\ x & y & z\end{pmatrix} \\ \hline \begin{pmatrix}1 & 0 & x\\ 0 & \frac{1}{\sqrt2} & y\\ 0 & \frac{1}{\sqrt2} & z\end{pmatrix} & \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\end{array}$$

$$\begin{pmatrix}1+x^2=1 & x\cdot y=0 & x\cdot z=0\\ x\cdot y=0 & \frac{1}{2}+y^2=1 & \frac{1}{2}+zy=0\\ z\cdot x=0 & \frac{1}{2}+z\cdot y=0 & \frac{1}{2}+z^2=1\end{pmatrix}$$

$\therefore$ Logo, $x=0$, $y=\frac{1}{\sqrt2}$ e $z=\frac{1}{\sqrt2}$.

$x^2=0$ $\qquad$ $y^2=\frac{1}{2}$ $\qquad$ $z^2=\frac{1}{2}$

$x=0$ $\qquad$ $y=\frac{1}{\sqrt2}$ $\qquad$ $z=\frac{1}{\sqrt2}$

7.b) $A=\begin{pmatrix}3 & 1\\ 5 & 2\end{pmatrix}$ $\quad$ $B=\begin{pmatrix}4 & 7\\ 1 & 2\end{pmatrix}$

$$\begin{array}{c|c} AB & \begin{matrix}4 & 7\\ 1 & 2\end{matrix} \\ \hline \begin{matrix}3 & 1\\ 5 & 2\end{matrix} & \begin{matrix}13 & 23\\ 22 & 39\end{matrix}\end{array}$$

$AB=\begin{pmatrix}13 & 23\\ 22 & 39\end{pmatrix}$

$A^{-1}=\dfrac{1}{3\cdot2-1\cdot5}\begin{pmatrix}2 & -1\\ -5 & 3\end{pmatrix}$ $\qquad$ $B^{-1}=\dfrac{1}{4\cdot2-7\cdot1}\cdot\begin{pmatrix}2 & -7\\ -1 & 4\end{pmatrix}$ $\qquad$ $AB^{-1}=\dfrac{1}{13\cdot39-23\cdot22}\begin{pmatrix}39 & -23\\ -22 & 13\end{pmatrix}$

$A^{-1}=\dfrac{1}{1}\cdot\begin{pmatrix}2 & -1\\ -5 & 3\end{pmatrix}$ $\qquad$ $B^{-1}=\dfrac{1}{1}\cdot\begin{pmatrix}2 & -7\\ -1 & 4\end{pmatrix}$ $\qquad$ $AB^{-1}=\dfrac{1}{1}\begin{pmatrix}39 & -23\\ -22 & 13\end{pmatrix}$

$A^{-1}=\begin{pmatrix}2 & -1\\ -5 & 3\end{pmatrix}$ $\qquad$ $B^{-1}=\begin{pmatrix}2 & -7\\ -1 & 4\end{pmatrix}$ $\qquad$ $AB^{-1}=\begin{pmatrix}39 & -23\\ -22 & 13\end{pmatrix}$