

Course of Study Bachelor Computer Science	Exercises Statistics WS 2022/23
Sheet IX	

Two Samples Tests

- Two machines produce screw-nuts. The diameters of the screw-nuts from machine 1 resp. 2 are normally distributed with standard deviation $\sigma_1 = 0.5$ resp. $\sigma_2 = 0.6$ and unknown means μ_1 resp. μ_2 . Two sample are drawn:

M1: 5.46, 5.34, 4.34, 4.82, 4.4, 5.12, 5.69, 5.53, 4.77, 5.82

M2: 5.45, 5.31, 4.11, 4.69, 4.18, 5.05, 5.72, 5.54, 4.62, 5.89, 5.6, 5.19, 3.31, 4.43, 5.3, 4.09

Test the hypotheses $\mu_1 \geq \mu_2$ at level $\alpha = 0.05$

- To test two training methods A and B for javelin throw, 60 untrained physical education students were randomly divided into two groups of $m=25$ and $n=35$ students, respectively. Before the start of the training phase First, a performance test was conducted and for each student the distance of the best of two throws was noted. After completion of the training phase, during which the students in group 1 were method A and the students of group 2 were trained according to method B, was trained, a corresponding performance test was performed. The following results were obtained for the differences between the values obtained in the second and the first performance test:

- Group 1:
7.06, 11.84, 9.28, 7.92, 13.5, 3.98, 3.82, 7.34, 8.7, 9.24, 4.86, 3.32, 12.78, 12, 5.24, 11.4, 6.56, 9.04, 7.72, 9.26, 7.88, 8.6, 9.3, 8.42, 8.54
- Group 2:
8.68, 6, 6.3, 10.24, 10.88, 5.36, 7.82, 4.7, 9.02, 9.78, 6.9, 5.8, 13.56, 10.32, 13.3, 11.38, 7.94, 10.74, 13.68, 14.92, 7.42, 10.36, 10.54, 5.22, 13.74, 12.98, 10.34, 10.02, 17.8, 13.04, 5.2, 9.4, 11.18, 12.68, 12.36

Which hypothesis do you have to test if you want to show that the Method B is better than Method A? Perform an corresponding test at

the level $\alpha = 0.05$. Assume that the values obtained are a realization of independent in the group i $N(\mu_i, \sigma_i^2)$ -distributed random variables ($i=1,2$).

3. A (hypothetical) experiment is conducted on the effect of alcohol on perceptual motor ability. Ten subjects are each tested twice, once after having two drinks and once after having two glasses of water. The two tests were on two different days to give the alcohol a chance to wear off. Half of the subjects were given alcohol first and half were given water first. The scores of the 10 subjects are shown below. The first number for each subject is their performance in the "water" condition. Higher scores reflect better performance. Test to see if alcohol had a significant effect. Report the t and p values.

person	1, 2, 3, 4, 5, 6, 7, 8, 9, 10
water	16, 15, 11, 20, 19, 14, 13, 15, 14, 16
alcohol	13, 13, 10, 18, 17, 11, 10, 15, 11, 16

Assume that the values obtained are a realization of independent in the group i $N(\mu_i, \sigma_i^2)$ -distributed random variables ($i=1,2$).

4. A company manager is considering whether to purchase a new type B scale on the market. A new acquisition should be made only if the type B scale is better than the type A scale used up to now. For the evaluation of the quality of a scale, the scatter of weighing results should be used. In weight measurements for one and the same weight the following results were obtained the following measured values for the individual scales:

A	102.4, 101.3, 97.6, 98.2, 102.3, 99.1, 97.8, 103.9, 101.6, 100.1
B	98.4, 101.7, 100.5, 99.3, 100.6, 99.6, 102.2, 101.1, 99.9, 101.0

Under suitable normal distribution assumptions, check with a test with $\alpha = 0.05$ to determine whether a new acquisition makes sense.

5. On one farm, 10 cattle were fed (group 1) were fed concentrates of composition A, and the remaining 10 cattle (group 2) were fed the conventional conventional feed of composition B. After a certain time weight gain was noted in both groups:

Group 1	7.2, 4.1, 5.5, 4.5, 5.7, 3.8, 4.6, 6.0, 5.2, 5.4
Group 2	5.3, 4.4, 5.0, 3.5, 3.9, 4.9, 5.6, 2.5, 4.0, 3.6

- (a) Assuming that weight gain can be described by independent random variables that are identically normally distributed in both cases, use an appropriate test at level $\alpha = 0.1$ to test whether to reject the assumption that weight gain from administration of composition A concentrate has the same dispersion as weight gain from administration of the conventional composition B diet.
- (b) Assuming that weight gain can be described by independent random variables with equal variance, identically normally distributed in each of the two cases, use an appropriate test at the level $\alpha = 0.025$ to test whether the hypothesis that weight gain with administration of concentrate of composition A is not greater than weight gain with administration of the conventional diet of composition B is correct.

6. Heumann, Schoemaker Aufgabe 10.5

A company producing clothing often finds deficient T-shirts among its production.

- (a) The company's controlling department decides that the production is no longer profitable when there are more than 10% deficient T-shirts. A sample of 230 shirts yield 32 shirts which contain deficiencies. Use the approximate binomial test and the exact binomial test to decide whether the shirt production is profitable or not ($\alpha = 0.05$).
- (b) The company is offered a new cutting machine. To test whether the change of machine helps to improve the production quality, 115 sample T-shirts are evaluated, 7 of which have deficiencies. Use the 2 sample binomial test to decide whether the new machine yields improvement or not ($\alpha = 0.5$)

- 7. In 380 randomly selected families with four children each it is investigated how many of them are girls. The result is the following findings:

Number of girls	families
0	25
1	95
2	150
3	80
4	30

Does this finding correspond to the hypothesis that the variable “number of girls in families with four children each” follows a Binomial distribution with $n = 4$ and $p = 0.5$? Test this hypothesis at a significance level of 0.1!

8. The hypothesis is to be tested that the height of adult of adult German men is normally distributed (significance level 10%). For this purpose, a random sample is collected, which leads to the following findings:

class	frequency
150 to 155	20
155 to 160	30
160 to 165	55
165 to 170	60
170 to 175	85
175 to 180	80
180 to 185	50
185 to 190	40
190 to 195	30
195 to 200	15
200 to 205	10

What is the test decision?

9. Some parents of the West Bay little leaguers think that they are noticing a pattern. There seems to be a relationship between the number on the kids’ jerseys and their position. These parents decide to record what they see. The hypothetical data appear below. Conduct a Chi Square test to determine if the parents’ suspicion that there is a relationship between jersey number and position is right.

	Infield	Outfield	Pitcher	Total
0-9	12	5	5	22
10-19	5	10	2	17
20+	4	4	7	15
Total	21	19	14	54

10. Two therapies for a specific febrile illness are to be compared. For this purpose, 4 and 6 randomly selected randomly selected patients and the duration of treatment in hours required for the patient to be necessary for the patient to be free of fever.

Therapie 1	Therapie 2
X	Y
89,75	89
94,5	91
98,75	94
101,5	96,75
	99,5
	101,25

It is assumed that the given measured values are a realization of independent random variables $X_1, \dots, X_4, Y_1, \dots, Y_6$ and these random variables have the continuous distribution function F and G , respectively. Test the hypothesis $H_0 : F = G$ at the level $\alpha = 0.05$ by applying an appropriate nonparametric test.

11. **Heumann, Schoemaker Aufgabe 10.3** Christian decide to purchase the new CD Bruce Springsteen. His first thought is to buy it online, via an online auction. He discovers that he can also buy the CD, without bidding at an auction, from the same online store. He also looks at the price at an internet book store which was recommended to him by a friend. He notes down the following prices in Euro.

- Internet book store: 16.95
 - Online store, no auction:
18.19, 16.98, 19.97, 16.98, 18.19, 15.99, 13.79, 15.90, 15.90, 15.90, 15.90, 19.97, 17.72
 - Online store, auction:
10.50, 12.00, 9.54, 10.55, 11.99, 9.30, 10.59, 10.50, 10.01, 11.89, 11.03, 9.52, 15.49, 11.02
- (a) Calculate and interpret the arithmetic mean, variance, standard deviation and the coefficient of variation for the online, both for the auction and non-auction.
 - (b) Test the hypothesis that the mean price at the online store (no auction) is unequal 16.95 Euro ($\alpha = 0.05$).
 - (c) Calculate a confidence interval for the mean price at the online store (no auction) and interpret your findings in the light of the hypothesis in b).
 - (d) Test the hypothesis that the mean price at the online store (auction) is less than 16.95 Euro ($\alpha = 0.05$).

- (e) Test the hypothesis that the mean non-auction price is higher than the mean auction price. Assume (i) that the variances are equal in both samples and (ii) that the variances are unequal.
- (f) Test the hypothesis that the variance of the non auction-price is unequal to the variance of the auction price ($\alpha = 0.05$).
- (g) Use the Wilcoxon-Mann-Whitney U-test to compare the location of the auction and non-auction prices.

12. Heumann, Schoemaker Aufgabe 10.6

Two friends play a computer game and each of them repeats the same level 10 times. The score obtained are:

	1	2	3	4	5	6	7	8	9	10
Player 1	91	101	112	99	108	88	99	105	111	104
Player 2	261	47	40	29	64	6	87	47	98	351

- (a) Player 2 insists that he is a better player and suggests to compare their performance. Use an appropriate test ($\alpha = 0.05$) to test this hypothesis.
- (b) Player 1 insists that he is a better player. He propose to not focus on the mean and to use the Wilcoxon-Mann-Whitney U-test for comparison ($\alpha = 0.05$). What are the advantages and disadvantages of using this test compared with a)?

13. Heumann, Schoemaker Aufgabe 10.8

The passengers rescued from the titanic depending on the travel classes is given in the following table

	1. Class	2. Class	3. Class	Staff	Total
Rescued	202	125	180	211	718
Not rescued	135	160	541	674	1510

Check with an appropriate test whether the “rescue status” and the “travel class” are independent and whether the conditional probabilities of “rescue status” given “travel class” differ by “travel class”.