Course of Study Bachelor Computer Science	Exercises Statistics WS 2020/21

Sheet IX

Hypothesis Testing

1. A tire manufacturer claims that its tires will last no less than an average of 50,000 km before they need to be replaced. A consumer group wishes to challenge this claim.

   (a) Clearly define the parameter of interest in this problem.

   (b) State H_0 and H_1 in terms of this parameter.

   (c) In the context of the problem, state what it means to make a type I and type II error.

   (d) Suppose we set the significance level of the test at 10%, what does this number mean?

2. Discus the following statement:
   "When test results are significant at the 5-percent level, this means that there is at least a 95% chance of being correct if you reject the null hypothesis."

3. A sample of lightbulbs is studied, to test the hypothesis that the mean lifetime of the bulbs is 200 hours. The sample data has a significance level of 1%, i.e. the hypothesis is rejected with significance level of 1%.

   Is the following statement true or false: If the mean lifetime in the population is indeed 200 hours, then a second sample (of the same size, analyzed similarly) has only one chance in a hundred of yielding a sample mean as far from 200 as the first sample mean.

4. A vaccine that is currently used to immunize people against a certain infection has an 80% success rate. That is, 80% of individuals who receive this vaccine will develop immunity against the infection. A manufacturer of a new vaccine claims that its vaccine has a higher success rate.

   (a) Define the parameter of interest.

(b) Suppose in a clinical trial, 200 people received the new vaccine. Of these, 172 became immune to the infection. Based on this, can we say that the new vaccine is indeed more effective than the current one? What is the corresponding Null-Hypothesis? Test at a 5% significance level and state your conclusion in the context of the problem.

(c) In making the above conclusion, which type of error are you risking, type I or type II?

(d) What is the probability of a type II error if the true success rate is 82%? What should be the minimal sample size if the probabiility of the type II error should be less than 5%?

Find the answers by using a normal approximation resp. without a normal approximation.

5. A magician uses a coin. You believe that the coin is biased, but you are not sure if it will come up heads or tails more often. You watch the magician flip the coin and record what percentage of the time the coin comes up heads.

   (a) Is this a one-tailed or two-tailed test?

   (b) Assuming that the coin is fair, what is the probability that out of 30 flips, it would come up one side 23 or more times?

   (c) Can you reject the null hypothesis at the 0.05 level? What about at the 0.01 level?

6. A bag of potato chips of a certain brand has an advertised weight of 250 grams. Actually, the weight (in grams) is a random variable. Suppose that a sample of 81 bags has mean 248 and standard deviation 5. At the 0.05 significance level, conduct the following tests and calculate the p-values.

   (a) $H_0 : \mu \geq 250$ versus $H_1 : \mu < 250$

   (b) $H_0 : \sigma \geq 7$ versus $H_1 : \sigma < 7$

   Hint: Assume that the data is approximately normally distributed.

7. The length of a certain machined part is supposed to be 10 centimeters. In fact, due to imperfections in the manufacturing process, the actual length is a random variable. The standard deviation is due to inherent

factors in the process, which remain fairly stable over time. From historical data, the standard deviation is known with a high degree of accuracy to be 0.3. The mean, on the other hand, may be set by adjusting various parameters in the process and hence may change to an unknown value fairly frequently. We are interested in testing

$$H_0 : \mu = 10 \quad \text{versus} \quad H_1 : \mu \neq 10$$

(a) Suppose that a sample of 100 parts has mean 10.1. Perform the test at the 0.1 level of significance.

(b) Compute the p-value for the data.

(c) Compute the probability of a type II error β of the test at $\mu = 10.05$.

(d) Compute the approximate sample size needed for significance level 0.1 and beta = 0.2 when mu = 10.05.

(e) Plot the probability of a type II error depending on the value of μ for different values of the sample size n = 50, 100, 150, 200, 250.

(f) Show that H_0 will be not rejected if the sample mean is 10.01. Determine the smallest n that the p-value for a sample with sample mean = 10.01 is less than 0.001. In general by increasing the sample size every small sample mean will become "highly significant - p value < 0.001".

Hint: Assume that the data is approximately normally distributed.

8. A coin is tossed 500 times and results in 302 heads. At the 0.05 level, test to see if the coin is unfair.

9. The processing time L of a stochastic searching algorithm at a workstation could be assumed to be normally distributed with expectation μ and variance $\sigma^2 = 40 sec^2$. The average of 50 independent measurements of the processing time is $\bar{l} = 121.9$ sec.

   (a) Find a suitable test which guarentees that the expectation is assumed erroneously to be bigger than 120 sec is at most 5%.

   (b) Perform the above test.

   (c) What is the upper bound of $\bar{l}$ in the above test that the null hypothesis will be not rejected?

   (d) What should be the value of the significance level α in the above test that the null hypothesis will be not rejected for the given value of $\bar{l}$?

(e) Assume that the true expectation μ is bigger than 122 sec. Find the lowest value of the sample size n that in the above test the probability to do not reject the null hypothesis is less than 5%?

(f) Sketch the the OC-function β of the above for n=25,50, 100.