Course of Study Bachelor Computer Science	Exercises Statistics WS 2020/21
Sheet VI	

Probability Spaces and Basic Rules

1. Consider a random experiment of tossing two dice. Let A denote the event that the first die score is 1 and B the event that the sum of the scores is 7.
   (a) Give the sample space Ω and find $|\Omega|$.
   (b) Explicitly list the elements of the following events:
   $$A, B, A \cup B, A \cap B, A^c \cap B^c$$
2. Suppose that A and B are events in an experiment with $P(A) = 1/3$, $P(B) = 1/4$, $P(A \cap B) = 1/10$. Express each of the following events verbally and find its probability:
   $$A \setminus B, A \cup B, A^c \cup B^c, A^c \cap B^c, A \cup B^c$$
3. Suppose that A, B, and C are events in an experiment with $P(A) = 0.3$, $P(B) = 0.2$, $P(C) = 0.4$, $P(A \cap B) = 0.04$, $P(A \cap C) = 0.1$, $P(B \cap C) = 0.1$, $P(A \cap B \cap C) = 0.01$. Express each of the following events in set notation and find its probability:
   (a) At least one of the three events occurs.
   (b) None of the three events occurs.
   (c) Exactly one of the three events occurs.
   (d) Exactly two of the three events occur.
4. **Urn Models**
   A large number of discrete probability spaces can be traced back to so-called urn models. An urn contains n balls, which do not all have to be different. From these urns r balls are drawn with or without replacement. For the result of the drawing, the order or only the quantity of

the drawn balls can be of importance.
Here an urn with 10 balls is considered. 5 of them are red, 3 balls are blue and 2 balls are green. 3 balls are drawn. The following 4 cases should be distinguished:

I Drawing with replacement with respect to the order

II Drawing with replacement without observing the order

III Drawing without replacement with respect to the order

IV Drawing without replacement without observing the order

Solve the following tasks.

(a) Load the library gtools and inspect the commands combinations() and permutations(). Consider the bags $b_1 = \{a, b, c\}$ and $b_2 = \{a, a, b, c\}$ = and list all combinations and all permutations of order 2 if duplicated elements in the outpot are allowd or not allowed.

(b) Use the function sample() to determine the result of 10 random draws.

(c) Determine a suitable event space Ω and its size to describe the random experiment.
Note that depending on whether the order of the drawn balls is important or not, the result of a drawing is considered as a r-variation or as a r-combination of the n set of balls. With the help of the permutations() and combinations() functions of the R-package gtools, the corresponding r-variations or r-combinations can be determined.

(d) Determine the probabilities of all elementary events in Ω using a Laplace model, i.e. as a determination of the ratio of the number of favorable cases by the number of all cases. The probabilities are first determined by counting methods and then by using the R function permutations().
Hint: To determine the probabilities with R, assume that the n balls are numbered consecutively, i.e. they are distinguishable, and that the order is first observed in a drawing. Every r-variation of the numbers 1 to n is equally probable. Determine the set of all these drawings with permutations(). Then map each such drawing to the corresponding elementary event. By dividing the number of drawings belonging to an elementary event and the number of all drawings, you then obtain the corresponding probabilities.

Independence and Conditional Probabilities

1. Suppose that A and B are events in an experiment with $P(A) = 1/3$, $P(B) = 1/4$, $P(A \cap B) = 1/10$. Find each of the following:

$$P(A|B), P(B|A), P(A^c|B), P(B^c|A), P(A^c|B^c)$$

2. In a certain population, 30% of the persons smoke and 8% have a certain type of heart disease. Moreover, 12% of the persons who smoke have the disease.

   (a) What percentage of the population smoke and have the disease?

   (b) What percentage of the population with the disease also smoke?

   (c) Are smoking and the disease positively correlated, negatively correlated, or independent?

3. Suppose that a bag contains 12 coins: 5 are fair, 4 are biased with probability of heads 1/3 and 3 are two-headed. A coin is chosen at random from the bag and tossed.

   (a) Find the probability that the coin shows head.

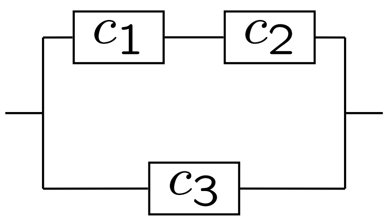
   (b) Given that the coin shows head, find the conditional probability of each coin type.

4. In a computer science course at an university we have the following data over a long time.
   10% of all students have attended the exercises in statistics regularly.
   2% of the students who have failed the statistics exam have attended the exercises regularly. 5% of the students who have attended the exercises regularly have failed the statistics exam.

   (a) Find the probability to fail the exam in statistics if the exercises in statistics are not attended regularly.

   (b) What is the effect of attending the exercises regularly to passing the exam?

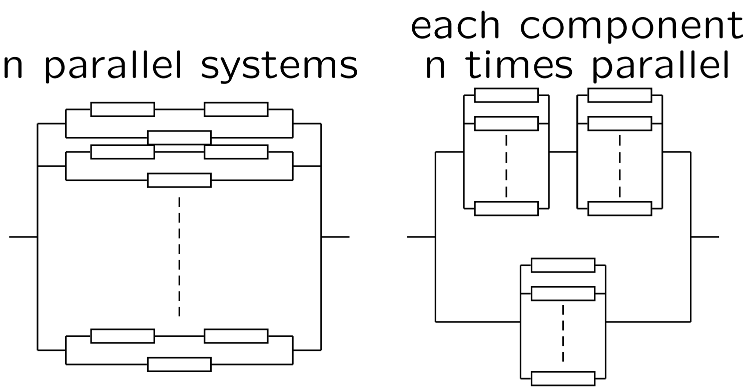
5. A representation of the reliability of a system with 3 components c_1, c_2, c_3 is given by.



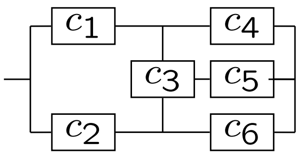
(a) Find the structure function of the system.

(b) Let α_i the failure probability of c_i, i=1,2,3. Find the failure probability and the reliability of the system.

(c) Let $\alpha_i = 0.9, i = 1, 2, 3$. Compare the reliability of the the following systems:



6. The reliability of a system with six components is described in the following diagram.



(a) Find the structure function of the system.

(b) Let p_i the probability that component c_i is functioning.

i	1	2	3	4	5	6
p_i	0.8	0.9	0.6	0.6	0.7	0.8

What is the reliability of the system?