Distortion Specifications

This article describes the numerous distortion specifications/ figures of merit an amplifier forces on the input signal. Ideally, an amplifier's transfer function is linear and memoryless.

$v_o = a_1 v_i$ Equation 1

However this does not describe the large behavior of the amplifier as the signal stimulates the large signal performance of the amplifier. The more realistic transfer function of an amplifier includes second and third order harmonics.This nonlinearity causes several different distortions based upon the input and its environment.

$v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3$ Equation 2

Harmonic Distortion (HD)

When one tone experiences a nonlinearity it will create other tones at the output that are multiples of the input's frequency. If the input is given by Equation 3 than that output is given by Equation 4.

$v_i = A\,cos(wt)$ Equation 3

$v_o = \frac{a_2}{2}A^2 + [a_1 A + \frac{3a_3}{4}A^3]cos(wt) + \frac{a_2}{2}A^2 cos(2wt) + \frac{a_3}{4}A^3 cos(3wt)$ Equation 4

The gain of the harmonic in reference to the linear gain is given by Equation 5 for 2nd order distortion and Equation 6 for 3rd order distortion.

$HD_2 = \frac{1a_2}{2a_1}A$ Equation 5

$HD_3 = \frac{1a_3}{4a_1}A^2$ Equation 6

Intermodulation (IM)

When two or more tones experience a nonlinearity they create other tones at the output that have frequencies equal to the sum and differences of the frequencies of the original inputs. If the input is given by Equation 7 than that output is given by Equation 8.

$v_i = A_1 cos(w_1 t) + A_2 cos(w_2 t)$ Equation 7

$v_o =$ Equation 8

Baseband $\frac{a_2}{2}A_1^2 + \frac{a_2}{2}A_2^2$

Linear Terms $+ [a_1 A_1 + \frac{3a_3}{4}A_1^3 + \frac{3a_3}{2}A_1 A_2^2]cos(w_1 t) + [a_1 A_2 + \frac{3a_3}{4}A_2^3 + \frac{3a_3}{2}A_1^2 A_2]cos(w_2 t)$

2nd Order Harmonics $+ \frac{a_2}{2}A_1^2 cos(2w_1 t) + \frac{a_2}{2}A_2^2 cos(2w_2 t)$

Square Intermodulation $+ a_2 A_1 A_2 cos([w_1 - w_2]t) + a_2 A_1 A_2 cos([w_1 + w_2]t)$

3rd Order Harmonics $+ \frac{a_3}{4}A_1^3 cos(3w_1 t) + \frac{a_3}{4}A_2^3 cos(3w_2 t)$

Cubic Intermodulation $+ \frac{3a_3}{4}A_1^2 A_2 cos([2w_1 - w_2]t) + \frac{3a_3}{4}A_1^2 A_2 cos([2w_1 + w_2]t)$

$+ \frac{3a_3}{4}A_1 A_2^2 cos([2w_2 - w_1]t) + \frac{3a_3}{4}A_1 A_2^2 cos([2w_2 + w_1]t)$

The gain of the third harmonic in reference to the linear gain is given by Equation 9 with the amplitudes of the two tones being equal.

$IM_3 = \frac{3a_3}{4a_1}A^2$ Equation 9

In-band Compression

The 3rd order harmonic distortion can cause a loss a loss in gain if a_3 is negative as shown in Equation 10. The 1dB gain compression point is the input level for which the small signal gain decreases by 1dB. This point is found in Equation 11 as the ratio between a_3 and a_1 .

$$v_o = [a_1 A + \frac{3a_3}{4} A^3] cos(wt) \qquad \text{Equation 10}$$

$$-1 = 20log(\frac{a_1 + \frac{3}{4}a_3 A^2}{a_1}) \rightarrow A_{-1dB} = \sqrt{0.11}\sqrt{\frac{4a_1}{3a_3}} \qquad \text{Equation 11}$$

Blocking

In the same way In-band Compression reduced the gain of the linear term, out of band signals can have a similar effect through 3rd order intermodulation given by Equation 12.

$$v_o(w_1) = [a_1 A_1 + \frac{3a_3}{4} {A_1}^3 + \frac{3a_3}{2} A_1 {A_2}^2] cos(w_1 t) \qquad \text{Equation 12}$$

The gain decrease due to the intermodulation ignoring the harmonic distortion of the second term is shown in Equation 13 for a compression of 3dB.

$$-3 = 20log(\frac{a_1 + \frac{3}{2}a_3 A^2}{a_1}) \rightarrow A_{-3dB} = \sqrt{0.3}\sqrt{\frac{2a_1}{3a_3}} \qquad \text{Equation 13}$$

3rd Order Intermodulation

If your desired signal is in an environment with evenly spaced interferers, than they can experience intermodulation to fall into your desired signal's bandwidth. Equation 14 shows how the sum and difference of carrier frequencies from 3rd order intermodulation results in interference at the frequency of interest.

$$2(w_0 + \Delta w) - (w_0 + 2\Delta w) = w_0 \qquad \text{Equation 14}$$

Input Third Intercept Point (IIP_3)

Input Third Intercept Point is the point where the linear and the IM_3 terms of the same input amplitude are equal for a given amplitude. With no further attenuation, the amplitude corresponding to this point is given by Equation 15. This metric raised the noise floor as given by SFDR.

$$A_{IIP3} = \sqrt{\frac{4a_1}{3a_3}} \qquad \text{Equation 15}$$

Spurious Free Dynamic Range (SFDR)

Spurious Free Dynamic Range is the ratio of maximum amplitude over smallest amplitude able to be sensed Equation 16 and Equation 17. IM will double the noise floor when Equation 18 is satisfied for an input amplitude with a matching network R_s to Z_{in} based on noise figure F from the matching network in linear units over a bandwidth B. A_{sens} is the signal to noise minimum requirement times the expected noise.

$$SFDR = \frac{A}{A_{sens}} \qquad \text{Equation 16}$$

$$A_{sens} = \sqrt{(SNR)_{min} FKTR_s B} \qquad \text{Equation 17}$$

$$(\frac{3a_3}{4} A^3)^2 = (a_1)^2 kTR_s B \rightarrow A = ({A^2}_{IIP3} \sqrt{FkTR_s B})^{1/3} \qquad \text{Equation 18}$$

2rd Order Intermodulation

The 2nd order intermodulation is problematic for AM interference because a signal at baseband will have the interfering spectrum placed on top of the signal of interest as the interferer expirances a form of demodulation with itself as shown in Equation 19.

$$(2w_0 + \Delta w) - (w_0 + \Delta w) = w_0 \qquad \text{Equation 19}$$

Input Second Intercept Point (IIP_2)

Input Third Intercept Point is the point where the linear and the IM_3 terms of the same input amplitude are equal for a given amplitude. With no further attenuation, the amplitude corresponding to this point is given by Equation 20. This metric raised the noise floor as well.

$$A_{IIP3} = \frac{a_1}{a_2} \qquad \text{Equation 20}$$

Cross-Modulation

Assume an AM interferer and a tone close to our desired signal experience a 3rd order nonlinearity. The resulting signal will spill into our desired spectrum if the AM's bandwidth is large enough to stretch from the tone into our desired signal's spectrum.

AM-PM Distortion

Phase can also be treated differently from the linear and higher order terms in the transfer equation as shown in Equation 21. A harmonic may have a phase delay associated with the nonlinearity diminishing both the amplitude and phase of communication symbols found in the popular QAM constellations which encode their data as both amplitude and phase.

$$v_o = a_1\, v_i + a_3(\theta)\, v_i^3 \qquad \text{Equation 21}$$