# LE-128/715 Segona prova

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# Continguts

- 1. Historial acadèmic i professional
- 2. Pla de treball
- 3. Lliçó

# Historial acadèmic i professional

### Educació

2008 - 2011 Diplomatura d'estadística UPC 2011 - 2013 Màster en estadística i IO (MEIO) **UPC-UB** 2013 - 2018 MS. PhD in Statistical Science Duke University Advisor: James O. Berger

## Trajectòria professional

2018 - 2022 Assistant Professor in Statistics Zicklin School of Business The City University of New York

2022 - present Becari María Zambrano UPC

### Transferència de coneixement

- Consultoria: Takata (amb David Banks a Duke), SYMYB (amb Xavier Tort-Martorell)
- Formació a empreses: ORICA (amb Xavier Tort-Martorell i Lluís Marco)

## Pla de treball

### Docència

- ► Més de 8 anys d'experiència entre doctorat, assistant professor i becari
- Estudiantat divers: des d'estudiants d'escola de negocis fins a CFIS, des d'estudiants de primer de grau fins a doctorat
- Assignatures diverses: estadística bàsica, multivariant, R, bayesiana, disseny, modelització, inferència, etc.
- Docència en anglès i català

#### Recerca

#### Especialitat: teoria i mètodes bayesians

- Proves d'hipòtesi i selecció de variables
- Mètodes no paramètrics bayesians
- Sèries temporals
- Disseny i anàlisi d'experiments
- Altres: privacitat, optimització, computació i fonaments

## Proves d'hipòtesi i selecció de variables

- Peña, V. & Barrientos, A.F. Differentially private methods for managing model uncertainty in linear regression models. JMLR (Q1/Q2: CS-AI).
- Peña, V. & Barrientos, A.F. (2023) Differentially Private Hypothesis Testing with the Subsampled and Aggregated Randomized Response Mechanism. Stat. Sinica (Q3).
- Mulder, J., Berger, J. O., Peña, V., & Bayarri, M. J. (2021). On the prevalence of information inconsistency in normal linear models. TEST (Q2).
- Peña, V. & Berger J.O. (2020). Restricted type II maximum likelihood priors on regression coefficients.
   Bayesian Analysis (Q1).

# Mètodes no paramètrics bayesians

- Jauch, M., Barrientos, A. F., Peña, V. & Matteson, D. Mixture representations and Bayesian nonparametric inference for likelihood ratio ordered distributions.
  Submitted to Bayesian Analysis (Q1).
- Barrientos, A. F. & Peña, V. (2020). Bayesian bootstraps for massive datasets. Bayesian Analysis (Q1).

# Sèries temporals

- ▶ Peña, V., & Irie, K. (2022). On the Relationship between Uhlig Extended and beta-Bartlett Processes. Journal of Time Series Analysis (Q4).
- Investigació concurrent i futura amb Kaoru Irie (Universitat de Tokyo): models dinàmics multivariants per a matrius de covariàncies

# Disseny i anàlisi d'experiments

- Attolini, C. S. O., Peña, V., & Rossell, D. (2015). Designing alternative splicing RNA-seq studies. Beyond generic guidelines. Bioinformatics (Q1: Comp. Bio.).
- Investigació concurrent i futura amb Gonzalo García-Donato (Universidad de Castilla y la Mancha): anàlisi de dissenys factorials fraccionals

## **Altres**

- Guo, Q., Barrientos, A.F. & Peña, V. Differentially Private Methods for Compositional Data. Submitted to JCGS (Q1).
- Jauch, M. & Peña, V. (2016). Bayesian optimization with shape constraints. NIPS Workshop on Bayesian Optimization.
- ▶ Peña, V. & Berger, J. O. A note on recent criticisms to Birnbaum's theorem. *arXiv:1711.08093*.
- Investigació sobre mètodes computacionals per a la distribució GIG i estimació amb SURE amb Michael Jauch (FSU)

## Finançament

- ➤ A CUNY: finançament intern pels estius i un projecte de recerca.
- Projecte del ministeri: "Métodos Bayesianos para la selección de variables en problemas de alta dimensionalidad y con datos perdidos" amb Gonzalo García-Donato (UCLM), Maria Eugenia Castellanos (URJC), Alicia Quirós (León), Stefano Cabras (UC3M) i Anabel Forte (UV)
- Presentarem proposta per la nova convocatòria de l'AEI (gener 2024)

# Lliçó

# p-values vs $P(H_0 \mid data)$

- ▶ In introductory statistics classes, you learned that the p-value is not  $P(H_0 \mid \text{data})$ .
- ▶ In Bayesian statistics, we can compute  $P(H_0 \mid \mathsf{data})$ .

### Question

How similar is  $P(H_0 \mid data)$  to the p-value?

# Testing a point null: normal mean

- ▶ We observe  $X_i \stackrel{\text{iid}}{\sim} N(\mu, \sigma^2)$  for  $i \in \{1, 2, ..., n\}$  with  $\sigma^2$  known.
- ▶ We want to do the hypothesis test

$$H_0: \mu = \mu_0 \qquad H_1: \mu \neq \mu_0$$

for a fixed  $\mu_0$ .

## Non-Bayesian solution: z-test

- ► From a non-Bayesian perspective, we can do a z-test.
- If  $\Phi$  is the cdf of the N(0,1) distribution and  $z = \sqrt{n}(\sum_{i=1}^{n} x_i/n \mu_0)/\sigma$ , the p-value is

$$2[1-\Phi(|z|)]=2[1-\mathtt{pnorm}(\mathtt{abs}(z))].$$

▶ If we reject  $H_0$  whenever the p-value is less than  $\alpha$ , then

$$P_{H_0}(\text{reject } H_0) = \alpha.$$

## Bayesian solution

- All unknowns have probability distributions.
- ▶ Need to specify  $P(H_0)$  and  $P(H_1)$ .
- ▶ Under  $H_0$ , we know that  $\mu = \mu_0$ .
- ▶ Under  $H_1$ , we know that  $\mu \neq \mu_0$ , but we don't know the value of  $\mu$  exactly: we need to specify a prior  $f(\mu \mid H_1)$ .

## Bayes theorem

#### Applying Bayes theorem, we can find

$$P(H_0 \mid x_{1:n}) = \frac{P(H_0)f(x_{1:n} \mid H_0)}{P(H_0)f(x_{1:n} \mid H_0) + P(H_1)f(x_{1:n} \mid H_1)}$$

$$f(x_{1:n} \mid H_0) = \prod_{i=1}^{n} N(x_i \mid \mu_0, \sigma^2)$$

$$f(x_{1:n} \mid H_1) = \int_{\mathbb{R}} \underbrace{\prod_{i=1}^{n} N(x_i \mid \mu, \sigma^2)f(\mu \mid H_1)}_{f(x_{1:n}, \mu \mid H_1)} d\mu.$$

## How to choose priors?

- ▶ How to choose  $P(H_0)$  and  $P(H_1)$ ?
  - In the absence of prior information, we can choose  $P(H_0) = P(H_1) = 1/2$ .
- ▶ How to choose  $f(\mu \mid H_1)$ ?
  - Use a default prior: for example,

$$\mu \mid H_1 \sim N(\mu_0, \sigma^2)$$
 (unit information prior)

We can also consider robust Bayes.

## Robust Bayes

- ▶ Instead of considering one prior, consider a class of priors  $\Gamma$ .
- ▶ Find  $\min_{\Gamma} P(H_0 \mid x_{1:n})$ : worst case for  $H_0$ .
- ► Following Berger and Sellke (1987), we will consider:

```
\Gamma_A = \{ \text{all distributions} \}

\Gamma_S = \{ \text{all distributions symmetric about } \mu_0 \}.
```

# p-values are not $P(H_0 \mid x_{1:n})$

- ▶ Robust Bayes is *aggressive* against  $H_0$ : we're picking the worst possible case over a class of priors.
- However, we'll see that

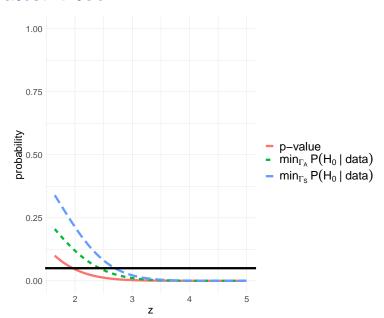
$$\min_{\Gamma} P(H_0 \mid x_{1:n}) \ge p\text{-value}.$$

▶ Interpreting a p-value as  $P(H_0 \mid x_{1:n})$  overestimates evidence against  $H_0$ .

## Results: Table

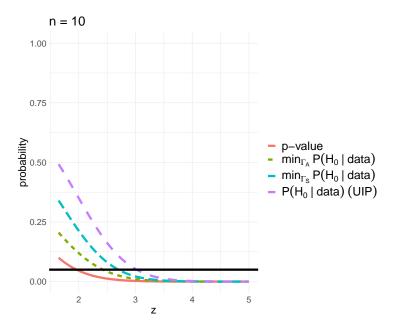
$\overline{z}$	p-value	$\min \Gamma_A$	$\min \Gamma_S$
1.645	0.100	0.205	0.340
1.960	0.050	0.128	0.227
2.576	0.010	0.035	0.068
3.291	0.001	0.004	0.009

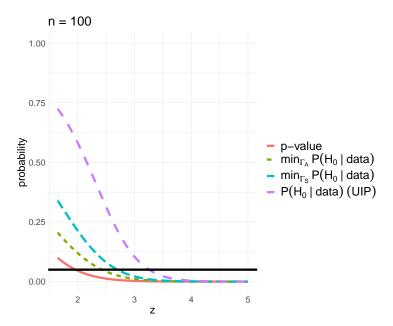
## Results: Plot

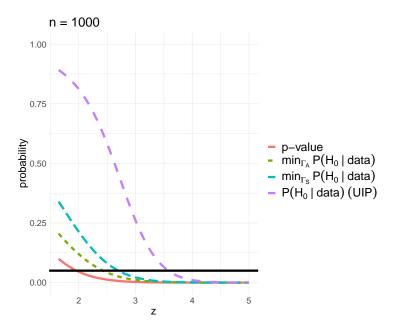


## Comparison with unit information prior

- ► How far are robust Bayesian answers to what we would obtain with a default prior?
- Add in the results for the unit information prior.
- ightharpoonup Results depend on z and n.
- ► The higher *n* is, the bigger the difference with *p*-values and robust Bayes.
- Similar results with other default priors: e.g., Cauchy or hyper-g.







## **Conclusions**

- ▶  $P(H_0 \mid x_{1:n})$  can be pretty different to p-values.
- In point null hypothesis testing of normal means,

$$P(H_0 \mid x_{1:n}) \geq p$$
-value.

▶ Robust Bayes can be far from proper Bayes for large n.

### Reference

Berger, J. O., & Sellke, T. (1987). Testing a point null hypothesis: The irreconcilability of p-values and evidence. Journal of the American Statistical Association, 82(397), 112-122.