Design of experiments: Lab 3

1.1. The patients are a blocking variable and the gas is the treatment. We only have $r = 1$, so we'll fit an additive model:

$$y_{ijk} = \mu + \beta_i + \tau_j + \varepsilon_{ijk}, \qquad \varepsilon_{ijk} \overset{\text{ind}}{\sim} N(0, \sigma^2)$$

with sum-to-zero restrictions

$$\sum_{i=1}^{7} \beta_i = \sum_{j=1}^{4} \tau_j = 0.$$

Here's the ANOVA table:

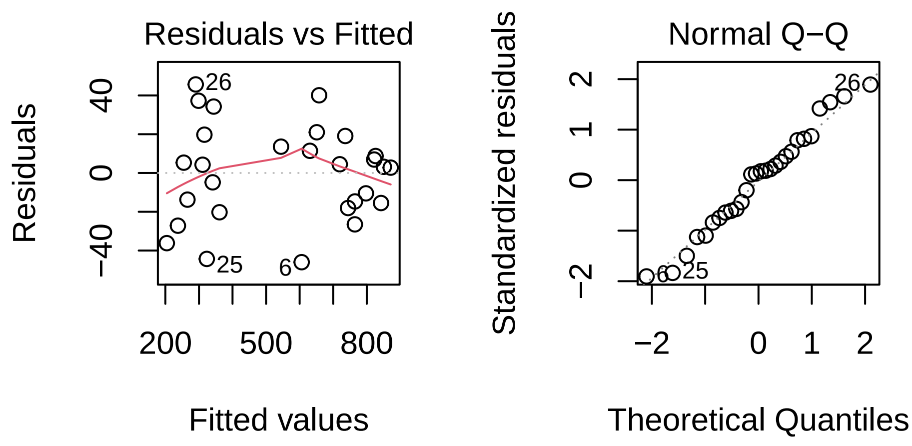
```
library(tidyverse)
gasos = read.csv2("http://vicpena.github.io/doe/lab3/Gases.csv")
gasos$Gas = factor(gasos$Gas)
gasos$Sujeto = factor(gasos$Sujeto)
options(contrasts = c("contr.sum", "contr.poly"))
mod_add = aov(Valor ~ Sujeto+Gas, data = gasos)
summary(mod_add)
```

```
##             Df  Sum Sq Mean Sq F value   Pr(>F)    
## Sujeto       6 1471772  245295  270.61  < 2e-16 ***
## Gas          3   44827   14942   16.48 2.11e-05 ***
## Residuals   18   16316     906                     
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Both the block and the treatment are significant.

Let's take a look at the residual plots:

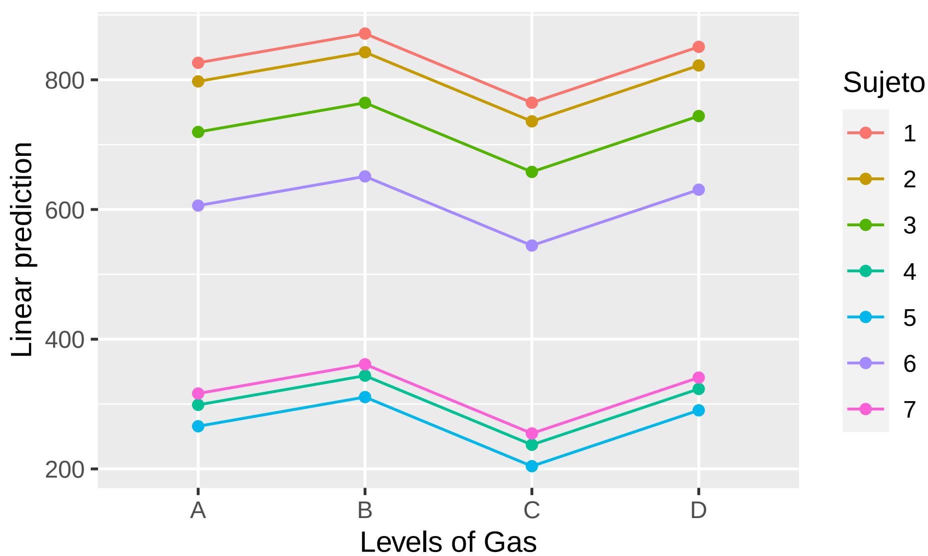
```
par(mfrow = c(1,2))
plot(mod_add, 1:2)
```



The residual plots look fine.

Let's take a look at the effect plots

```
library(emmeans)
emmip(mod_add, Sujeto ~ Gas)
```



There are obvious differences between subjects. It seems that gas C might be significantly worse than the others (the response is distance walked in 12 minutes). We can compare the gases with `TukeyHSD`:

```
TukeyHSD(mod_add, which = "Gas")
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = Valor ~ Sujeto + Gas, data = gasos)
##
## $Gas
```

```
##              diff          lwr       upr     p adj
## B-A    45.00000   -0.4840367  90.48404 0.0530703
## C-A   -61.57143 -107.0554653 -16.08739 0.0061872
## D-A    24.57143  -20.9126082  70.05547 0.4429649
## C-B  -106.57143 -152.0554653 -61.08739 0.0000178
## D-B   -20.42857  -65.9126082  25.05547 0.5929753
## D-C    86.14286   40.6588204 131.62689 0.0002338
```

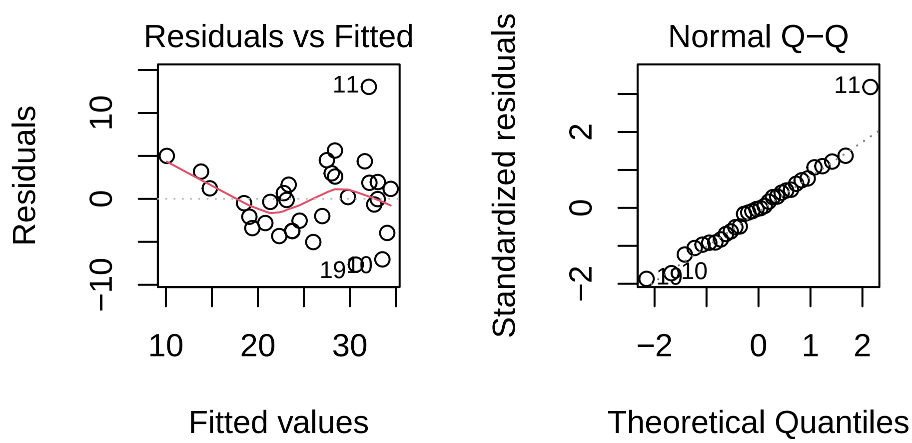
The p-values for the tests comparing gas C to the others are significant, confirming our initial intuition.

1.2. The rats are blocks and the zones are treatments. We only have one replicate ($r = 1$), so we fit an additive model with both variables. The p-values are all significant. The residuals look fine, with the exception of observation 11, which seems to be badly predicted by the model (large residual).

```r
rates = read.csv2("http://vicpena.github.io/doe/lab3/Rates.csv")
rates$Sujeto = factor(rates$Sujeto)
rates$Zona = factor(rates$Zona)
mod_add = aov(Cobre ~ Sujeto+Zona, data = rates)
summary(mod_add)
```

```
##             Df Sum Sq Mean Sq F value  Pr(>F)
## Sujeto       7  703.3  100.48   3.938 0.00678 **
## Zona         3  565.9  188.63   7.393 0.00146 **
## Residuals   21  535.8   25.51
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
par(mfrow = c(1, 2))
plot(mod_add, 1:2)
```



Since we're interested in comparing zones, let's run `TukeyHSD`:

```r
TukeyHSD(mod_add, which = "Zona")
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = Cobre ~ Sujeto + Zona, data = rates)
```

```
##
## $Zona
##          diff        lwr       upr     p adj
## Z1-N  10.7125   3.672793 17.752207 0.0019145
## Z2-N   9.3000   2.260293 16.339707 0.0069970
## Z3-N   4.6750  -2.364707 11.714707 0.2786427
## Z2-Z1 -1.4125  -8.452207  5.627207 0.9429572
## Z3-Z1 -6.0375 -13.077207  1.002207 0.1098481
## Z3-Z2 -4.6250 -11.664707  2.414707 0.2872316
```

There are significant differences between Z1 and N and also between Z2 and N.

1.3. We'd put the loaves randomly to avoid systematic biases (for example, some parts of the oven might be hotter than other). The batches are a block effect and the recipes are the treatment. This is another complete block design with $r = 1$ – we'll fit an additive model. There are significant batch and recipe effects. The residuals look fine (not including them for concreteness).

```r
pa = read.csv2("http://vicpena.github.io/doe/lab3/Pan.csv")
pa$Receta = factor(pa$Receta)
pa$Hornada = factor(pa$Hornada)
mod_add = aov(Densidad ~ Receta + Hornada, data = pa)
summary(mod_add)
```

```
##             Df  Sum Sq Mean Sq F value Pr(>F)
## Receta       2 0.08657 0.04329   8.137 0.0118 *
## Hornada      4 0.09884 0.02471   4.645 0.0312 *
## Residuals    8 0.04256 0.00532
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Let's compare the recipes with `TukeyHSD`:

```r
TukeyHSD(mod_add, which = "Receta")
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = Densidad ~ Receta + Hornada, data = pa)
##
## $Receta
##       diff        lwr         upr     p adj
## B-A -0.088 -0.2198146  0.04381462 0.1983163
## C-A -0.186 -0.3178146 -0.05418538 0.0093756
## C-B -0.098 -0.2298146  0.03381462 0.1460142
```

There are significant differences between recipes A and C.

1.4. Now we have $r = 2$, so we can fit a model with an interaction and see if we need it

```r
options(contrasts = c("contr.sum", "contr.poly"))
aigua = read.csv2("http://vicpena.github.io/doe/lab3/Aigua.csv")
mod_inter = aov(Reduccio ~ Accio*Densitat, data = aigua)
summary(mod_inter)
```

```
##                Df Sum Sq Mean Sq F value   Pr(>F)
## Accio           2 182.36   91.18  32.242 1.49e-05 ***
## Densitat        3 260.32   86.77  30.685 6.55e-06 ***
## Accio:Densitat  6  30.39    5.07   1.791    0.184
## Residuals      12  33.94    2.83
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

It turns out that the interaction isn't significant, so we go ahead and fit an additive model:

```r
mod_add = aov(Reduccio ~ Accio + Densitat, data = aigua)
summary(mod_add)
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)    
## Accio        2 182.36   91.18   25.51 5.58e-06 ***
## Densitat     3 260.32   86.77   24.28 1.50e-06 ***
## Residuals   18  64.33    3.57                     
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Our final model is

$$y_{ijk} = \mu + \alpha_i + \tau_j + \varepsilon_{ijk}, \qquad \varepsilon_{ijk} \overset{\text{iid}}{\sim} N(0, \sigma^2),$$

where τ_i represents the effect of the "action" and τ_j the population density. The model has sum-to-zero restrictions on the effects, as usual. The hypothesis tests are

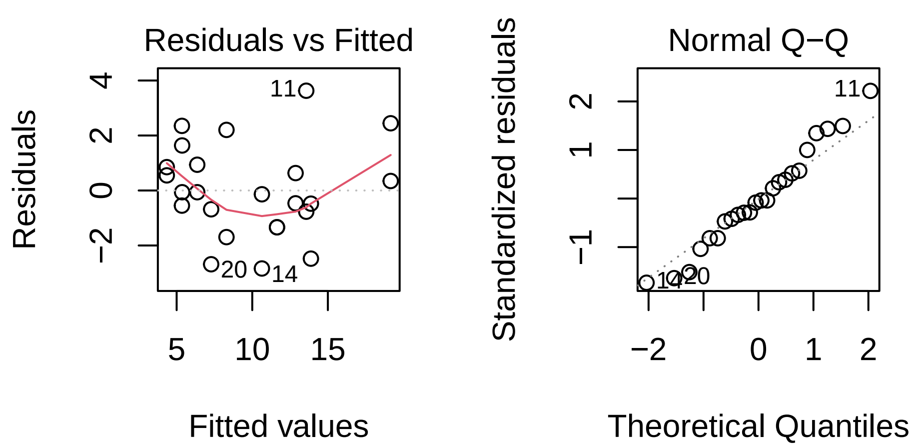
$$H_{0,\alpha} : \alpha_i = 0 \text{ for all } i, \qquad H_{1,\alpha} : \alpha_i \neq 0 \text{ for at least one } i,$$

and

$$H_{0,\tau} : \tau_j = 0 \text{ for all } j, \qquad H_{1,\tau} : \tau_j \neq 0 \text{ for at least one } j.$$

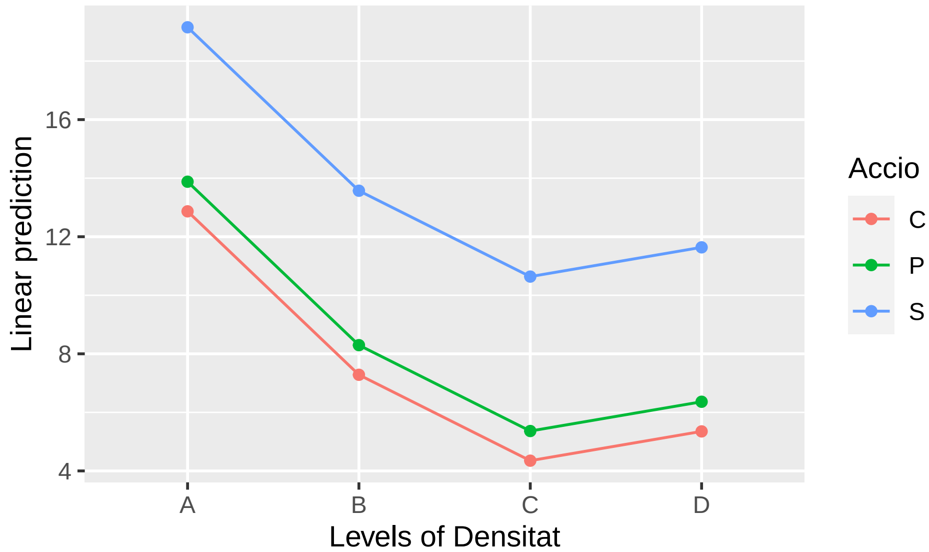
Here are the residual plots, which look fine:

```r
par(mfrow = c(1, 2))
plot(mod_add, which = c(1,2))
```



And here's the effects plot:

```r
emmip(mod_add, Accio ~ Densitat)
```



It seems that action S (a subsidy for changing old equipment) works best. This can be confirmed with `TukeyHSD`. If we want to find intervals for the actions, we can use `emmeans`:

```
emmeans(mod_add, ~ Accio)
```

```
##  Accio emmean    SE df lower.CL upper.CL
##  C       7.46 0.668 18     6.06     8.87
##  P       8.47 0.668 18     7.07     9.88
##  S      13.75 0.668 18    12.35    15.15
##
## Results are averaged over the levels of: Densitat
## Confidence level used: 0.95
```

The 95% confidence interval for S goes from 12.35 to 15.15.

Finally, we can find the estimated mean for action S and density A with `emmeans`, which is 19.15:

```
emmeans(mod_add, ~ Accio + Densitat)
```

```
##  Accio Densitat emmean    SE df lower.CL upper.CL
##  C     A         12.87 0.945 18    10.88    14.85
##  P     A         13.88 0.945 18    11.89    15.87
##  S     A         19.15 0.945 18    17.17    21.14
##  C     B          7.28 0.945 18     5.30     9.27
##  P     B          8.30 0.945 18     6.31    10.28
##  S     B         13.57 0.945 18    11.58    15.56
##  C     C          4.35 0.945 18     2.36     6.34
##  P     C          5.36 0.945 18     3.38     7.35
##  S     C         10.64 0.945 18     8.65    12.62
##  C     D          5.35 0.945 18     3.36     7.34
##  P     D          6.36 0.945 18     4.38     8.35
##  S     D         11.64 0.945 18     9.65    13.62
##
## Confidence level used: 0.95
```