LE-128/715
Segona prova

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Continguts

Historial acadèmic i professional

Educació

2008 - 2011   Diplomatura d'estadística
              UPC

2011 - 2013   Màster en estadística i IO (MEIO)
              UPC-UB

2013 - 2018   MS, PhD in Statistical Science
              Duke University
              Advisor: James O. Berger

Trajectòria professional

2018 - 2022   Assistant Professor in Statistics
Zicklin School of Business
The City University of New York

2022 - present   Becari María Zambrano
UPC

Transferència de coneixement

- Consultoria: Takata (amb David Banks a Duke), SYMYB (amb Xavier Tort-Martorell)

- Formació a empreses: ORICA (amb Xavier Tort-Martorell i Lluís Marco)

Pla de treball

Docència

- Més de 8 anys d'experiència entre doctorat, assistant professor i becari

- Estudiantat divers: des d'estudants d'escola de negocis fins a CFIS, des d'estudants de primer de grau fins a doctorat

- Assignatures diverses: estadística bàsica, multivariant, R, bayesiana, disseny, modelització, inferència, etc.

- Docència en anglès i català

Recerca

Especialitat: teoria i mètodes bayesians

- ▸ Proves d'hipòtesi i selecció de variables

- ▸ Mètodes no paramètrics bayesians

- ▸ Sèries temporals

- ▸ Disseny i anàlisi d'experiments

- ▸ Altres: privacitat, optimització, computació i fonaments

Proves d'hipòtesi i selecció de variables

- Peña, V. & Berger J.O. (2020). Restricted type II maximum likelihood priors on regression coefficients. **Bayesian Analysis (Q1).**

- Mulder, J., Berger, J. O., Peña, V., & Bayarri, M. J. (2021). On the prevalence of information inconsistency in normal linear models. **TEST (Q2)**.

- Peña, V. & Barrientos, A.F. (2023) Differentially Private Hypothesis Testing with the Subsampled and Aggregated Randomized Response Mechanism. **Stat. Sinica (Q3)**.

- Peña, V. & Barrientos, A.F. (2024+) Differentially private methods for managing model uncertainty in linear regression models. **JMLR (Q1/Q2: CS-AI)**.

Mètodes no paramètrics bayesians

- Barrientos, A. F. & Peña, V. (2020). Bayesian bootstraps for massive datasets. **Bayesian Analysis (Q1)**.

- Jauch, M., Barrientos, A. F., Peña, V. & Matteson, D. Mixture representations and Bayesian nonparametric inference for likelihood ratio ordered distributions. **Submitted to Bayesian Analysis (Q1)**.

Sèries temporals

- Peña, V., & Irie, K. (2022). On the Relationship between Uhlig Extended and beta-Bartlett Processes. **Journal of Time Series Analysis (Q4)**.

- Investigació concurrent i futura amb Kaoru Irie (Universitat de Tokyo): models dinàmics multivariants per a matrius de covariàncies

Disseny i anàlisi d'experiments

- Attolini, C. S. O., Peña, V., & Rossell, D. (2015).
  Designing alternative splicing RNA-seq studies. Beyond
  generic guidelines. **Bioinformatics (Q1: Comp. Bio.)**.

- Investigació concurrent i futura amb Gonzalo
  García-Donato (Universidad de Castilla y la Mancha):
  anàlisi de dissenys factorials fraccionals

Altres

- Jauch, M. & Peña, V. (2016). Bayesian optimization with shape constraints. **NIPS Workshop on Bayesian Optimization.**

- Peña, V. & Berger, J. O. A note on recent criticisms to Birnbaum's theorem. *arXiv:1711.08093.*

- Guo, Q., Barrientos, A.F. & Peña, V. Differentially Private Methods for Compositional Data. **Submitted to JCGS (Q1)**.

- Investigació sobre mètodes computacionals per a la distribució GIG i estimació amb SURE amb Michael Jauch (FSU)

Finançament

- A CUNY: finançament intern pels estius i un projecte de recerca.

- Projecte del ministeri: "Métodos Bayesianos para la selección de variables en problemas de alta dimensionalidad y con datos perdidos" amb Gonzalo García-Donato (UCLM), Maria Eugenia Castellanos (URJC), Alicia Quirós (León), Stefano Cabras (UC3M) i Anabel Forte (UV)

- Presentarem proposta per la nova convocatòria de l'AEI (gener 2024)

Lliçó

p-values vs $P(H_0 \mid \text{data})$

- In introductory statistics classes, you learned that the p-value is not $P(H_0 \mid \text{data})$.

- In Bayesian statistics, we can compute $P(H_0 \mid \text{data})$.

Question

How similar is $P(H_0 \mid \text{data})$ to the p-value?

Testing a point null: normal mean

- We observe $X_i \overset{\text{iid}}{\sim} N(\mu, \sigma^2)$ for $i \in \{1, 2, ..., n\}$ with σ^2 known.

- We want to do the hypothesis test

$$H_0 : \mu = \mu_0 \qquad H_1 : \mu \neq \mu_0$$

for a fixed μ_0.

Non-Bayesian solution: z-test

- From a non-Bayesian perspective, we can do a z-test.

- If Φ is the cdf of the $N(0,1)$ distribution and $z = \sqrt{n}(\sum_{i=1}^{n} x_i/n - \mu_0)/\sigma$, the p-value is

$$2[1 - \Phi(|z|)] = 2[1 - \texttt{pnorm}(\texttt{abs}(z))].$$

- If we reject H_0 whenever the p-value is less than α, then

$$P_{H_0}(\text{reject } H_0) = \alpha.$$

Bayesian solution

- All unknowns have probability distributions.

- Need to specify $P(H_0)$ and $P(H_1)$.

- Under H_0, we know that $\mu = \mu_0$.

- Under H_1, we know that $\mu \neq \mu_0$, but we don't know the value of μ exactly: we need to specify a prior $f(\mu \mid H_1)$.

Bayes theorem

Applying Bayes theorem, we can find

$$P(H_0 \mid x_{1:n}) = \frac{P(H_0)f(x_{1:n} \mid H_0)}{P(H_0)f(x_{1:n} \mid H_0) + P(H_1)f(x_{1:n} \mid H_1)}$$

$$f(x_{1:n} \mid H_0) = \prod_{i=1}^{n} \mathrm{N}(x_i \mid \mu_0, \sigma^2)$$

$$f(x_{1:n} \mid H_1) = \int_{\mathbb{R}} \underbrace{\prod_{i=1}^{n} N(x_i \mid \mu, \sigma^2) f(\mu \mid H_1)}_{f(x_{1:n}, \mu \mid H_1)} \, \mathrm{d}\mu.$$

How to choose priors?

- How to choose $P(H_0)$ and $P(H_1)$?
  - In the absence of prior information, we can choose $P(H_0) = P(H_1) = 1/2$.

- How to choose $f(\mu \mid H_1)$?
  - Use a default prior: for example,

    $$\mu \mid H_1 \sim N(\mu_0, \sigma^2) \qquad \text{(unit information prior)}$$

  - We can also consider robust Bayes.

Robust Bayes

- Instead of considering one prior, consider a class of priors Γ.

- Find $\min_\Gamma P(H_0 \mid x_{1:n})$: worst case for H_0.

- Following Berger and Sellke (1987), we will consider:

  $\Gamma_A = \{\text{all distributions}\}$

  $\Gamma_S = \{\text{all distributions symmetric about } \mu_0\}$.

p-values are not $P(H_0 \mid x_{1:n})$

- Robust Bayes is *aggressive* against H_0: we're picking the worst possible case over a class of priors.
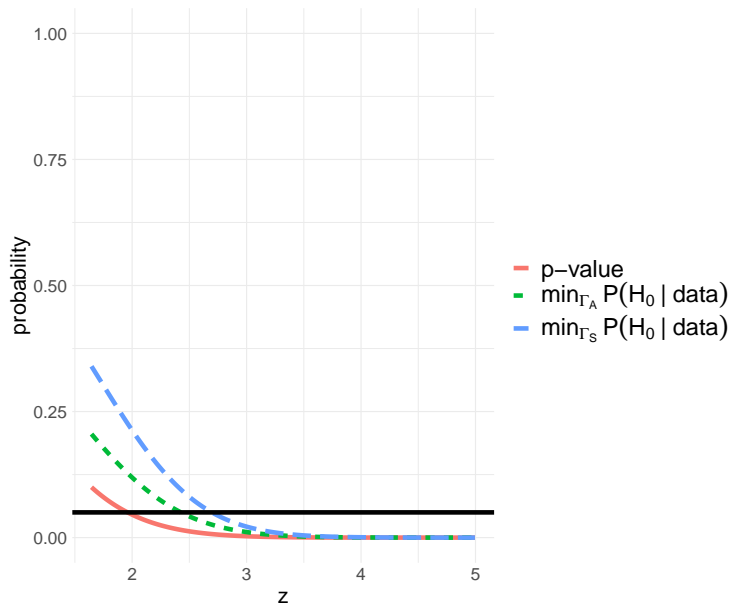
- However, we'll see that

$$\min_{\Gamma} P(H_0 \mid x_{1:n}) \geq p\text{-value}.$$

- Interpreting a p-value as $P(H_0 \mid x_{1:n})$ overestimates evidence against H_0.

Results: Table

| z	p-value	$\min \Gamma_A$	$\min \Gamma_S$
1.645	0.100	0.205	0.340
1.960	0.050	0.128	0.227
2.576	0.010	0.035	0.068
3.291	0.001	0.004	0.009

Results: Plot

Comparison with unit information prior

- How far are robust Bayesian answers to what we would obtain with a default prior?

- Add in the results for the unit information prior.

- Results depend on z and n.

- The higher n is, the bigger the difference with p-values and robust Bayes.

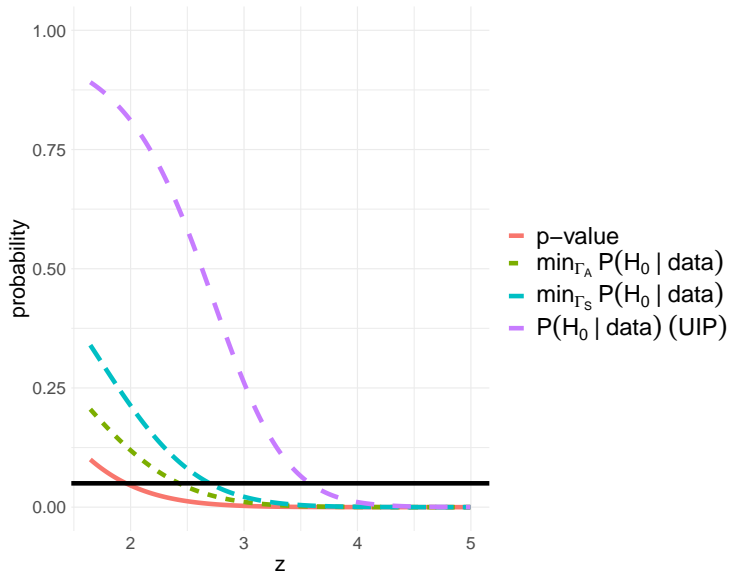- Similar results with other default priors: e.g., Cauchy or hyper-g.
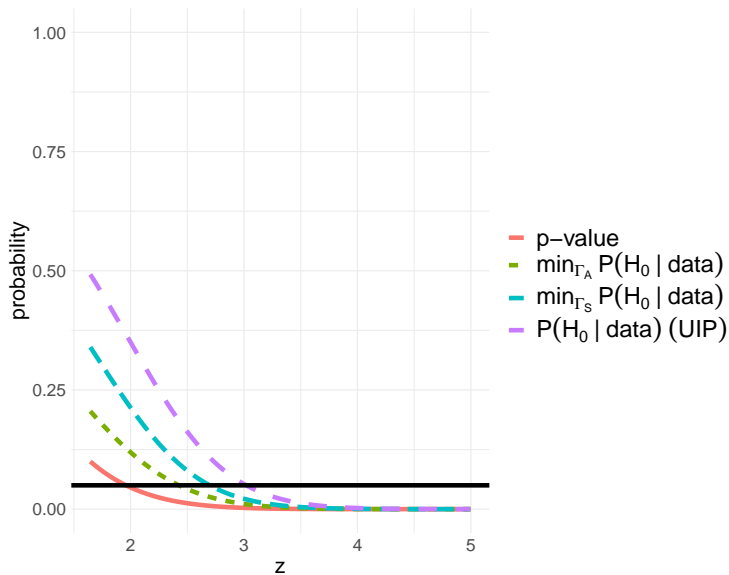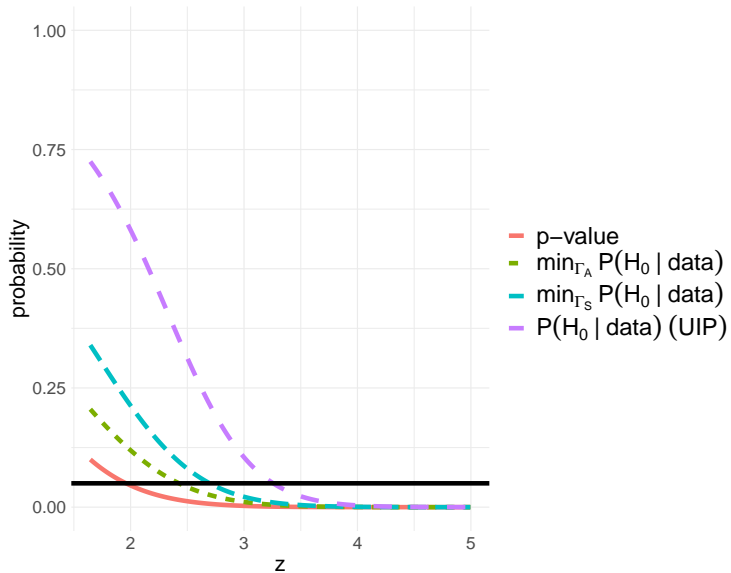
Conclusions

- $P(H_0 \mid x_{1:n})$ can be pretty different to p-values.

- In point null hypothesis testing of normal means,
$$P(H_0 \mid x_{1:n}) \geq p\text{-value}.$$

- Robust Bayes can be far from proper Bayes for large n.

Reference

Berger, J. O., & Sellke, T. (1987). Testing a point null hypothesis: The irreconcilability of p-values and evidence. Journal of the American Statistical Association, 82(397), 112-122.