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1 Laplaciano na dim N+1

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We use the (N+1)-dimensional ellipsoidal coordinates

$$\begin{aligned}
x_{N+1} &= a\sinh\chi\cos\theta_N \\
x_N &= a\cosh\chi\sin\theta_N\cos\theta_{N-1} \\
x_{N-1} &= a\cosh\chi\sin\theta_N\sin\theta_{N-1}\cos\theta_{N-2} \\
& ... \\
x_3 &= a\cosh\chi\sin\theta_N ... \sin\theta_3\cos\theta_2 \\
x_1 &= a\cosh\chi\sin\theta_N ... \sin\theta_3\sin\theta_2\cos\theta_1 \\
x_2 &= a\cosh\chi\sin\theta_N ... \sin\theta_3\sin\theta_2\sin\theta_1 \\
0 &\le \chi < \infty,\ 0 \le \theta_i \le \pi,\ 2 \le i \le N,\ 0 \le \theta_1 < 2\pi
\end{aligned}$$

they describe a family of N-hiperboloids and N-ellipsoids:

$$\frac{1}{a^2\sin^2\theta_N}\sum_{i=1}^{N} x_i^2 - \frac{1}{a^2\cos^2\theta_N}x_{N+1}^2 = 1,\ \frac{1}{a^2\cosh^2\chi}\sum_{i=1}^{N} x_i^2 + \frac{1}{a^2\sinh^2\chi}x_{N+1}^2 = 1$$

The Laplacian in the ellipsoidal coordinates in N+1 dimensions is given by the expression:

$$\nabla^2 u = \frac{1}{a^2\left(\sinh^2\chi + \cos^2\theta_N\right)}\nabla^2_{\theta_N,\chi}u + \frac{1}{a^2\cosh^2\chi\sin^2\theta_N}\nabla^2_{S^{N-1}}u$$

where $\nabla^2_{S^{N-1}}$ is the Laplacian on the (N-1)-dimensional unit sphere and

$$\nabla^2_{\theta_N,\chi} = \left(\frac{1}{\sin^{N-1}\theta_N}\frac{\partial}{\partial\theta_N}\left(\sin^{N-1}\theta_N\frac{\partial u}{\partial\theta_N}\right) + \frac{1}{\cosh^{N-1}\chi}\frac{\partial}{\partial\chi}\left(\cosh^{N-1}\chi\frac{\partial u}{\partial\chi}\right)\right)$$

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2 proximo passo

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separação de variáveis, ainda para 4d

aplicar o troco de variáveis

$$\xi = \cos\theta_3,\ \zeta = \sinh\chi,\ \theta_1, \theta_2$$

$$-1 \leq \xi \leq 1,\ 0 \leq \zeta < \infty,\ 0 \leq \theta_1 < 2\pi,\ 0 \leq \theta_2, \theta_3 < \pi$$

e separar a parte com (θ_1, θ_2) e a parte com (θ_3, χ)

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