Imaging focusing of coherent electron waves

LELEC2710

ARIB Sofiane – TRINQUET Victor

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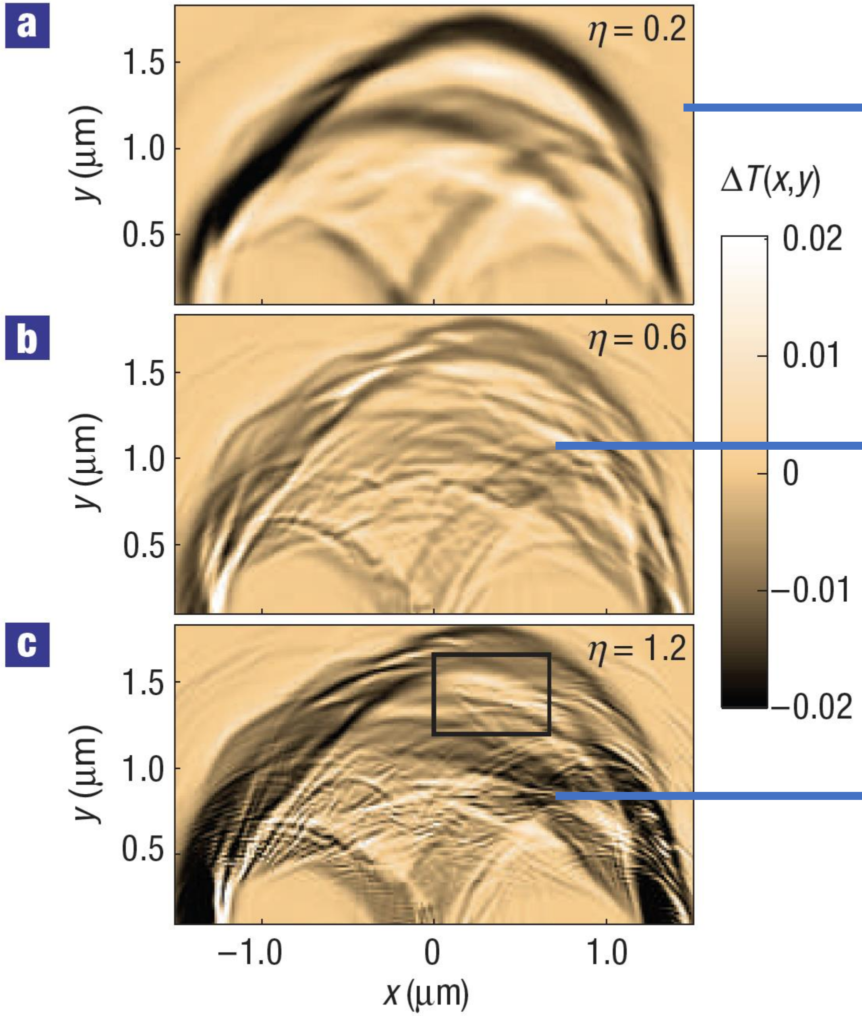
Goal of the project

Aidala, K., Parrott, R., Kramer, T. *et al.* Imaging magnetic focusing of coherent electron waves. *Nature Phys* **3,** 464–468 (2007). https://doi.org/10.1038/nphys628

Quantum simulations of SPM images including small-angle scattering



Weak focusing ($\eta = \frac{V_0}{E_F} < 1$) : classical cyclotron orbit

Weak focusing but fringes are visible

Strong focusing ($\eta > 1$): quantum interference fringes are clearly visible because backscattering is possible

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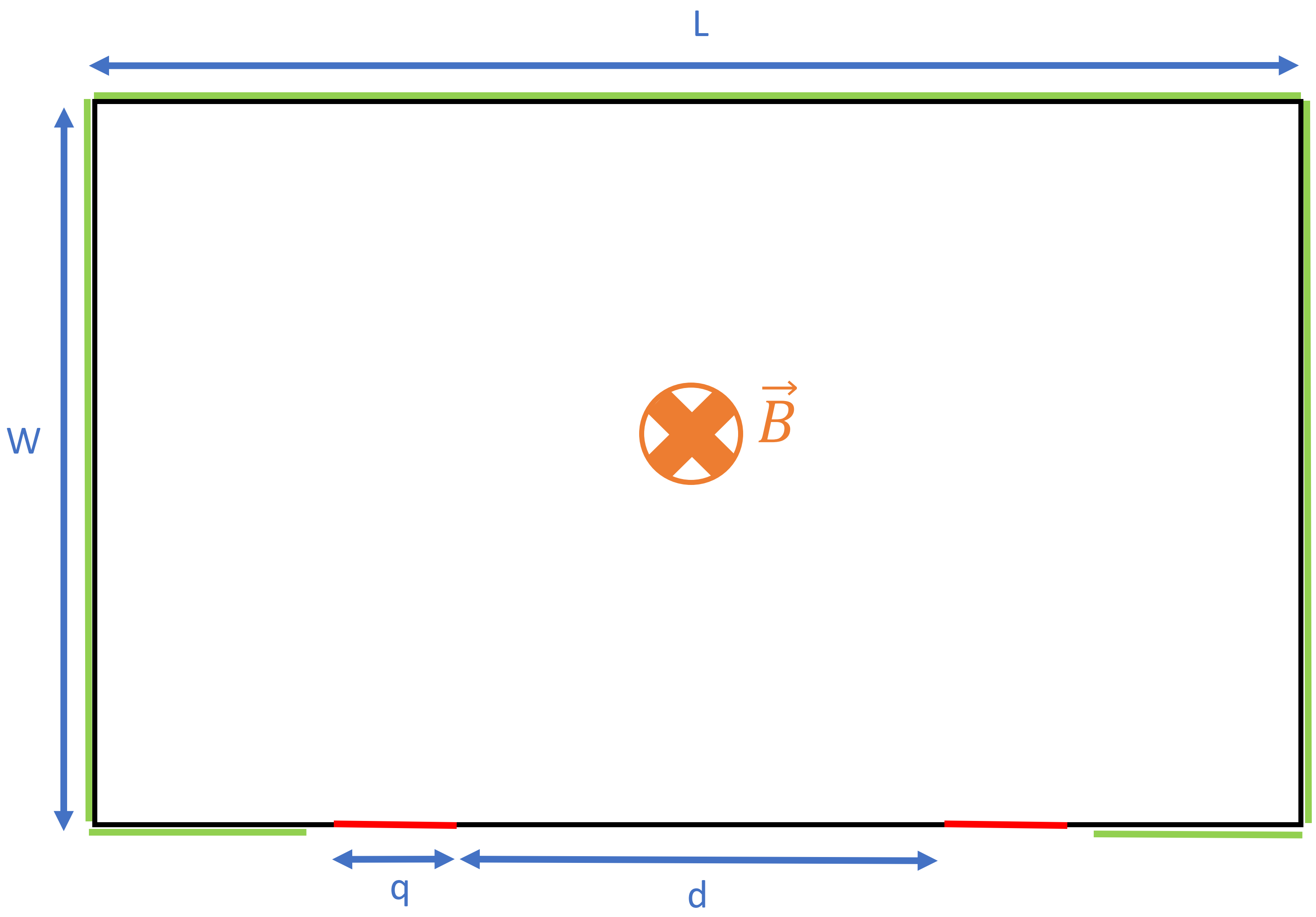
Geometry of the system

System of square lattice:

- L = 250 nm
- W = 160 nm
- q = 20 nm
- d = 120 nm
- a = 1 nm
- t = 1 eV

Leads for the current of interest // QPCs

Leads to avoid reflections on walls // open device



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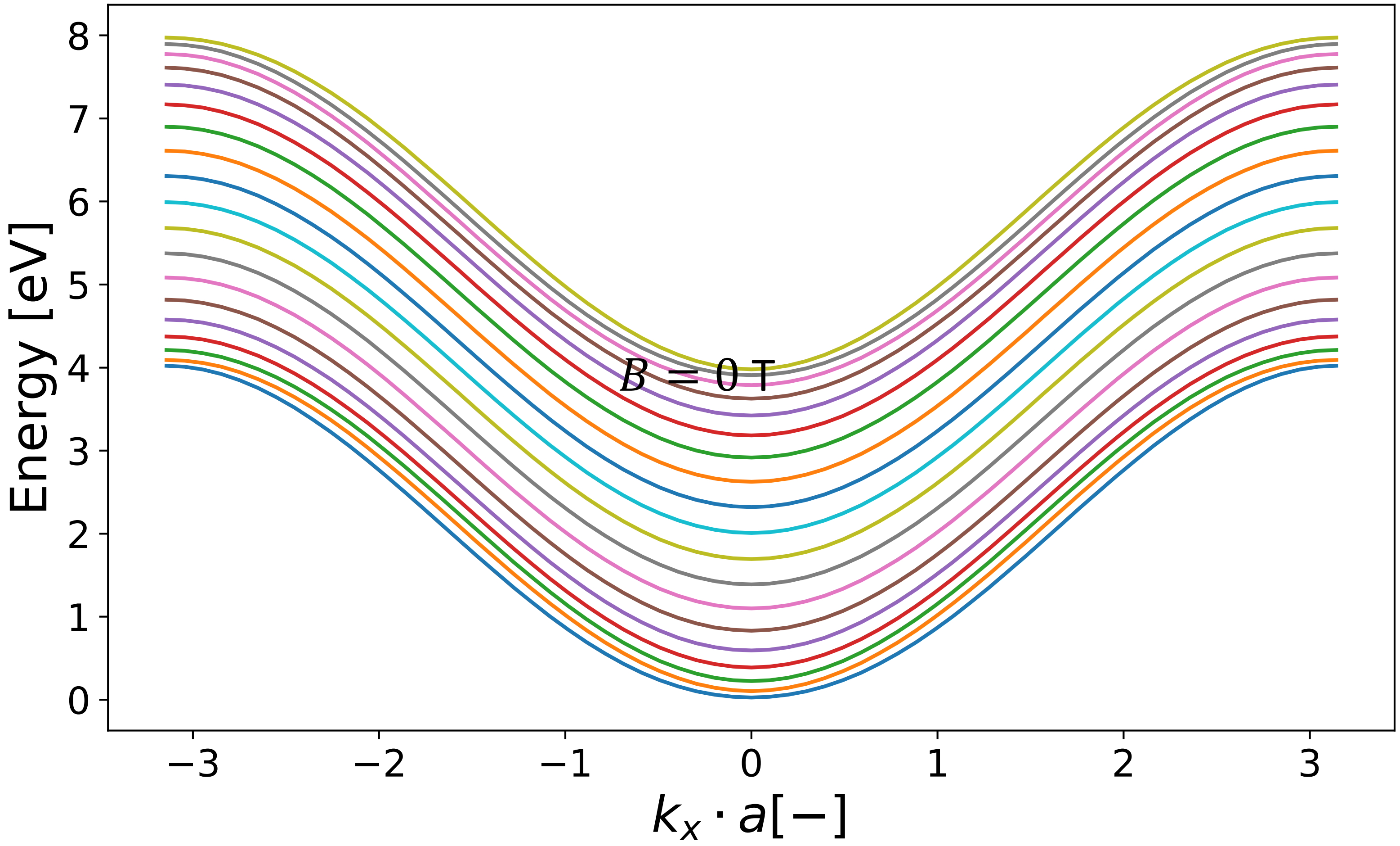
Modes in the leads

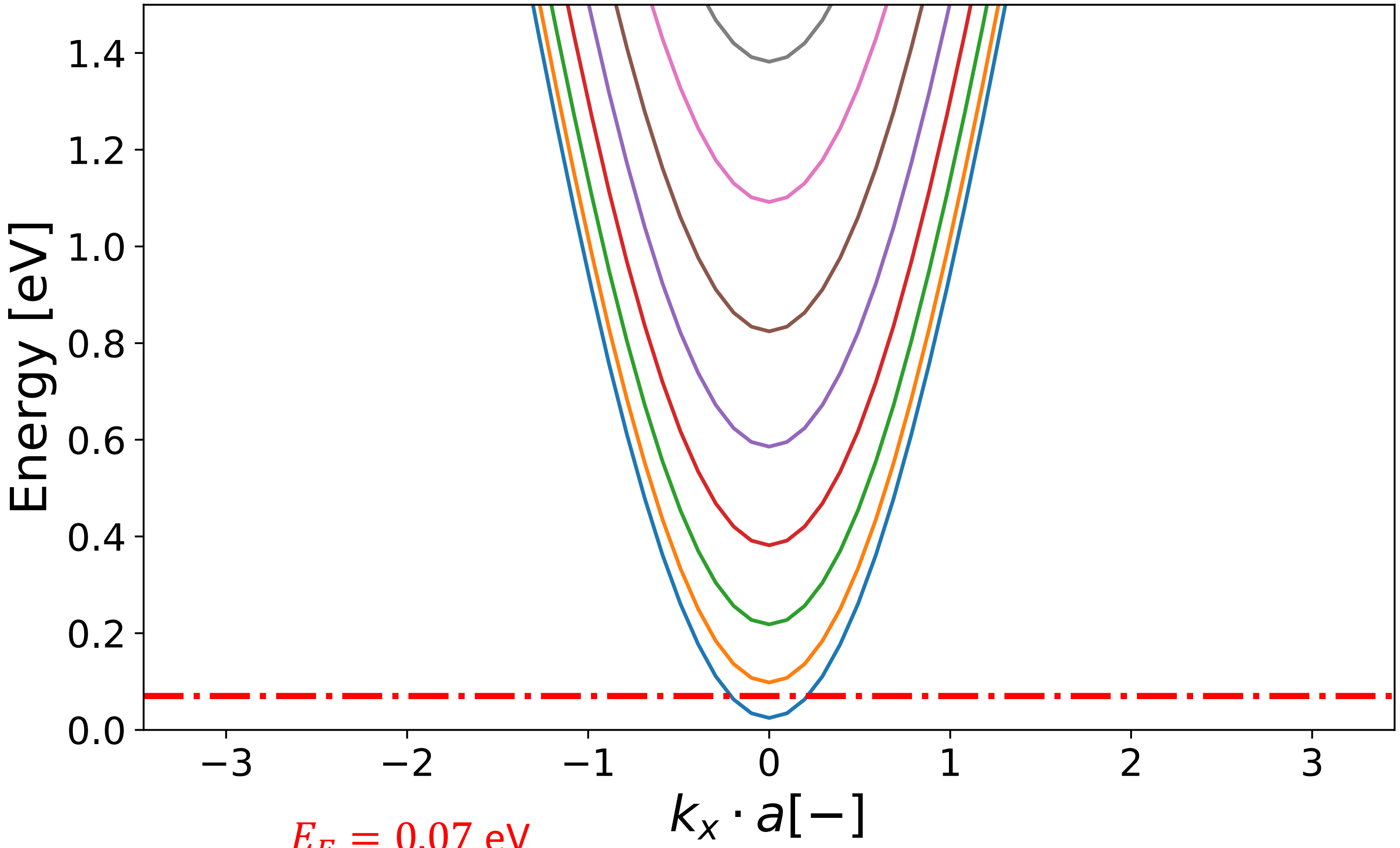
The number of modes in the leads depends on :

- The number of atoms along the width of the lead, fixed to 20.
- The magnetic field B
  - ➔ Varying to reach the resonances of the system
- The Fermi level E_F
  - ➔ Tuned in order to have 1 or 2 modes



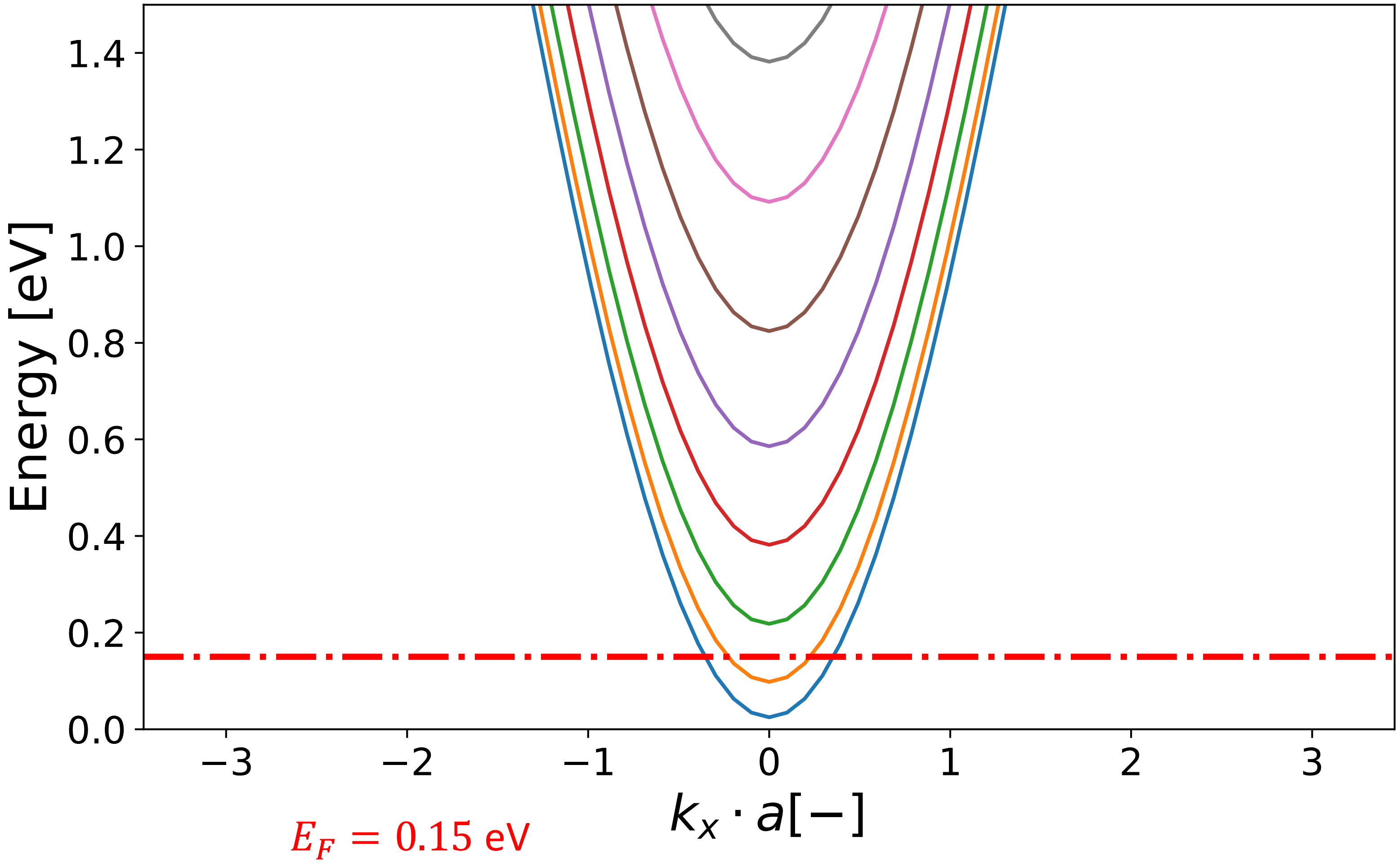
$B = 10$ T

➔ The magnetic field slightly influences the bandstructure



$E_F = 0.07$ eV

$B = 10$ T



$B = 10$ T

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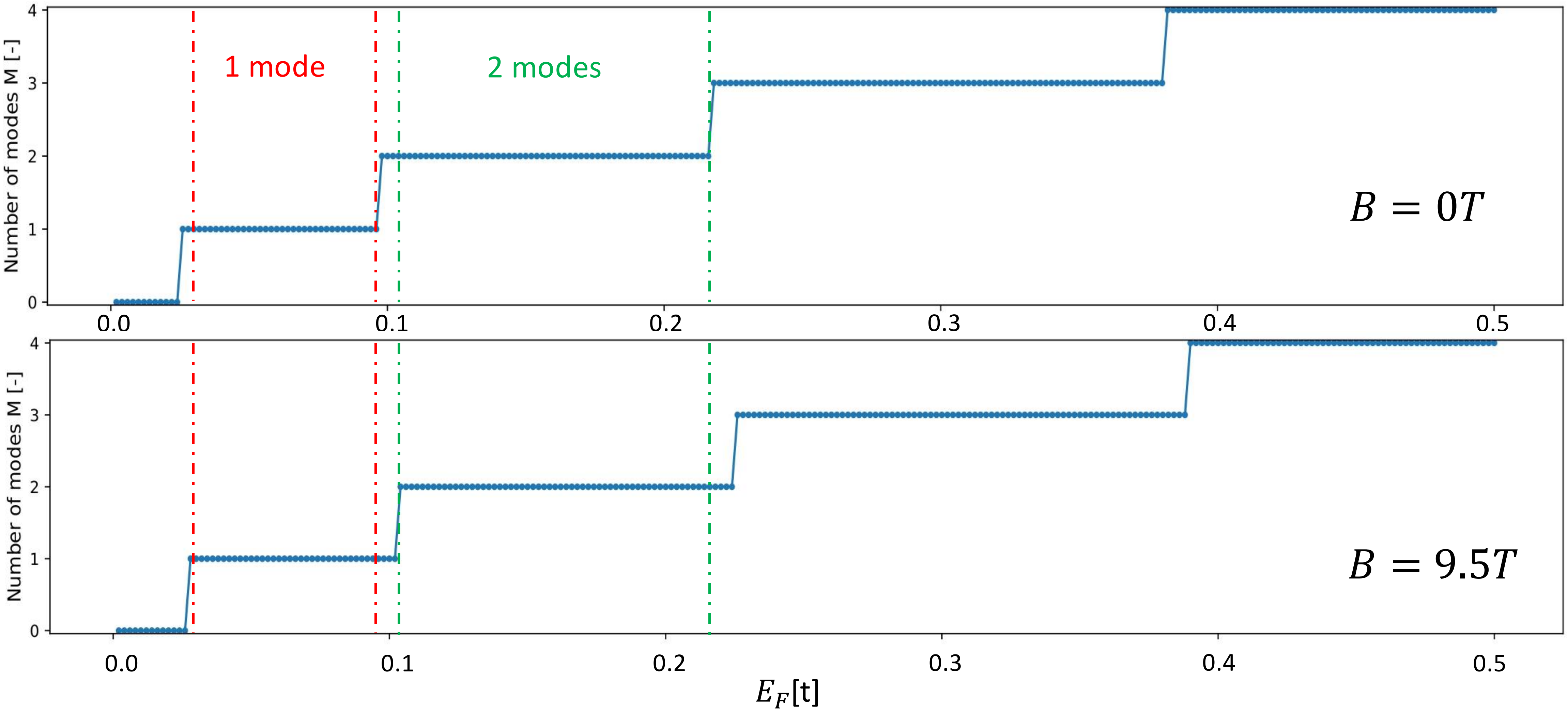
Effect of E_F



Modes in the leads

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Range of magnetic fields

The cylotron radius of electrons in a magnetic field is given by :

$$r_c = \frac{\hbar k_F}{eB}$$

Since magnetic focusing occurs when $r_c = \frac{d}{2n}$, the resonant magnetic fields are :

$$B_n = n\frac{2\hbar}{ed}\sqrt{\frac{E_F}{t}}$$

Using $E_F = \frac{\hbar^2 k^2}{2m^*}$ and $m^* = \frac{\hbar^2}{2a^2 t}$

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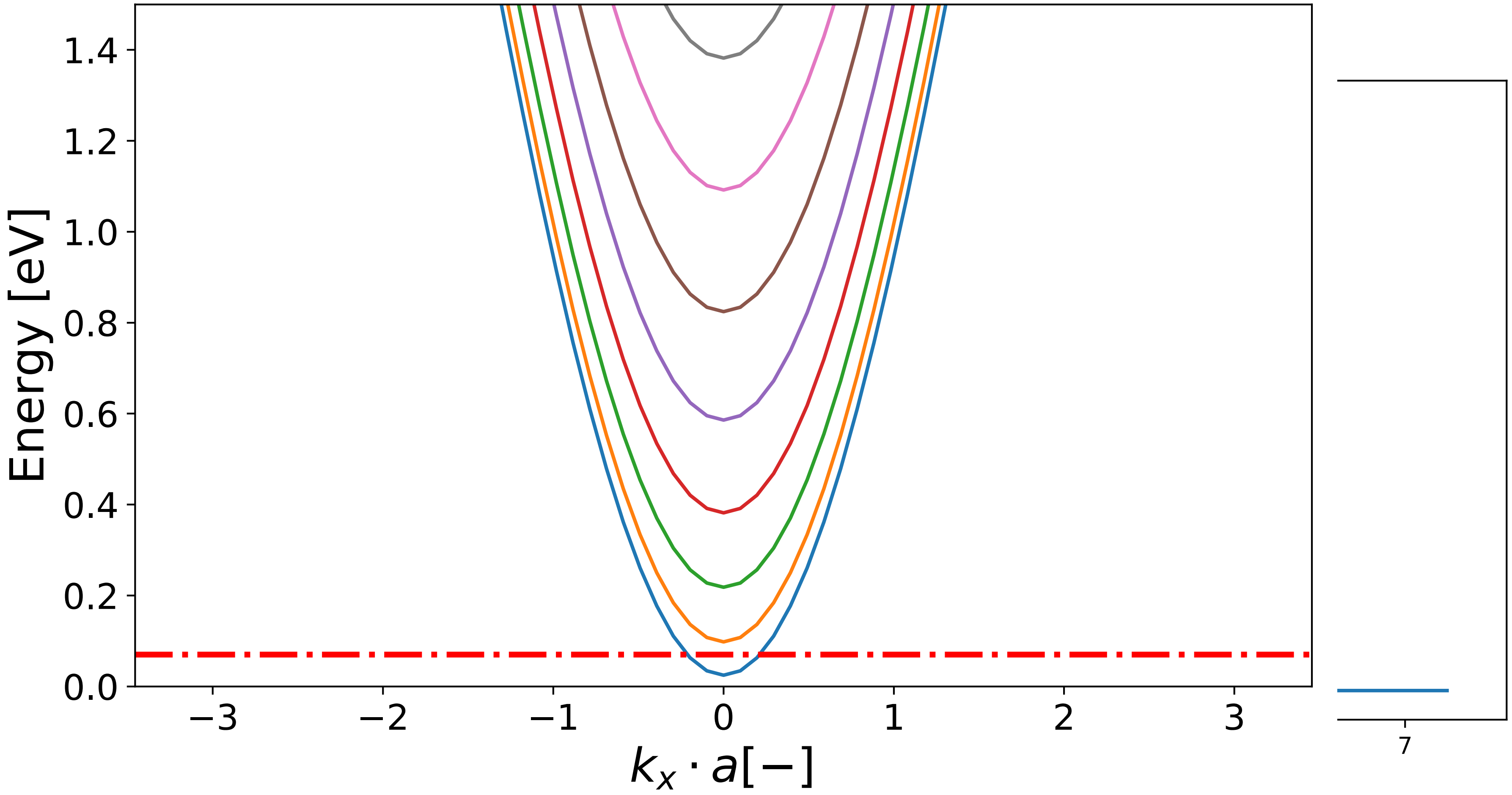
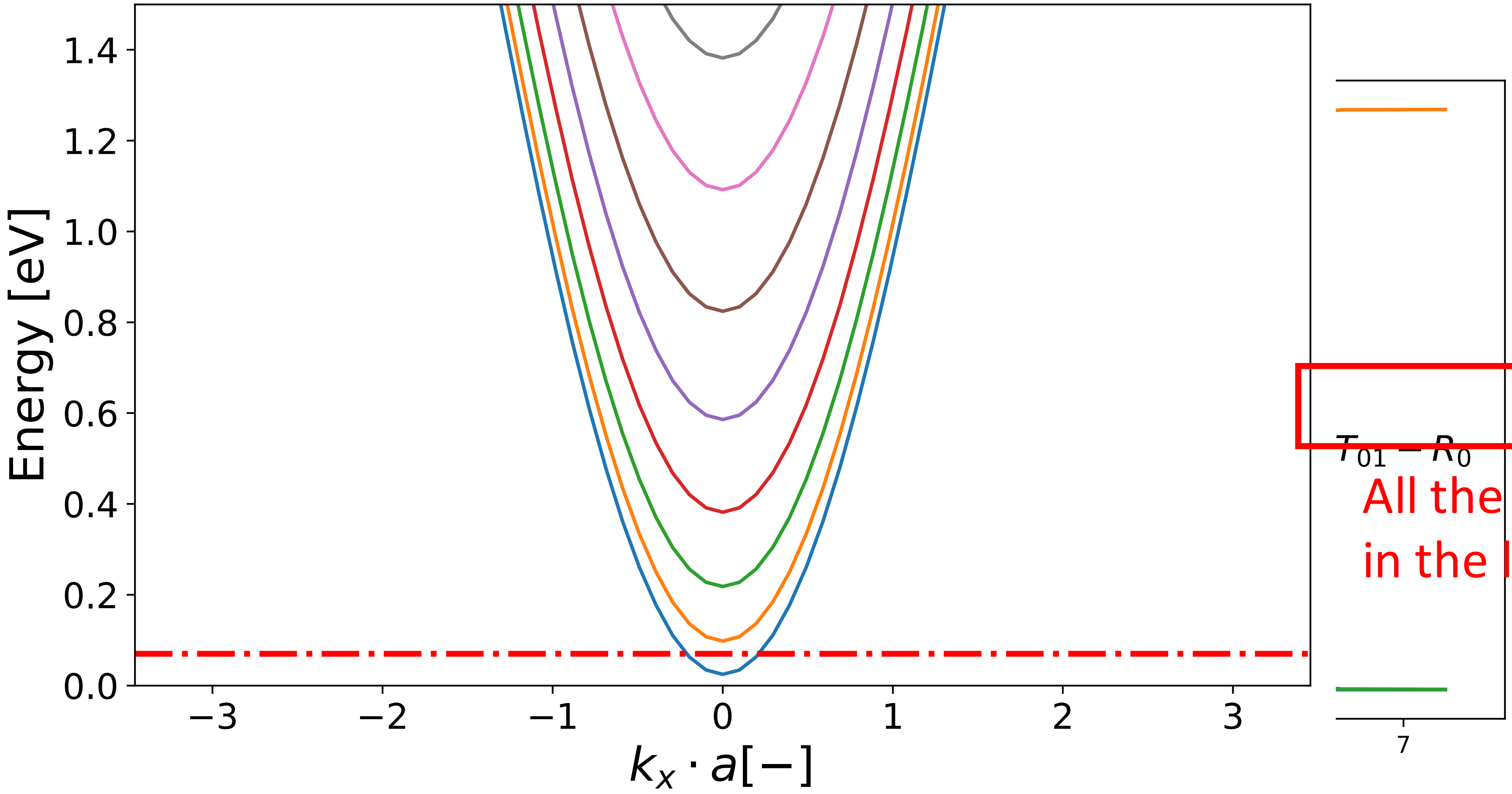


$E_F = 0.07$ eV

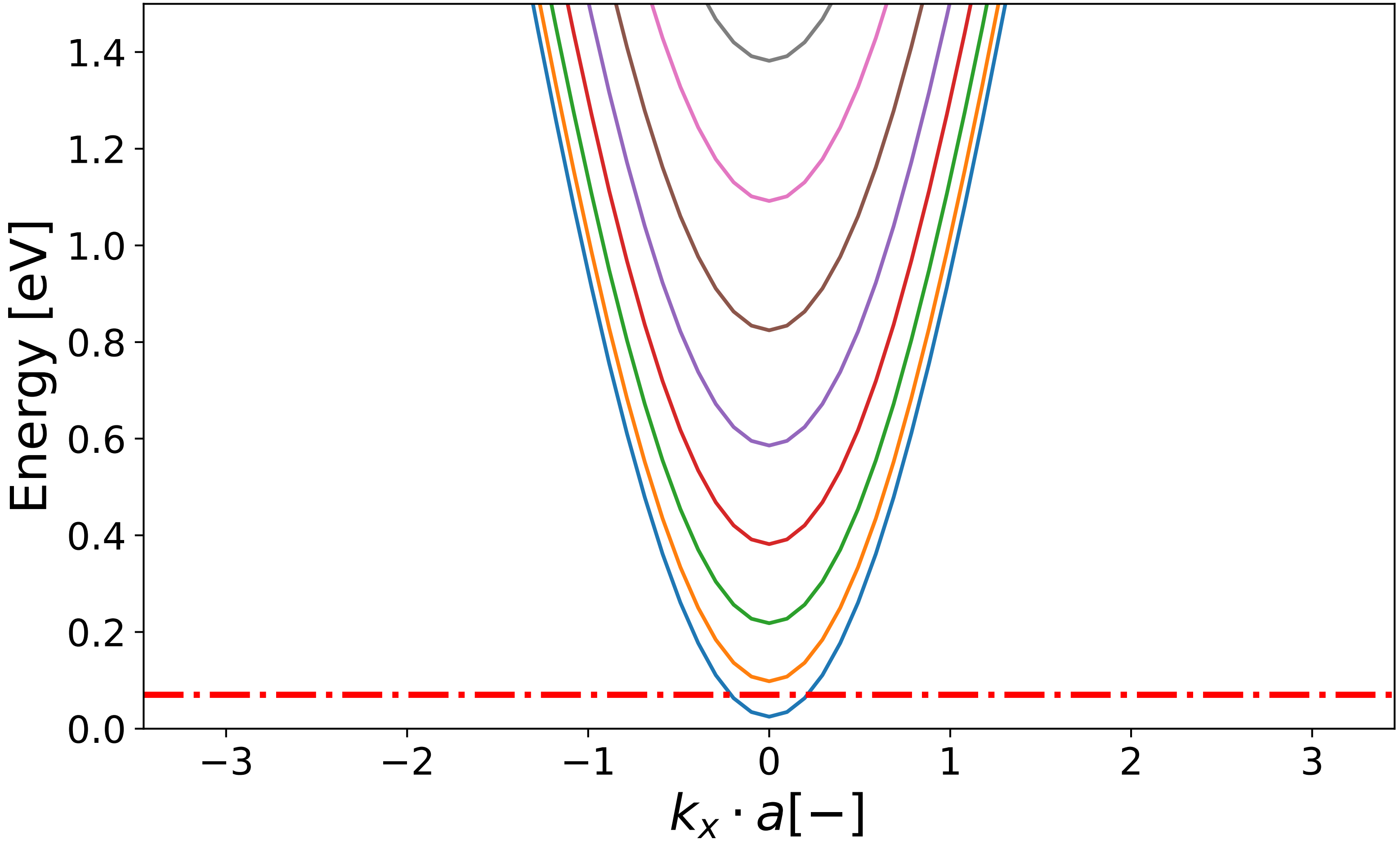
No transmission?

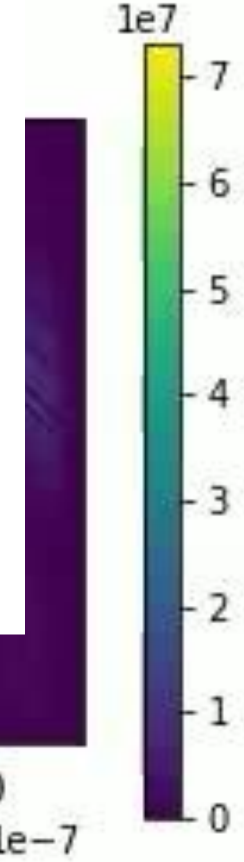


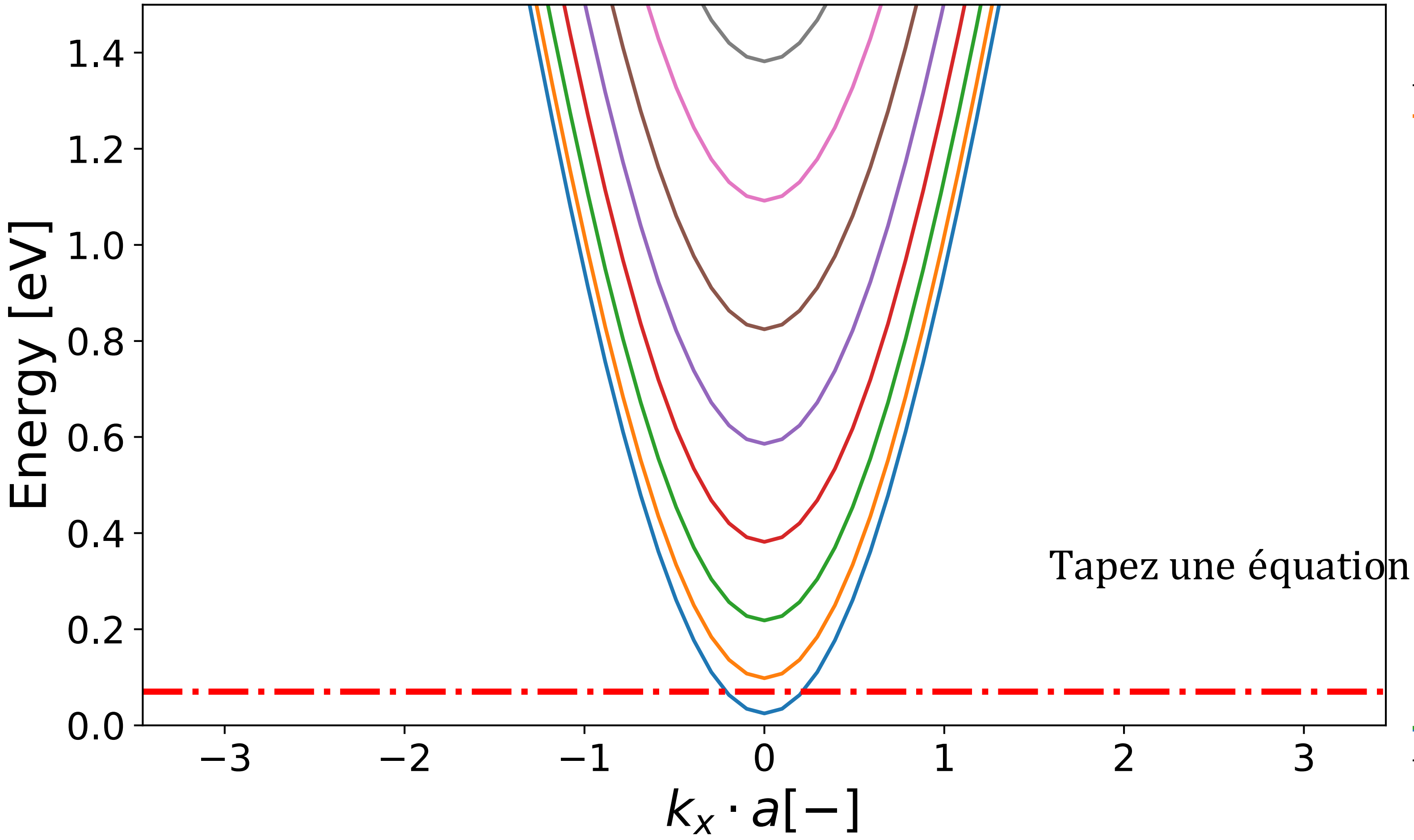
$E_F = 0.07$ eV

All the current is reflected in the left QPC he leads

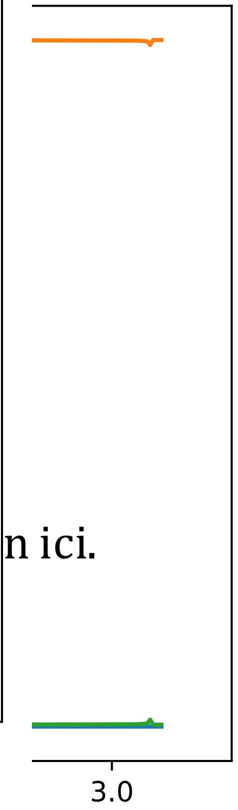
$E_F = 0.07$ eV







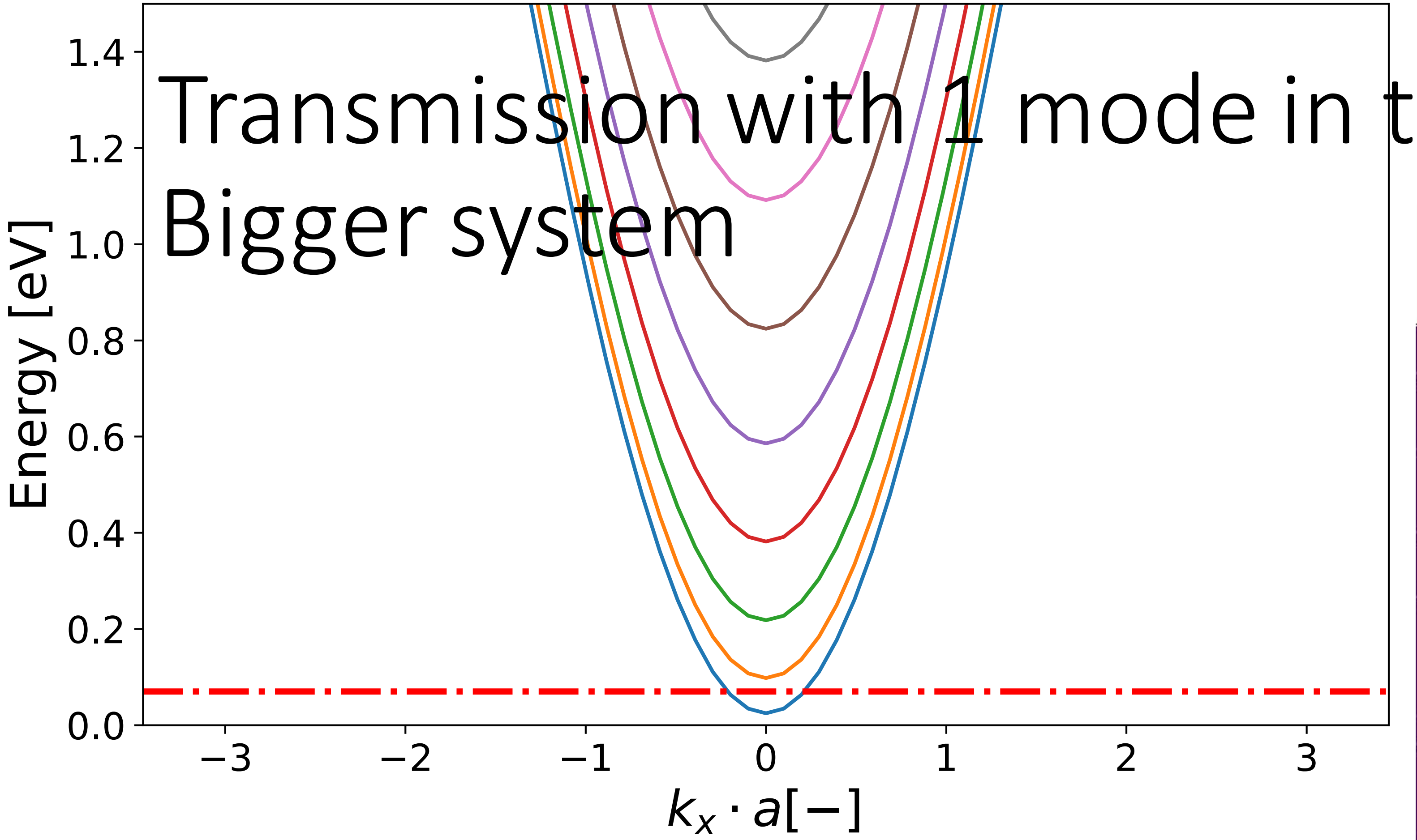
Tapez une équation ici.



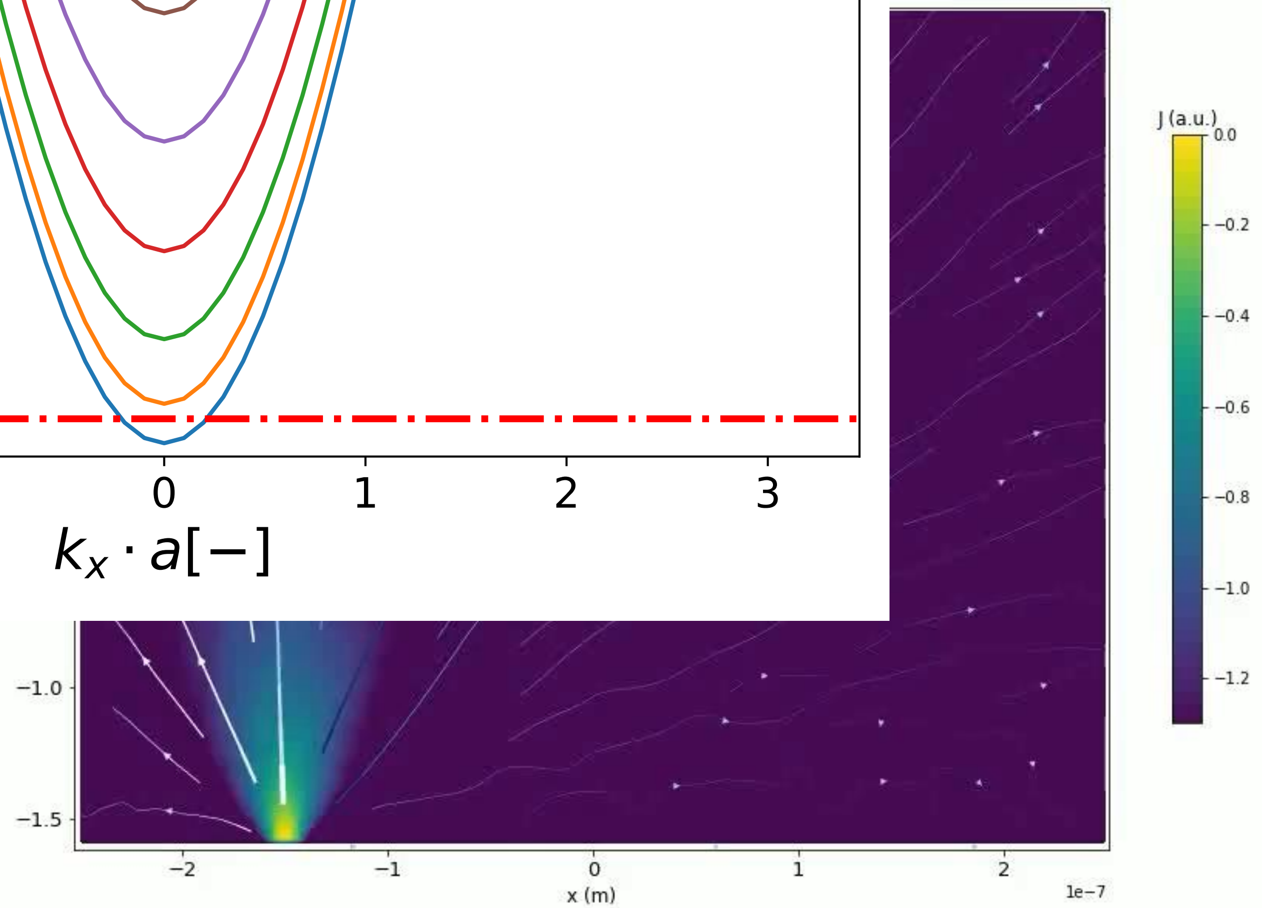
$E_F = 0.07$ eV

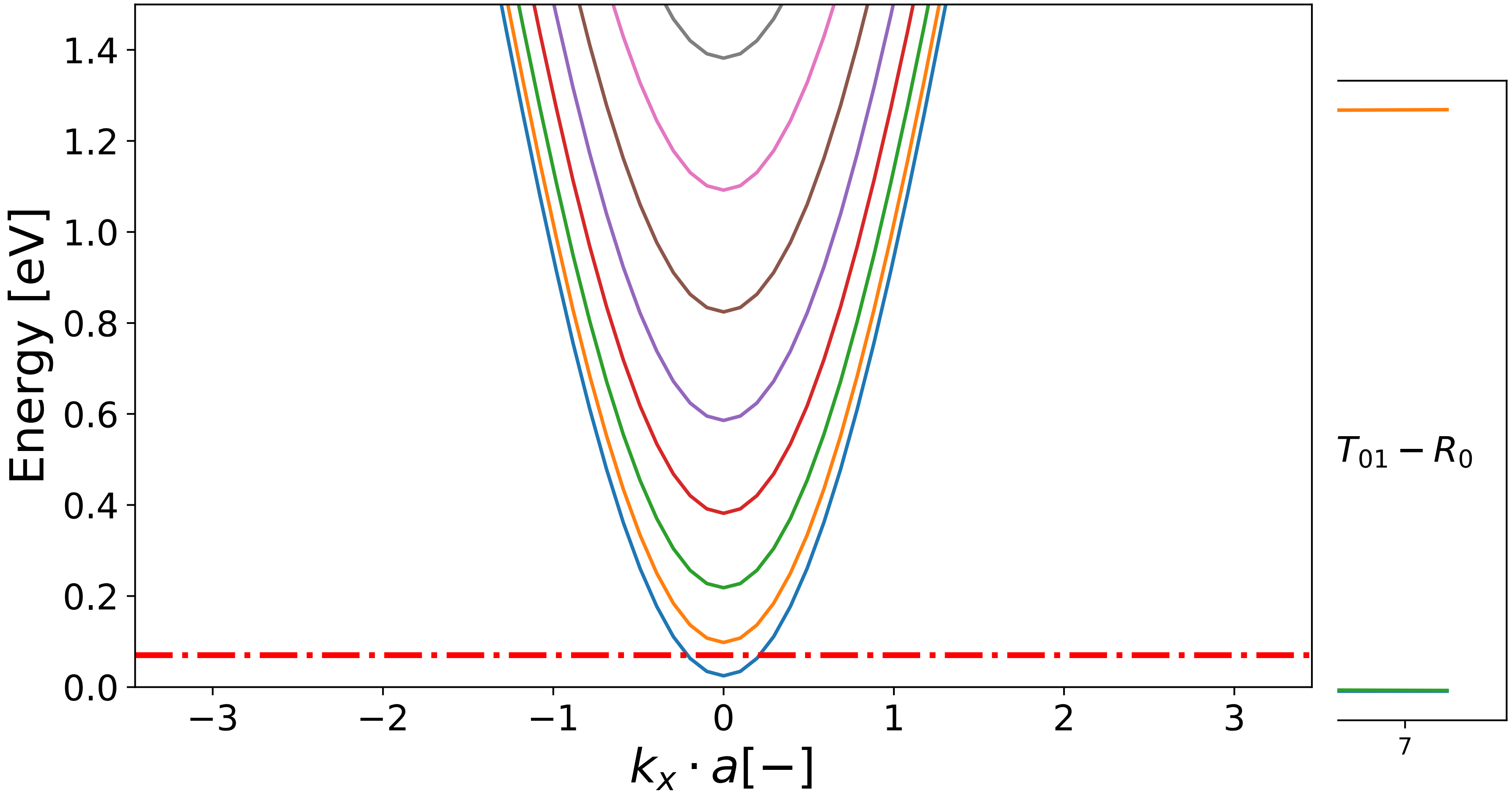
- L = 500 nm
- W = 320 nm
- d = 280 nm
- q = 20 nm

Transmission with 1 mode in the leads
Bigger system



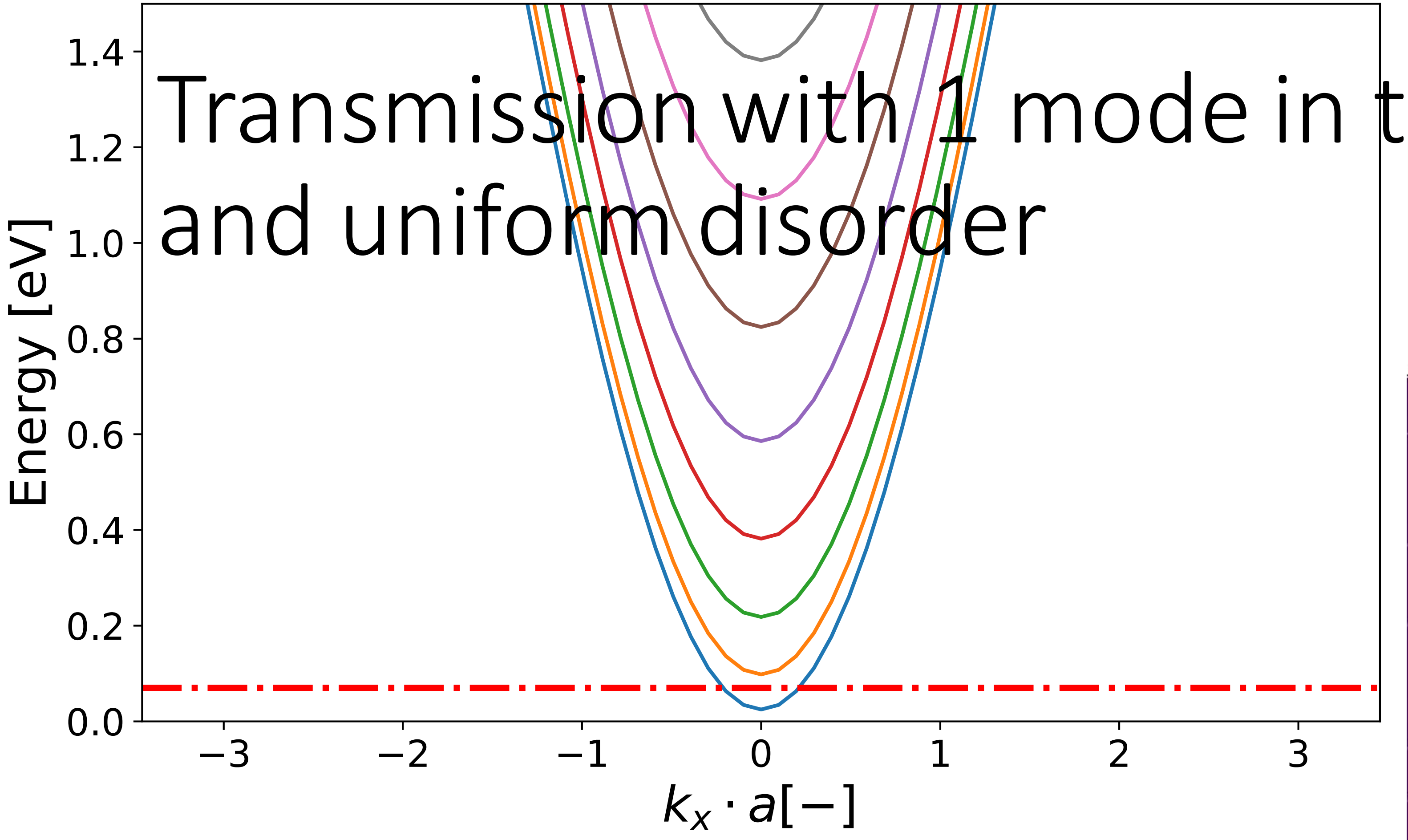
$E_F = 0.07$ eV



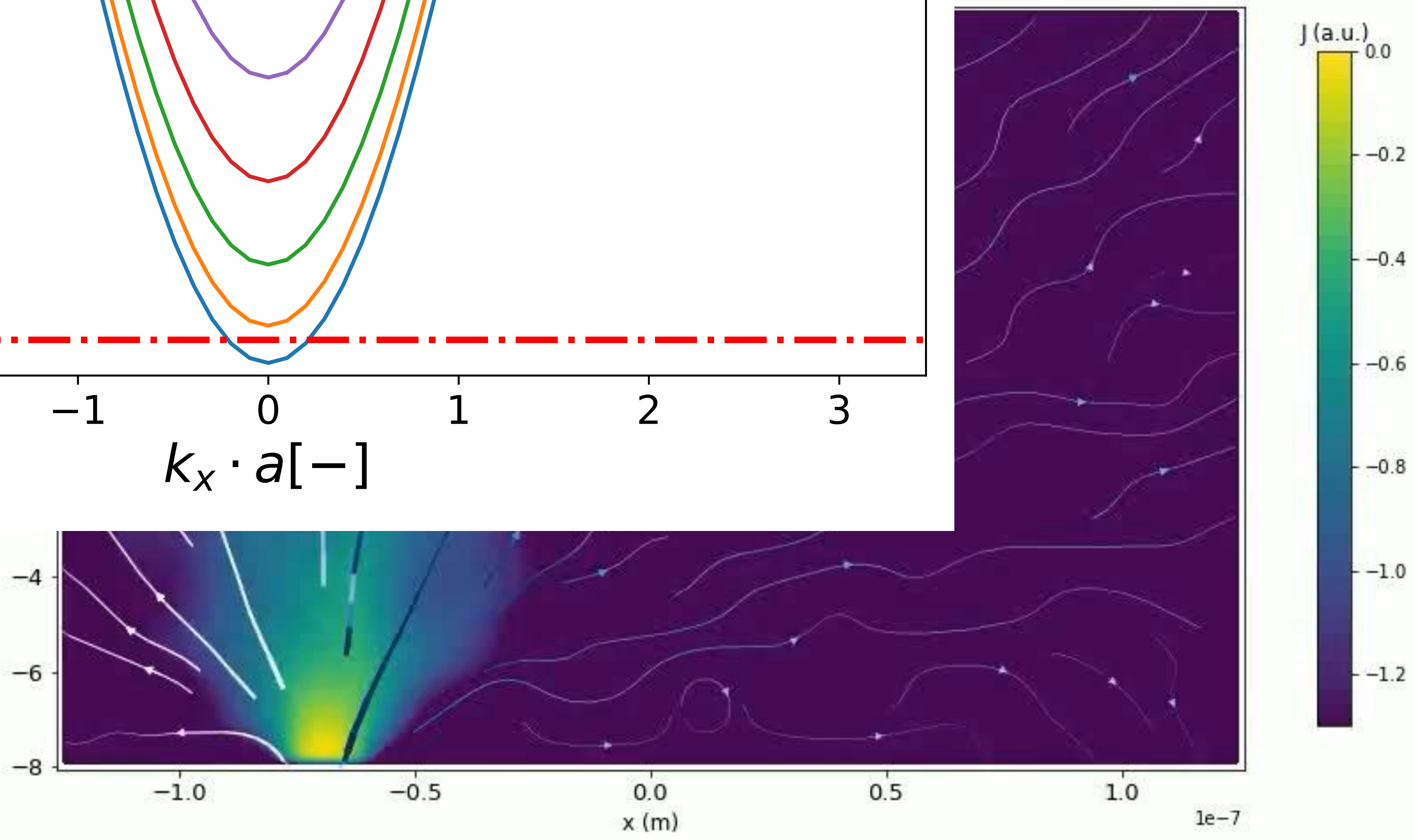


$E_F = 0.07$ eV

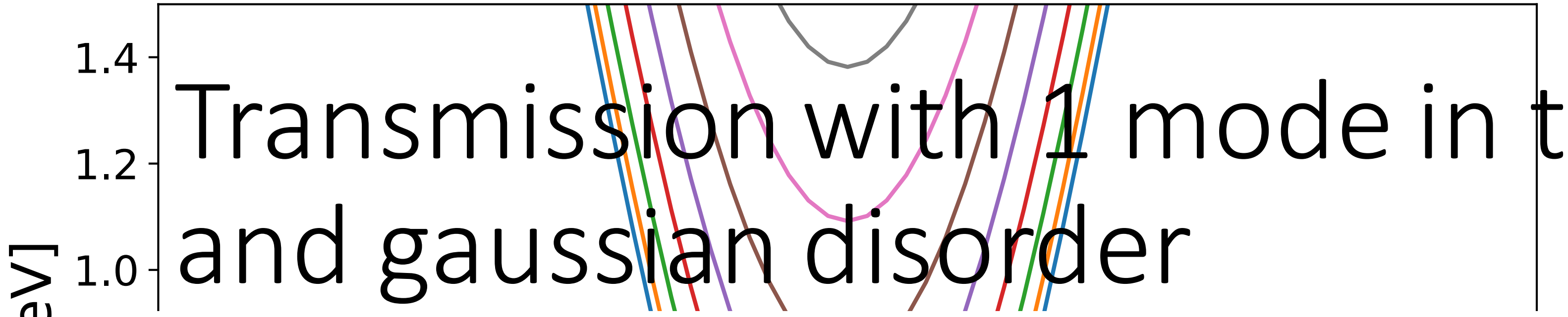
Transmission with 1 mode in the leads and uniform disorder



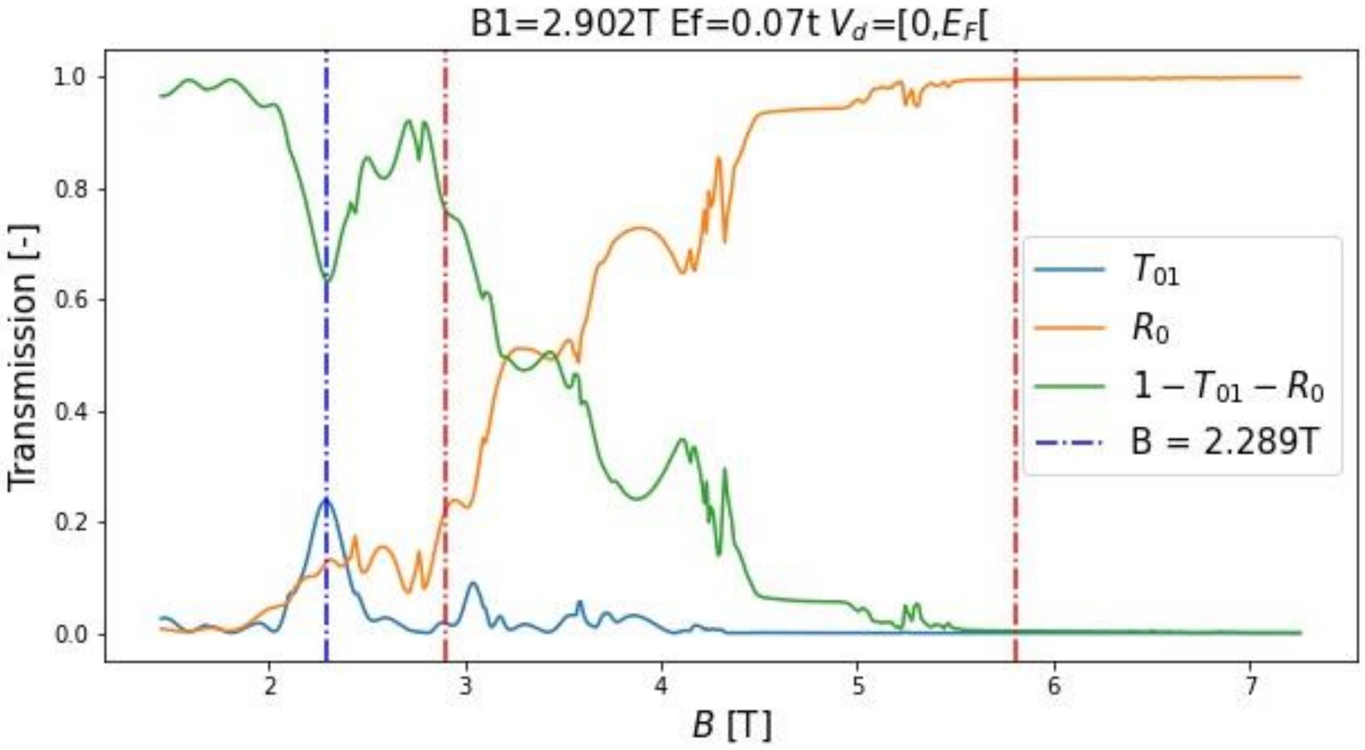
$E_F = 0.07$ eV

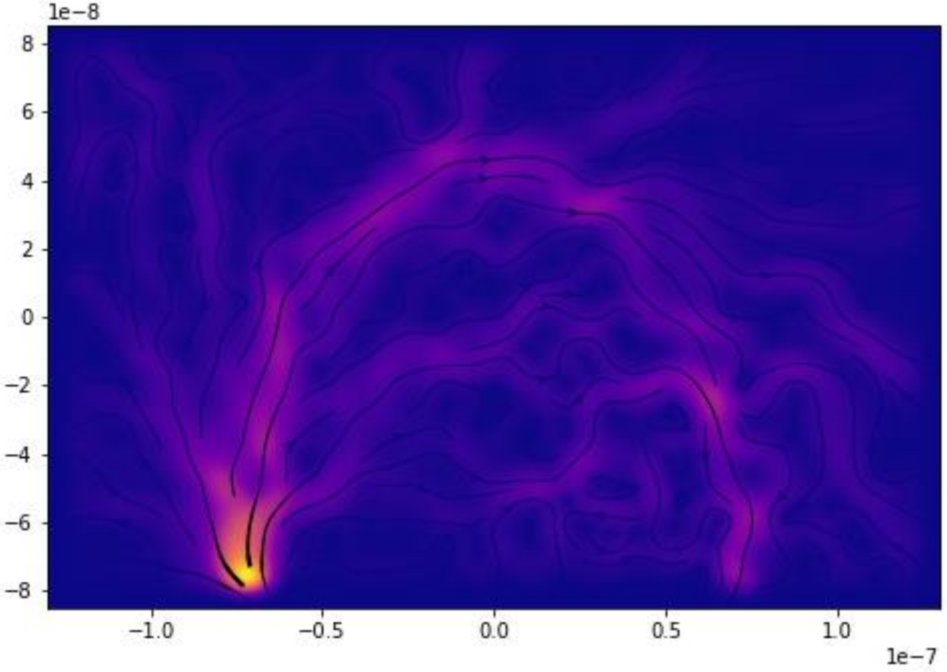


Transmission with 1 mode in the leads and gaussian disorder



$E_F = 0.07$ eV





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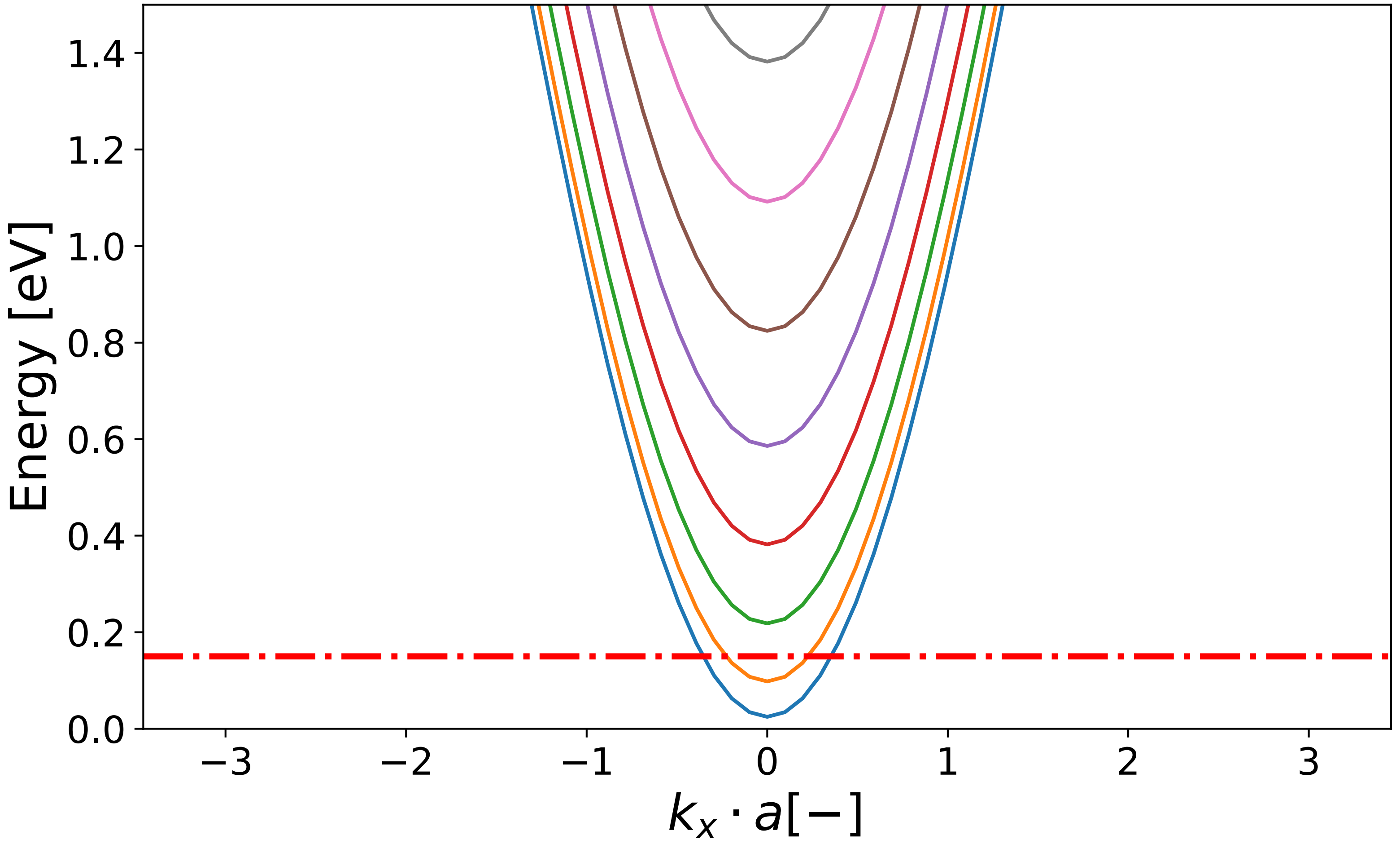
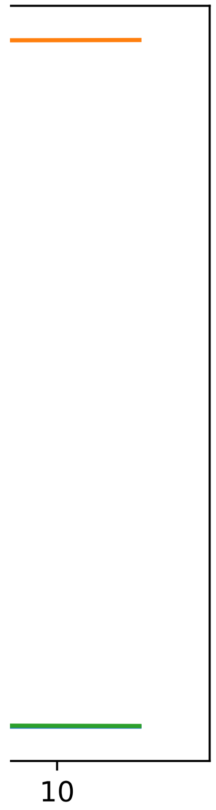


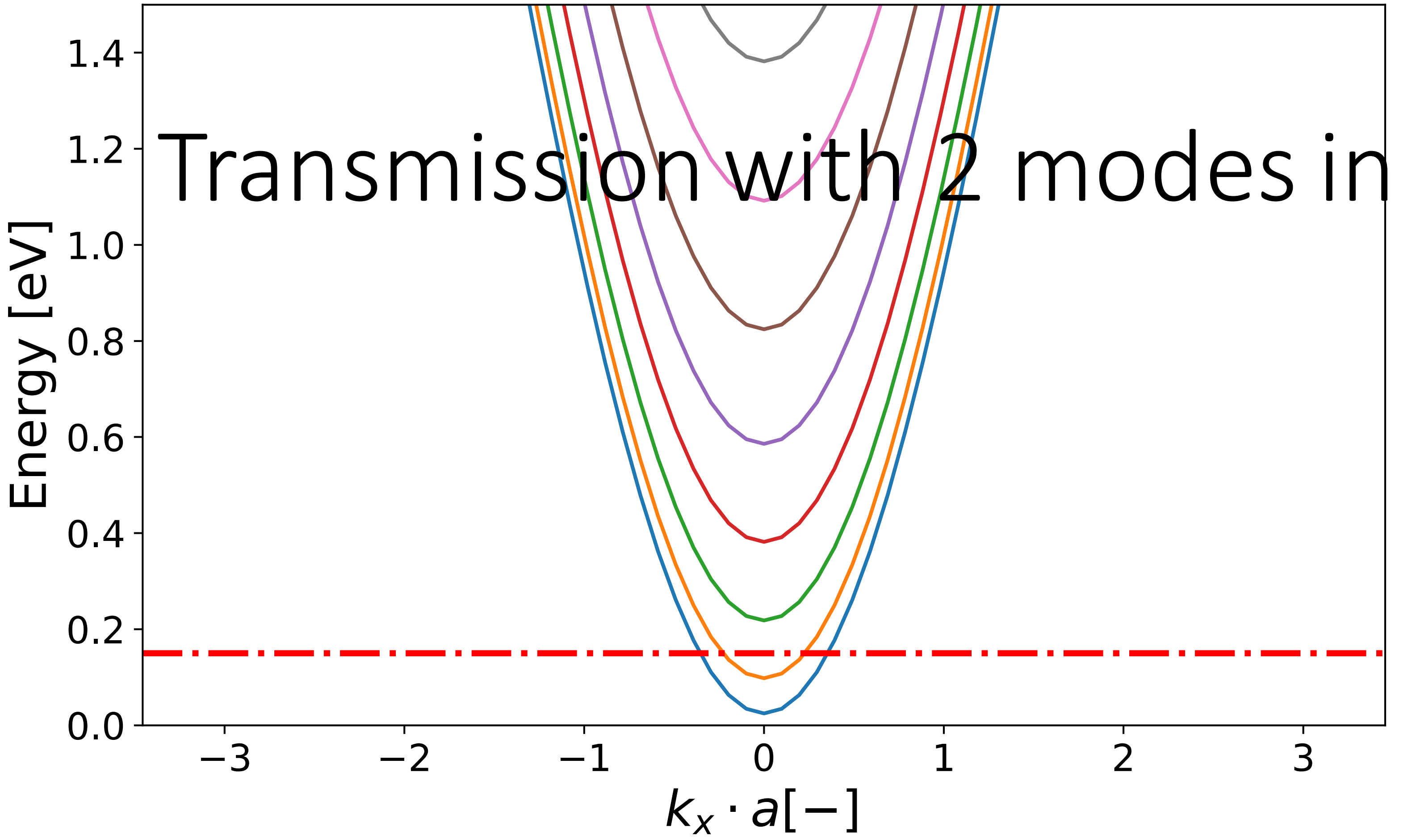


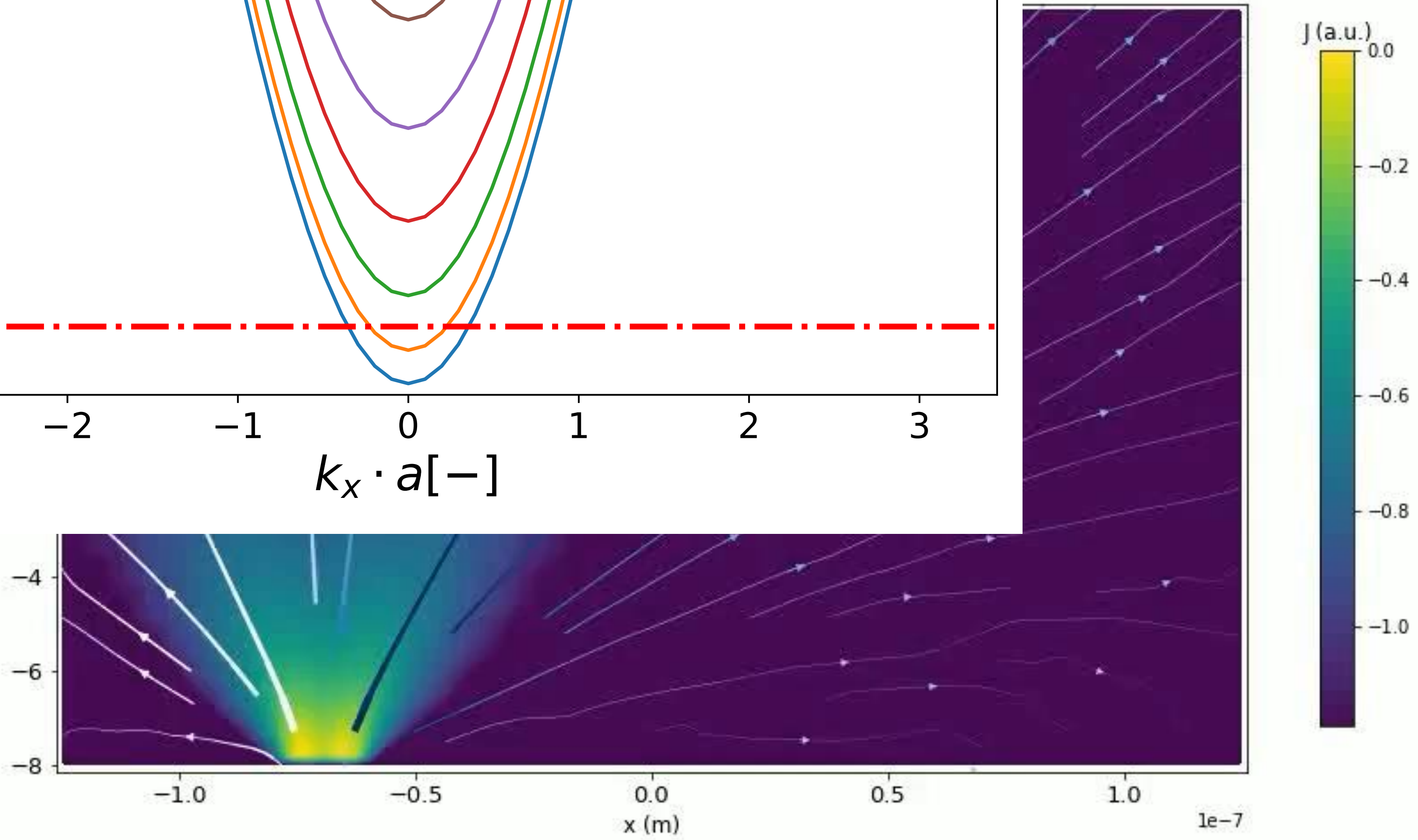
S

$E_F = 0.15$ eV

Transmission with 2 modes in the leads

$E_F = 0.15$ eV





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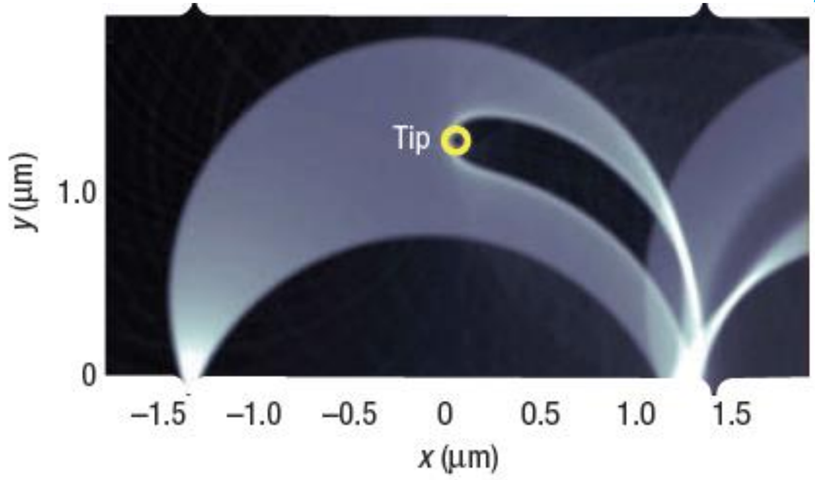
SPM images - approach

1. Choose magnetic field: $B_{1,exp}$
2. Calculate transmission without tip: T_{noTip}

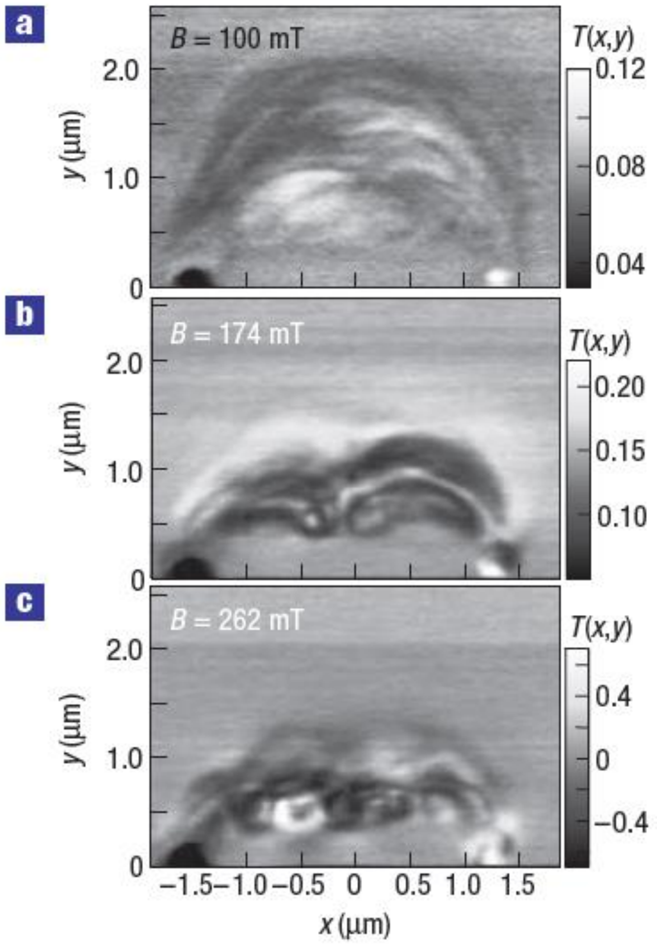
Iterate over whole system

3. Add tip to system at (x_{tip}, y_{tip}): positive Gaussian potential: $V = \eta E_F$
4. Calculate transmission with tip: T_{Tip}

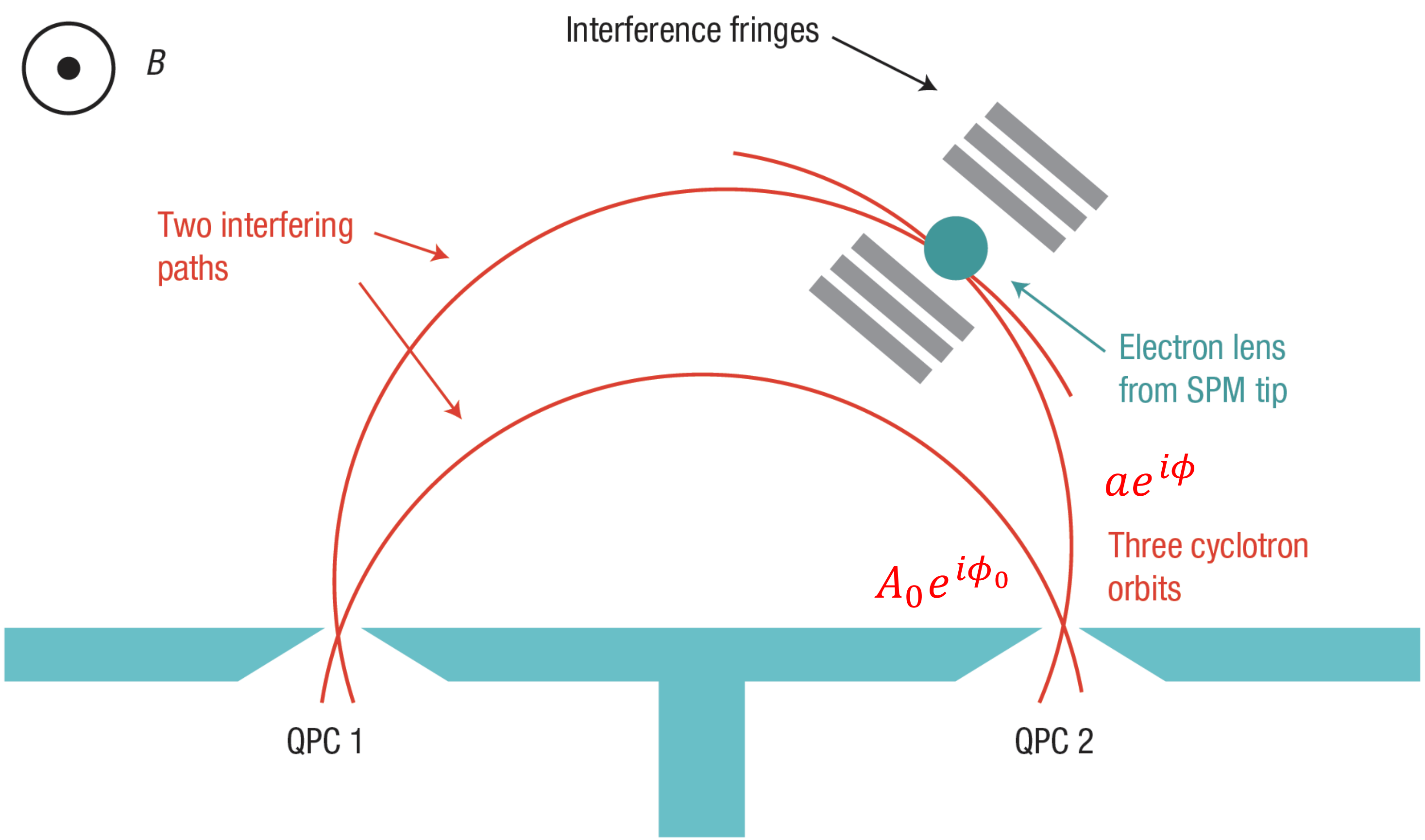


5. Store the difference in transmission in a 2D-matrix: $\Delta T_{array}[i,j] = \mathrm{T_{Tip}} - \mathrm{T_{noTip}}$

6. Plot ΔT_{array} with colorbar



SPM images - expectations



$$\Rightarrow \Delta T \propto \cos(\phi - \phi_0)$$

- Phase of the deflected trajectory is proportional to the classical action accumulated along the trajectory :

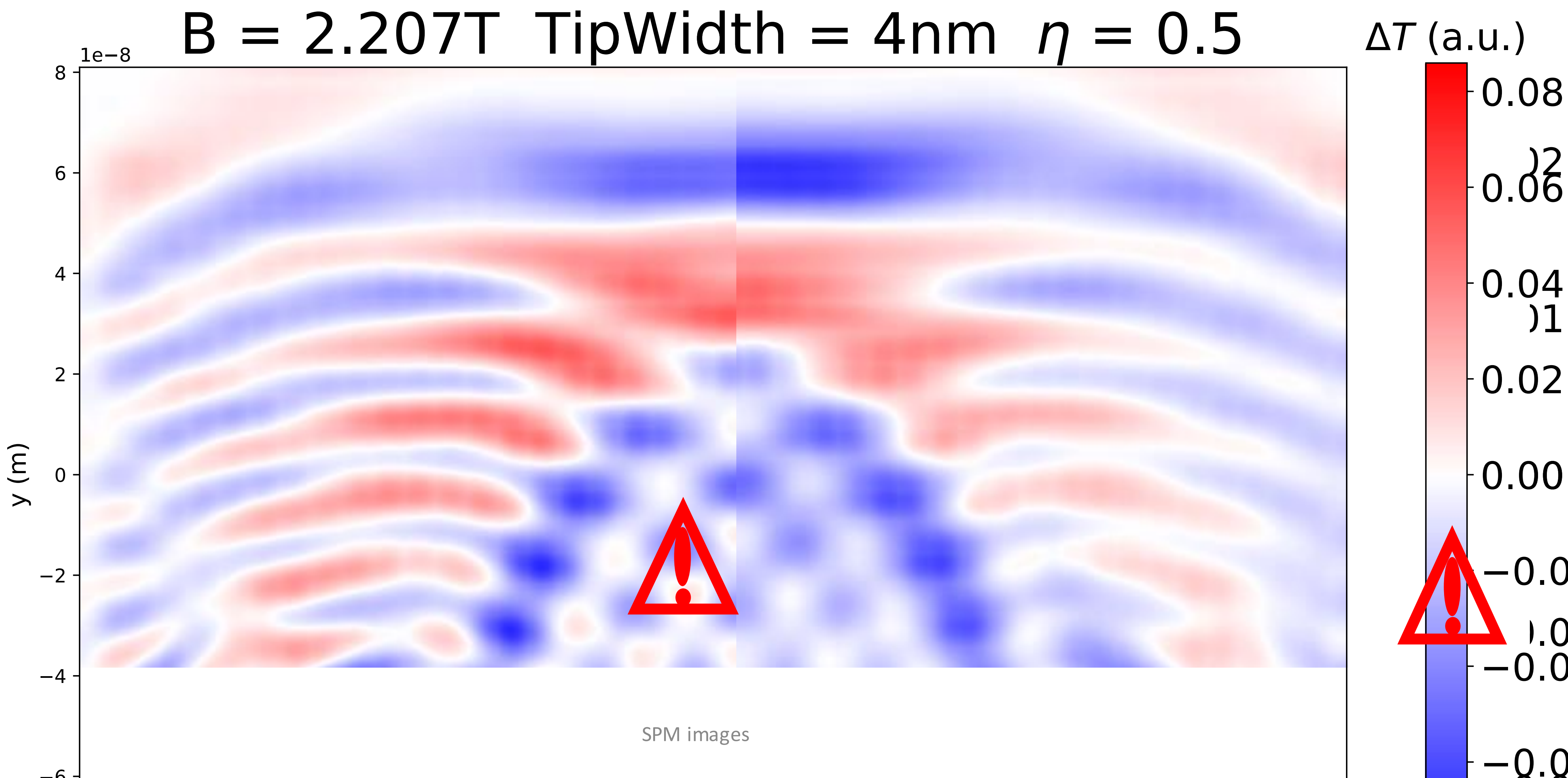
$$\phi = \frac{S}{\hbar}$$

- This leads to an equation for the fringe spacing :

$$d = \frac{\lambda_F}{2} \csc\left(\frac{\theta}{2}\right)$$

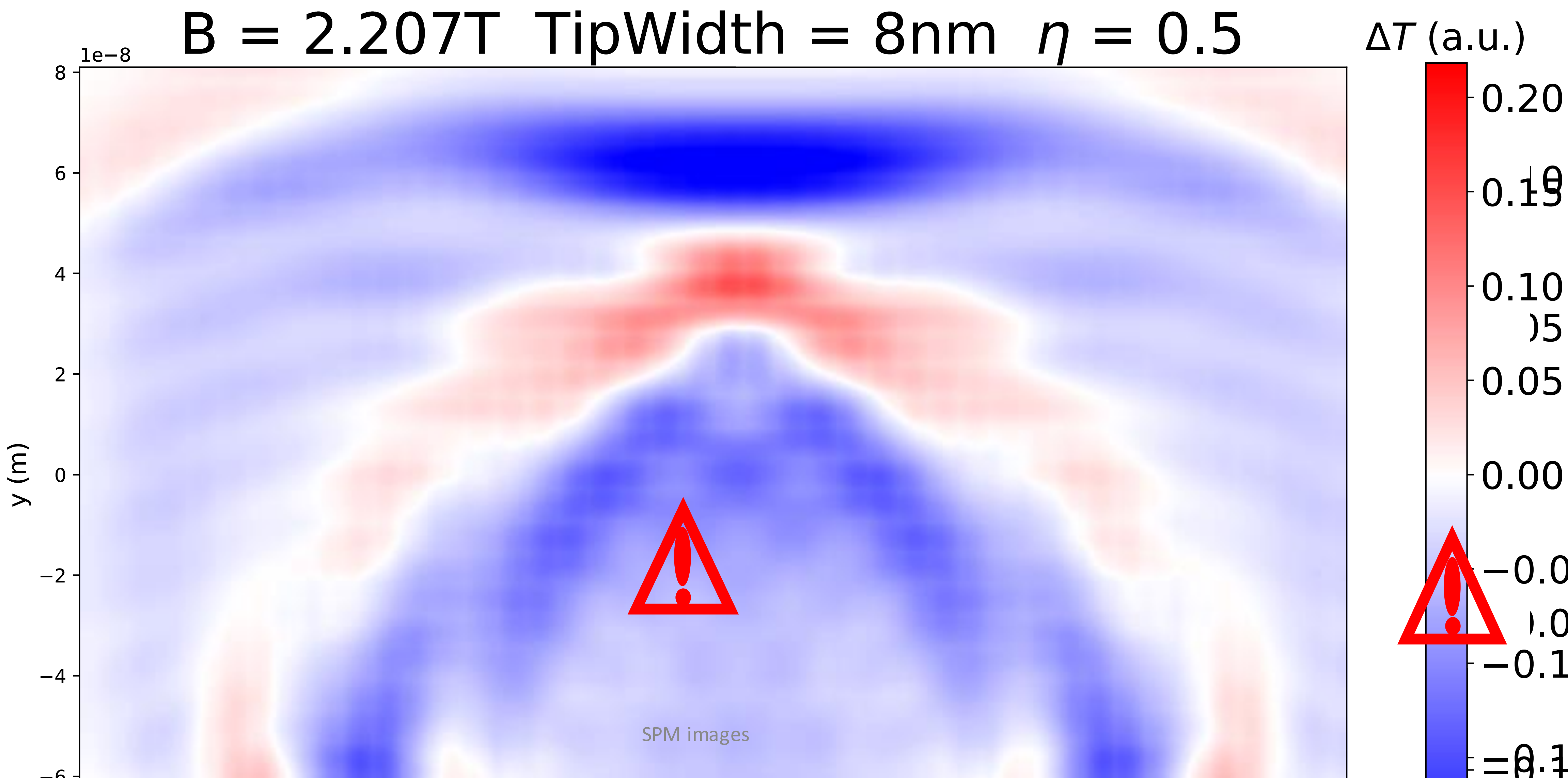
$$\Rightarrow d_{min} = \frac{\lambda_F}{2} \text{ when } \theta = \pi$$

Influence of the tip width



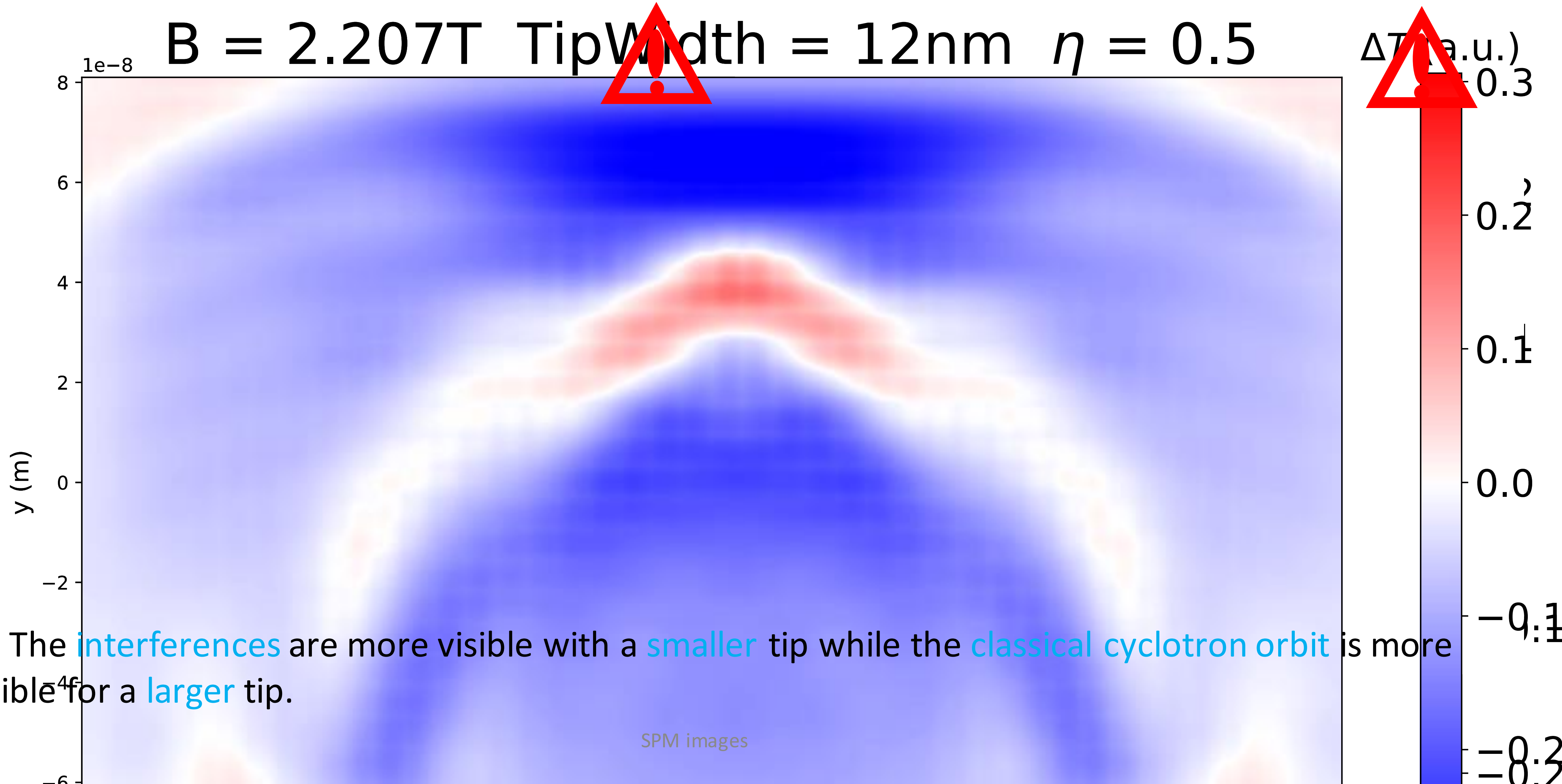
SPM images

Influence of the tip width



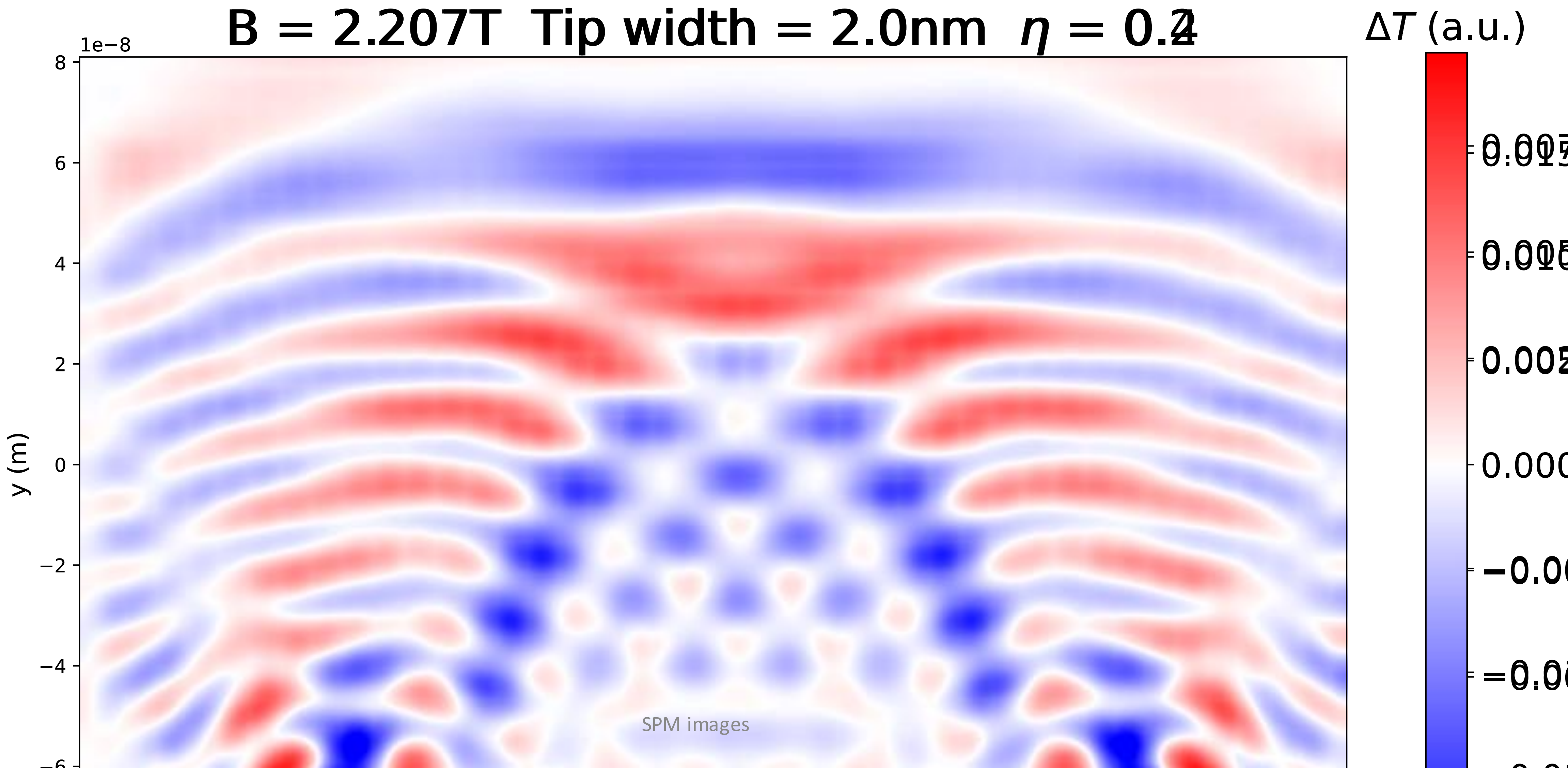
SPM images

Influence of the tip width



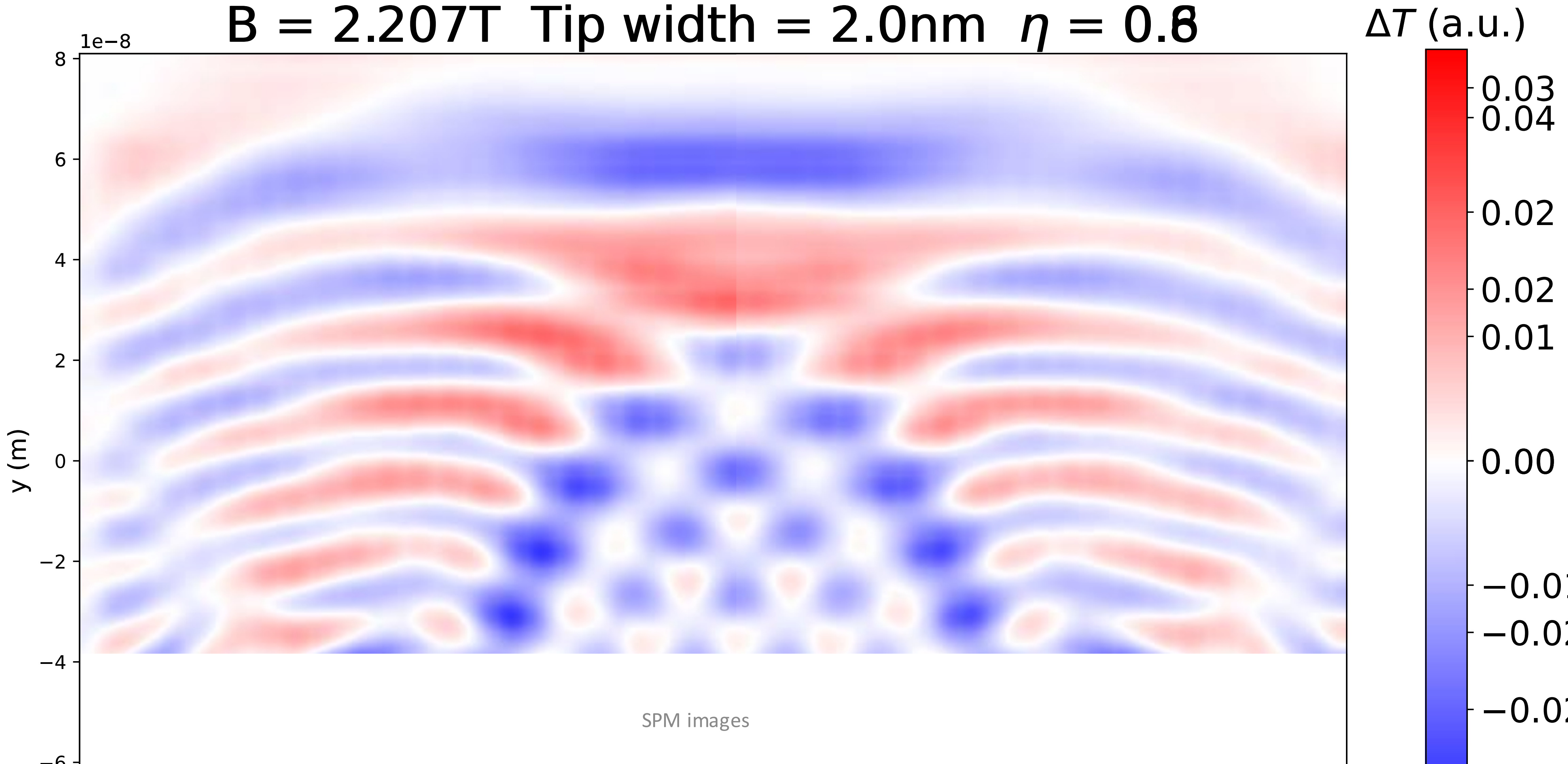
➔ The interferences are more visible with a smaller tip while the classical cyclotron orbit is more visible for a larger tip.

Influence of η – tip width of 2nm

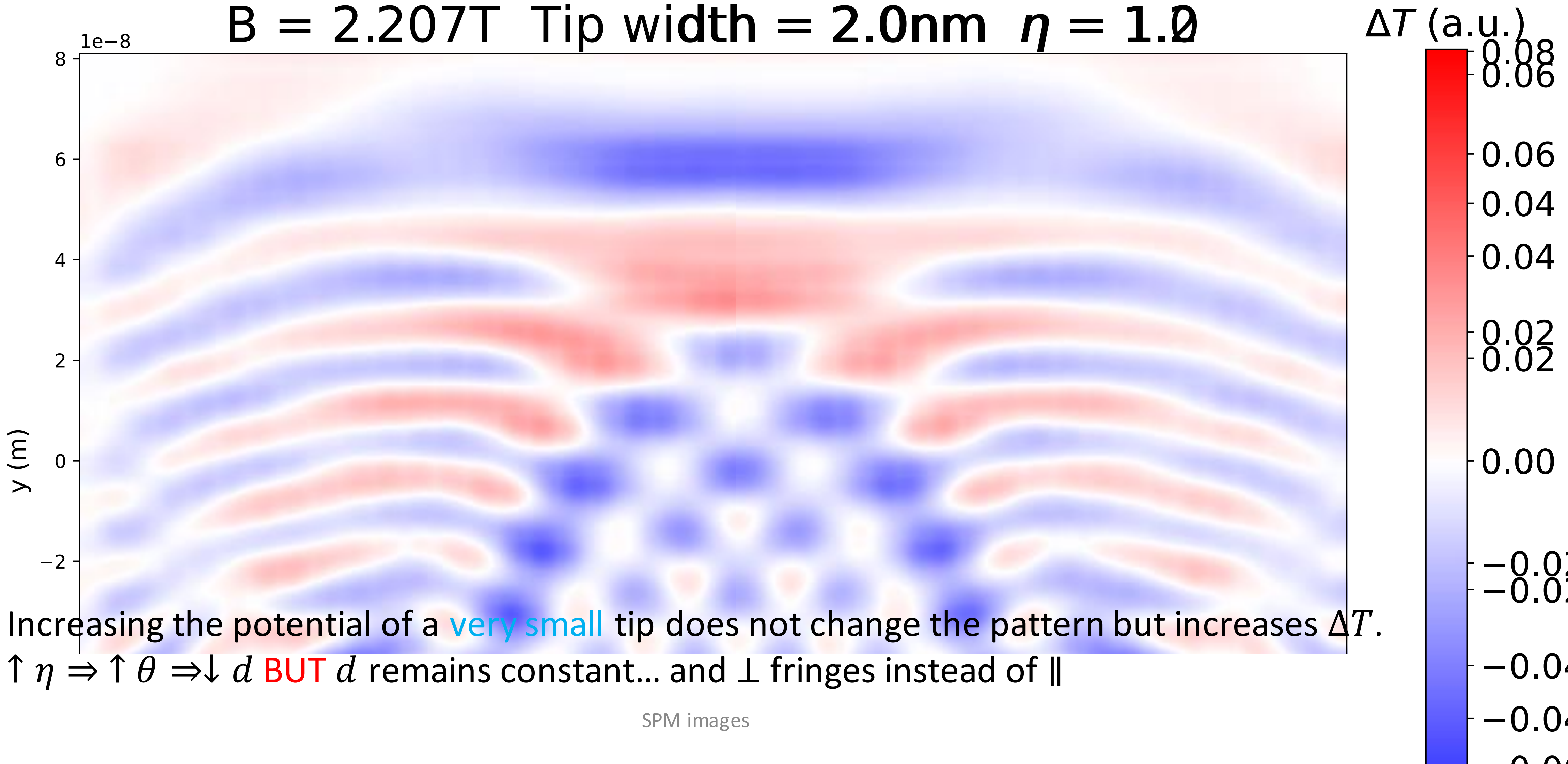


SPM images

Influence of η – tip width of 2nm



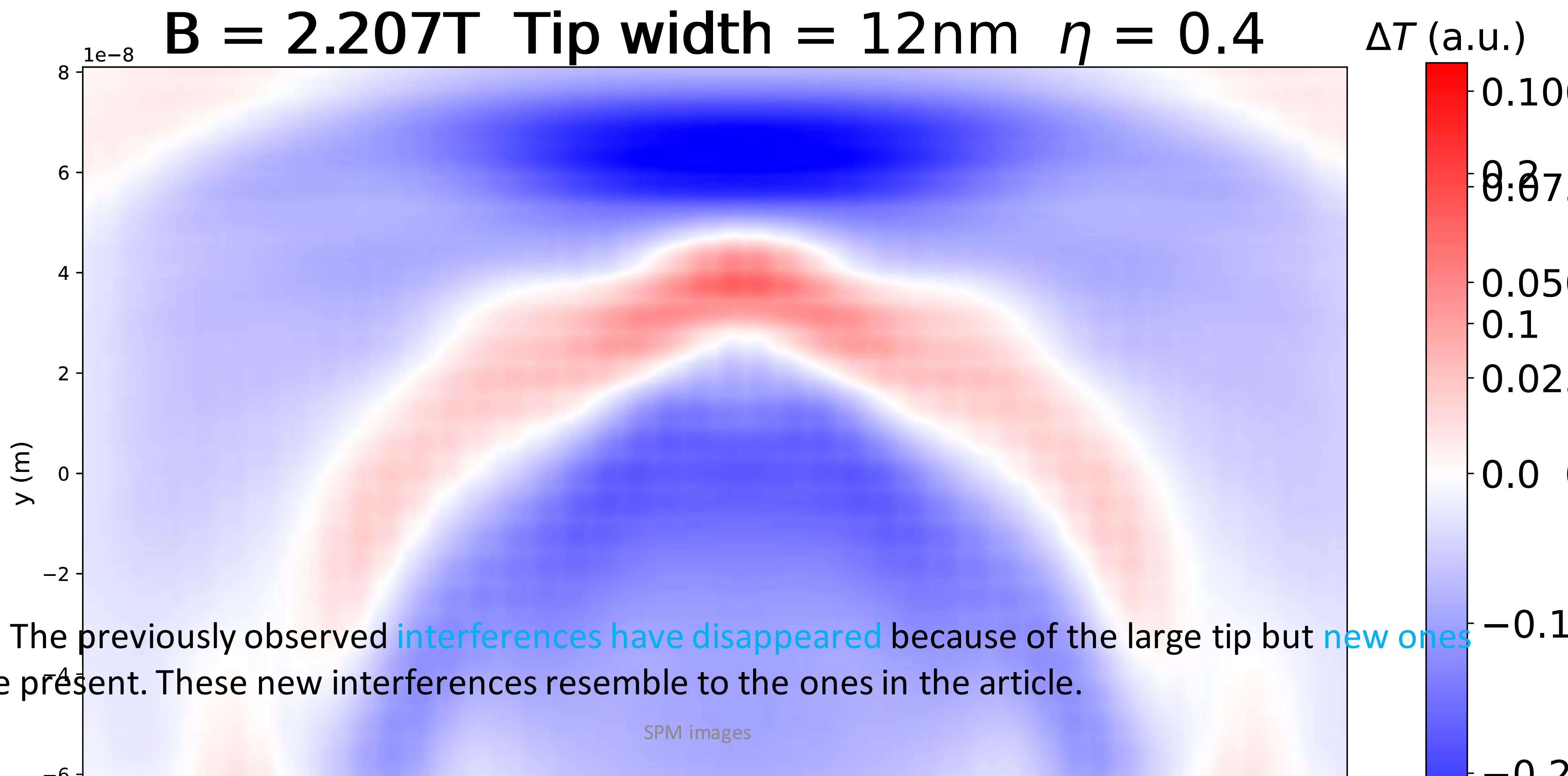
Influence of η – tip width of 2nm



➔ Increasing the potential of a very small tip does not change the pattern but increases ΔT.

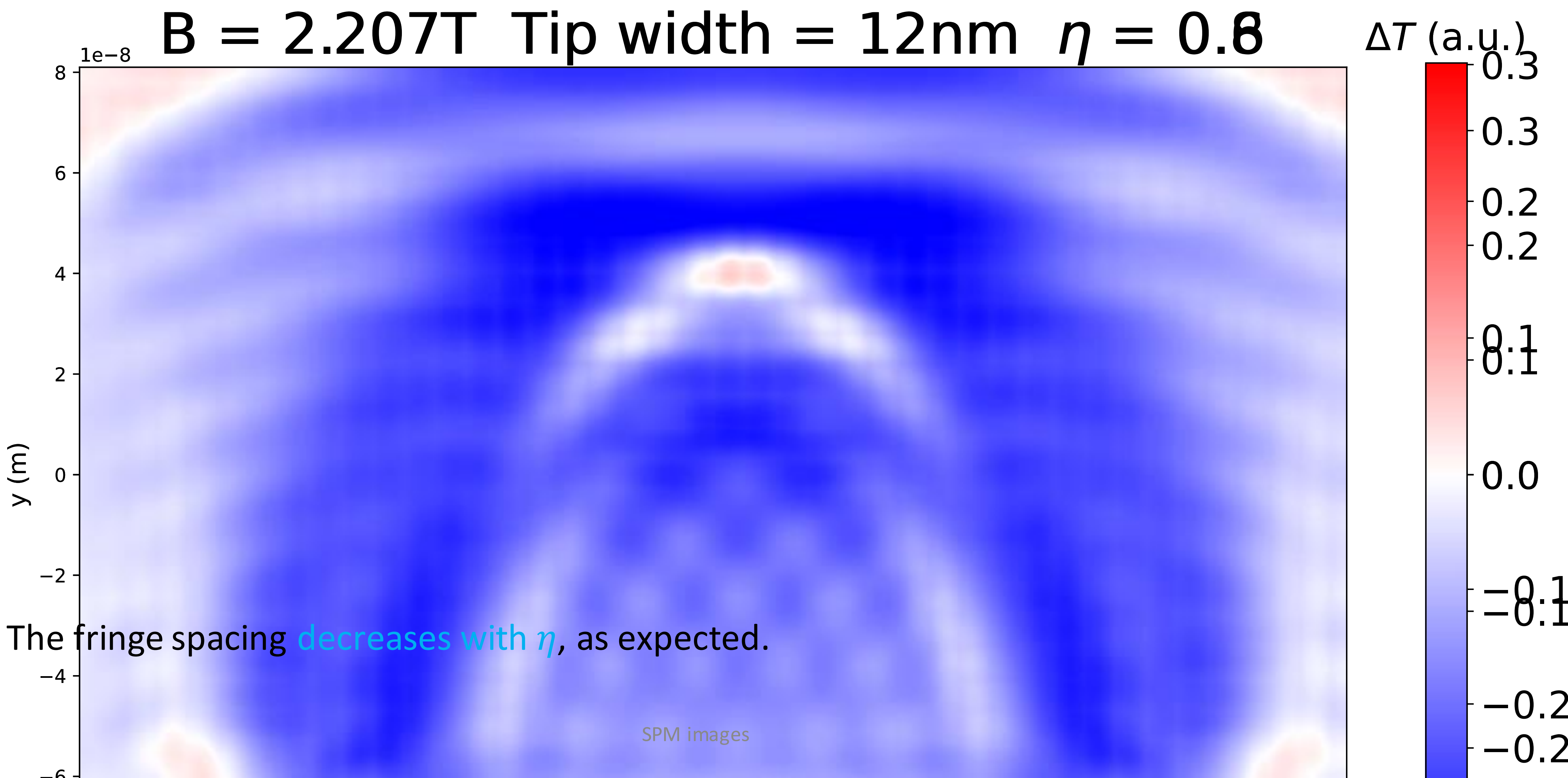
➔ $\uparrow \eta \Rightarrow \uparrow \theta \Rightarrow \downarrow d$ BUT d remains constant… and ⊥ fringes instead of ∥

Influence of η – tip width of 12nm



➔ The previously observed interferences have disappeared because of the large tip but new ones are present. These new interferences resemble to the ones in the article.

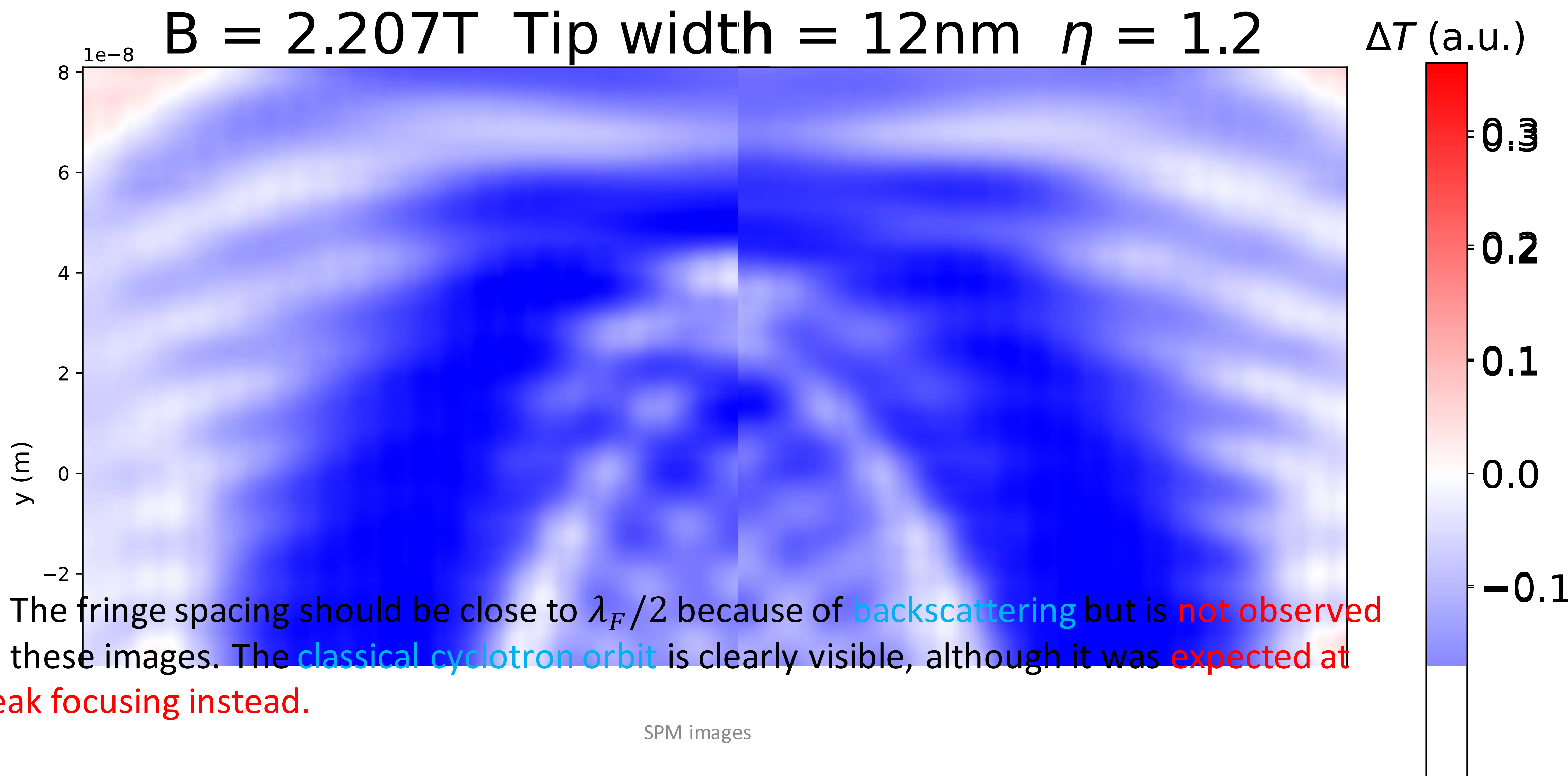
Influence of η – tip width of 12nm



➔ The fringe spacing decreases with η, as expected.

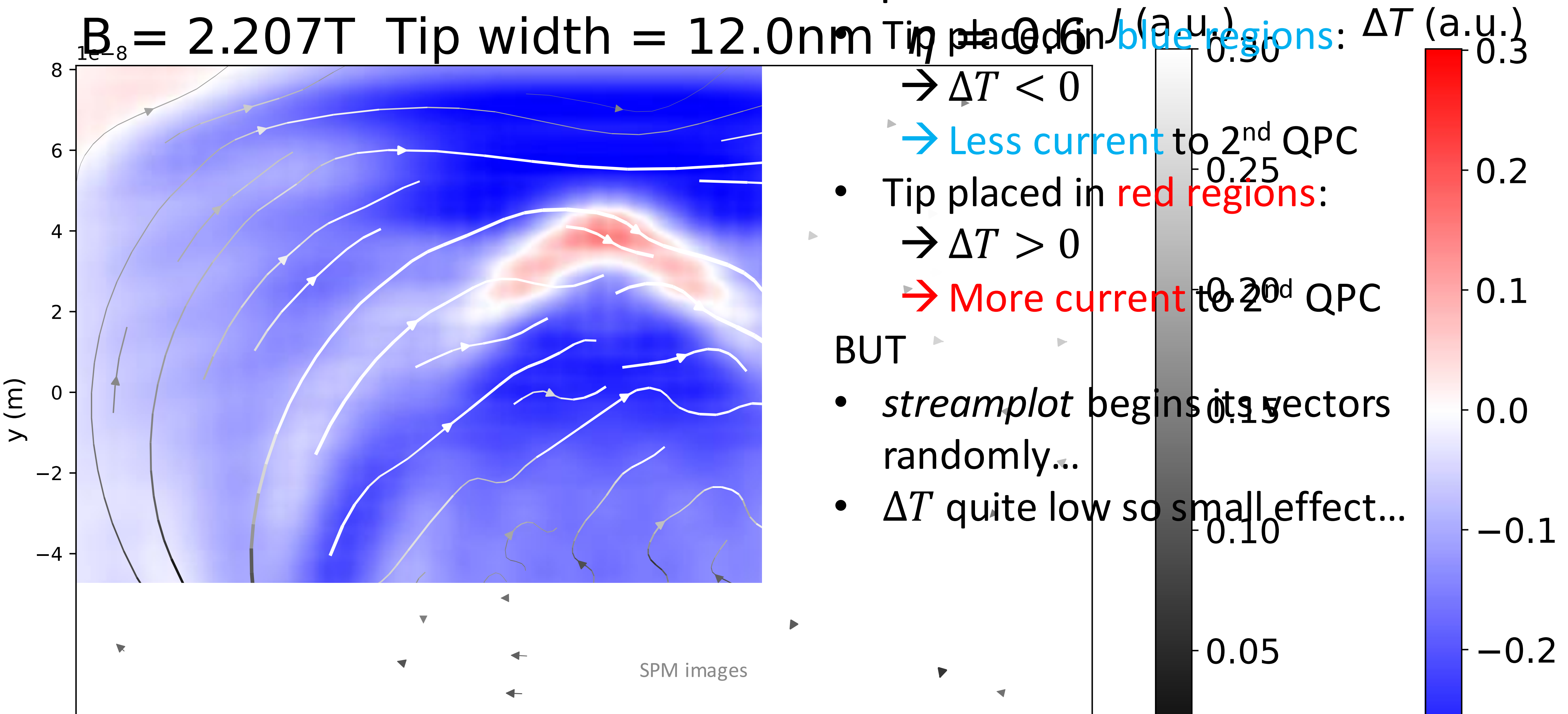
SPM images

Influence of η – tip width of 12nm



➔ The fringe spacing should be close to $\lambda_F/2$ because of backscattering but is not observed on these images. The classical cyclotron orbit is clearly visible, although it was expected at weak focusing instead.

Visualizing the current without tip



Expectations

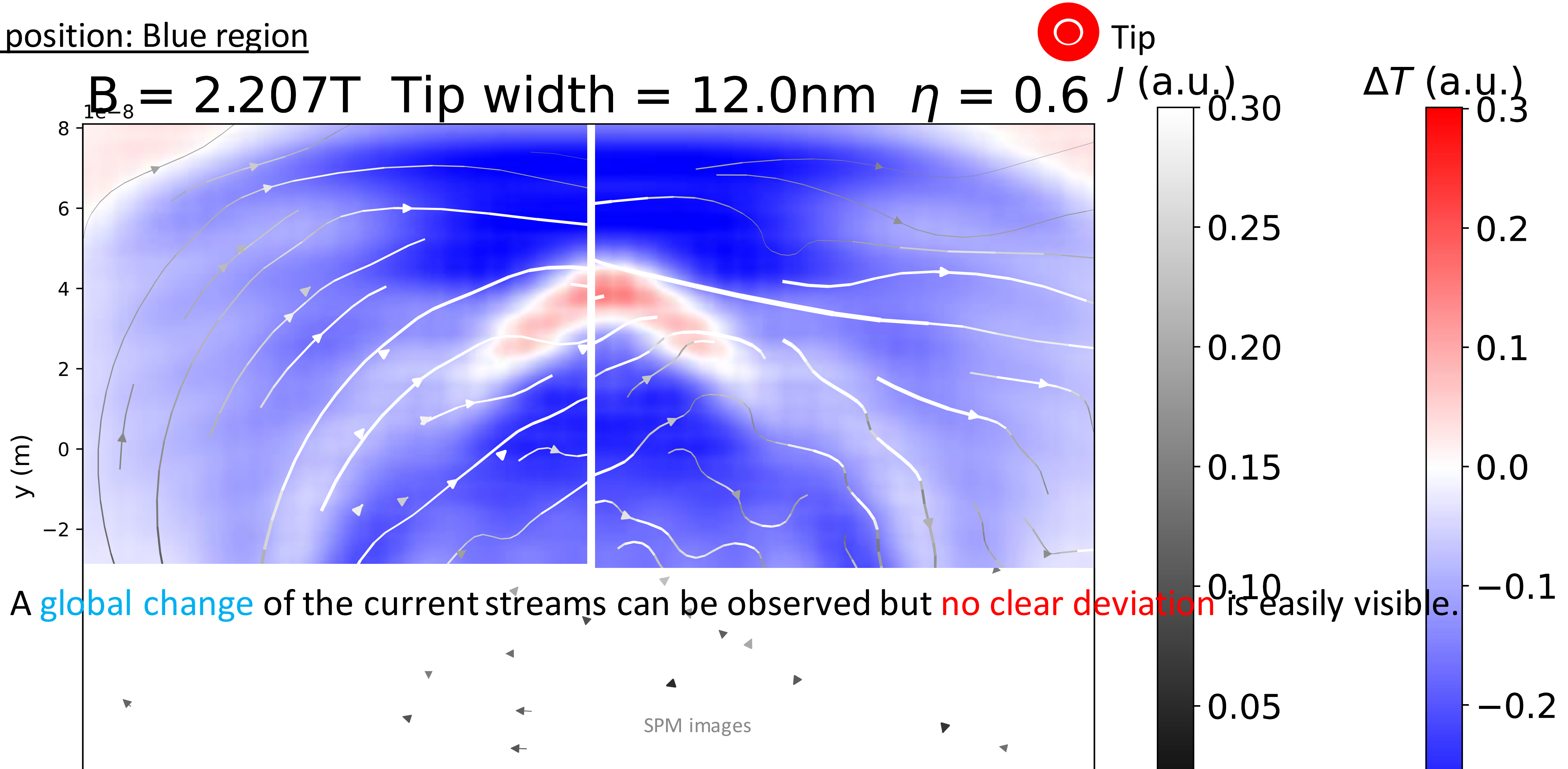
- Tip placed in blue regions:
  → $\Delta T < 0$
  → Less current to 2nd QPC
- Tip placed in red regions:
  → $\Delta T > 0$
  → More current to 2nd QPC

BUT

- *streamplot* begins its vectors randomly…
- ΔT quite low so small effect…

Visualizing the current with tip

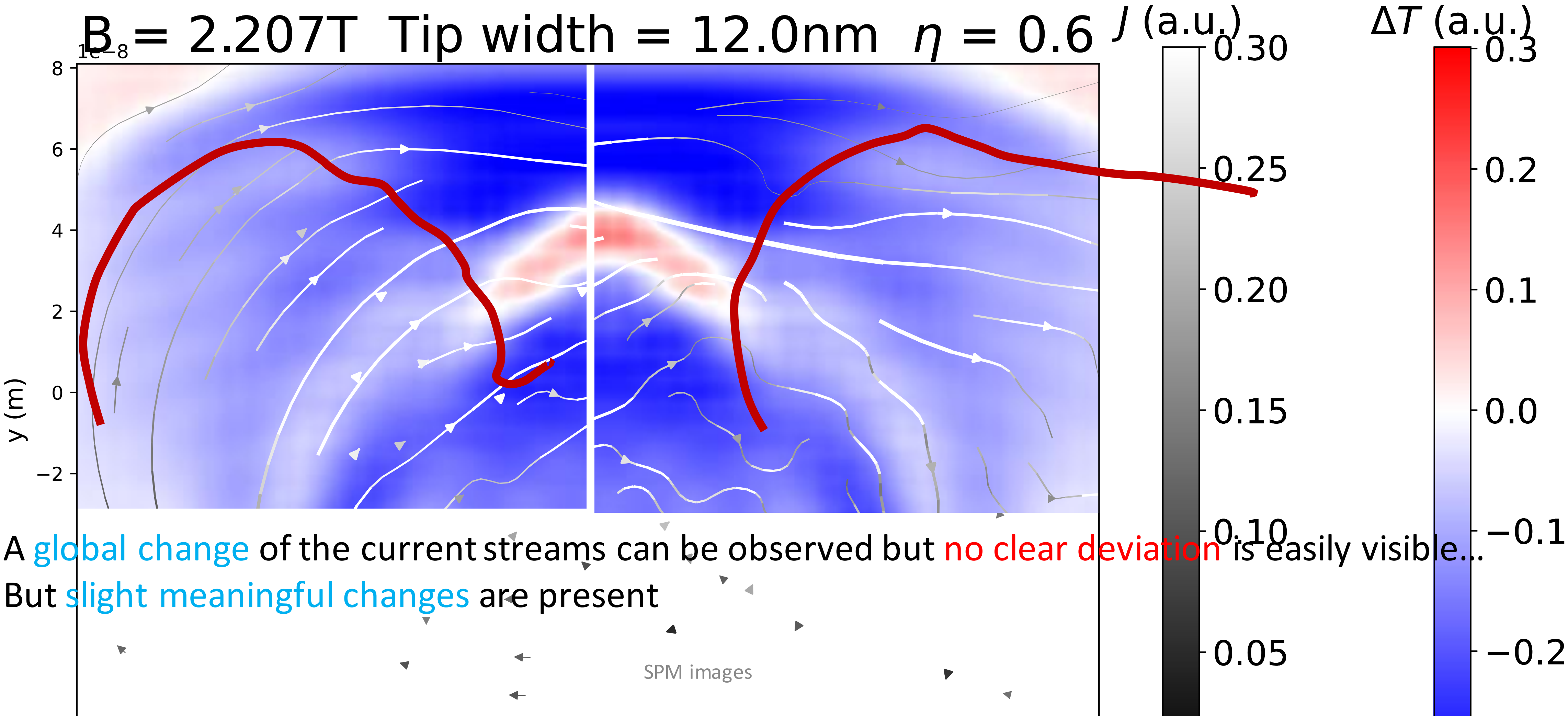
<u>1st position: Blue region</u>



➔ A global change of the current streams can be observed but no clear deviation is easily visible.

Visualizing the current with tip

<u>1st position: Blue region</u>

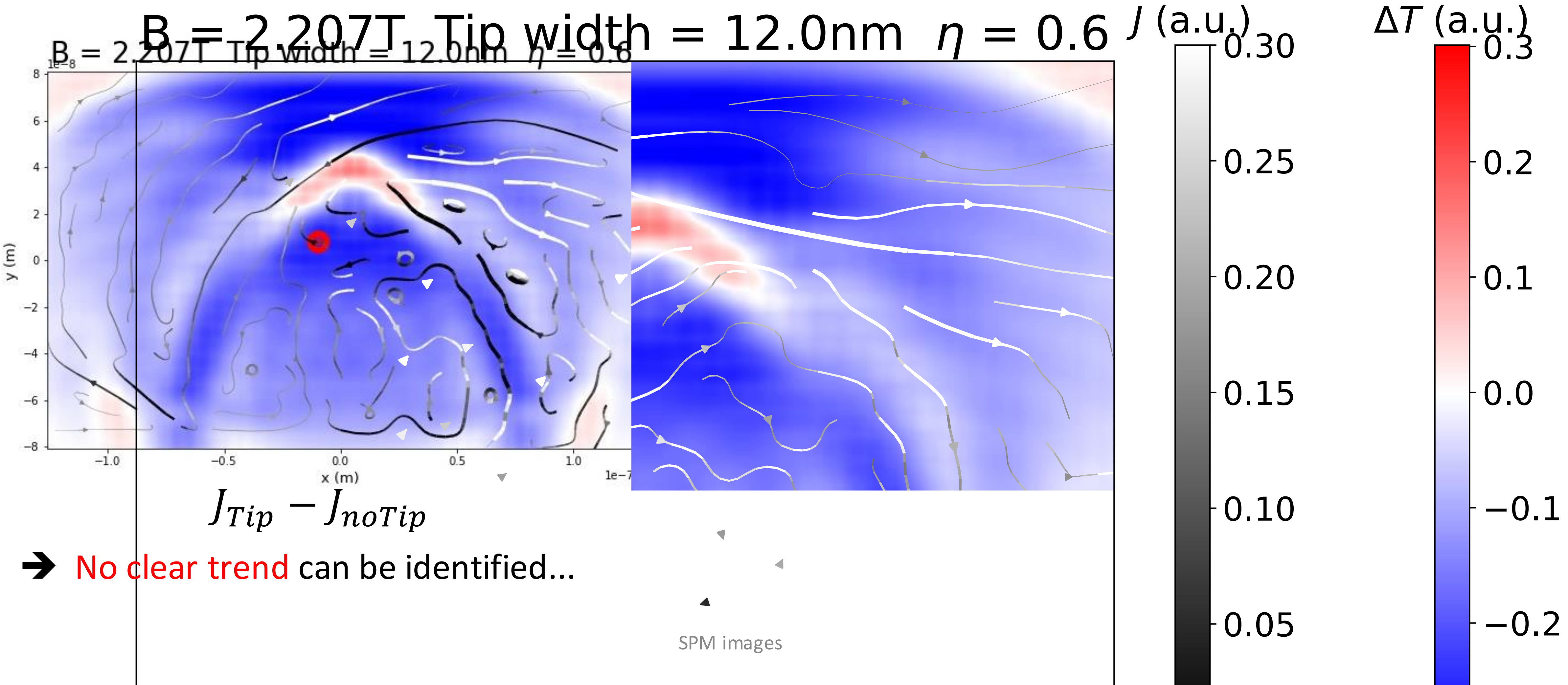


➔ A global change of the current streams can be observed but no clear deviation is easily visible…

➔ But slight meaningful changes are present

Visualizing the difference of current with tip

1st position: Blue region

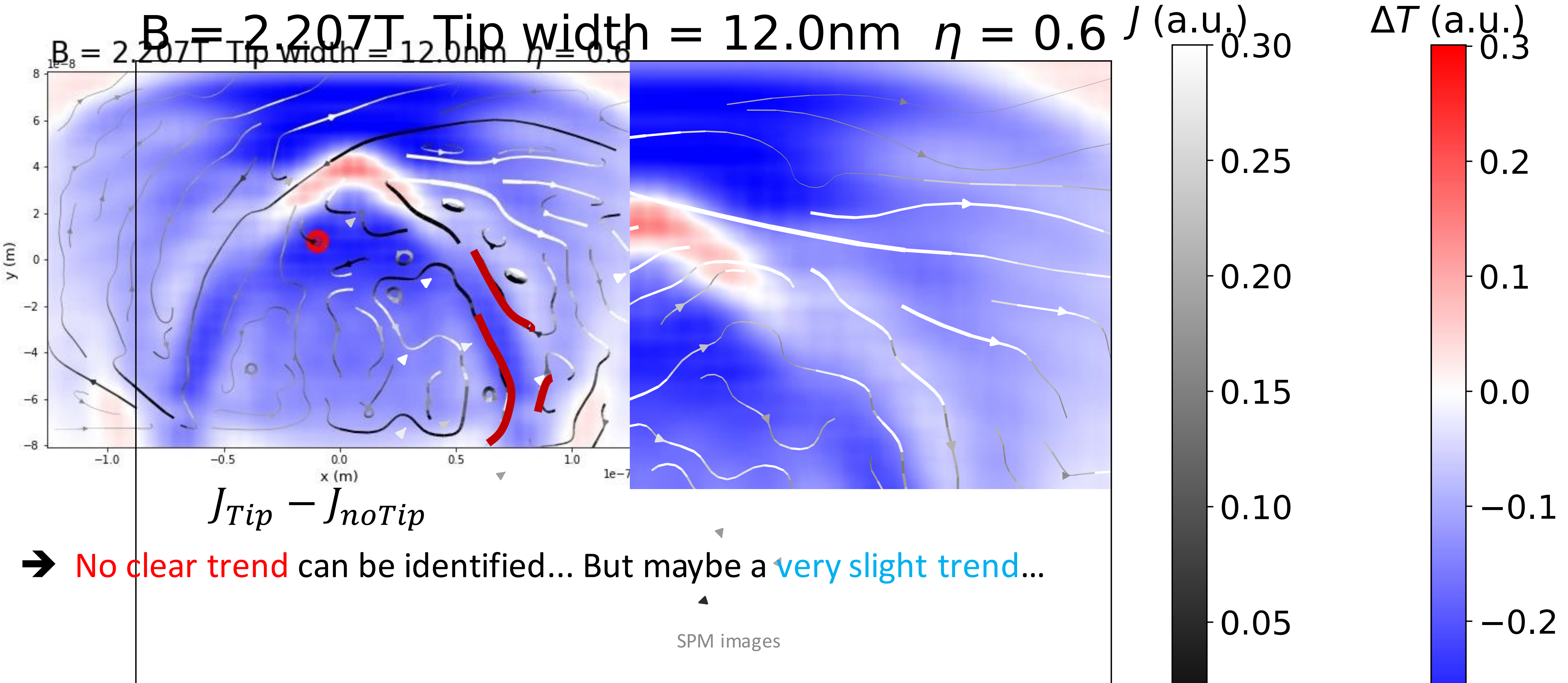


$J_{Tip} - J_{noTip}$

➔ No clear trend can be identified…

Visualizing the difference of current with tip

<u>1st position: Blue region</u>



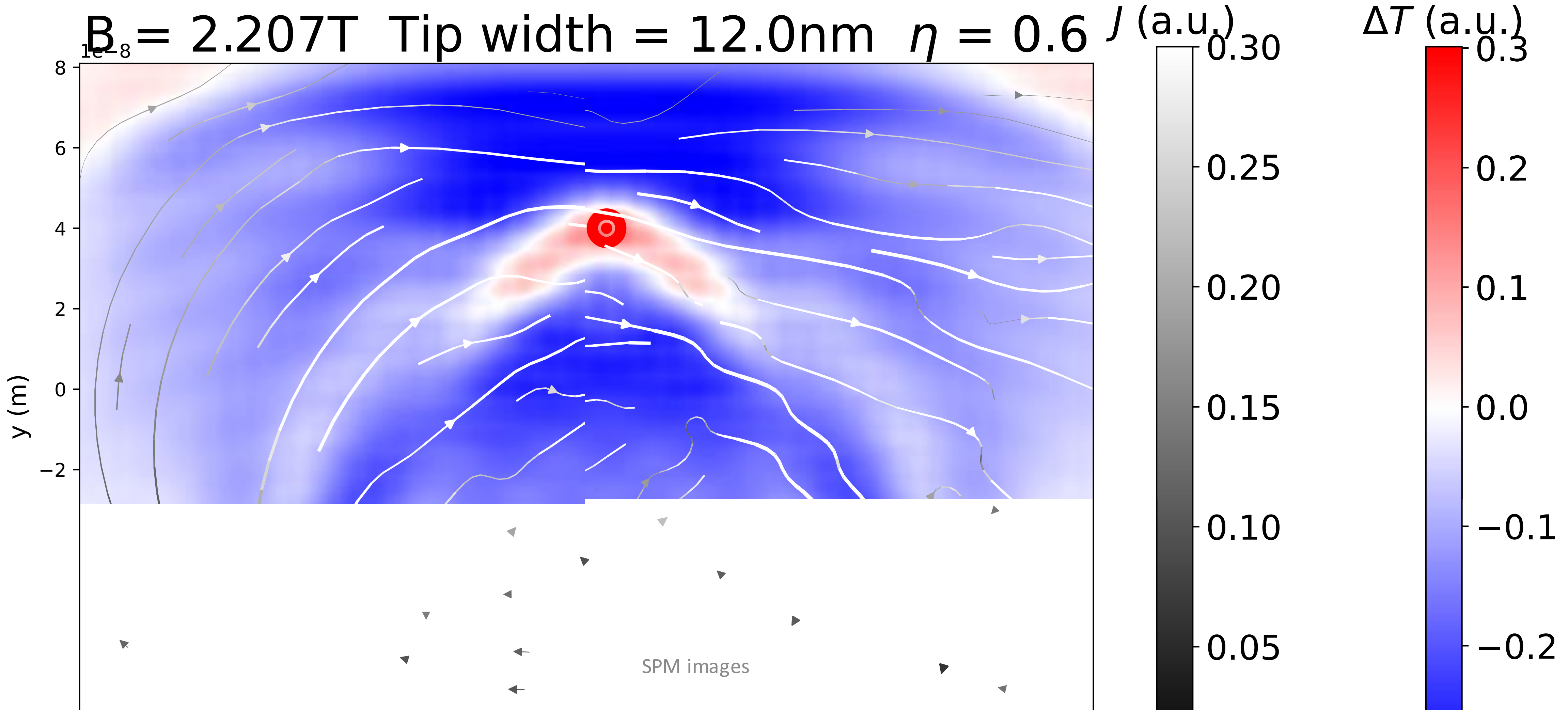
$J_{Tip} - J_{noTip}$

➔ No clear trend can be identified… But maybe a very slight trend…

SPM images

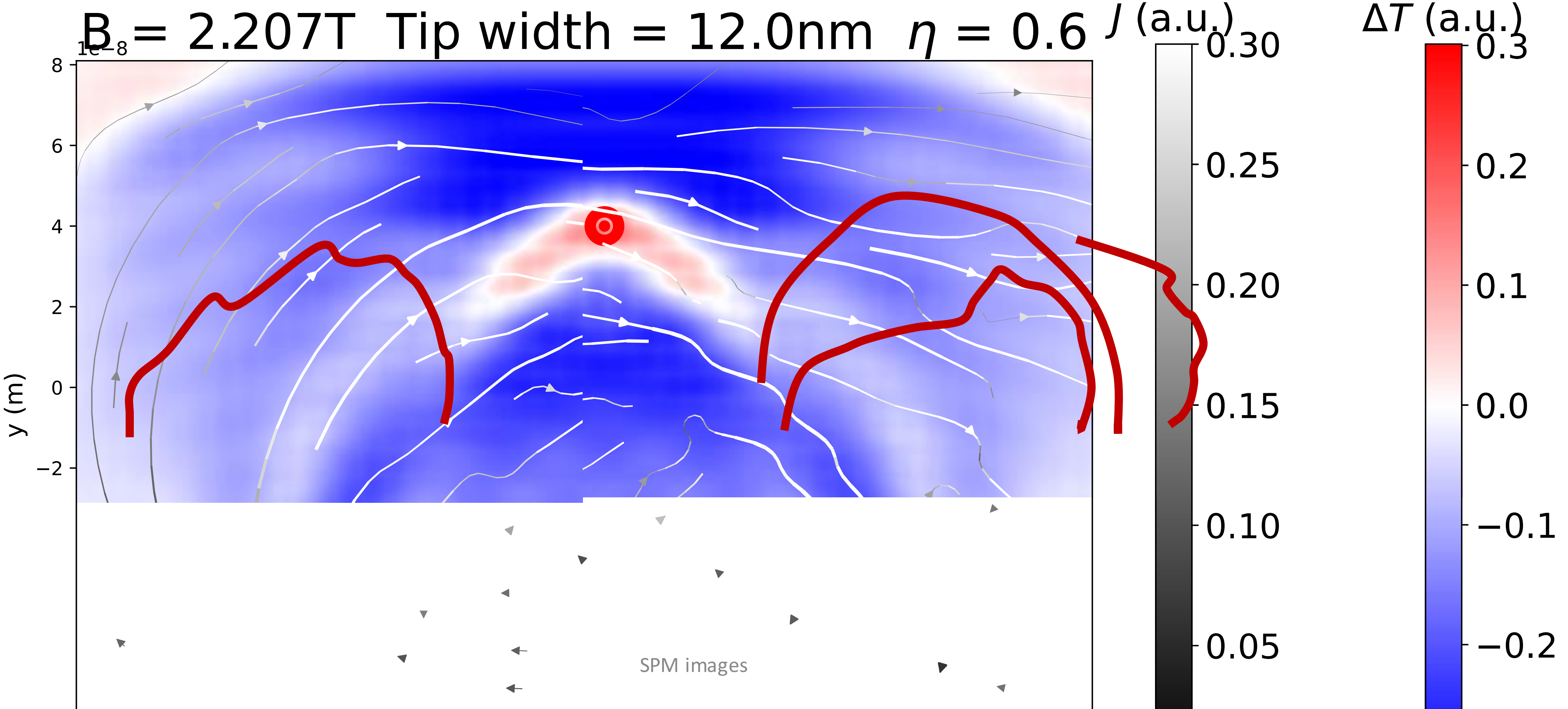
Visualizing the current with tip

2nd position: red region



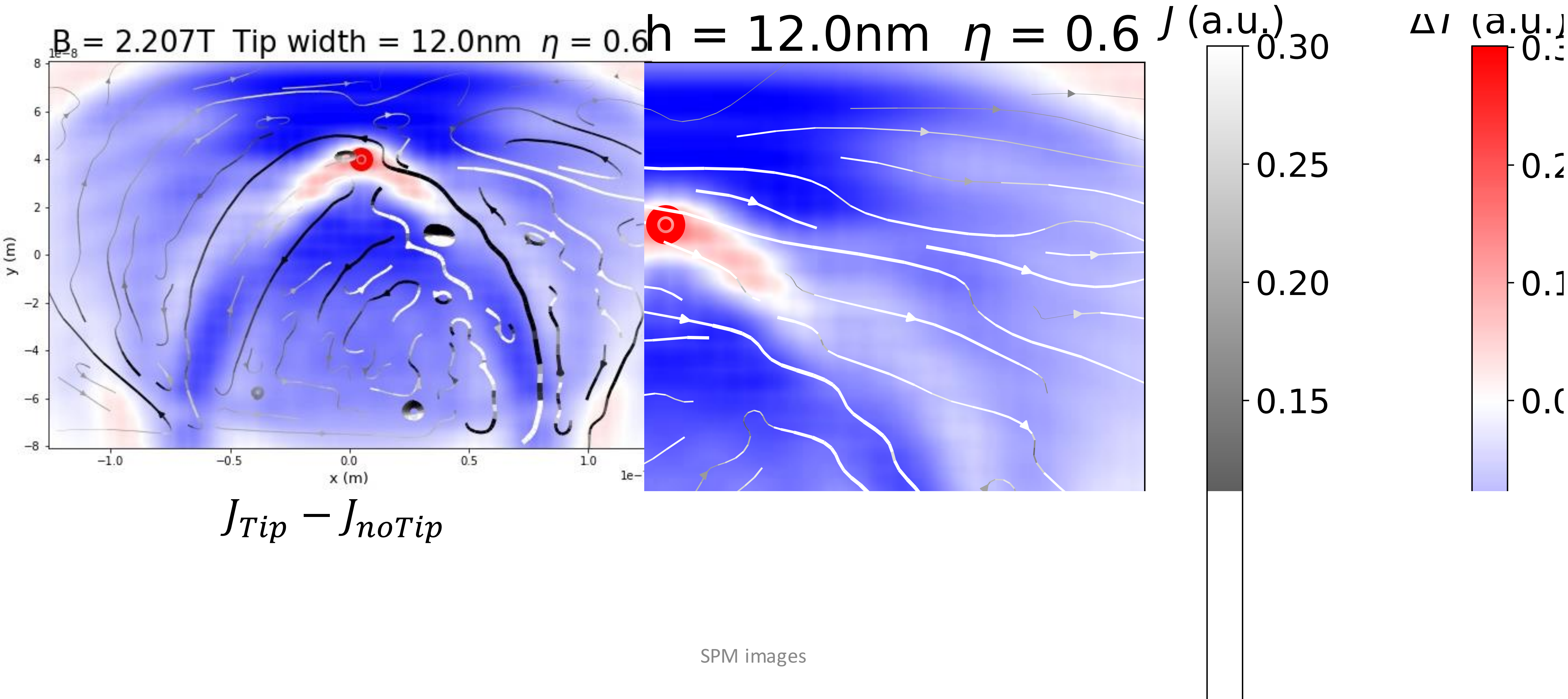
Visualizing the current with tip

<u>2nd position: red region</u>



Visualizing the difference of current with tip

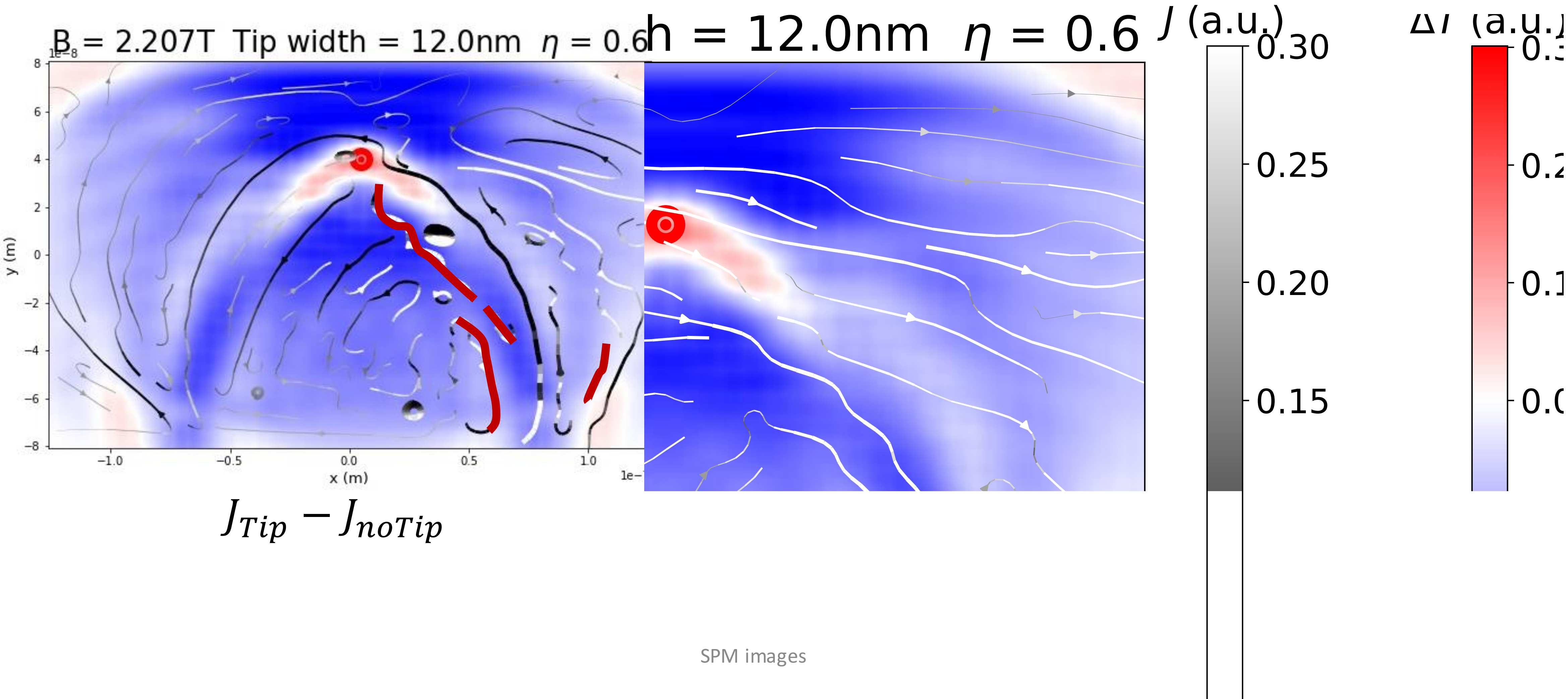
2nd position: red region



$J_{Tip} - J_{noTip}$

Visualizing the difference of current with tip

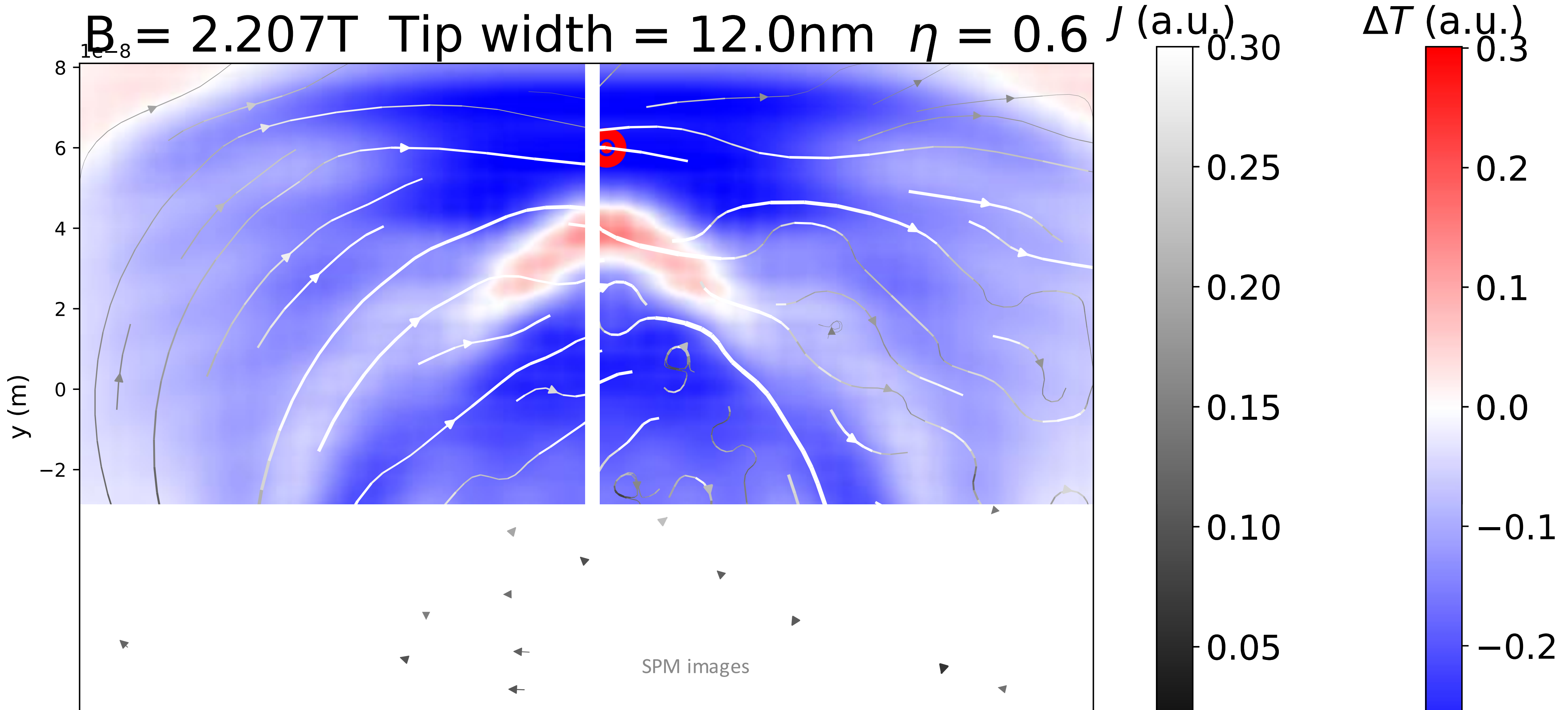
<u>2nd position: red region</u>



$J_{Tip} - J_{noTip}$

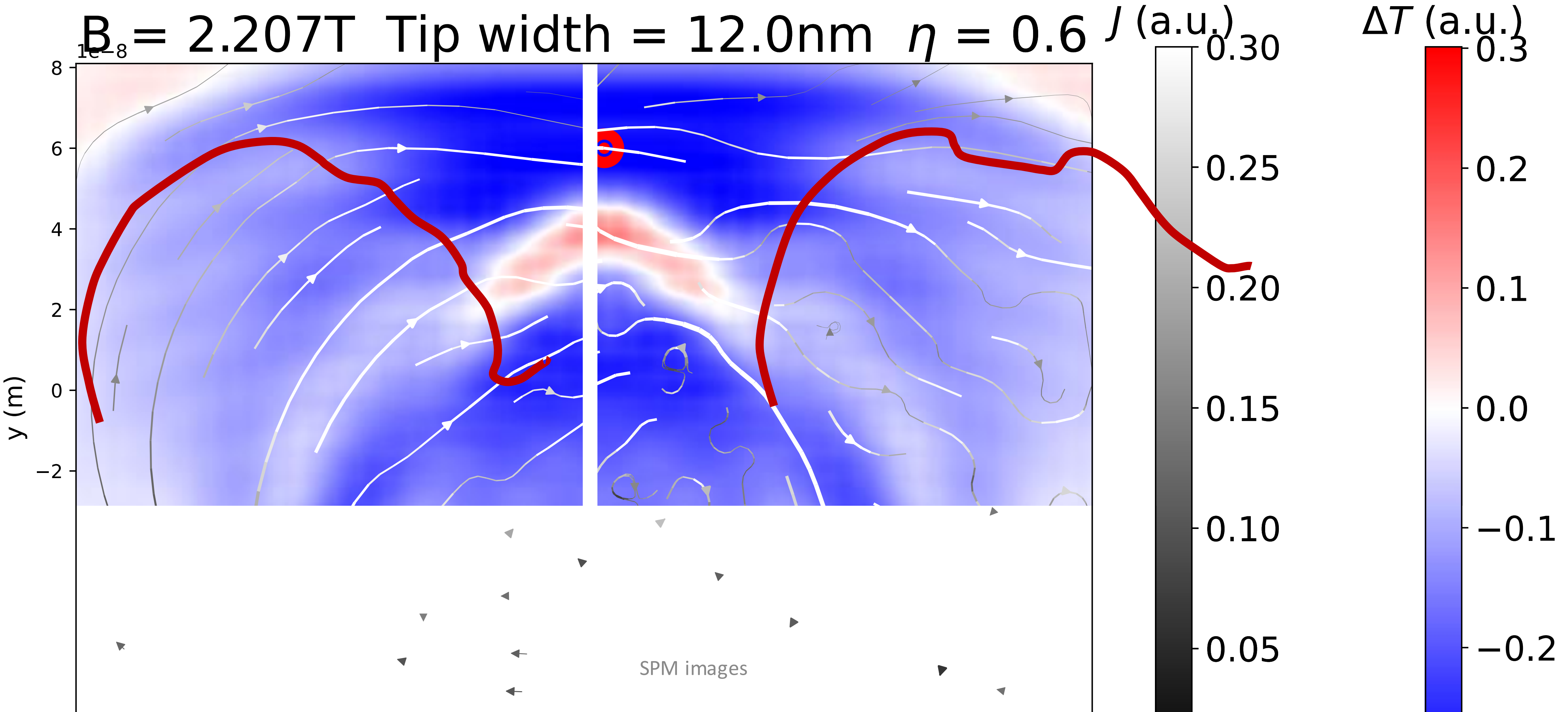
Visualizing the current with tip

3rd position: blue region



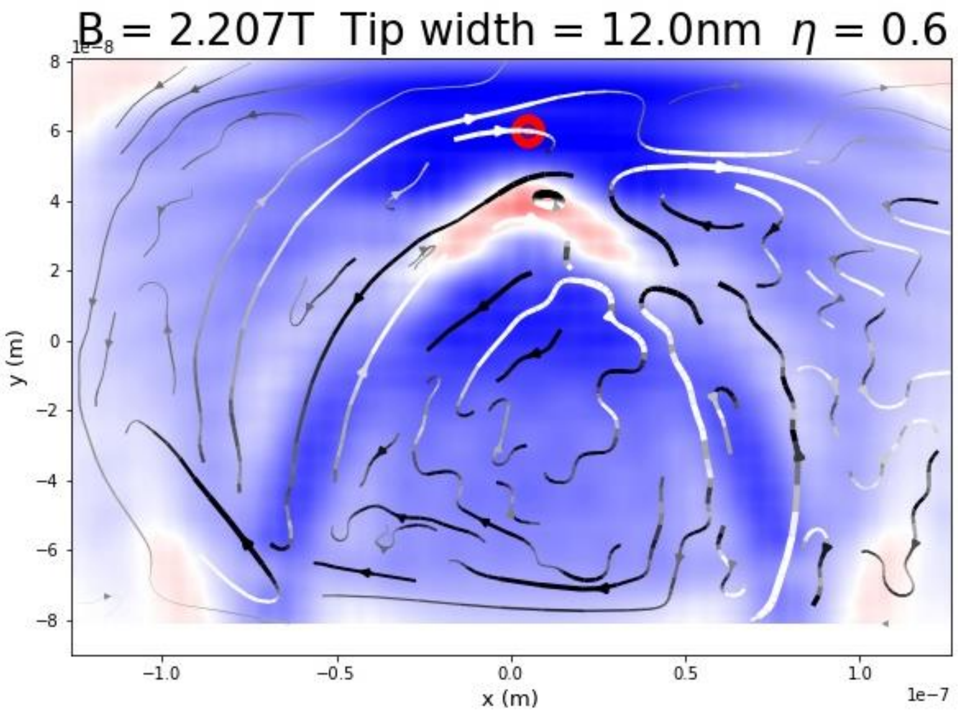
Visualizing the current with tip

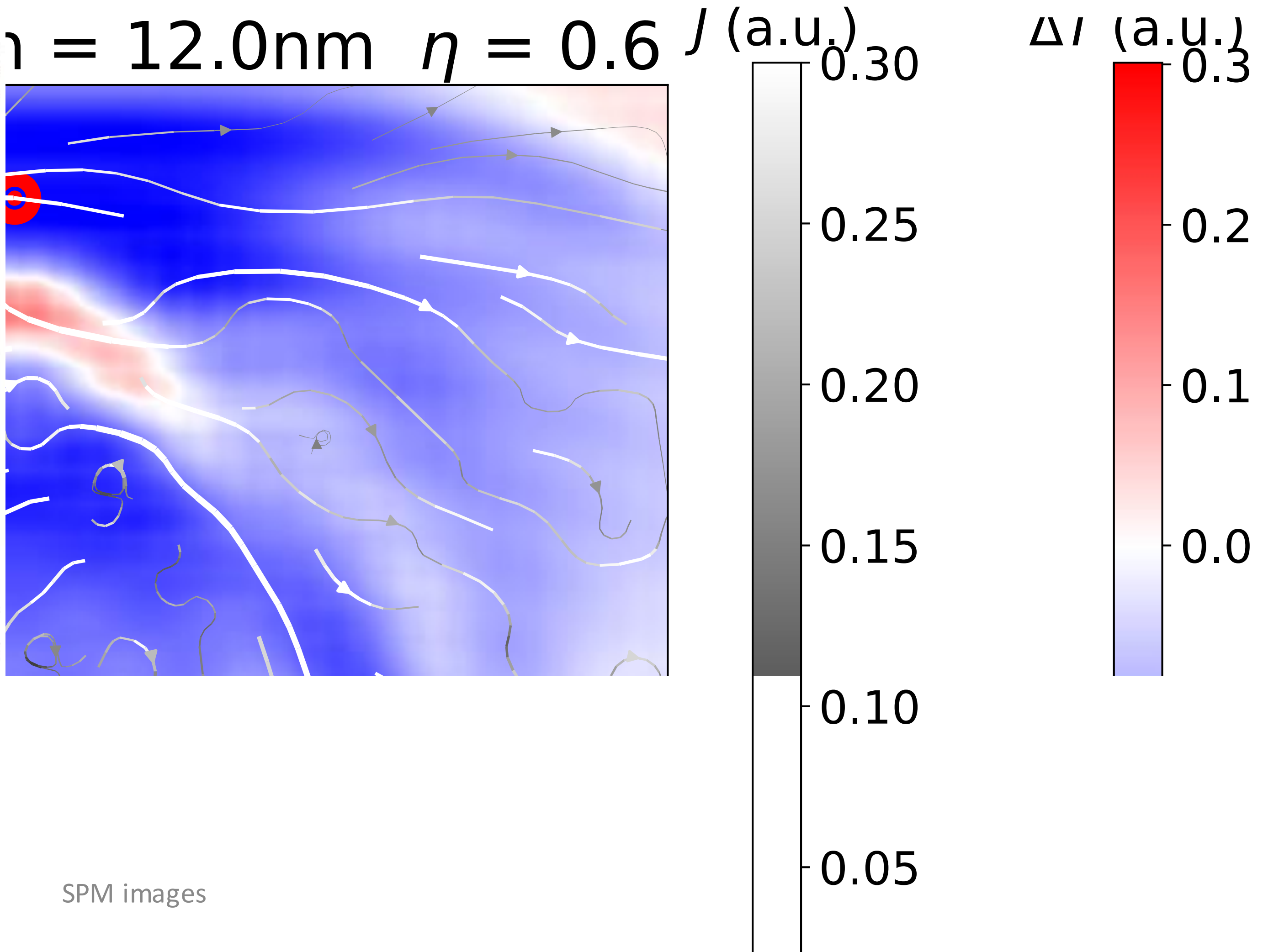
<u>3rd position: blue region</u>



Visualizing the difference of current with tip

<u>3rd position: blue region</u>

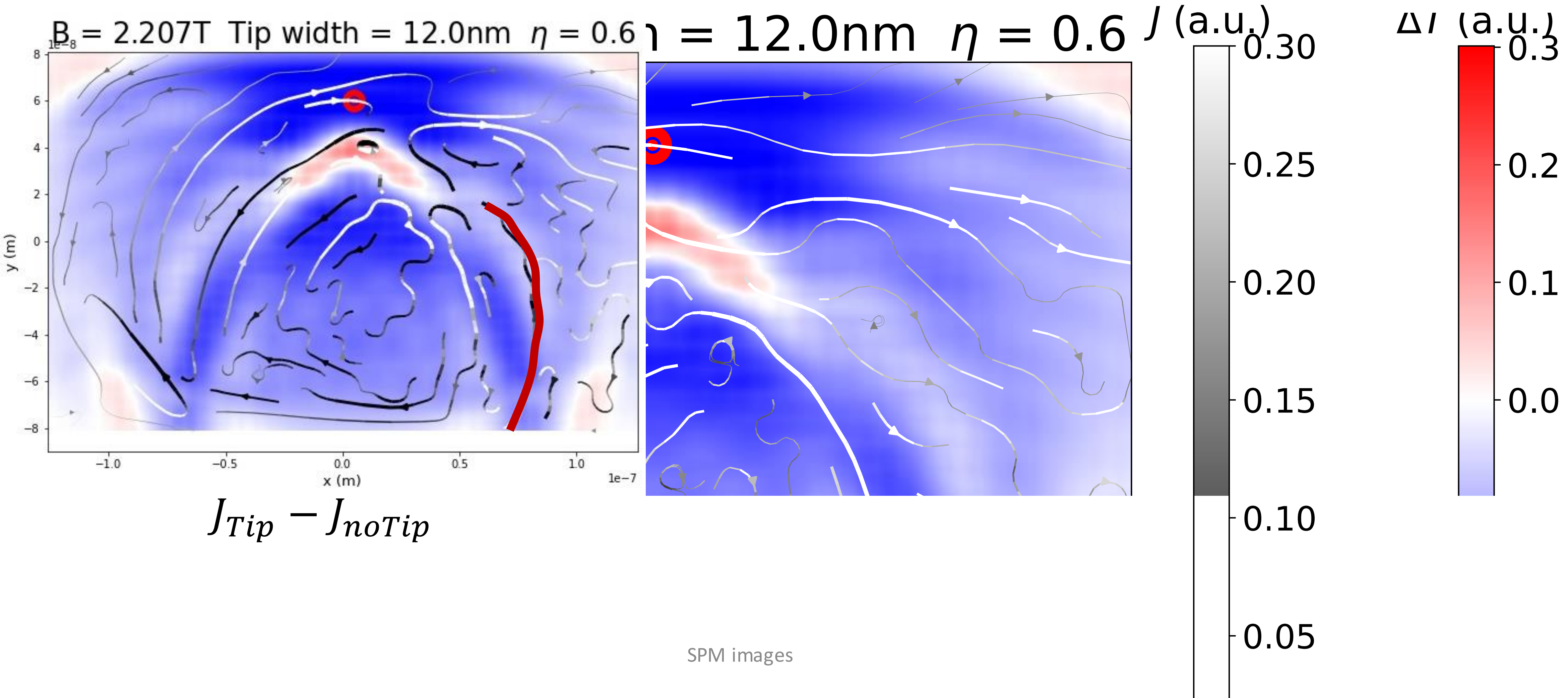




$J_{Tip} - J_{noTip}$

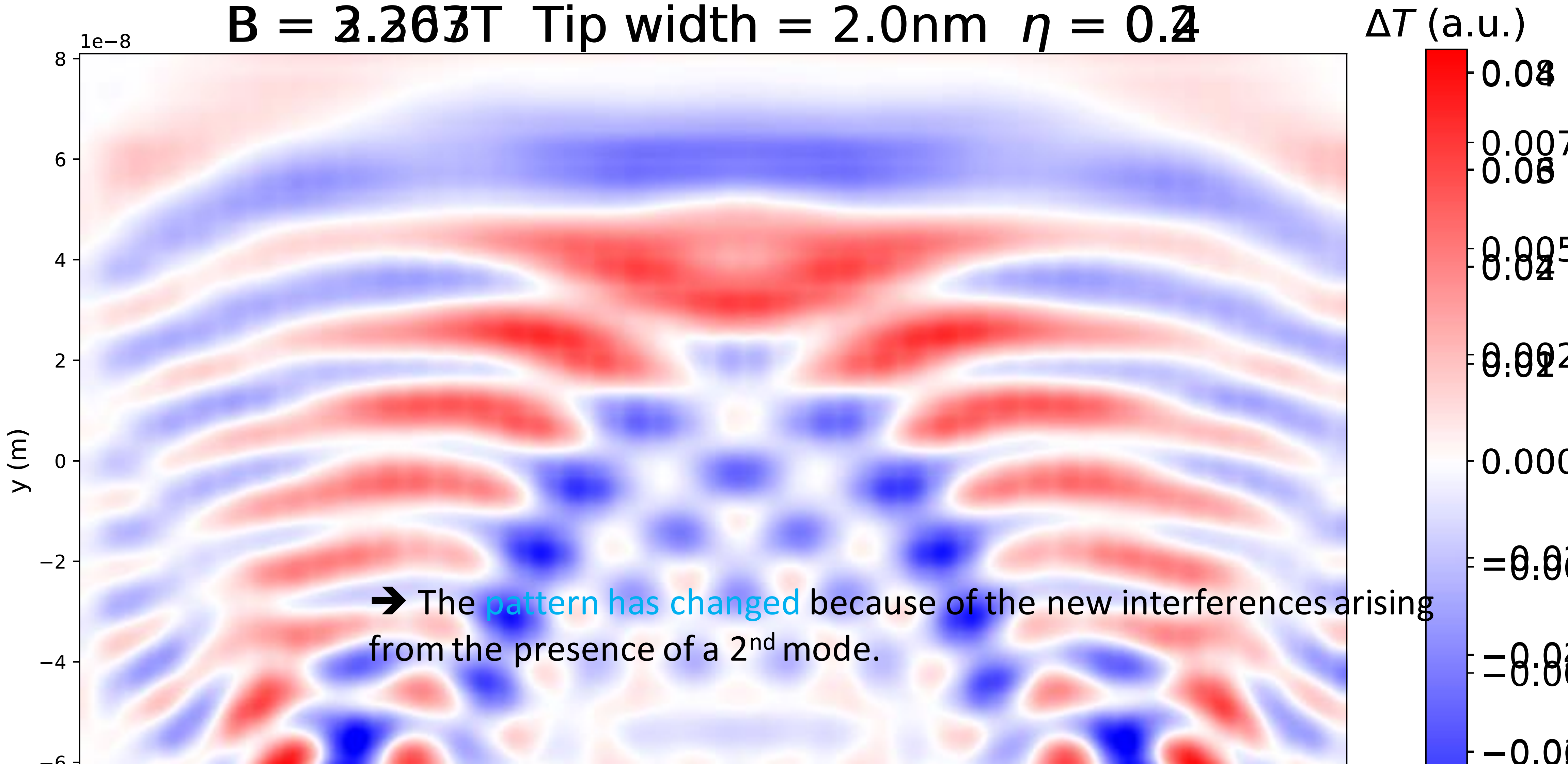
Visualizing the difference of current with tip

3rd position: blue region



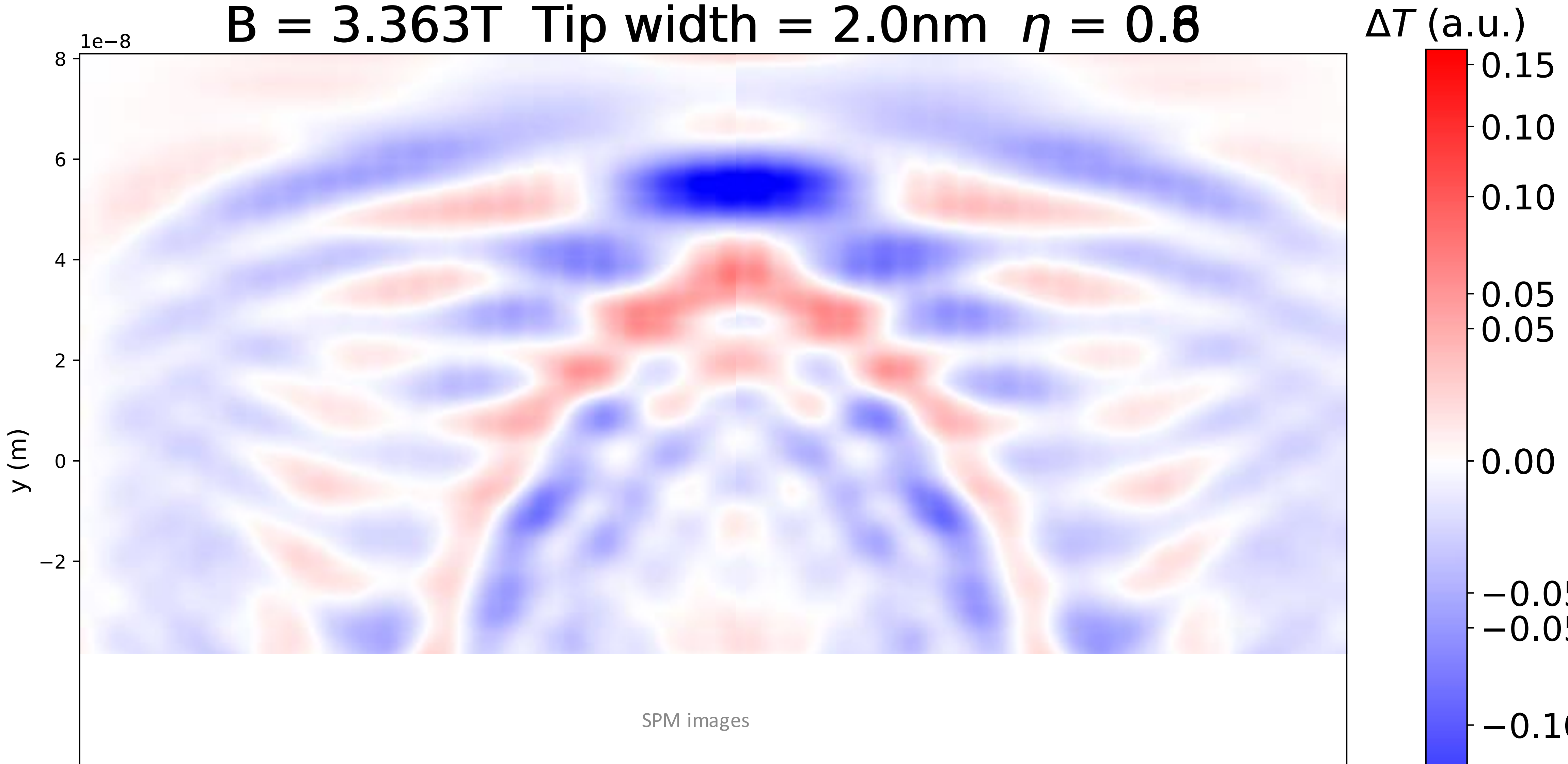
$J_{Tip} - J_{noTip}$

Influence of η for 2 modes

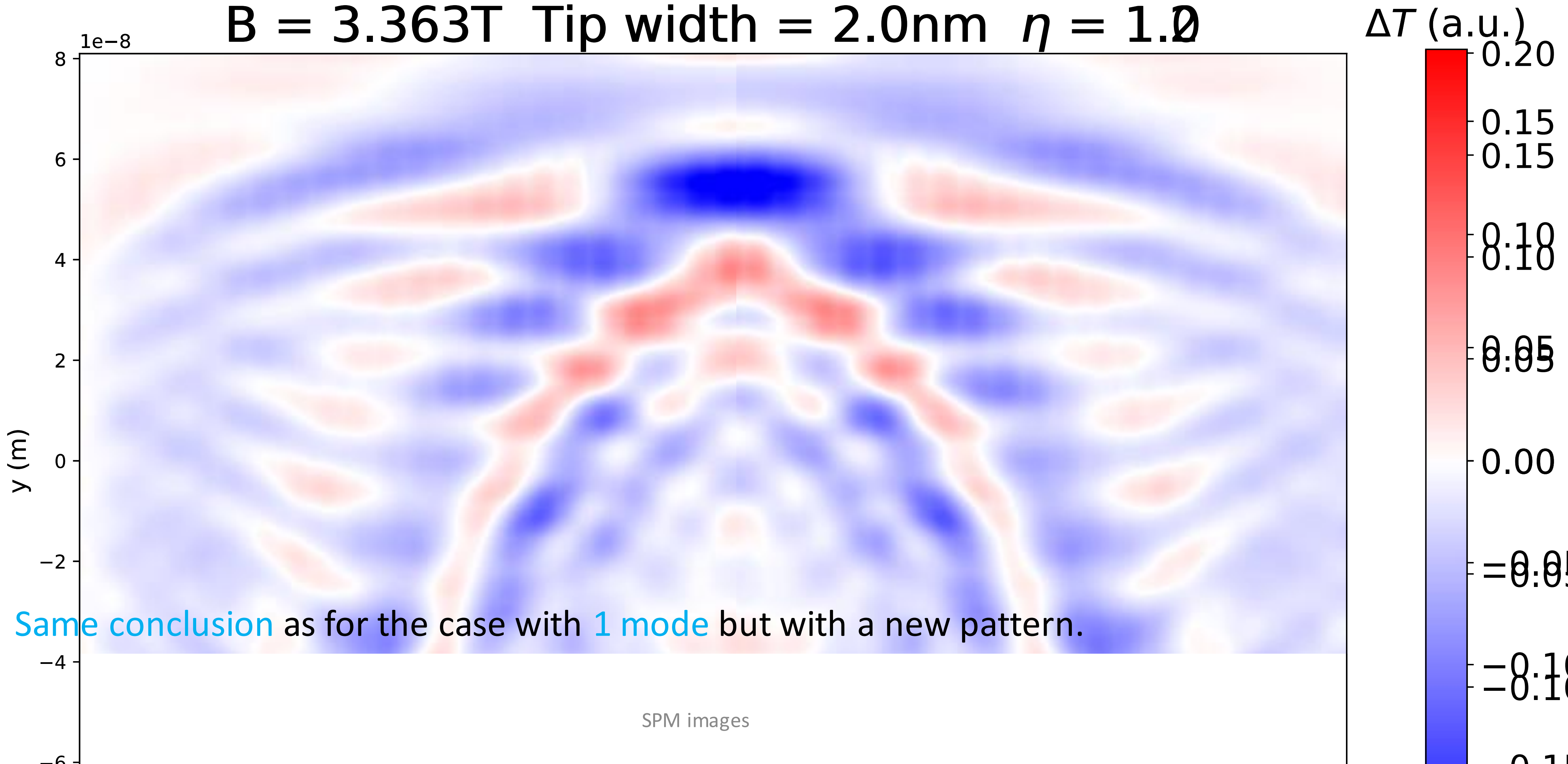


➔ The pattern has changed because of the new interferences arising from the presence of a 2nd mode.

Influence of η for 2 modes

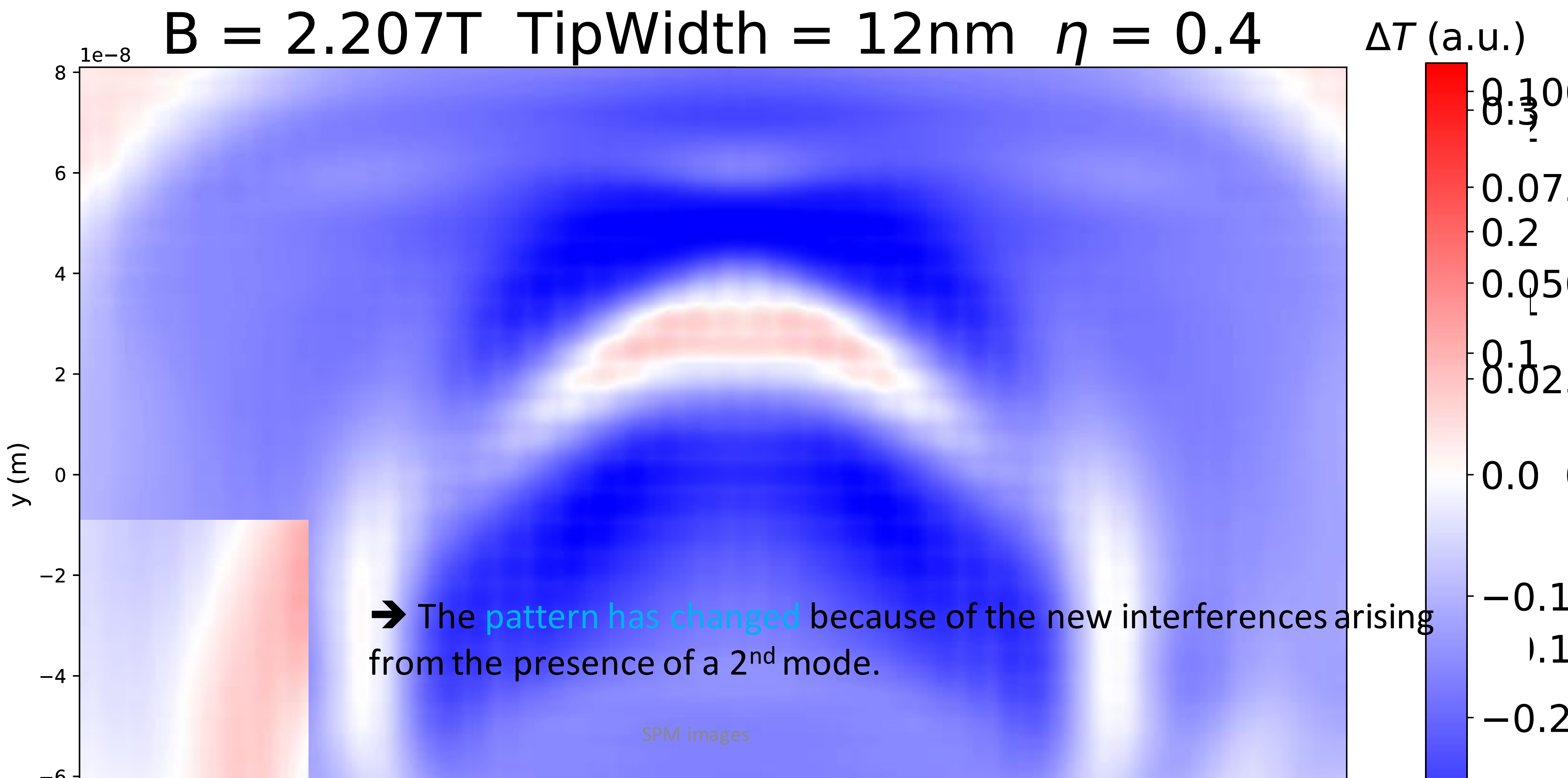


Influence of η for 2 modes



➔ Same conclusion as for the case with 1 mode but with a new pattern.

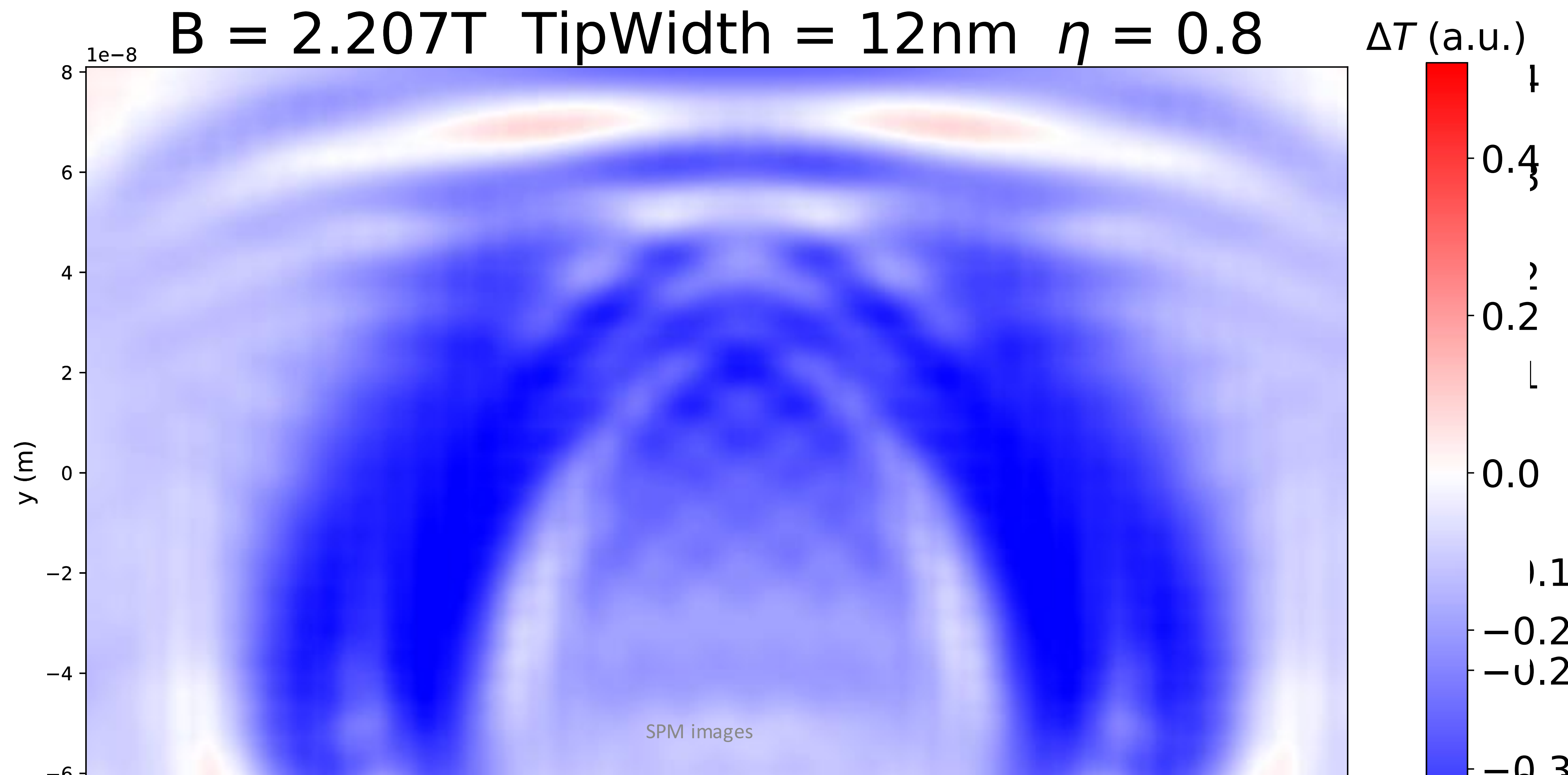
Influence of η for 2 modes



➔ The pattern has changed because of the new interferences arising from the presence of a 2nd mode.

SPM images

Influence of η for 2 modes



SPM images

Influence of η for 2 modes

➔ Same conclusion as for the case with 1 mode but with a new pattern.

The hopping function

The hopping function we used :

```
def hopping(site_i, site_j, Bfield):
    """
    Returns the hopping between two sites when a magnetic field Bfield is applied
    """
    xi, yi = site_i.pos
    xj, yj = site_j.pos

    hop = -tScaled * np.exp(1j*e_C/hbar_Js * Bfield * (yi+yj)/2 * (xj-xi))
    return hop
```

➔ The hopping is varying along y (as in the Hall bar from session 4) although it should vary along x

The hopping function

The hopping functions we should have used:

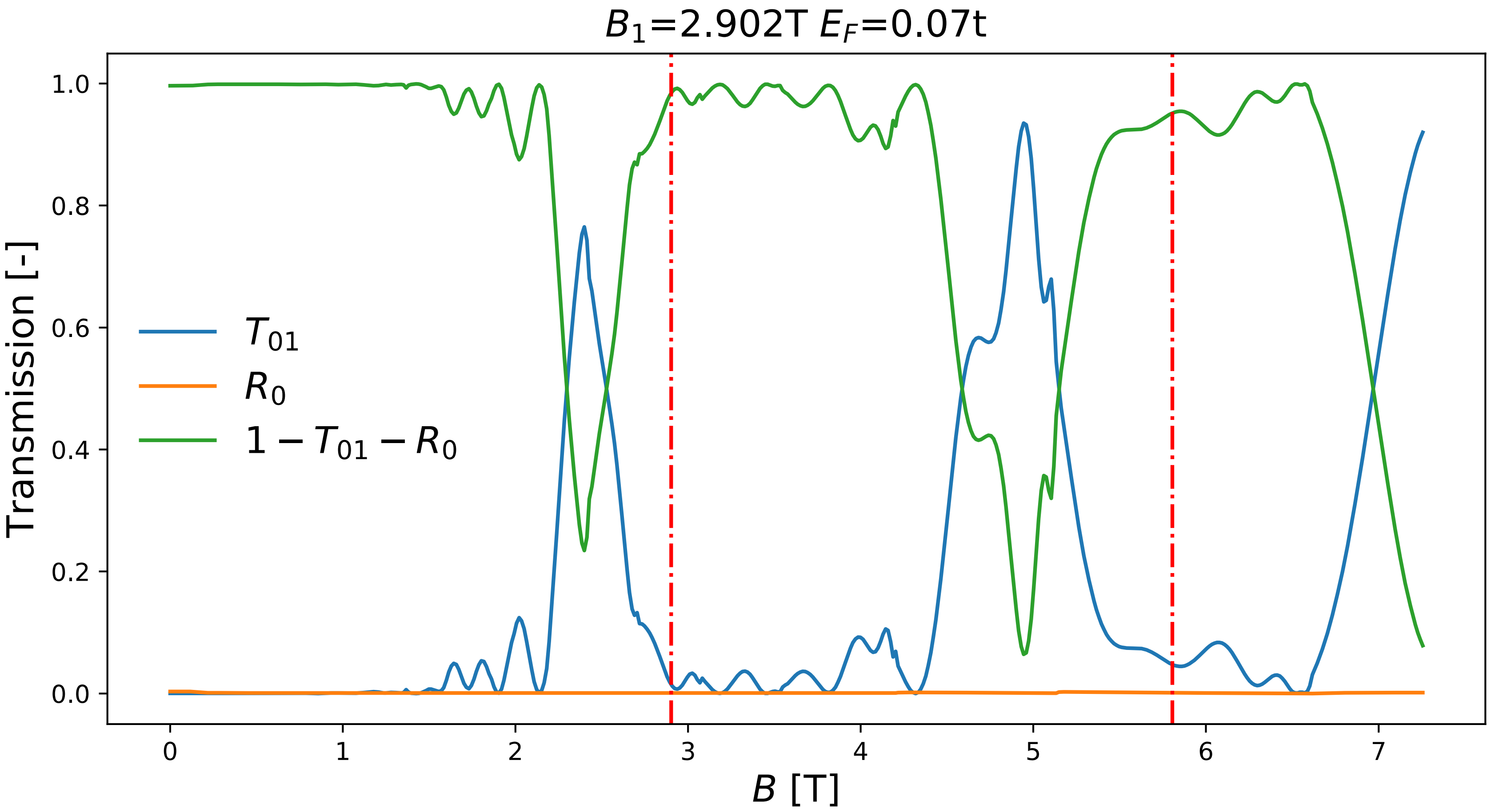
```python
def hopping(site_i, site_j, Bfield):
    """
    Returns the hopping between two sites when a magnetic field Bfield is applied
    """
    xi, yi = site_i.pos
    xj, yj = site_j.pos

    hop = -tScaled * np.exp(1j*e_C/hbar_Js * Bfield * (xi+xj)/2 * (yi-yj))
    return hop

def hopping2(site_i, site_j, Bfield): # Hopping function for the leads on the left and on the right
    """
    Returns the hopping between two sites when a magnetic field Bfield is applied
    """
    xi, yi = site_i.pos
    xj, yj = site_j.pos

    hop = -tScaled * np.exp(1j*e_C/hbar_Js * Bfield * l/2 * (yi-yj))
    return hop
```

➔ The hopping is now varying along x but is kept constant for the leads on the left and on the right


B_1=2.902T E_F=0.07t
1.0
0.8
0.6
0.4
0.2
0.0
Transmission [-]
T_{01}
R_0
$1 - T_{01} - R_0$
0
1
2
3
4
5
6
7
B [T]

The hopping function