Power-Law Temporary Impact Modeling for Optimal Order Execution

Victory Ochei

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Abstract

We address the problem of optimal order execution in limit order book markets, focusing on minimizing temporary market impact. A power-law model is proposed to capture the nonlinear relationship between order size and slippage. Using empirical analysis of three stock tickers, we estimate model parameters and develop a dynamic programming framework for optimal execution strategies that balance the trade-off between immediate execution costs and market timing risk.

1 Introduction

Executing large orders in financial markets involves a trade-off between speed and cost. Traders can use market orders for immediate execution at the expense of slippage or limit orders that carry execution risk. We develop a quantitative framework to minimize total temporary market impact.

Let $g_t(x)$ denote the slippage incurred when executing x shares at time t. Modeling this function accurately enables more cost-effective execution strategies.

2 Modeling Temporary Impact

2.1 Power-Law Specification

We model the temporary impact as:

$$g_t(x) = a_t x^\beta \tag{1}$$

where:

- x: shares executed at time t
- a_t: time-varying scaling parameter
- β: impact curvature parameter ($0 < \beta < 1$ for concavity)

2.2 Rationale

As x increases, deeper order book levels are consumed, increasing marginal costs. This nonlinear cost structure justifies a concave power-law form. Prior studies and market microstructure theory support this relationship.

2.3 Greedy Fill Illustration

Under a greedy fill approximation, execution fills as:

$$x_i = \min\left(N_i, T - \sum_{k=0}^{i-1} x_k\right)$$

with slippage cost:

$$\begin{aligned} S_B(T,t) &= \frac{1}{T}\sum_i x_i(P_i - s_t) \\ &= \frac{1}{T}\sum_i \min\left(N_i, T - \sum_{k=0}^{i-1} x_k\right)(P_i - s_t) \end{aligned}$$

Assuming $C_i = P_i - s_t = C + \frac{i}{100}$ and $\sum x_i = T$, then:

$$S_B(T,t) \approx C + \frac{1}{T}\sum_i \frac{i x_i}{100}$$

Only when x_i is constant is $S_B(T,t)$ linear in T. Thus, $x_i \sim \alpha T^\beta$ is more flexible and appropriate.

3 Parameter Estimation

We use order book data from three stocks.

Step 1: Slippage Computation

For a given x, slippage is:

$$\text{Slippage}(x) = \frac{\sum_i q_i p_i}{\sum_i q_i} - p_{\text{mid}} \tag{2}$$

Step 2: Log-Linear Regression

Taking logs of Eq. (1):

$$\log(g_t(x)) = \log(a_t) + \beta \log(x) \tag{3}$$

Estimate β and $\log(a_t)$ using OLS for each time interval and stock.

Step 3: Analysis

We assess the stability of β and model a_t as a function of spread, order book depth, and volatility.

4 Optimal Execution Framework

4.1 Problem Formulation

Let S be the total shares to execute over N intervals. Let x_i be the number of shares at time i. Minimize:

$$\min_{x_1,\dots,x_N} \quad \sum_{i=1}^{N} g_i(x_i) \tag{4}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} x_i = S, \quad x_i \geq 0 \ \forall i \tag{5}$$

4.2 Dynamic Programming

Define the value function:

$$V_t(R) = \min_{0 \leq x \leq R} [g_t(x) + V_{t+1}(R - x)] \tag{6}$$

with $V_{N+1}(0) = 0$ and $V_{N+1}(R > 0) = \infty$. Using the power-law form:

$$V_t(R) = \min_{0 \leq x \leq R} \left[a_t x^{\beta} + V_{t+1}(R - x)\right] \tag{7}$$

4.3 First-Order Condition

For interior optimal x:

$$\beta a_t x^{\beta - 1} = V'_{t+1}(R - x) \tag{8}$$

This equates current marginal cost to marginal future value.

4.4 Algorithm

Algorithm 1 Dynamic Optimal Execution

Initialize $V_{N+1}(R) = \infty$ for $R > 0$, $V_{N+1}(0) = 0$
for $t = N$ to 1 **do**
  for each R **do**
    $V_t(R) = \min_{0 \leq x \leq R} \left[a_t x^{\beta} + V_{t+1}(R - x)\right]$
    Store $x_t^*(R)$ that achieves the minimum
  end for
end for
Forward simulate: $x_t = x_t^*(R_t)$

5 Empirical Analysis

5.1 Data

We use intraday order book snapshots from three stocks. For each interval i:

- x_i: volume executed
- $g_t(x_i)$: computed slippage from order book