ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

MGT-482 Principles of Finance
Assignment 1

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Exercice 1

Following the rule of **time value of money**, the present value is calculated with the following formula:

$$PV(CF) = CF \cdot discountfactor \tag{1}$$

Then the present of value is calculated with

$$PV = 10'000_{1year} + 10'000_{3years} = \frac{10'000}{1.07} + \frac{10'000}{(1.07)^3} = 17'508.773 \tag{2}$$

Exercice 2

The investment is

$$PV(costs) = 1'000 + 5'000_{2years} = 1'000 + \frac{5'000}{1.02^2} = 5'805.84 \tag{3}$$

$$PV(benefits) = 4'000_{1year} + 4'000_{2years} + 4'000_{3years} = \frac{4'000}{1.02} + \frac{4'000}{1.02^2} + \frac{4'000}{1.02^3} = 11'535.53 \tag{4}$$

Then the **Net Present Value** of this project is

$$NPV = PV(benefits) - PV(costs) = 11'535.53 - 5'805.84 = 5'729.69 \tag{5}$$

The NPV is positive so we would undertake the project.

Exercice 3

First we convert CHF into US Dollars: $250'000CHF * 0.9 = \$225'000$
The investment is

$$PV(costs) = \$225'000 \tag{6}$$

The **Present Value** of the benefit is

$$PV(benefits) = 310'000_{1year} = \frac{310'000}{1.05} = 295'238.09 \tag{7}$$

Then the **Net Present Value** of this project is

$$NPV = PV(benefits) - PV(costs) = 295'238.09 - 250'000 = 45238.09 \tag{8}$$

The NPV is positive so we would undertake the project.

Exercice 4

Under the **Law of One Price**, securities that produce the same cash flows must have the same price

$$P(A) + P(B) = P(C) \tag{9}$$

As $P(A) = \$80$ and $P(C) = \$180$, $P(B)$ must be equal to

$$P(B) = P(C) - P(A) = 180 - 80 = \$100 \tag{10}$$

Following this rule, there is no arbitrage opportunity.

Exercice 5

$$\begin{aligned} NPV(A) = -150 + 50_{1year} + 75_{2year} + 100_{3year} = \\ -150 + \frac{50}{1.1} + \frac{75}{1.1^2} + \frac{100}{1.1^3} = 32.56 \end{aligned} \tag{11}$$

$$\begin{aligned} NPV(B) = -250 + 150_{1year} + 150_{2year} - 50_{3year} = \\ -250 + \frac{150}{1.1} + \frac{150}{1.1^2} + \frac{-50}{1.1^3} = -27.23 \end{aligned} \tag{12}$$

You should by project A but not B.

Exercice 6

First we calcule the present value of the loan we have to pay to the bank

$$PV = \frac{1000}{1.05} + \frac{1000}{1.05^2} + \frac{1000}{1.05^3} = 2723.24 \quad (13)$$

Then we calculate what should the bank ask you to pay at the end of the third year to have exactly the same present value

$$PV_{offer1} == PV_{offer2} = \frac{x}{1.05^3} = 3152.49 \quad (14)$$

$$x = 3152.49 \quad (15)$$

Then you should not accept a payment of more than 3152.49.

Exercice 7

From your 18th birthday to your 25th, there is 7 years where you can let the money.

$$V_{7years} = V_{now} * (1.08)^7 = 39'960 * (1.08)^7 = 68'484.41 \quad (16)$$

a) You will have \$ 68'484.41 at your 25th birthday
Originally, 18 years ago, the value of this money was

$$V_{0year} = \frac{V_{now}}{(1.08)^{18}} = \frac{39'960}{(1.08)^{18}} = 9'999.95 \quad (17)$$

Your parents originally put \$ 9'999.95 in the account (if they are normal people, I guess they actually put \$10'000).

Exercice 8

The couple's spendings are represented as an outgoing cashflow. The present value of a cash flow is calculated as follows:

$$PV = CF \cdot \text{DiscoutRate} \quad (18)$$

One can determine the following table (the time scale starts at 0 due to payables at the beginning of the year):

Time	0	1	2	3	...
Cash flow [$]	10000	$10000 \cdot 1.05$	$1000 \cdot 1.05^2$	$1000 \cdot 1.05^3$	...
Present value added [$]	10000	$\frac{10000 \cdot 1.05}{1.05}$	$\frac{10000 \cdot 1.05^2}{1.05^2}$	$\frac{10000 \cdot 1.05^3}{1.05^3}$	...

Summing up all the present values, one gets the value of the initial investment of **$13'000**.

Exercice 9

The table below describes the savings made each year thanks to the new machine:

Time	1	2	3	...	i	...
Savings [$]	1000	$\frac{1000}{1.02}$	$\frac{1000}{1.02^2}$	...	$\frac{1000}{1.02^i}$	...

In this problem, the constant payment is called the perpetuity C. Th present value of a growing perpetuity can easily be determined with the following formula:

$$PV(\text{Savings}) = \frac{C}{r-g} \tag{19}$$

where $r = 0.05$ is the interest rate and $g = -0.02$ is the growth rate. The final present value of my saving therefore is **$14'285.7**.

Exercice 10

In the first part of the problem, we will use the present value formula for an annuity C for N periods with an interest rate r:

$$PV = C \cdot \frac{1}{r} \cdot \left(1 - \frac{1}{(1+r)^N}\right) \tag{20}$$

$$300'000 = C \cdot \frac{1}{0.07} \cdot \left(1 - \frac{1}{(1+0.07)^{30}}\right) \tag{21}$$

$$C = \frac{300'000 \cdot 0.07}{1 - \frac{1}{(1.07)^{30}}} = 24'175.92 \tag{22}$$

The annual payment will be **$24'175.92**.

The second part of the problem consists in determine the present value for 30 years if only $23'500 can be afforded per year. We have:

$$PV = 23500 \cdot \frac{1}{0.07} \cdot \left(1 - \frac{1}{1.07^{30}}\right) = 29'1612.5 \tag{23}$$

The difference compared to the first situation is $300'000 - 291'612.5 = 8'387.5$ representing the present value of the balloon payment, adding the interests for 30 years, we have $8'387.5 \cdot 1.07^{30} = 63'848$

The balloon payment in 30 years will be **$63'848**.

Exercice 11

In this case we can use two different types of present values PV_1 and PV_2. Let PV_1 be the annuity with payments that grow at the rate g when the discount rate is r. The calculation is as follows:

$$PV_1 = C_1 \cdot \frac{1}{r-g} \cdot \left(1 - \left(\frac{1+g}{(1+r)}\right)^N\right) + C_1 \tag{24}$$

It is important not to forget the first payment done today (at t=0). Now, let PV_2 be the present value of \$2 million kept in an account during 35 years. We have:

$$PV_2 = \frac{C_2}{r^N} \tag{25}$$

The trick in this exercise is to equalize both PV_1 and PV_2 to determine the initial payment to make.

$$PV_1 = PV_2 \tag{26}$$

$$C_1 \cdot \frac{1}{r-g} \cdot \left(1 - \left(\frac{1+g}{1+r}\right)^N\right) + C_1 = \frac{C_2}{(1+r)^N} \tag{27}$$

$$C_1 = \frac{C_2 \cdot \frac{1}{(1+r)^N}}{1 + \frac{1}{r-g}\left(1 - \left(\frac{1+g}{1+r}\right)^N\right)} = \frac{2000000 \cdot \frac{1}{1.05^{35}}}{1 + \frac{1}{0.02}\left(1 - \left(\frac{1.03}{1.05}\right)^{35}\right)} = 14222.3 \tag{28}$$

I will need to put **\$14'222.3** today into my account to reach the \$2 million in 35 years.

Exercice 12

In this exercise, at time $t = 0$, my grandmother spends \$200'000 (negative cash flow). Then at time $t = 1$, she receives \$25'000(positive cash flow), at time $t = 2$ \$25'000, and so on until she dies. The present value of her annuity is:

$$PV = C \cdot \frac{1}{r} \cdot \left(1 - \frac{1}{(1+r)^N}\right) = 25'000 \cdot \frac{1}{0.05} \cdot \left(1 - \frac{1}{1.05^N}\right) \tag{29}$$

Isolating N in the above relation we get and setting the upper limit of PV to \$200'000 we get

$$N \geq \log\left(\frac{1}{1 - \frac{200'000 \cdot 0.05}{25'000}}\right) = 10.46 \tag{30}$$

In order to reach the desired amount, she must live at least **11 years** after the day she retired.