Report of Lab 11

Question 1

(1) Formulation

min $C_1 P_1 + C_4 P_4 + C_5 P_5$

s.t. $P_1 + P_4 + P_5 = -P_2 - P_3$

$P_i \geq 0; \quad P_i \leq 1 \ (for\ i = 1,4,5)$

(2) Matlab input

f = [100.0000, 33.3300, 66.6700]

Aeq = [1, 1, 1] , beq = 1.6, lb = $[0, 0, 0]^T$, ub = $[1, 1, 1]^T$

(3) Result

P = $[0.0000, 1.0000, 0.5000]^T$

Best price: 66.6650 CHF.

Marginal price for bus 2 and 3: 66.67 CHF/p.u.

Question 2

(1) Formulation

min $C_1 P_1 + C_4 P_4 + C_5 P_5$

s.t. $P_1 + P_4 + P_5 = -P_2 - P_3$

$P_{ij} = B_{ij}(\theta_i - \theta_j)$

$P_i = \sum_{i \neq j} P_{ij}$

$0 \leq P_i \leq 1$ for i=1, 4, 5;

$-S_{max} \leq P_{ij} \leq S_{max}\ (i \neq j)$

$\theta_1 = 0;$

(2) Matlab input

f = [100.0000 33.3300 66.6700 zeros(1,10)];

Aeq =

[1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	1	-1	0	0	0	-1	0	0	0	0
0	0	0	1	0	-1	0	0	0	-1	0	0	0
0	0	0	0	0	1	0	-1	0	0	-1	0	0
0	0	0	0	1	0	-1	0	0	0	0	-1	0
0	0	0	0	0	0	1	-1	0	0	0	0	-1
-1	0	0	0	0	0	0	0	1	1	0	0	0
0	-1	0	0	0	0	0	0	0	0	0	-1	1
0	0	-1	0	0	0	0	0	0	0	-1	0	-1
0	0	0	1	0	0	0	0	0	0	0	0	0]

beq = [2.5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0];

lb = [0, 0, 0, -3.1416, -3.1416, -3.1416, -3.1416, -3.1416, -1.0000, -1.0000, -1.0000, -1.0000 -1.0000];

ub = [1.0000, 1.0000, 1.0000, 3.1416, 3.1416, 3.1416, 3.1416, 3.1416, 1.0000, 1.0000, 1.0000 1.0000, 1.0000];

Note: this formulation input corresponds to the case when S_{max}=1p.u.

The upper bound and lower bound of angles are set to $\pm\pi$ because it is more convenient for formulation in Matlab.

(3) Result

(i) S_{max} = 1

Optimal price: 150 CHF;

Power generations:

$P_1 = 0.5\ p.u.\quad P_2 = 1\ p.u.\quad P_3 = 1\ p.u.$

Power angles:

$\theta_1 = 0$; θ_2 =-0.25; θ_3 =-0.25; θ_4 =-0.75; θ_5 =-0.75;

Power flows:

$P_{12} = 0.25\ p.u.;\quad P_{13} = 0.25\ p.u.;\quad P_{35} = -1\ p.u.;\quad P_{24} = -1\ p.u.;\quad P_{45} = 0\ p.u.;$

(ii) S_{max} = 0.8

Optimal price: 163.332 CHF;

Power generations:

$P_1 = 0.9\ p.u.\quad P_2 = 1\ p.u.\quad P_3 = 0.6\ p.u.$

Power angles:

$\theta_1 = 0$; θ_2 =-0.35; θ_3 =-0.55; θ_4 =0.45; θ_5 =0.25;

Power flows:

$P_{12} = 0.35\ p.u.;\quad P_{13} = 0.55\ p.u.;\quad P_{35} = -0.8\ p.u.;\quad P_{24} = -0.8\ p.u.;$

$P_{45} = 0.2\ p.u.;$

Question 3

Formulation at time slot t

min $\sum_i (C_i P_i + C_i^{SU} w_i(t)\big(1 - w_i(t-1)\big) + C_i^{SD}(1 - w_i(t))w_i(t-1)))\qquad (i = 1, 4, 5)$

s.t. $P_1 + P_4 + P_5 = -P_2 - P_3$

$0 \le P_i \le w_i$ for i=1, 4, 5

$w_i \in \{0,1\}$

Question 4

(1) Core Matlab codes (iteration part)

```
% iteration
intcon = [4;5;6];
w_last = [0;1;0];
x = zeros(6,4);price = zeros(1,4);
for i = 1:1:4
    fw = Csu.*(1-w_last)-Csd.*w_last;
    f(4:6,:) = fw;
    [x(:,i),fval] = intlinprog(f,intcon,A,b,Aeq,beq,lb,ub);
    price(i) = fval+sum(Csd.*x(4:6,i));
```

```
    w_last = x(4:6,i);
end
```

(2) Optimization input

The weighting vector f has the same first 3 components as in previous case. The last three components are calculated by:

$$C_i^{SU} w_i(t)\big(1 - w_i(t-1)\big) + C_i^{SD}(1 - w_i(t))w_i(t-1)$$
$$= \Big(C_i^{SU}\big(1 - w_i(t-1)\big) - C_i^{SD} w_i(t-1)\Big)\, w_i + C_i^{SD} w_i(t-1)$$

The corresponding weighting factor of generator i is:

$$f_i = C_i^{SU}\big(1 - w_i(t-1)\big) - C_i^{SD} w_i(t-1)$$

Another term only related to previous on/off state should be added to the total price.

For the time step 1 of the first case:

$f = [100.0000, 33.3300, 66.6700, 0, -20, 30]^T$

The other changes to the input matrix/vectors only concern the upper bound of power input. It has to be w_i.

(3) Result

(i) Case 1

For all time steps, $P_1 = 0.5$, $P_4 = 1$, P_5=0.

prices: fval(t=1) = 93.33 CHF; fval for other t = 83.33 CHF.

(ii) Case 2

For all time steps, $P_1 = 0$, $P_4 = 1$, P_5=0.5.

prices: fval(t=1) = 76.66 CHF; fval for other t = 63.33 CHF.

Comparison:

In case 1, start-up cost of generator 5 is higher than generator 1, but its operational cost is lower. The algorithm compares and finally decides that it is more economical to start up 1. In case 2, start-up cost of both generator 5 and 1 are the same. Obviously the algorithm would choose to turn on generator 5 for lower operational cost.

Question 5

(1) Formulation

$$\min \ \sum_t \sum_i \big(C_i P_i(t) + C_i^{SU} u_i(t) + C_i^{SD} v_i(t) \big) \qquad (i = 1, 4, 5;\ t = 1,2,3,4)$$

s.t. $P_1(t) + P_4(t) + P_5(t) = -P_2 - P_3$

$0 \le P_i(t) \le w_i(t)$

$w_i(t) - w_i(t-1) \le u_i(t)$

$w_i(t-1) - w_i(t) \le v_i(t)$

$w_i(t) \in \{0,1\} \quad u_i(t) \ge 0 \quad v_i(t) \ge 0$

$(\text{for any } i = 1, 4, 5;\ t = 1,2,3,4)$

(2) Results and discussion

The optimal cost for all periods in total: price = 296.66 CHF.

State of operation: generator 5 is started ever since the first period. It produces 0.5 p.u. of power while generator 4 is responsible for 1 p.u. of power.

The algorithm has chosen to start generator 5, despite it is more expensive to turn on 5 than 1. The reason is that the operational cost of generator 5 is cheaper than generator 1. In the long run (4 periods), the money saved by cheaper operational cost compensates the difference in start-up price between generator 1 and 5.
This proves that a multistage economical dispatch planning has better estimation than doing several single-stage planning.

Question 6

Formulation :

min $-2u_9 - 2(\overline{P_4} + \overline{P_{11}})v_9 \; - P_5 - 3\sum_{i\notin \mathrm{ind}} P_i^t - 3\overline{P_4} - 3\overline{P_{11}})$

(control variables are in red)

s.t. $\sum_{i\notin \mathrm{ind}} P_i^t + P_4^t + P_{11}^t + P_5 + u_9^t + (P_4^t + P_{11}^t)v_9^t \geq -1$ (1)

$\sum_{i\notin \mathrm{ind}} P_i^t + P_4^t + P_{11}^t + P_5 + u_9^t + (P_4^t + P_{11}^t)v_9^t \leq 1$ (2)

$0 \leq P_5 \leq 2$ (3)

$-1 \leq u_9^t + (P_4^t + P_{11}^t)v_9^t \leq 1$ (4)

$0 \leq \mathrm{E}_9^t \leq 1$ (5)

$\mathrm{E}_9^t = \mathrm{E}_9^{t-1} - 0.1(u_9^t + (P_4^t + P_{11}^t)v_9^t)$ (6)

$-1 \leq P_{11}^t + P_{13} + P_{10} \leq 1$ (7)

$-1 \leq u_9^t + (P_4^t + P_{11}^t)v_9^t + P_8 \leq 1$ (8)

$-1 \leq P_{11}^t + P_{13} + P_{10} + u_9^t + (P_4^t + P_{11}^t)v_9^t + P_8 \leq 1$ (9)

$-1 \leq P_4^t + P_3 \leq 1$ (10)

$\mathrm{P}_{4,\min}^t \leq P_4^t \leq P_{4,max}^t \qquad \mathrm{P}_{11,\min}^t \leq P_{11}^t \leq P_{11,max}^t$ (11)

Note: all the control variables are in red, all disturbance variables are in blue, state variables are in green.

$\mathrm{ind} = \{1,4,5,9,11\}$.

Constraints (7) to (10) are branch power boundary constraints.

Question 7

Consider the control variable vector to be in the form of:

$$\mathrm{u} = (\mathrm{P}_5^t, \mathrm{u}_9^\mathrm{t}, \mathrm{v}_9^\mathrm{t})^\mathrm{T}$$

Consider the disturbance variable vector to be in the form of:

$$\mathrm{d} = (\mathrm{P}_4^\mathrm{t}, \mathrm{P}_{11}^\mathrm{t})^\mathrm{T}$$

We can reorganize the formulation to be:

$$\min f^T u + h$$

$$\text{s.t.} \qquad \mathrm{a}_\mathrm{j}^T u + b_j^T d + d^T B_j u \leq \gamma_j \quad \mathrm{j} = 1{:}\,\mathrm{J}$$

$$\forall \mathrm{d}\ \mathrm{s.t.}\ \mathrm{Dd} \leq \mathrm{c}$$

$$\mathrm{Eu} \leq \mathrm{e}$$

1. for constraints (1) and (2), at time t:

$\mathrm{a}_{1,\mathrm{t}}^T = [-1, -1, 0]$ (the second -1 appears at 3t-1th place)

$\mathrm{b}_{1,\mathrm{t}}^T = [-1, -1]$ (the first -1 appears at 2t-1th place)

$\mathrm{B}_{1,\mathrm{t}}$ is a matrix with elements (1, 3) and (2, 3) equal to -1, others equal to zero.

$\gamma_{1,\mathrm{t}} = \sum_{i\notin \mathrm{ind}} P_i^t + 1.$

$a_{2,t}^T, b_{2,t}^T, B_{2,t}$ have elements of 1 in the same places of -1 in j=1 case.
$\gamma_{2,t} = 1 - \sum_{i \notin \text{ind}} P_i^t$.

2. for constraints (4) at time t:
$a_{3,t}^T = [0, \ -1, 0]$ (-1 appears at 3t-1th place)
$b_{3,t}^T$ is a zero vector.
$B_{3,t}$ is a matrix with elements (1, 3) and (2, 3) equal to -1, others equal to zero.
$\gamma_{3,t} = 1$.
$a_{4,t}^T, b_{4,t}^T, B_{4,t}$ have elements of 1 in the same places of -1 in j=3 case.
$\gamma_{4,t} = 1$.

3. for constraints (5), (6) at time t:
Two constraints can be represented by:
$-0.1\left(u_9^t + \left(P_4^t + P_{11}^t\right) v_9^t\right) \le 1 - E_9^{t-1}$
$0.1\left(u_9^t + \left(P_4^t + P_{11}^t\right) v_9^t\right) \le E_9^{t-1}$
$a_{5,t}^T = [-0.1,0,0]$
$b_{5,t}^T = [0,0]$
$B_{5,t}$ is a matrix with elements (1, 3) and (2, 3) equal to -0.1, others equal to zero.
$\gamma_{5,t} = 1 - E_9^{t-1}$.
$a_{6,t}^T, b_{6,t}^T, B_{6,t}$ have negative elements of j=5 case.
$\gamma_{6,t} = E_9^{t-1}$

4. for constraints (8) at time t:
$a_{7,t}^T = [0, -1\ , 0]$ (-1 appears at 3t-1th place)
$b_{7,t}^T$ is a zero vector.
$B_{7,t}$ is a matrix with elements (1, 3) and (2, 3) equal to -1, others equal to zero.
$\gamma_{7,t} = P_8^t + 1$.
$a_{8,t}^T, b_{8,t}^T, B_{8,t}$ have elements of 1 in the same places of -1 in j=7 case.
$\gamma_{8,t} = 1 - P_8^t$.

5. for constraints (9) at time t:
$a_{9,t}^T = [0, -1\ , 0]$ (-1 appears at 3t-1th place)
$b_{9,t}^T = \ [0, -1]$ (-1 appears at 2tth place)
$B_{9,t}$ is a matrix with elements (1, 3) and (2, 3) equal to -1, others equal to zero.
$\gamma_{9,t} = P_8^t + P_{10}^t + P_{13}^t + 1$.
$a_{10,t}^T, b_{10,t}^T, B_{10,t}$ have elements of 1 in the same places of -1 in j=9 case.
$\gamma_{10,t} = 1 - (P_8^t + P_{10}^t + P_{13}^t)$.

6. For constraints only related to disturbance variables d (constraint (7), (10), (11)):
D is a 8x2 matrix that has elements (1, 1), (3, 1), (5, 2) and (7, 2) equal to -1;
elements (2, 1), (4, 1), (6, 2) and (8, 2) equal to 1, all other elements are 0.
For time t,

$$c_t^T = [P_3^t + 1, 1 - P_3^t, -P_{4,\min}^t, P_{4,\max}^t, P_{10}^t + P_{13}^t + 1, 1 - P_{10}^t - P_{13}^t, -P_{11,\min}^t, P_{11,\max}^t]$$

7. For constraints only related to control variables u (constraint (3)):
E = [-1, 0, 0; 1, 0, 0];
e = [0, 2];

8. For the cost function
$f^T = [-T_{max}, -2, -2(\overline{P_4} + \overline{P_{11}}), 0, \cdots, -2, -2(\overline{P_4} + \overline{P_{11}}), 0]$
(3t+1 elements in total)
h = $\sum_{t=1}^{Tmax}(-3\sum_{i\notin \text{ind}} P_i - 3\overline{P_4} - 3\overline{P_{11}})$

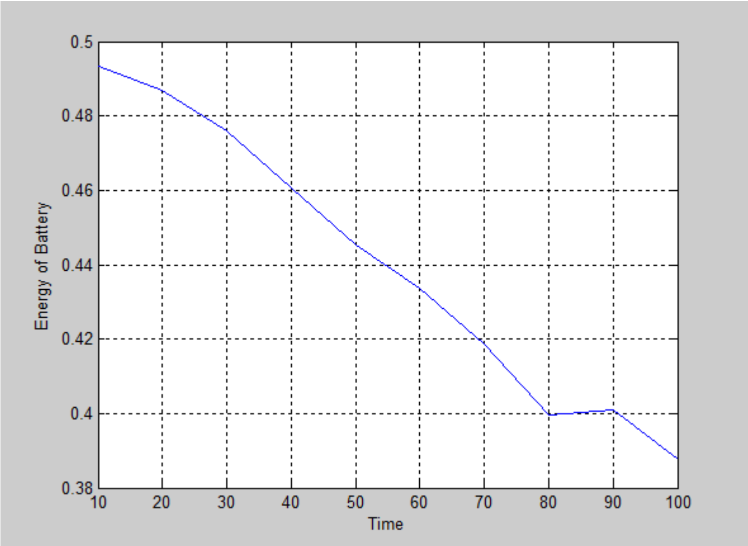
The formulation of non-robust problem at each time step:

$$\min f^T u$$
$$\text{s.t. } a_j^T u + c^T\lambda_j \leq \gamma_j$$
$$D^T\lambda_j = b_j + B_j u$$
$$Eu \leq e$$
$$\lambda_j \geq 0$$
$$(j=1:J)$$

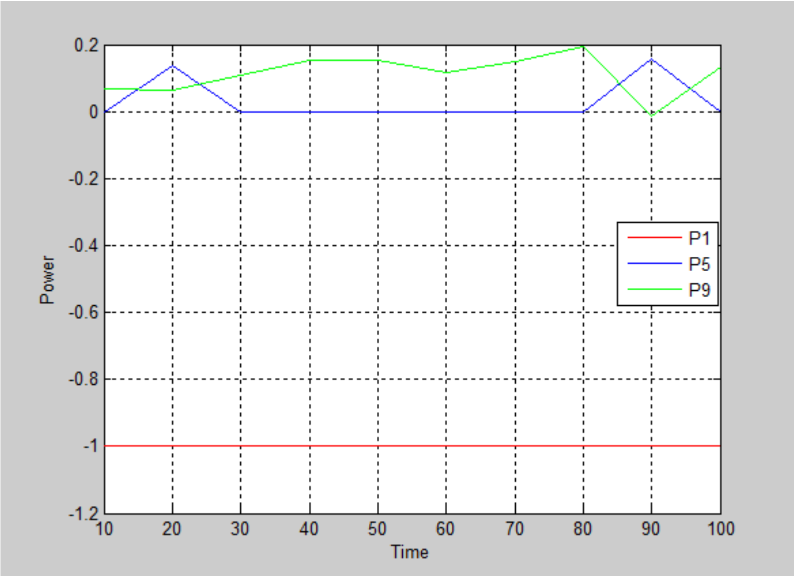
with all coefficient vectors & matrices defined above.
J=10; $a_j^T: 1 \times 3$; $c^T: 1 \times 8$; $D^T: 2 \times 8$; $b_j: 2 \times 1$; $B_j: 2 \times 3$; $E: 2 \times 3$; $e: 2 \times 1$.

Question 8

Solve the non-robust optimization problem and plot the state of energy $E_{9;t}$.



The energy of the battery keeps decreasing, because the power from the battery has relatively small penalty cost. It wants to transfer energy to node 1 for smaller total cost, since power to node 1 has largest penalty.



From the power profile, it is to see that P1 always remains as -1, because this is the lower bound for creating smallest cost;
P5 and P9 are positive because they have relatively smaller penalty coefficients and want to supply power to node 1;
At most times, P5 remains 0 as it has larger penalty cost than P9. When t=20 and t=90, certain power flow limits are reached so that P9 is not able to generate power. The rest is therefore covered by P9.

Question 9 AC Optimal Power Flow

Write the equivalent problem that is without absolute values.

$$\min \sum_{i=2}^{13} u_{i,t}$$

$$\text{s.t} \quad \mathrm{V_{i,t}} - 1 \le u_{i,t}$$

$$-\mathrm{V_{i,t}} + 1 \le u_{i,t}$$

$$0 \le \mathrm{P_{5,t}} \le 0.5$$

$$\mathrm{Q_{5,t}} \le 0.2P_{5,t}$$

$$-\mathrm{Q_{5,t}} \le 0.2P_{5,t}$$

$$-1 \le \mathrm{P_{9,t}} \le 1$$

$$0 \le \mathrm{E_{9,t-1}} - 0.1P_{9,t} \le 1$$

$$0.95 \le \mathrm{V_i} \le 1.05$$

and load flow equations

(for all $\mathrm{i} \neq 1$)

For each time t, load flow equations in the formulation constraints are:

$$\mathrm{J}_{P_5,P_9,Q_5}^{-1} \cdot \left[P_5 - P_5^0, P_9 - P_9^0, Q_5 - Q_5^0\right]^T = [V - V_0; \Theta - \Theta_0]$$

N.B. $\mathrm{J}_{P_5,P_9,Q_5}^{-1}$ refers to the corresponding columns of the inverse jacobian matrix.
The decision variable vector is chosen to be:

$$\mathrm{x^T} = [u^T, V^T, \Theta^T, P_5, P_9, Q_5]$$

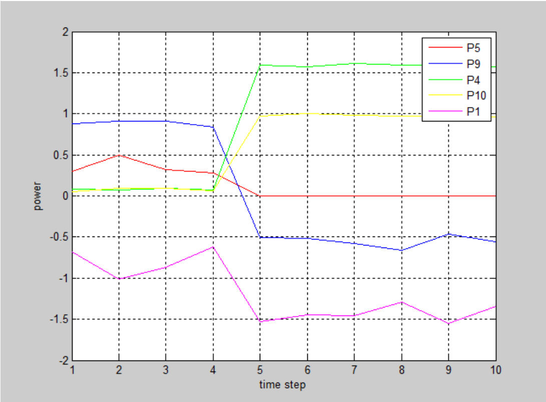
which has size 39×1.

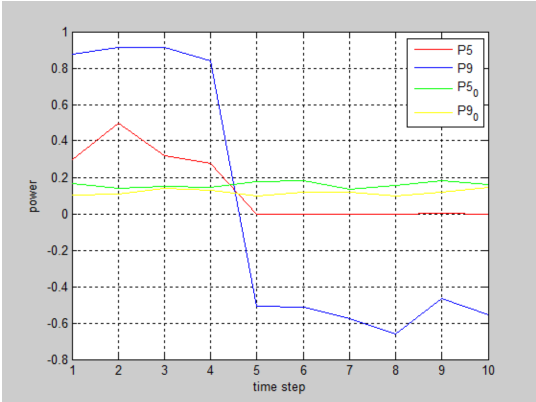
Question 10

Write the voltage control problem in each time step of this period, using our computed voltage and one-step SLP. Formulate the problem as a linear program in each time step. Solve it and plot the adjusted values of $P_{5;t}$; $P_{9;t}$ and analyze their trajectories.

For the formulation, please refer to the previous question.

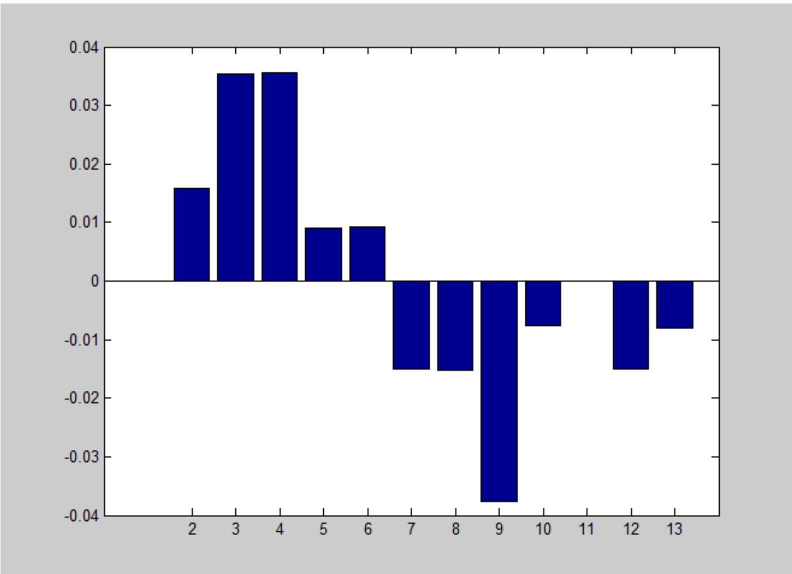
Power revolution:





At time 5, we see an obvious increase in PV and wind turbine power output. In response, diesel generator stops supplying power to the system, and the battery starts absorbing power. This result makes sense, because the remaining power is consumed locally by the battery in order to avoid too much power flow from node 7 to 2 and 2 to 1, which will cause serious voltage rise at nodes 7 and 2.

On the other hand, we can see that not all power gap is absorbed by the battery. Some is transferred out of the system (P 1). This is because, an over-large power flow to the battery will create big voltage drop on node 8. The cost function has equal weighting factors of all nodes' voltage, so the program gives a quite even distribution of extra power's flow among different routes.



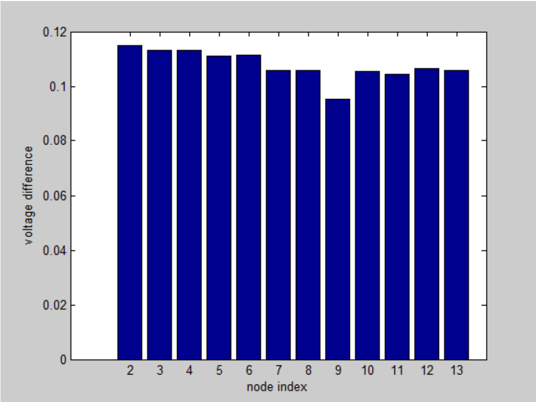
The graph above shows the voltage change from time 4 to time 5 of nodes 2 to 13.

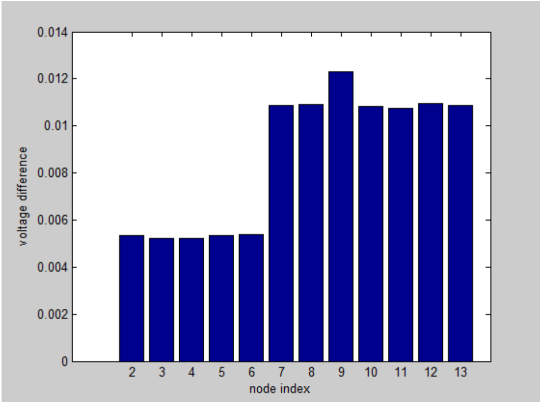
The voltages at node 3 and node 4 see greatest increase, since they're the only possible path

of PV node's power flow.
The voltage at node 9 sees greatest decrease, because it is the node for the battery and it is far away from the slack node where voltage is fixed to be 1. Large power from node 7 to node 9 pulls down the voltage at node 9.

Question 11 (bonus)

Do you think one-step SLP is sufficient? What will happen if $P_{4;t}$; $P_{11;t}$ are very large for long period?





The left graph shows the voltage magnitude difference of all nodes at time 5 between original values and values after first optimization. The right graph shows the same quantity, but between the first and second optimization.
While the first optimization makes voltages around 0.1 p.u. closer to exact solution, the second optimization pushes voltages 0.01 p.u. further. Therefore, one step SLP is usually enough. If there is more demand of accuracy, another step can be added upon as well.

If $P_{4,t}$ and $P_{11,t}$ stays large for a long time, battery would be fully charged and it is not able to absorb power anymore. Therefore, the power difference would all flow to node 1, leading to serious voltage rise along the route. If the power reaches certain limits, voltage could not be confined within upper bound, and there would be no feasible solution to the problem.