Water Resources Engineering Exercise

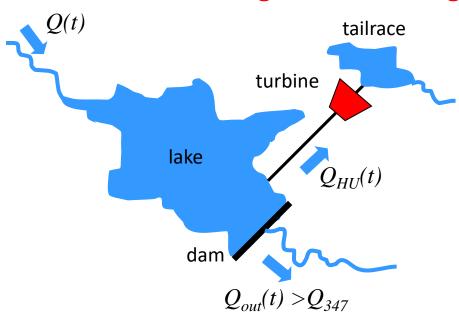
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lecture: Tuesday 8.15-10 in room GR B3 30

exercise: Thursday 10.15-12 in room INF 3

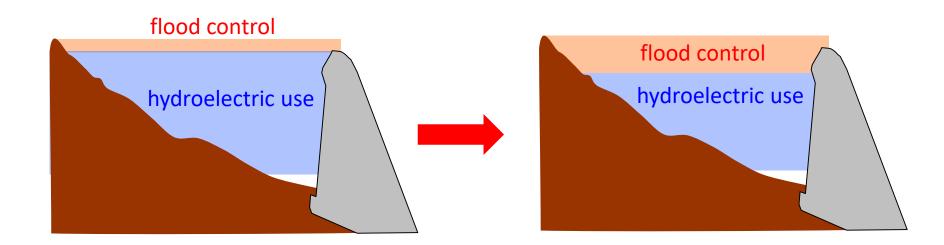
office hours: Thursday 12-14 (GR C1 532 – GR C1 564)

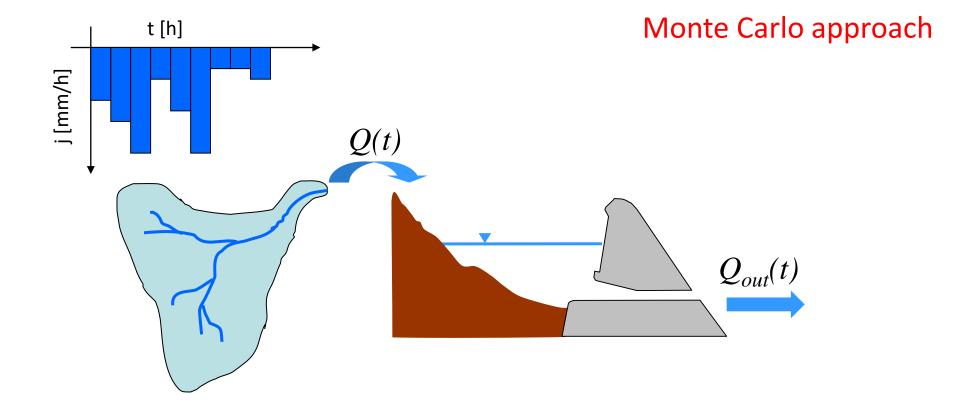
Assignment: management of a multipurpose reservoir



Evaluate the feasability of improving the flood control operations of an exixting reservoir of a hydropower plant.

A larger fraction of the storage needs to be reserved for flood control and cannot be used for hydroelectric purpose

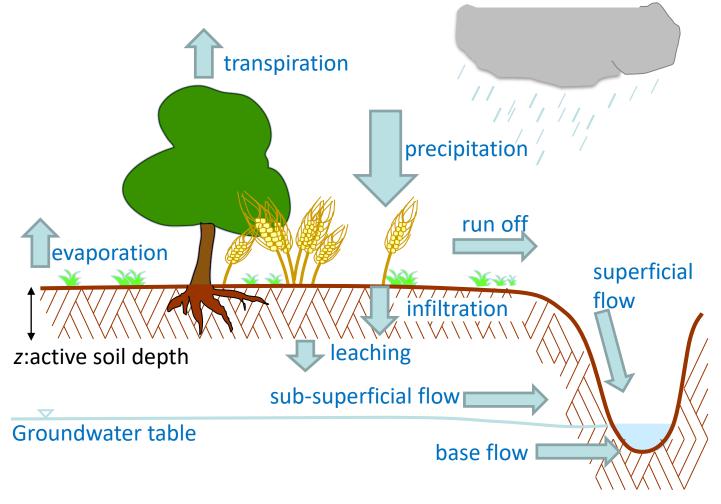




Step

- Develop and calibrate a lumped continuous hydrological model to transform rainfall into discharge.
 The model is fitted based on the available dataset.
- Generate rainfall time series with the same statistical properties of the observed ones.
- Transform generated rainfall into a generated time-series of input discharge.
- Simulate the reservoir routing and the flood control operations for different maximum level for hydroelectric use.
- Measure the energy produced and the flooding probability

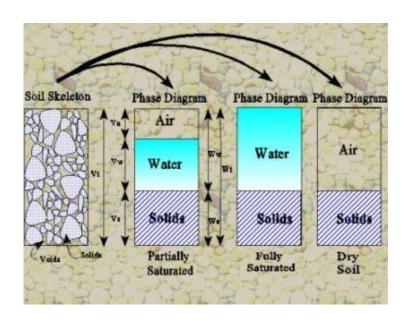
assigment, part 1: continuous lumped hydrological model



Lumped hydrological model does not consider explicitly the spatial distribution of soil properties and precipitation. The model is lumped in a representative soil column. The geometry of the basin is accounted for in the routing scheme that transforms run-off into superficial flow and leaching into sub-superficial flow. Continuous as it accounts for evapotranspiration and can thus be simulated for long time windows.

Opposite of lumped: distributed. Opposite of continuous: event based

soil properties



$$V_{\rm s}$$
 volume of solids

$$V_{_{\scriptscriptstyle{\mathcal{W}}}}$$
 volume of water

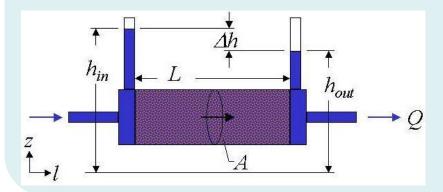
$$V_a$$
 volume of air

$$V = V_a + V_w + V_s$$
 total volume

$$n = \frac{V_a + V_w}{V}$$
 porosity

soil moisture
$$s = \frac{V_w}{V_a + V_w}$$
 dry $0 < s < 1$ saturation

Hydraulic conductivity at saturation K_{sat} [LT⁻¹]

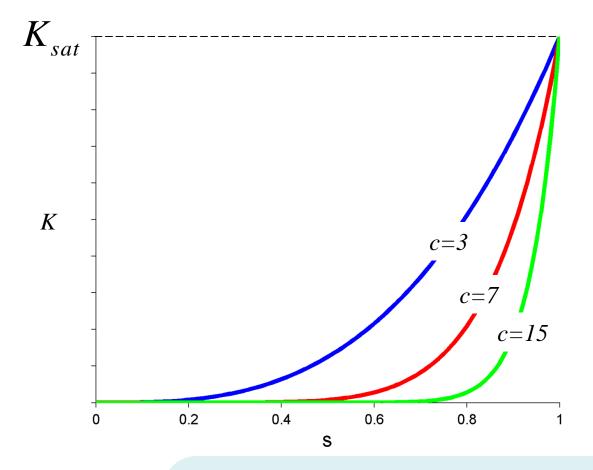


Darcy Law

$$\frac{Q}{A} = K_{sat} \frac{\Delta h}{L}$$

soil properties

unsaturated hydraulic conductivity K [LT⁻¹]

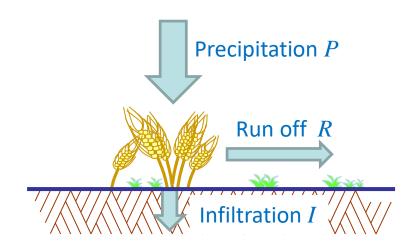


$$K(s) = K_{sat} \cdot s^c$$

K: unsaturated hydraulic conductivity

 $K_{\rm gat}$: saturated hydraulic conductivity

Precipitation/Infiltration/Runoff



$$P(t) = R(t) + I(t)$$

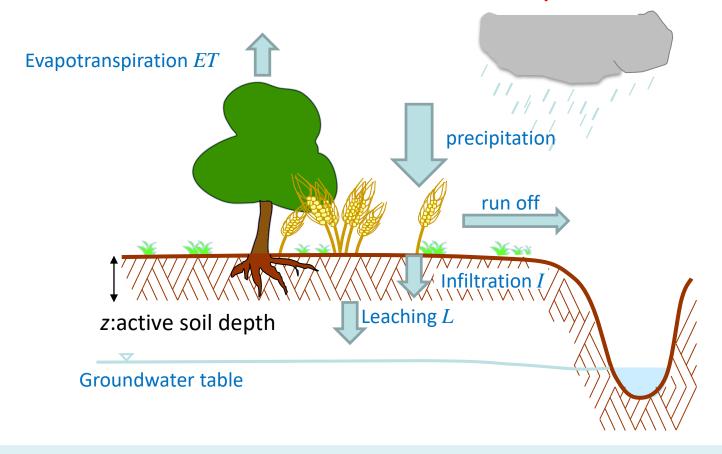
RUNOFF PRODUCTION MECHANISM

•Infiltration Excess (saturation from above, Horton mechanism): rate of rainfall exceeds the rate at which water can infiltrate into the ground. The maximum infiltration rate is equal to the saturate hydraulic conductivity

$$I(t) = \min(P(t), K_{sat})$$

$$R(t) = P(t) - I(t)$$

Soil moisture dynamic in the root zone



$$nz\frac{ds}{dt} = I(t) - \underbrace{ET(s,t)} - L(s,t)$$

$$L(s,t) = K(s) = K_{sat} \cdot s(t)^{c}$$

n: porosity [-]

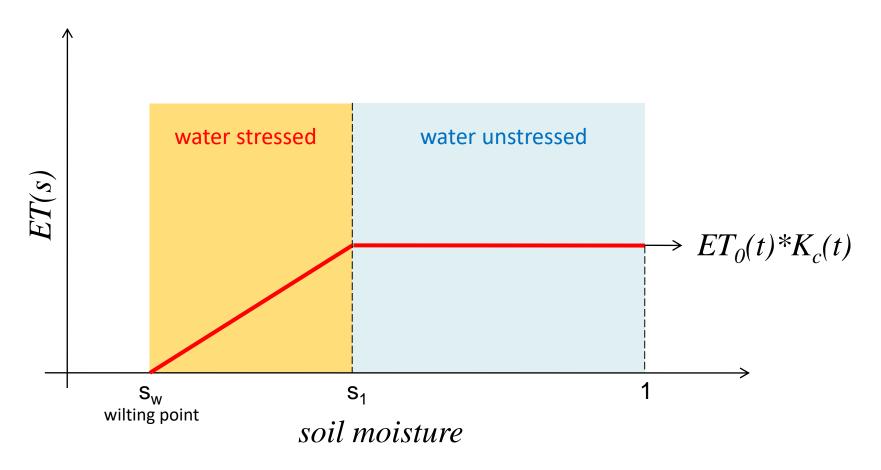
s: soil moisture 0 < s < 1 [-]

z: root zone thickness [L]

nsz: root zone water storage [L]

L: leaching [LT⁻¹]

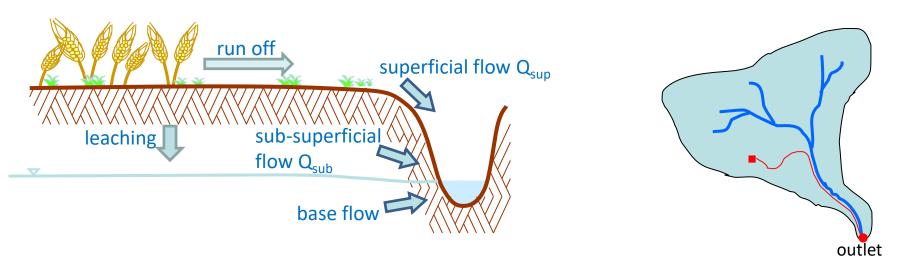
Evapotranspiration



 $ET_0(t)$: reference crop evapotranspiration (computed with the Thornthwaite method)

- $K_{c}(t)$: crop coefficient representative of the whole catchment
- s,, wilting point, soil moisture content at which plants stop transpiring
- s₁: soil moisture above which plants transpire at the maximum rate

routing



routing model accounts for the delay imposed by the drainage path of water (inside the hillslope and in the channel) from where it is produced to the outlet

runoff
$$R(t)$$
 superficial flow $Q_{sup}(t)$

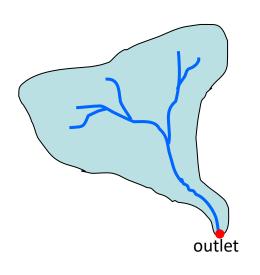
leaching $L(t)$ sub-superficial flow $Q_{sub}(t)$

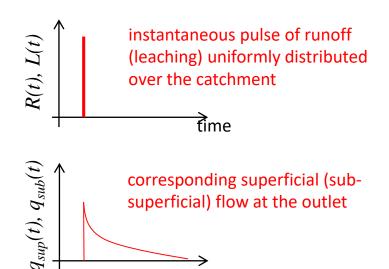
baseflow $Q_b(t)$ long term drainage of groundwater (not directly coming from precipitation)

Total discharge at the outlet $Q(t) = Q_{sup}(t) + Q_{sub}(t) + Q_b(t)$

routing

Linear reservoir scheme equivalent to assuming an exponential instantaneous unit hydrograph (IUH) often used when the time spent in the channel is negligible with respect to that spent in the hillslope





$$\frac{dV_{sup}}{dt} = R(t) - q_{sup}(t)$$

$$q_{sup}(t) = V_{sup}(t) / t_{sup}$$

$$\frac{dV_{sub}}{dt} = L(t) - q_{sub}(t)$$

$$q_{sub}(t) = V_{sub}(t) / t_{sub}$$

 $V_{\it sup}$: water stored in the superficial layer [L]

 $V_{\it sub}$: water stored in the sub-superficial layer [L]

time

 t_{sup} : mean superficial residence time [T]

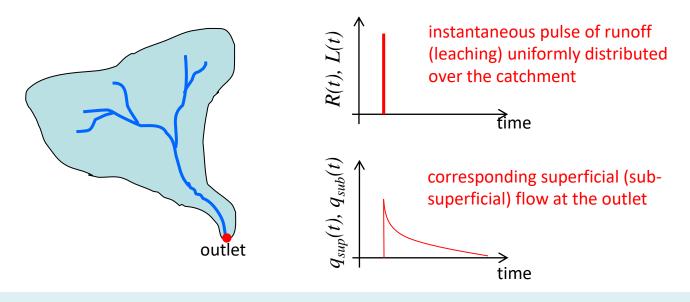
 t_{sub} : mean sub-superficial residence time [T]

 q_{sup} : superficial specific discharge (discharge per unit of area) [LT⁻¹]

 q_{sub} : sub-superficial specific discharge (discharge per unit of area) [LT $^{-1}$]

routing

Linear reservoir scheme equivalent to assuming an exponential instantaneous unit hydrograph (IUH) often used when the time spent in the channel is negligible with respect to that spent in the hillslope

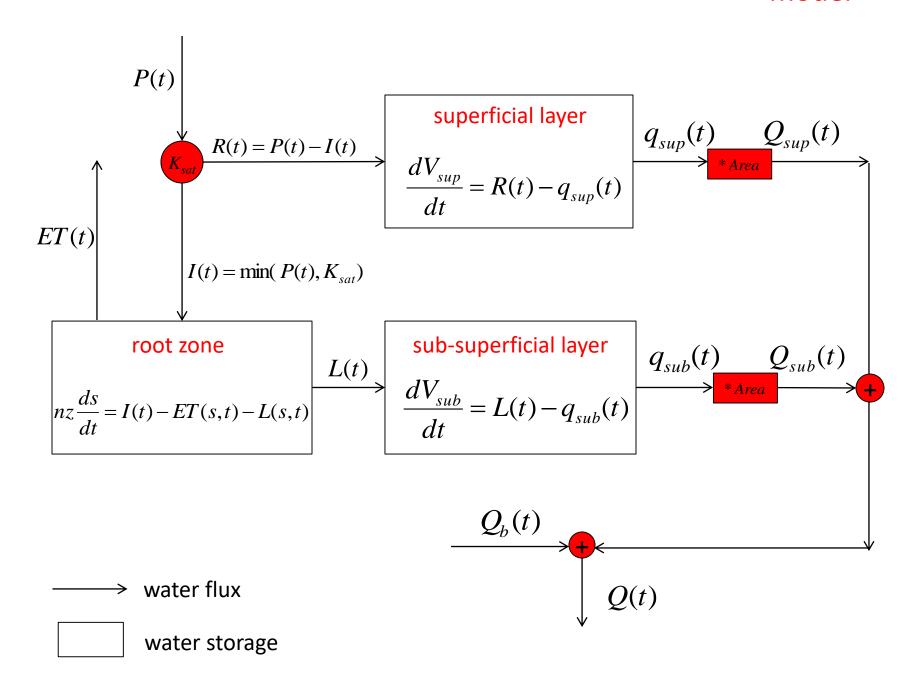


$$Q_{sup}(t) = Area * q_{sup}(t)$$
$$Q_{sub}(t) = Area * q_{sub}(t)$$

$$Q_{sub}(t) = Area * q_{sub}(t)$$

Area: Area of the catchment

model



numerical integration: Euler explicit method

The dynamics of each storage (root zone, superficial and subsuperficial storage) can be generically written as:

$$\frac{dV(t)}{dt} = F_{in}(t) - F_{out}(t)$$

where F_{in} and F_{out} represent input and output fluxes, respectively.

Numerical integration

Taylor
$$V(\bar{t} + \Delta t) = V(\bar{t}) + \Delta t \frac{dV}{dt}\Big|_{t=\bar{t}} + o(\Delta t^2)$$

$$V(\bar{t} + \Delta t) \approx V(\bar{t}) + F_{in}(\bar{t})\Delta t - F_{out}(\bar{t})\Delta t$$

Time step of integration $\Delta t=1$ hour

model input and parameters

model inputs

some parameters can be estimated from literature values and analysis of the discharge time series

```
0.25
                  Wilting point [-]
S_{w}
                  Soil moisture above which plants transpire at k<sub>r</sub>*ET<sub>0</sub> [-]
        0.4
S_{1}
         0.3
                  Porosity [-]
n
         7
                  Baseflow [m<sup>3</sup>/s]
Q_b
         22
                  Superficial residence time [h]
t_{sup}
A
        4000
                  Area of the basin [km<sup>2</sup>]
         38
                   Latitude of the basin [degrees]
φ
```

the remaining parameters (k_{sat}, c, t_{sub}, z) need to be calibrated

calibration

Goodness of fit: Nash-Sutcliffe

$$NS = 1 - \frac{\sum_{t} (Q_o(t) - Q(t))^2}{\sum_{t} (Q_o(t) - \overline{Q_o(t)})^2}$$

 ${\it Q}$: modelled discharge

 Q_o : observed discharge

 \overline{Q}_o : average observed discharge

Nash-Sutcliffe index ranges from $-\infty$ to 1:

NS = 0: the model is as predictive as the mean of the observed values

NS = 1: perfect fit ($Q = Q_o$ for every t)

Free parameters for calibration: k_{sat} , c, t_{sub} , z

A (meta-heuristic) calibration algorithm must be implemented (next week!)

Build function Lastname_HydroModel.m that, given the time series of precipitation and a parameter set, computes the time series of discharge that results from the implementation of the hydrological model.

Function Lastname_HydroModel.m is called by the main script Lastname CalibrateHM.m, in which:

- Inputs and known parameters are read;
- Potential evapotranspiration is computed;
- The hydrological model is tested (check on water balance);
- The algorithm for calibration is implemented (next week!).

For this week, attribute arbitrary values to the parameters that require calibration in order to test the model.

Possible ranges:
$$10^{-7} \le K_{sat} \le 10^{-5} \text{ [m/s]}$$
 $1 \le c \le 20 \text{ [-]}$

$$1 \le t_{sub} \le 400 \text{ [h]}$$
 $1 \le z \le 2000 \text{ [mm]}$