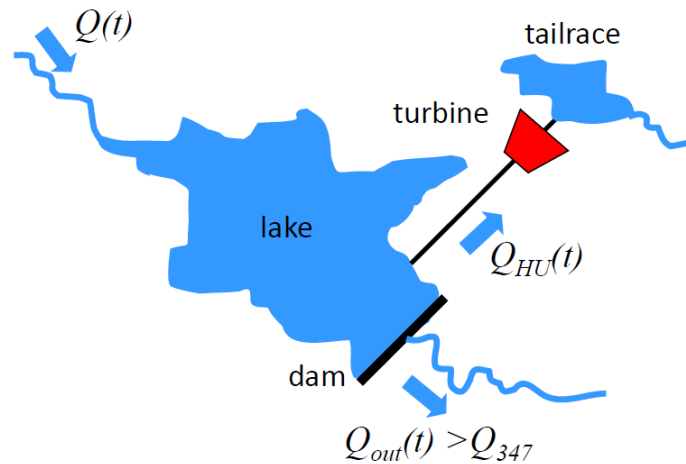


# Assignment: Management of a multipurpose reservoir

Deadline for submission: 24<sup>th</sup> December



## Synopsis

Evaluate the feasibility of improving the flood control operations of an existing reservoir of a hydropower plant. A larger fraction of the storage needs to be reserved for flood control and cannot be used for hydroelectric generation. The downstream part of the river should be protected from floods larger than  $Q_{lim} = 150 \text{ m}^3/\text{s}$ . The larger the volume for hydroelectric use, the higher the energy production. The larger the volume for flood control, the more efficiently floods can be attenuated. The two uses (hydroelectric and flood control) are therefore in competition. The goal of the assignment is to estimate the trade off between energy production and probability that the annual maximum released flow  $Q_{out}$  exceeds  $Q_{lim}$ , as a function of the volume reserved for flood control. Note that the discharge used for hydropower generation is then released into a different river system.

## Tasks

1. Develop a continuous lumped hydrological model for the drainage basin of the reservoir to transform rainfall into discharge (inflow in the reservoir).
2. Fit the hydrological model using as training set the available time series of precipitation and discharge.
3. By using a Poisson pulse model, generate a 100-years-long rainfall time series with the same statistical properties as the observed rainfall.
4. Transform generated rainfall into a generated time-series of discharge by means of the fitted hydrological model.
5. Simulate the reservoir routing and the flood control operations for different maximum levels for hydroelectric use.
6. Evaluate the average annual energy produced and the flooding probability.

7. (*facultative*) Repeat the simulations of point 5. by using seasonally varying maximum levels for hydroelectric use and different values of the discharge for hydroelectric use  $Q_{HU}$ . Find the subset of non-dominated solutions and analyse their properties.

### Input data

- Six-years-long series of measured rainfall and discharge (files `P.txt`, `Q_obs.txt`);
- Monthly mean series of temperature and crop factor (files `temperature.txt` and `kc.txt`)
- Parameters for the hydrological model: wilting point  $s_w$ , soil moisture for maximum plant transpiration  $s_1$ , porosity  $n$ , base flow  $Q_b$ , mean superficial residence time ( $t_{sup}$ ), area of the basin  $A$  and latitude (see Table 2);
- Relation between area of reservoir lake and level of the reservoir (file `area_rating_curve.txt`);
- Reservoir parameters: discharge coefficients for sluice gate and spillway ( $C_{q,sl}$ ,  $C_{q,sp}$ ) spillway length and height ( $L$ ,  $p$ ) (see Table 2);
- The design discharge of the turbine  $Q_T = 65 \text{ m}^3/\text{s}$ ;
- Power plant parameters: pipe diameter, length and sand equivalent roughness ( $D$ ,  $L_p$ ,  $k_s$ ), turbine efficiency  $\eta$ , altitude gap between the bottom of the reservoir and the tailrace  $\Delta h$ , minimum level for hydropower production (see Table 2).

### Procedure

**Hydrological model.** Implement the hydrological model on the 6-years-long series of precipitation and discharge.

1. Compute the potential evapotranspiration  $ET_0$  by means of Thornthwaite method.
2. Attribute arbitrary values to the parameters  $K_{sat}$ ,  $c$ ,  $t_{sub}$ .
3. Use an Euler explicit scheme to integrate the system of equations at hourly scale (see Table 3).
4. Test the mass balance of your system in order to ensure a correct implementation
5. Evaluate the goodness of fit of the chosen parameter set by calculating the Nash-Sutcliffe index  $NS$ .

$$NS = 1 - \frac{\sum_{t=t_0}^{t_{end}} [Q_{obs}(t) - Q_{mod}(t)]^2}{\sum_{t=t_0}^{t_{end}} [Q_{obs}(t) - \overline{Q_{obs}}]^2};$$

where  $Q_{obs}$  and  $Q_{mod}$  are the measured and modelled discharges, respectively.  $\overline{Q_{obs}}$  is the average measured discharge.

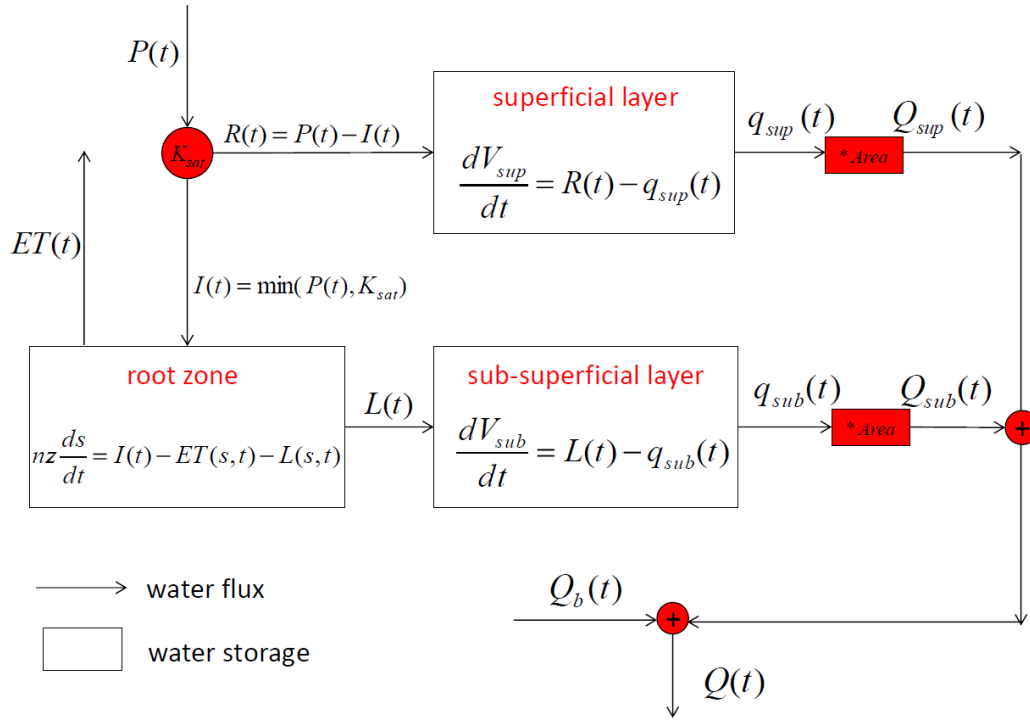


Figure 1: Scheme of the hydrological model. Boxes indicate water storages, arrows indicate water fluxes.

**Model calibration.** Find the set of parameters  $\{K_{sat}, c, t_{sub}, z\}$  that maximizes the Nash-Sutcliffe index. Use a simulated annealing strategy in order to find the parameter set that maximizes NS.

1. Define a functional form for the temperature

$$T_{SA}(i) = \exp(-c_r \cdot i)$$

where  $i$  counts the iterations of the calibration procedure, while  $c_r$  is a cooling rate (see Table 2)

2. Attribute arbitrary values to the parameters  $K_{sat}, c, t_{sub}, z$ . Run the hydrological model and evaluate  $NS_{old}$ .
3. Select a new parameter set by drawing from a truncated normal distribution (function `TruncNormRnd.m`).
4. Run the hydrological model with the new parameter set and evaluate  $NS_{new}$ .
5. If  $NS_{new} > NS_{old}$ , accept the new parameter set.
6. Otherwise, accept the new parameter set with probability

$$\exp\left(\frac{NS_{new} - NS_{old}}{T_{SA}(i)}\right)$$

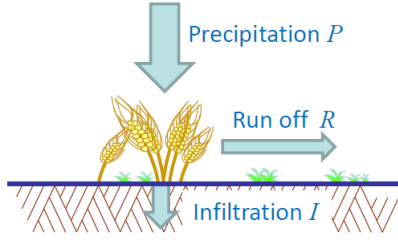
7. Repeat from 3. until convergence. A good fitting should be around  $NS = 0.89$ .

Variable	Symbol	Unit
Precipitation	$P$	$[LT^{-1}]$
Infiltration	$I$	$[LT^{-1}]$
Runoff	$R$	$[LT^{-1}]$
Soil moisture	$s$	$[-]$
Evapotranspiration	$ET$	$[LT^{-1}]$
Leaching	$L$	$[LT^{-1}]$
Superficial storage	$V_{sup}$	$[L]$
Superficial specific discharge	$q_{sup}$	$[LT^{-1}]$
Superficial discharge	$Q_{sup}$	$[L^3T^{-1}]$
Sub-superficial storage	$V_{sub}$	$[L]$
Sub-superficial specific discharge	$q_{sub}$	$[LT^{-1}]$
Sub-superficial discharge	$Q_{sub}$	$[L^3T^{-1}]$

Table 1: List of variables for the hydrological model

Parameter	Symbol	Value	Unit
<i>Hydrological model: to be calibrated</i>			
Hydraulic conductivity for saturated soil	$K_{sat}$	-	$[LT^{-1}]$
Exponent for power-law relation $L(s)$	$c$	-	$[-]$
Root zone depth	$z$	-	$[L]$
Mean sub-superficial residence time	$t_{sub}$	-	$[T]$
<i>Hydrological model: other</i>			
Wilting point	$s_w$	0.25	-
Soil moisture stress threshold	$s_1$	0.4	-
Porosity	$n$	0.3	-
Baseflow	$Q_b$	8	$m^3/s$
Mean superficial residence time	$t_{sup}$	18	h
Catchment area	$A$	4000	$km^2$
Latitude	$\phi$	40	$^\circ$
<i>Calibration of the hydrological model</i>			
Cooling rate	$c_r$	1/1200	-
<i>Reservoir</i>			
Discharge coefficient for sluice gate	$C_{q,sl}$	0.6	-
Discharge coefficient for spillway	$C_{q,sp}$	0.5	-
Length of the spillway	$L$	100	m
Level of the spillway crest	$p$	18	m
<i>Power plant</i>			
Discharge to the turbine	$Q_T$	65	$m^3/s$
Pipe diameter	$D$	3	m
Pipe length	$L_p$	900	m
Sand equivalent roughness	$k_s$	0.3	mm
Turbine efficiency	$\eta$	0.85	-
Altitude gap between the bottom of the reservoir and the tailrace	$\Delta h$	60	m
Minimum level for hydroelectric production	$l_{min,HU}$	2	m

Table 2: List of parameters

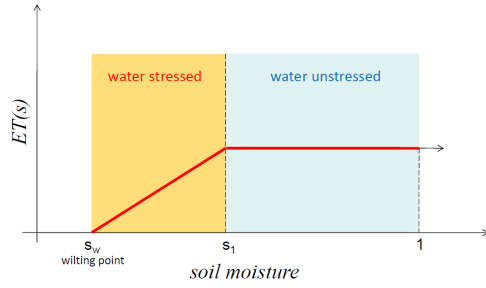


### Runoff generation (Horton mechanism):

$$P(t) = R(t) + I(t);$$

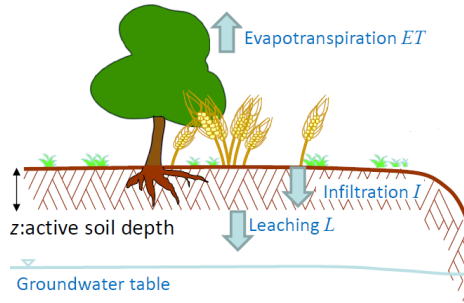
$$I(t) = \min(P(t), K_{sat});$$

$$R(t) = P(t) - I(t);$$



### Evapotranspiration computation:

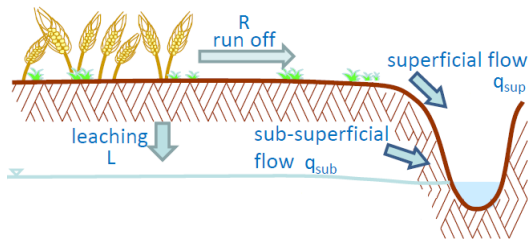
$$ET(t) = \begin{cases} 0, & \text{if } 0 \leq s(t) \leq s_w, \\ k_c ET_0 \cdot \frac{s(t) - s_w}{s_1 - s_w}, & \text{if } s_w < s(t) \leq s_1. \\ k_c ET_0, & \text{if } s_1 < s(t) \leq 1. \end{cases}$$



### Soil moisture dynamics:

$$nz \frac{ds}{dt} = I(t) - ET(s, t) - L(s, t);$$

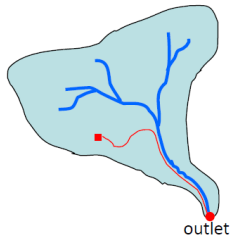
$$L(t) = K(s) = K_{sat} \cdot s(t)^c;$$



### Linear reservoir scheme:

$$\frac{dV_{sup}}{dt} = R(t) - q_{sup}(t); \quad q_{sup}(t) = t_{sup}^{-1} V_{sup}(t);$$

$$\frac{dV_{sub}}{dt} = L(t) - q_{sub}(t); \quad q_{sub}(t) = t_{sub}^{-1} V_{sub}(t).$$



### Discharge computation:

$$Q_{sup}(t) = \mathcal{A} \cdot q_{sup}(t);$$

$$Q_{sub}(t) = \mathcal{A} \cdot q_{sub}(t);$$

$$Q(t) = Q_{sup}(t) + Q_{sub}(t) + Q_b.$$

Table 3: System of equations constituting the hydrological model.

**Simulated rainfall sequence.** Generate 100 years of rainfall (at daily timescale) with the parameters (mean daily rainfall depth  $\alpha$  and rainfall frequency  $\lambda$ ) estimated from the observed precipitation.

1. Up-scale the given hourly rainfall series to average daily rainfall. The resulting time series should have one value for each day with mm/day as unit.
2. Evaluate the parameters  $\alpha$  (mean precipitation intensity) and  $\lambda$  (frequency of rainfall events). Account for seasonality by computing different values of the parameters for each month.
3. Generate the 100-years-long rainfall series. For any generic day of month  $m$ :
  - a rainfall event occurs with probability  $\lambda(m)\Delta t$ , where  $\Delta t = 1$  day.
  - If a rainfall events occurs, the rainfall depth is extracted from an exponential distribution with mean  $\alpha(m)$ . Use the inverse transformation method to generate exponentially distributed random variables.

Points 2. and 3. are repeated for all the days and all the months of the generation period.

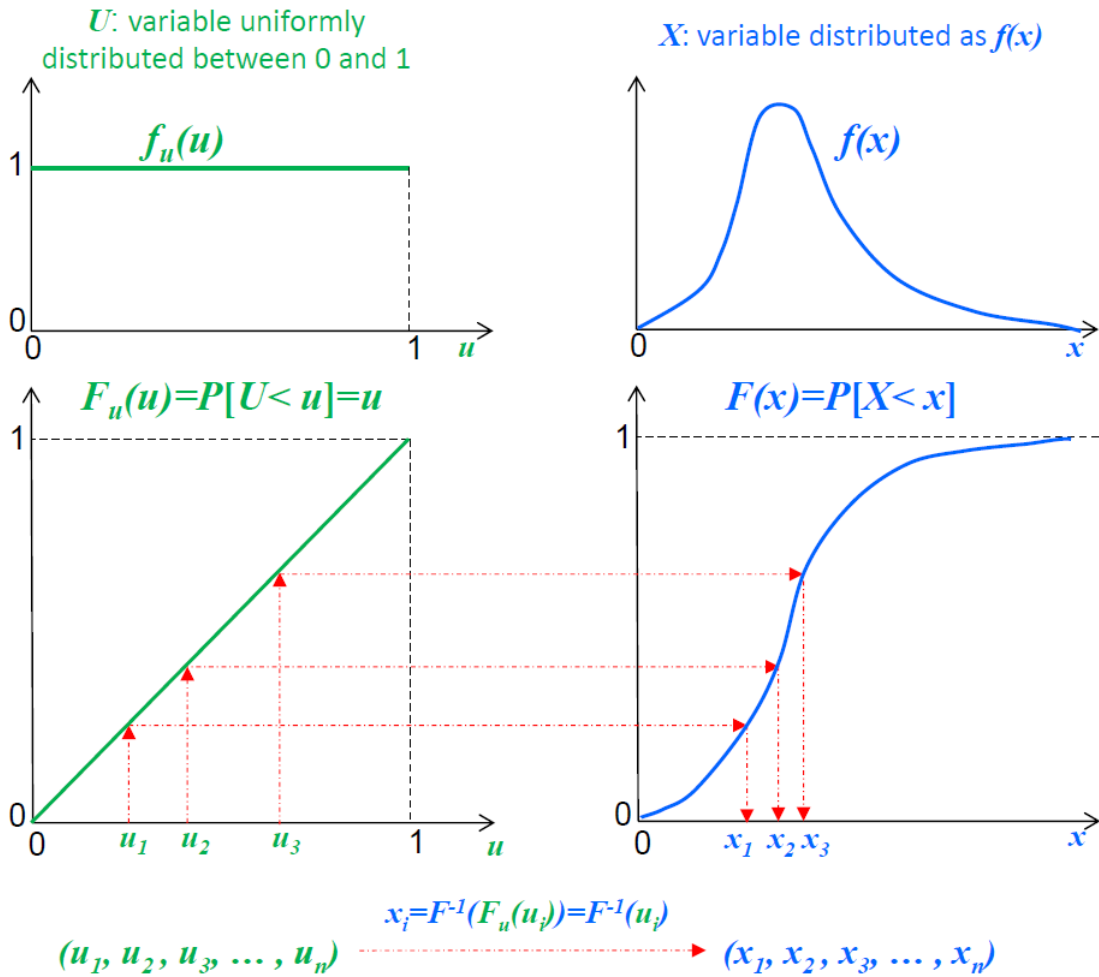


Figure 2: Inverse transformation method.

4. Use the function `downscaling.m` to down-scale the daily generated rainfall to hourly rainfall.

**Simulated discharge sequence.** Run the calibrated hydrological model forced by the generated rainfall and obtain the discharge time series that will constitute the input of the reservoir.

**Flood control.** Implement the flood control practice and the reservoir routing. Assume that the power plant works only during peak hours (from 12.00 to 18.00, 6 hours). If at the beginning of the day (midnight) the level in the reservoir is above the minimum level for hydroelectrical use, the plant will work during that day, otherwise it will not. When the turbine is working,  $Q_{HU}$  is always equal to the turbine design discharge  $Q_T$ . To compute the net head of the turbine, account for frictional head losses along the pipe and entrance head loss (half of the kinematic term). Exit head losses are negligible. The efficiency  $\eta$  of the turbine can be assumed as constant for the range of head experienced.

1. Compute the minimum flow target  $Q_{347}$  (discharge that is exceeded 95% of the time) from the 100-years-long generated input discharge time series.
2. Starting from the area rating curve, compute the volume rating curve (use trapezoidal approximation).
3. Assume a value for the maximum level for hydroelectric use (e.g. 15 m) and compute the corresponding volume.
4. For each time step of integration ( $\Delta t = 1$  hour):
  - Evaluate the level of the reservoir (use the function `level_volume.m` which is faster than `interp1`).
  - Compute the discharge  $Q_{HU}$  routed to the power plant.  $Q_{HU} = Q_T$  during peak hours if the power plant is working in that particular day, otherwise  $Q_{HU}=0$ .
  - Compute the sluice opening area  $A$ . The opening of the sluice gate is operated so that:
    - the discharge through the gate  $Q_g(t)$  is larger than the minimum flow  $Q_{347}$  and lower than  $Q_{lim}$ ;
    - the level is kept, if possible (i.e. if the gate discharge is within the aforementioned limits), at the maximum level for hydroelectrical use;
    - if during floods the maximum level for hydroelectrical use is exceeded, the reservoir is emptied as quick as possible.

The above practice can be implemented as follows. At each time-step, the opening of the sluice gate is computed so that volume at the end of the time-step equals (if possible, i.e. if the gate discharge is within the aforementioned limits) the volume corresponding to the maximum level for hydroelectrical use ( $V_{max,HU}$ ).

$$Q_g(t) = \max \begin{cases} Q_{347} \\ \min \left\{ \frac{V(t) + [Q(t) - Q_{HU}(t)] \cdot \Delta t - V_{max,HU}}{\Delta t}, Q_{lim} \right\} \end{cases}$$

where  $V$  is the volume stored in the reservoir.

- Compute the total output discharge  $Q_{out}$ :

$$Q_{out} = \begin{cases} C_{q,sl} A \sqrt{2gl} & \text{if } l \leq p, \\ C_{q,sl} A \sqrt{2gl} + C_{q,sp} L \sqrt{2g(l-p)^3}, & \text{if } l > p. \end{cases}$$

where  $l$  is the level in the reservoir (with respect to the empty pool),  $p$  and  $L$  are the level of the spillway crest and the spillway length, respectively;  $C_{q,sl}$  and  $C_{q,sp}$  are the discharge coefficients for sluice gate and spillway, respectively;  $g$  is the gravity acceleration.

- Integrate the storage equation:

$$\frac{dV(t)}{dt} = Q(t) - Q_{out}(l(V(t))) - Q_{HU}(t),$$

via an Euler explicit scheme. Other fluxes (e.g. evaporation from the lake, input of rainfall into the lake, deep percolation) are negligible.

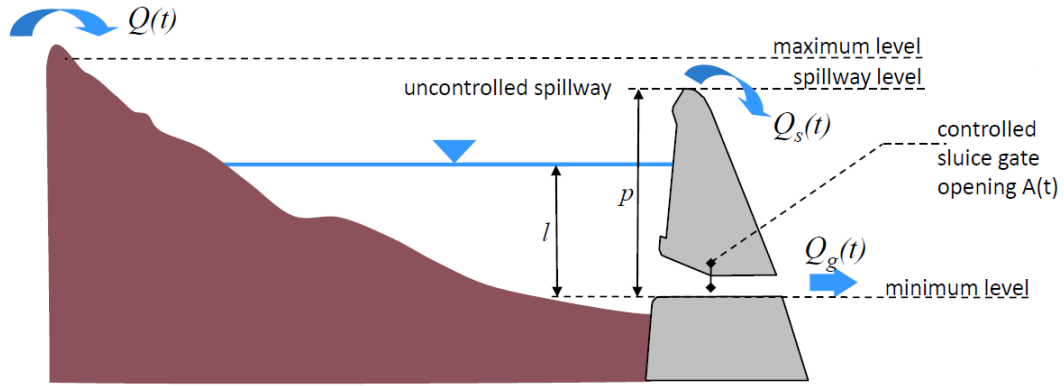


Figure 3: Cross-section of the reservoir

**Energy production and flooding probability.** Repeat the flood control practice and the routing of the reservoir for different values (from 10 to 18 m, with step size of 1 m) of maximum level for hydroelectrical use  $l_{max,HU}$ . Compute the average annual energy production in [GWh] and the probability that the annual maximum released discharge  $Q_{out}$  is larger than  $Q_{lim} = 150 \text{ m}^3/\text{s}$  (for numerical reasons, the condition should read: maximum annual  $Q_{out} > 151 \text{ m}^3/\text{s}$ ) as a function of  $l_{max,HU}$ . Use the same sequence of generated discharge (100 years long) to simulate the reservoir routing for different values of  $l_{max,HU}$ .

**Facultative: Multi-criteria optimization** In a separate script (Lastname\_MultiCriteria.m), modify the previous reservoir routing by accounting for seasonally varying maximum levels for hydroelectric use. Moreover, consider different values for the design discharge to the turbine  $Q_T$ . Using as input a discharge time series of shorter length, run the reservoir routing and plot flooding probability against average annual energy production for all possible combinations of seasonal  $l_{max,HU}$  and  $Q_T$ . Identify all non-dominated solutions. The template for this file is not given.

Suggested settings:



- Length of the discharge time series: 30 years.
- Use one value of  $l_{max,HU}$  from April to October and a second one from November to March.
- For both seasons,  $l_{max,HU}$  ranges from 13 m to 18 m with a step size of 0.5 m.
- $Q_T$  ranges from 60 m<sup>3</sup>/s to 80 m<sup>3</sup>/s, with a step size of 5 m<sup>3</sup>/s.
- Suppose that the turbine efficiency  $\eta$  does not change as a function of  $Q_T$ .

Warning: with the suggested settings, it might take around one hour to run the code! It is suggested to start by running the script with a reduced set of decision variables.

## Report

1. Report the best fit parameters for the hydrological model.
2. Plot the time series of observed discharge and of the discharge as simulated by the hydrological model with the best fit parameters (Figure 101 of the template Lastname\_Calibrate\_HM.m).
3. Plot Markov chains of the calibrated parameters and sequences of values of  $NS$  and  $T_{SA}$  as a function of the number of iterations (Figure 102 of the template Lastname\_Calibrate\_HM.m).
4. Plot the comparison between the statistics of the observed and generated precipitation (Figure 1 of the template Lastname\_Main.m).
5. Plot the time series (100 years long) of generated precipitation in [mm/h], run off in [mm/h], infiltration in [mm/h], soil moisture, leakage in [mm/h] and evapotranspiration in [mm/h] (as Figure 103 of the template Lastname\_Calibrate\_HM.m but for the calibrated hydrological model forced by the generated precipitation).
6. Plot the area and the volume rating curves (Figure 1001 of the template Lastname\_Main.m).
7. Report the minimum flow (discharge that is equalled or exceeded 95% of the time) of the generated discharge.
8. For a maximum level for hydroelectric use of 15 m, plot the time series (100 years long) of input discharge, output discharge, volume within the reservoir and level (Figure 1002 of the template Lastname\_Main.m).
9. Plot the average annual energy production in [GWh] and the probability that  $Q_{out}$  exceeds 150 m<sup>3</sup>/s for different values of the maximum level for hydroelectrical use (see Figure 4).
10. (facultative) For each combination of  $l_{max,HU}$  (in summer and winter) and  $Q_T$ , plot the average annual energy production against the flooding probability (1 point for each combination). Highlight all non-dominated solutions.
11. (facultative) Elaborate on the influence of decision variables on the determination of non-dominated solutions. Do not exceed 400 words. Figures are allowed.

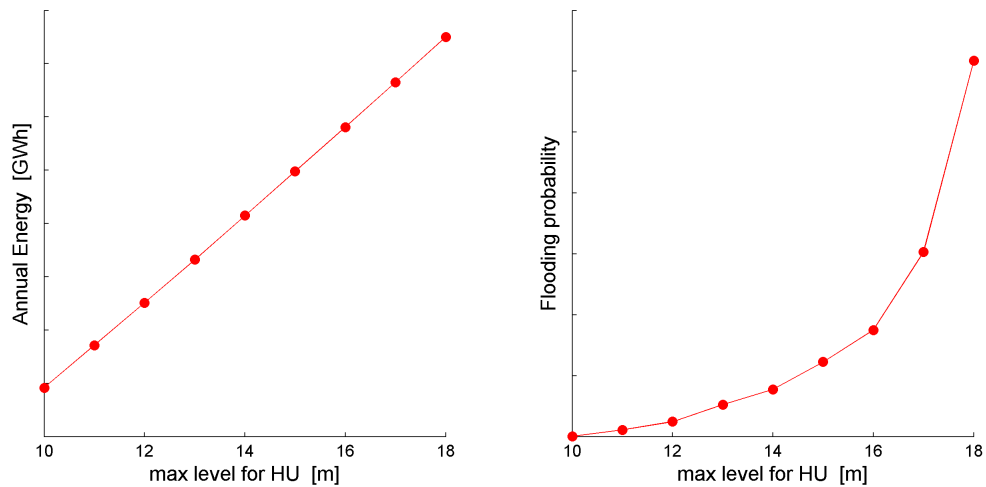


Figure 4: Annual average energy production and flooding probability as a function of  $l_{max,HU}$ .

### Required files

- `Lastname_HydroModel.m`: function that executes the hydrological model.
- `Lastname_Calibrate_HM.m`: calibration of the hydrological model.
- `Lastname_Main.m`: generation of rainfall; calibrated hydrological model forced by the generated rainfall; implementation of the flood control practice and reservoir routing; computation of the annual energy production and flooding probability for different values of the maximum level for hydroelectrical use.
- (facultative) `Lastname_MultiCriteria.m`: modification of `Lastname_Main.m` that accounts for different levels  $l_{max,HU}$  in summer and winter and different values for  $Q_T$ ; plot of all possible combinations in an energy vs. flooding probability graph; algorithm for identification of non-dominated solutions; graphs used for the answer to question 11.
- `Lastname.Report.pdf`: report providing the answers to the previous questions and contains the figures described above (no description nor discussion, except for question 11).

### Notes

- The facultative part may let you gain up to 15 points. The maximum grade for the assignment is 100.
- Everyone must submit her/his own files through Moodle.
- Do not upload the input files nor the auxiliary functions (`level_volume.m`, `down-scaling.m`, `TruncNormRnd.m`).
- Do not upload compressed files.
- Scripts must work without errors and produce the same figures that are shown in the report. Commented parts of the script will not be read.
- Please report the unit of measure in all the figures' axes and in all the answers.