Water Resources Engineering
Exercise

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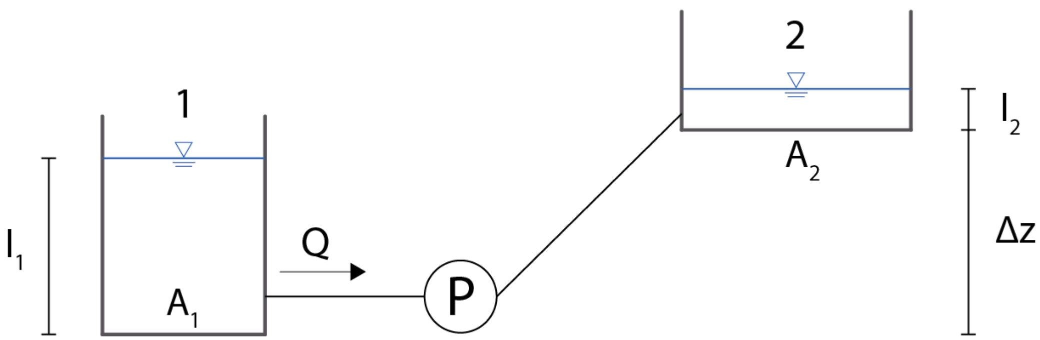
lecture: Tuesday 8.15-10 in room GR B3 30
exercise: Thursday 10.15-12 in room INF 3
office hours: Thursday 12-14 (GR C1 532 – GR C1 564)

Exercise: Emptying of a reservoir (analytical)

Water is pumped from reservoir **1** to reservoir **2**, as sketched. Both reservoirs are prismatic, with surfaces equal to A_1 and A_2, respectively. The pump works until reservoir **1** is emptied. The difference in altitude between the bottoms of reservoir **2** and reservoir **1** is Δz. Before the pump starts working, the level in reservoir **1** is $l_1(t_0) = \Delta z$, while reservoir **2** is empty ($l_2(t_0) = 0$).

The characteristic curve of the pump is $h_P = h_S - k_C{*}Q^2$, where h_P is the head provided by the pump, h_S is the shutoff head, k_C a constant coefficient and Q is the discharge along the pipeline. Pump efficiency η is assumed to be constant. The pipeline has a diameter D, total length L and friction factor f. Assume fully turbulent flow. Fitting head loss at the entrance is equal to a fraction k_{en} of the kinematic term; exit loss is equal to a fraction k_{ex} of the kinematic term.

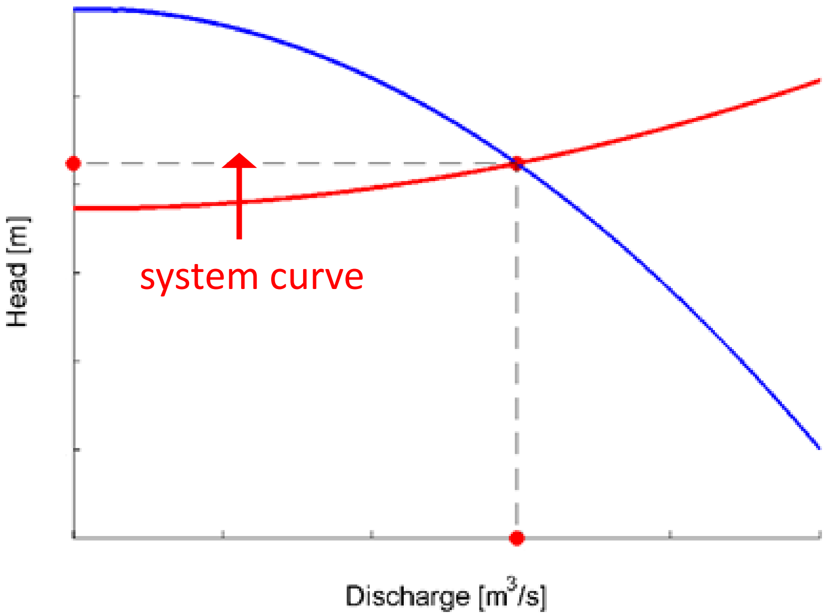


Use the following numerical values:

$A_1 = 300\ m^2$	$h_S = 60\ m$	$L = 200\ m$
$A_2 = 600\ m^2$	$k_C = 100\ s^2m^{-5}$	$D = 0.4\ m$
$\Delta z = 30\ m$	$\eta = 0.75$	$f = 0.028$
$k_{en} = 0.5$	$k_{ex} = 1$	

- Draw the qualitative behaviour of the energy grade line.
- Calculate the time required for emptying reservoir **1**.
- Calculate the total energy required by the pump.

Exercise: Emptying of a reservoir (analytical)



As reservoir 1 empties

- the level in reservoir 1 decreases, while the level in reservoir 2 increases,
- the difference in elevation between the two reservoirs increases,
- the system curve moves upwards,
- the discharge delivered by the pump changes.

The discharge outflowing from reservoir **1** depends on the level, which, in turn, depends on the volume stored in the reservoir.

Storage equations: $$\frac{dV_1}{dt} = Q_{in,1}(t) - Q_{out,1}(t) \qquad \frac{dV_2}{dt} = Q_{in,2}(t) - Q_{out,2}(t)$$

where: $Q_{in,1}(t) = 0; \quad Q_{out,2}(t) = 0; \quad Q_{out,1}(t) = Q_{in,2}(t) = Q(l_1(V_1(t))) = Q(l_2(V_2(t))).$

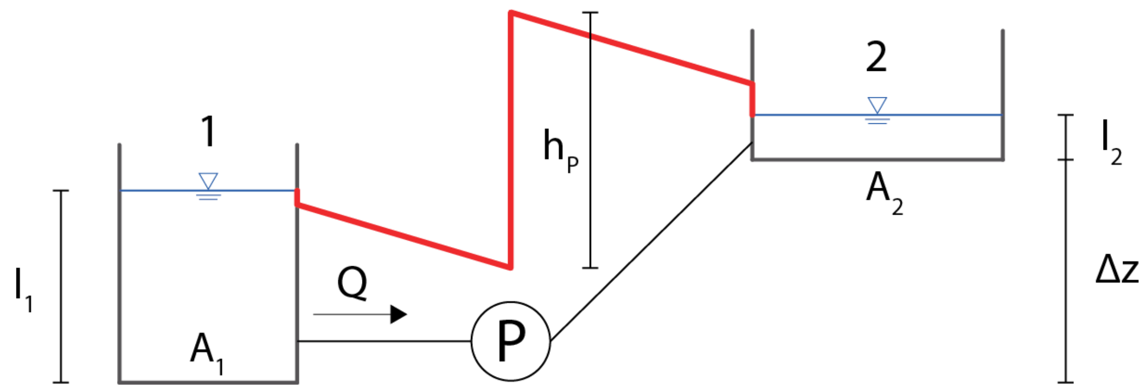
We therefore need to:

- specify the relationship between level, volume and time (*l(V(t))*) in reservoirs **1** and **2***;*
- integrate the storage equations.

In simple cases like the one here presented, the integration can be done **analytically**.

Solution

- Draw the qualitative behaviour of the energy grade line.



- Calculate the time required for emptying reservoir **1**.

First of all, let's find out the relationship between l_2 and l_1. Let's write the storage equations in differential form:

$$dV_1 = A_1 dl_1 = -Q dt = -A_2 dl_2 = -dV_2$$

Let's now integrate the 2nd and 4th members of the previous equation from t_0 to a generic state:

$$A_1 \int_{\Delta z}^{l_1} dl_1' = -A_2 \int_0^{l_2} dl_2' \quad \rightarrow \quad l_2(t) = -\frac{A_1}{A_2} l_1(t) + \frac{A_1}{A_2}\Delta z \qquad (1)$$

Solution

The system curve reads:

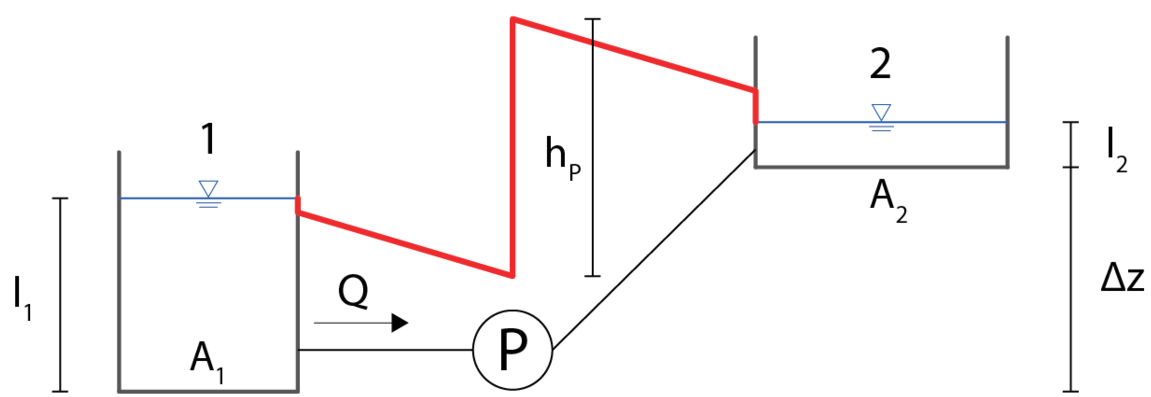
$$l_2(t) - l_1(t) + \Delta z = h_P(t) - \left[k_{en} + k_{ex} + \frac{fL}{D}\right] \frac{8Q(t)^2}{g\pi^2 D^4} \qquad (2)$$

By combining (1), (2) and the characteristic curve of the pump, one gathers:

$$Q = \sqrt{\frac{h_S - \left(1 + \frac{A_1}{A_2}\right)\Delta z + \left(1 + \frac{A_1}{A_2}\right) l_1}{\frac{8}{g\pi^2 D^4}\left(k_{ex} + k_{en} + \frac{fL}{D}\right) + k_C}} \qquad (3)$$

With the values given, (3) becomes:

$$Q = \sqrt{\frac{15 + 1.5 l_1}{150.02}} \approx \sqrt{0.1 + 0.01 l_1} \qquad (4)$$



Solution

Eq. (4) relates Q(t) to l_1(t), but in order to answer the question we need an equation relating l_1 to time. Therefore, let's get back to the storage equation (2nd and 3rd members):

$$300 dl_1 = -\sqrt{0.1 + 0.01 l_1} dt$$

By separating variables and integrating from t_0 to a generic state:

$$-300 \int_{\Delta z}^{l_1} \frac{dl_1'}{\sqrt{0.1 + 0.01 l_1'}} = \int_0^t dt' \quad \rightarrow \quad -300 \frac{2\sqrt{0.1 + 0.01 l_1'}}{0.01} \Bigg|_{\Delta z}^{l_1} = t$$

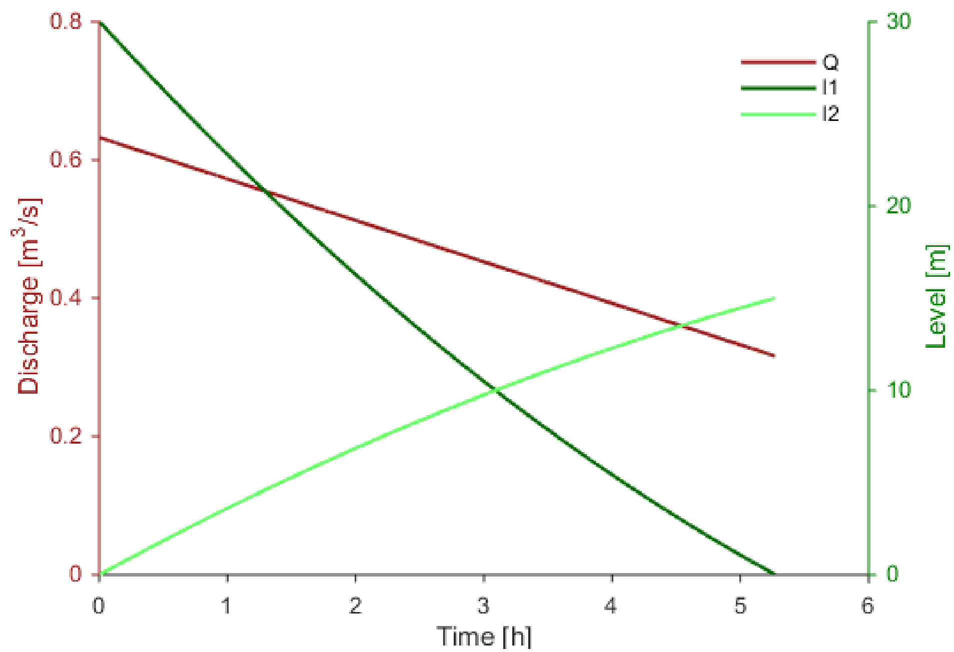
from which

$$t = 60000 \left(0.6325 - \sqrt{0.1 + 0.01 l_1} \right) \quad (5)$$

In order to empty reservoir **1** (l_1 = 0), the time required is **t_e = 18978 s ≈ 5 h 15'**.

Note that, by combining (4) and (5), one gathers:

$$Q = 0.6325 - \frac{t}{60000} \quad (6)$$



Solution

- Calculate the total energy required by the pump.

Energy is given by: $E = \int_0^{t_e} P dt$ (7)

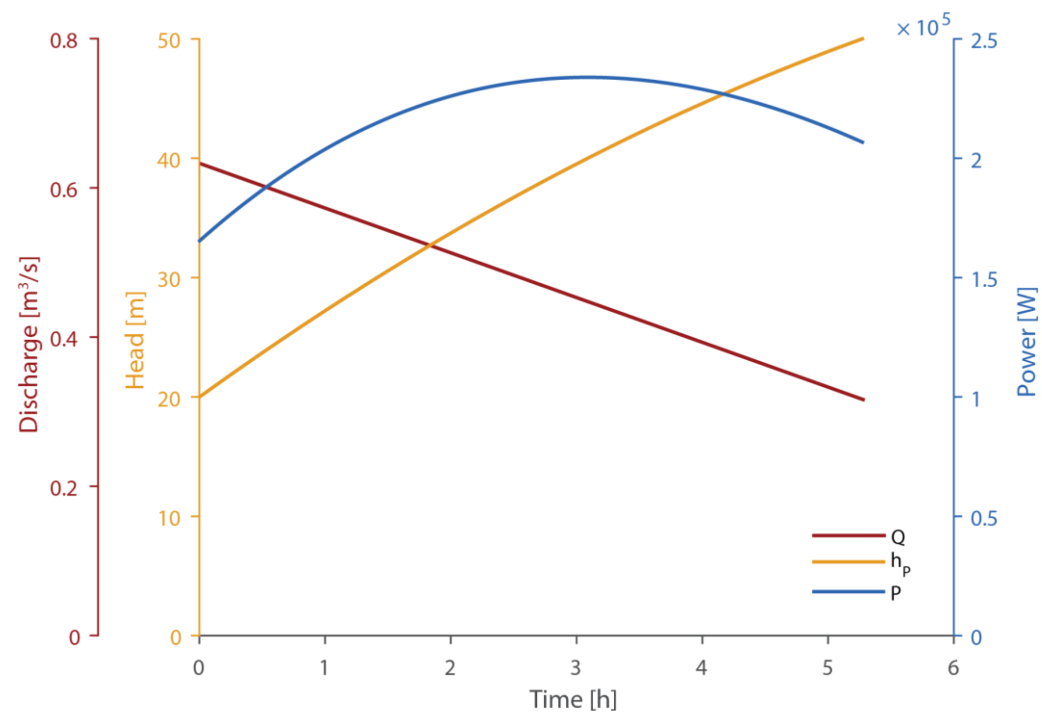
where power P can be calculated as (using (6)):

$$P = \frac{\gamma}{\eta} h_P Q = \frac{\gamma}{\eta}(h_S - K_C Q^2)Q = \frac{9806}{0.75}\left[60 - 100\left(0.6325 - \frac{t}{60000}\right)^2\right]\left(0.6325 - \frac{t}{60000}\right)$$

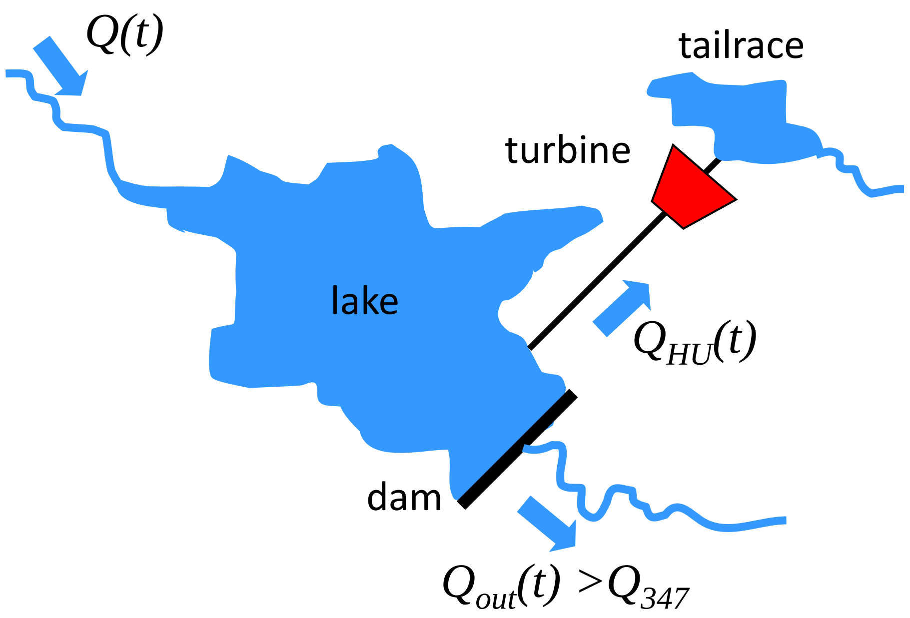
By expanding the calculations one obtains:

$$P = 6.05 \cdot 10^{-9} t^3 - 6.88 \cdot 10^{-4} t^2 + 13.08t + 165348$$

Now we can calculate the integral (7), obtaining **E = 4.12 GJ = 1.1 MWh.**

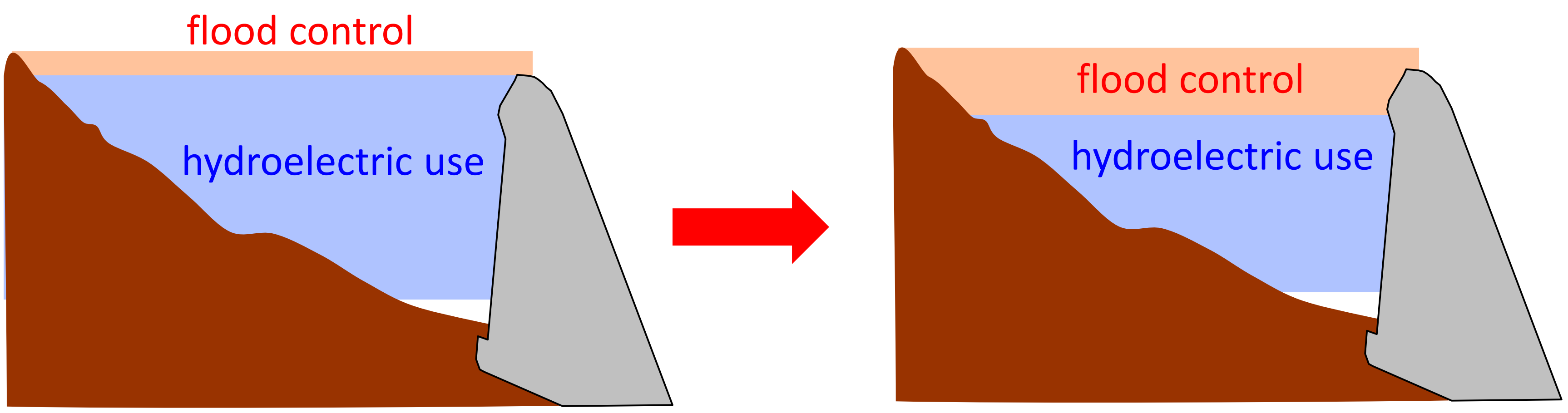


Assignment: management of a multipurpose reservoir

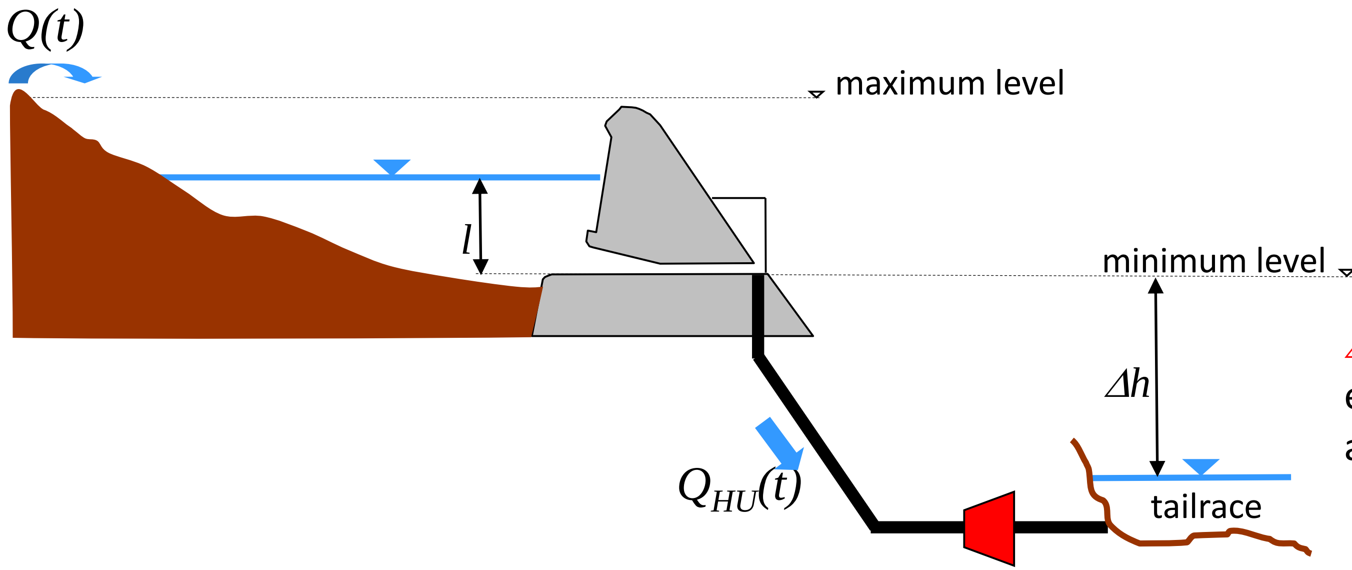


Evaluate the feasability of improving the flood control operations of an existing reservoir of a hydropower plant.

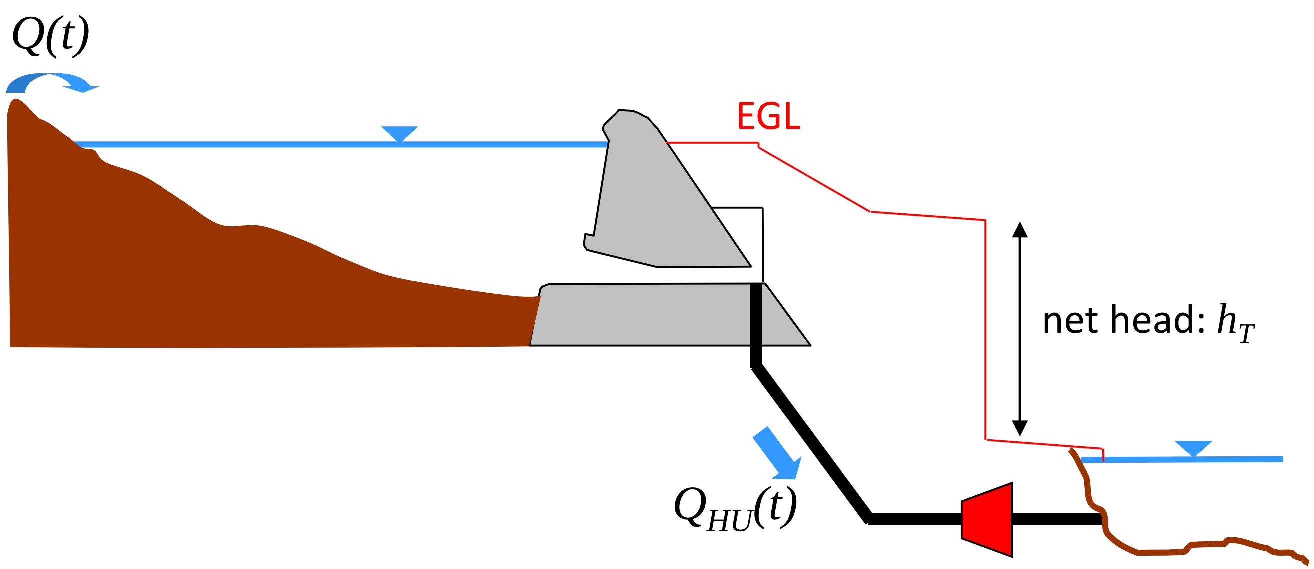
A larger fraction of the storage needs to be preserved for flood control and cannot be used for hydroelectric generation.



hydropower production



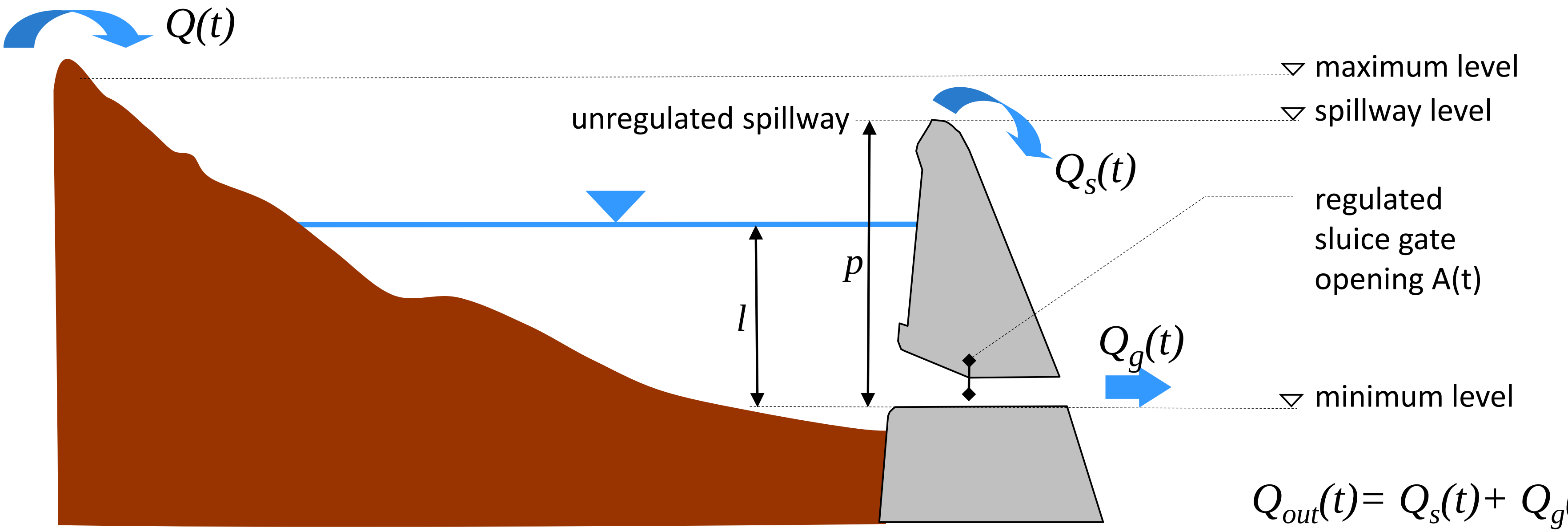
Δh: difference between the elevation of the empty pool and the tailrace



turbine power generation

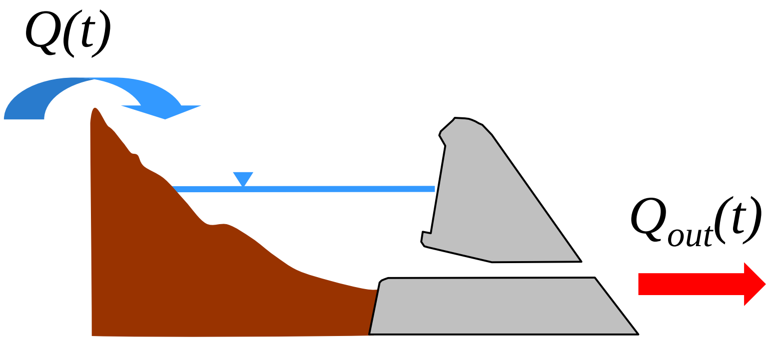
$$P = \eta \gamma Q_{HU} h_T$$

devices for water release

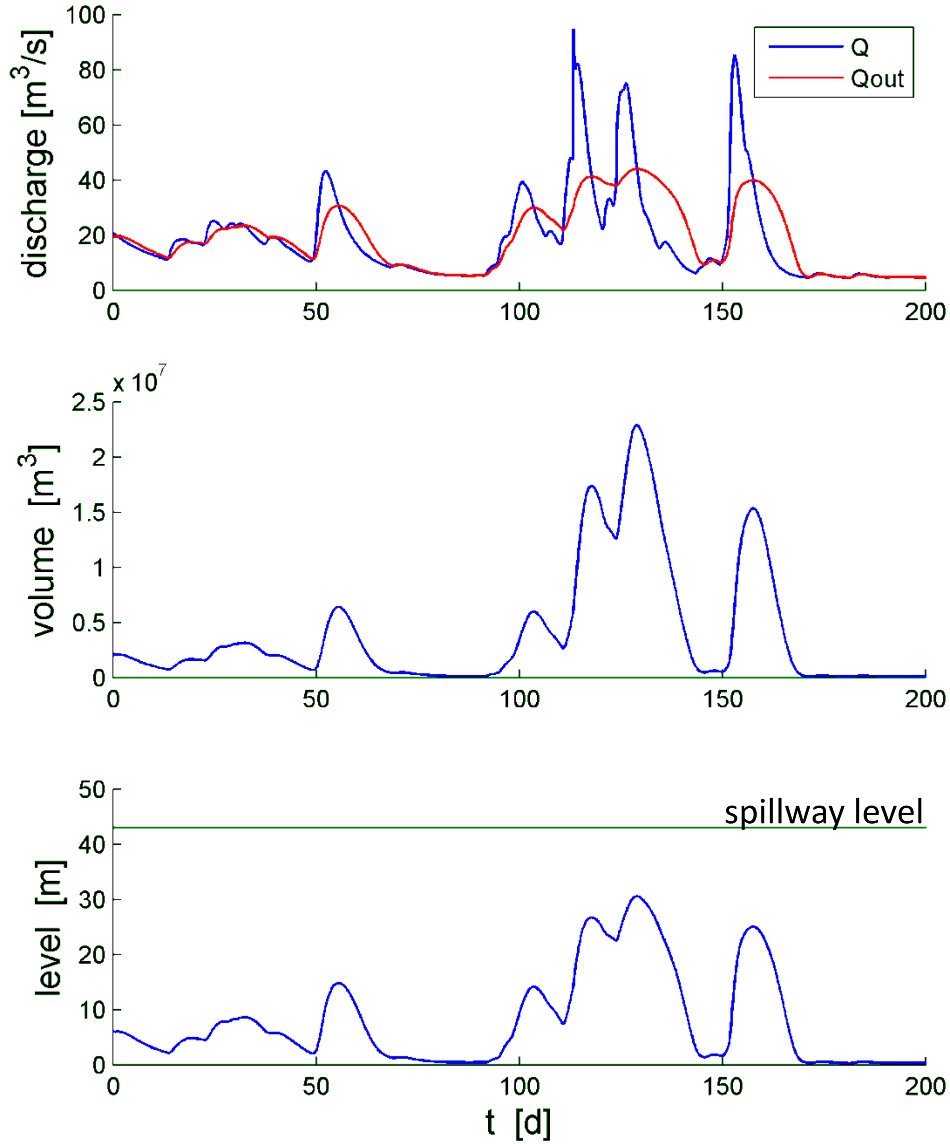


$$Q_{out}(t) = Q_s(t) + Q_g(t)$$

flood control

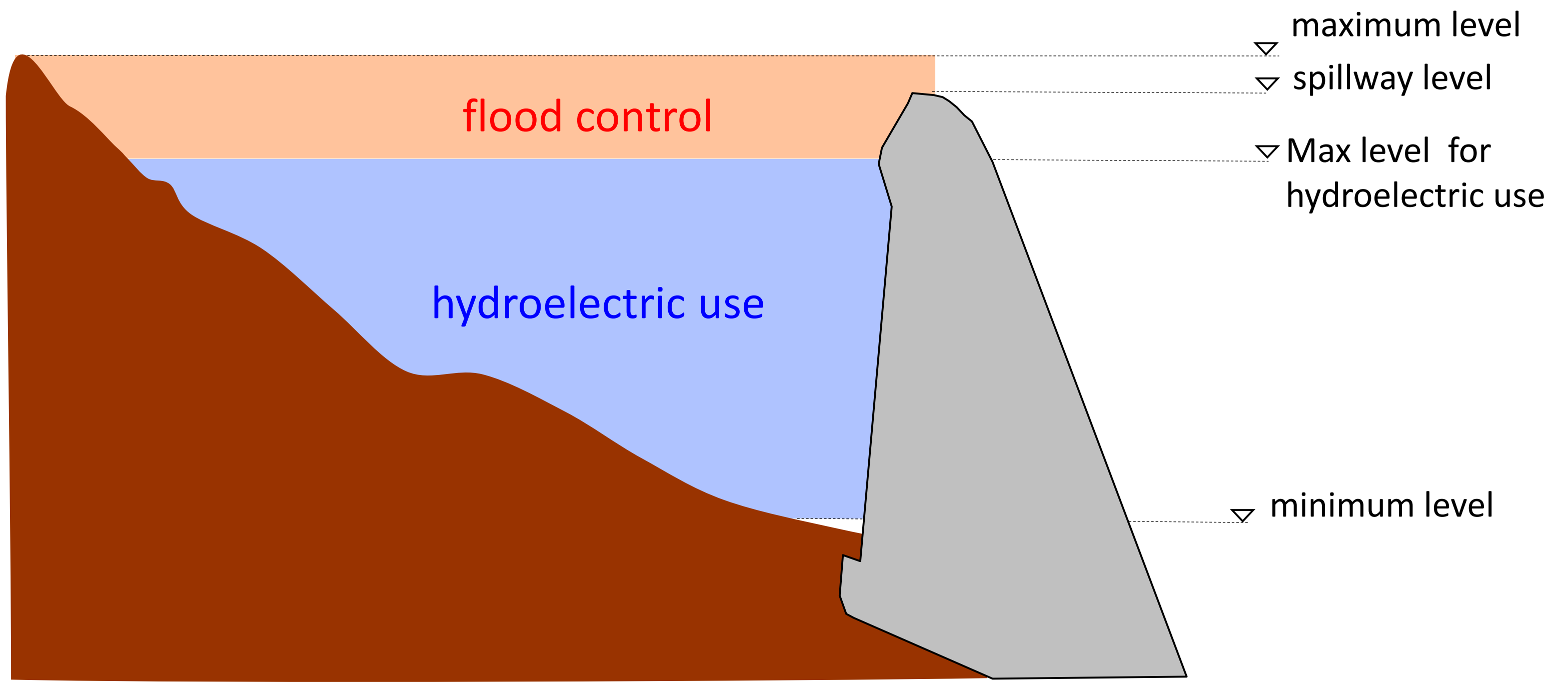


During high flows the water is stored in the reservoir and it is released after the peak



NB:plots refer to another case

Hydroelectric vs flood control: competitive uses



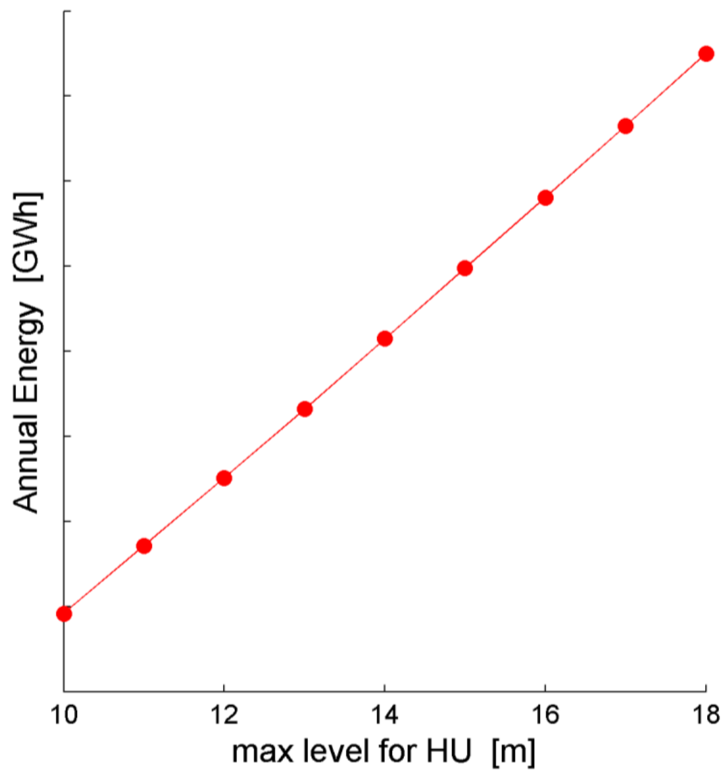
- The higher the level, the higher the energy production
- The more the volume available for flood control, the more efficiently floods are attenuated

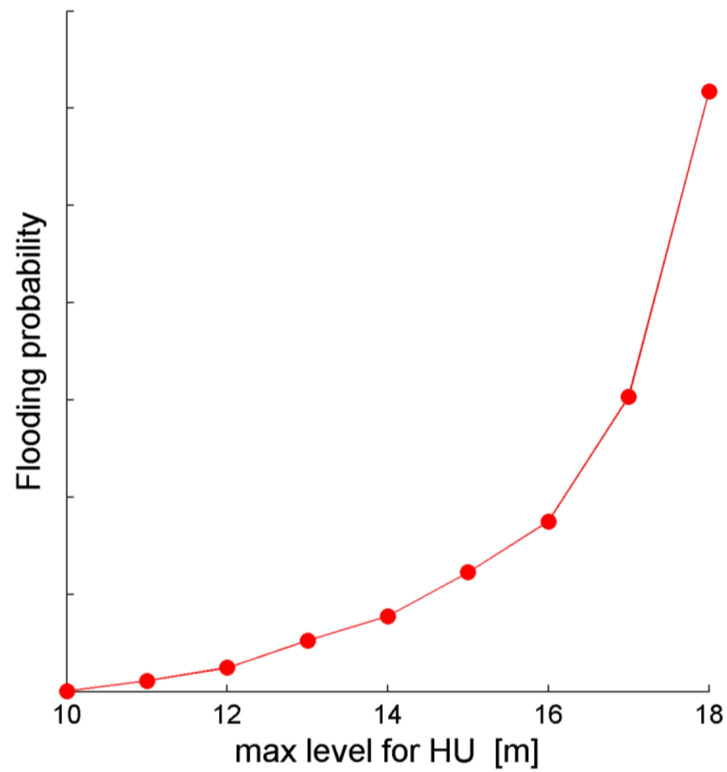
The two uses are in competition

goal

Improving the flood control operations to protect the downstream part of the river from floods larger than 150 m^3/s.

We need to estimate the trade off between the energy production and the probability of released flow (Q_{out}) larger than 150 m^3/s as a function of the volume (maximum level for hydroelectric use) reserved for flood control.

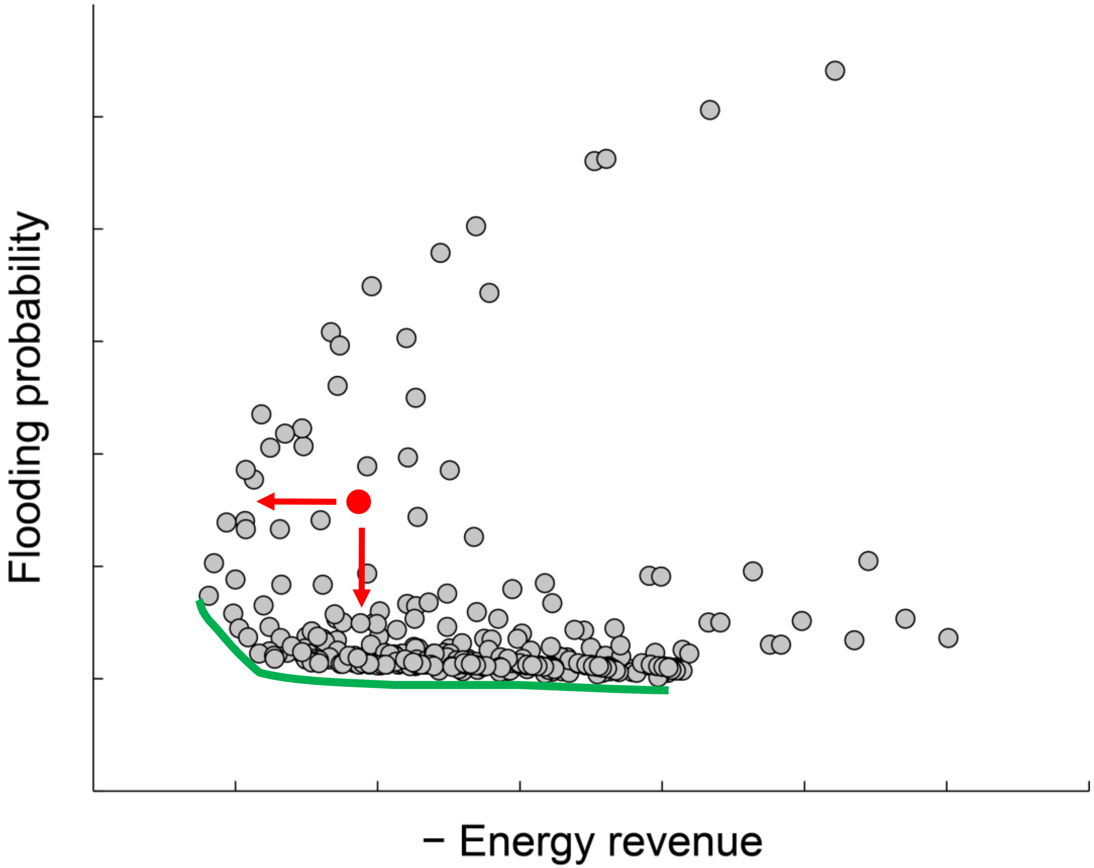




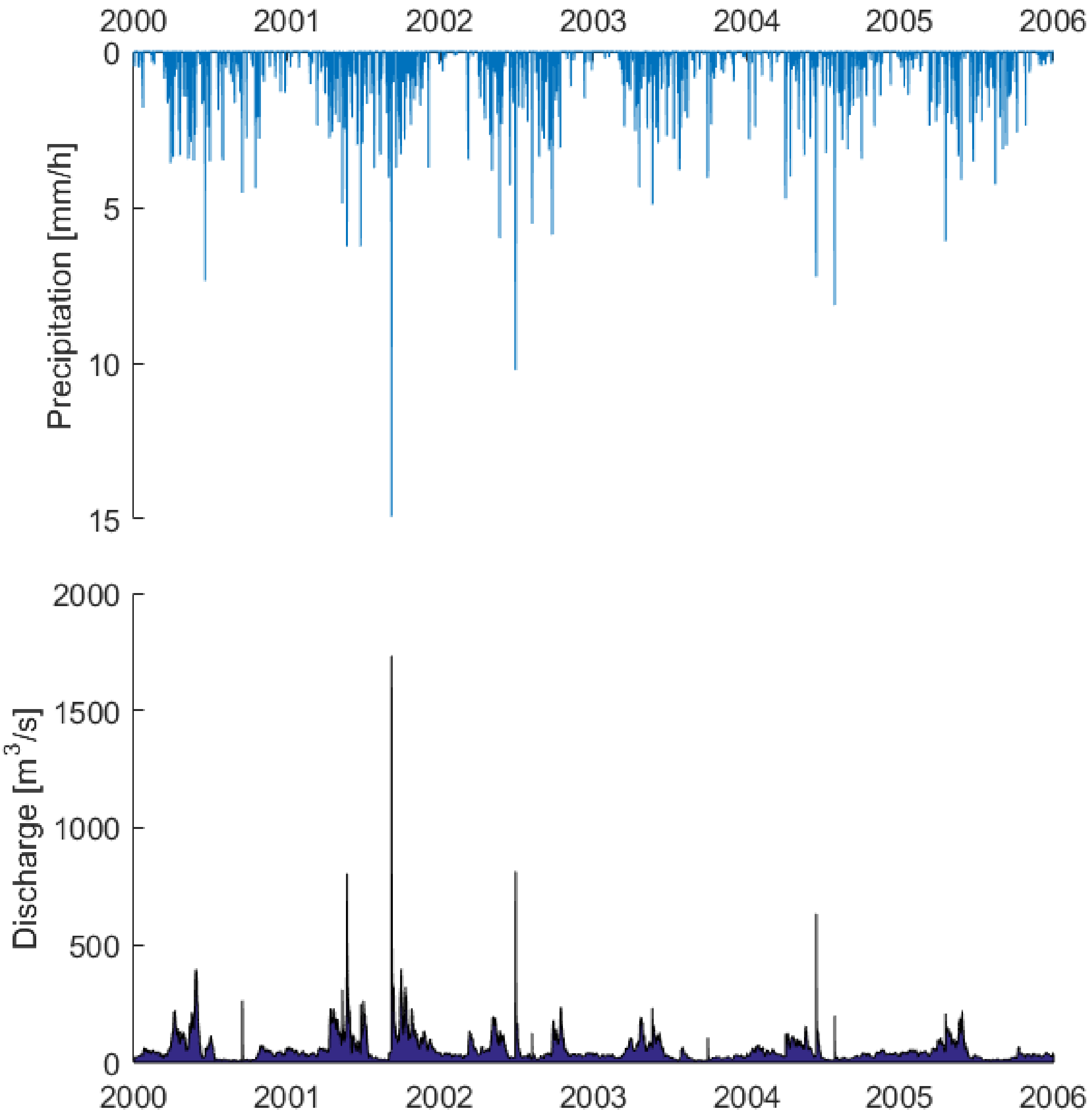
It is not possible to both maximize energy revenue and minimize flooding probability.

However, some solutions are sub-optimal (dominated), i.e. there exists a solution which outperforms them with regards to both costs (flooding probability) and benefits (energy revenue).

Pareto front: boundary that identifies all non-dominated solutions.

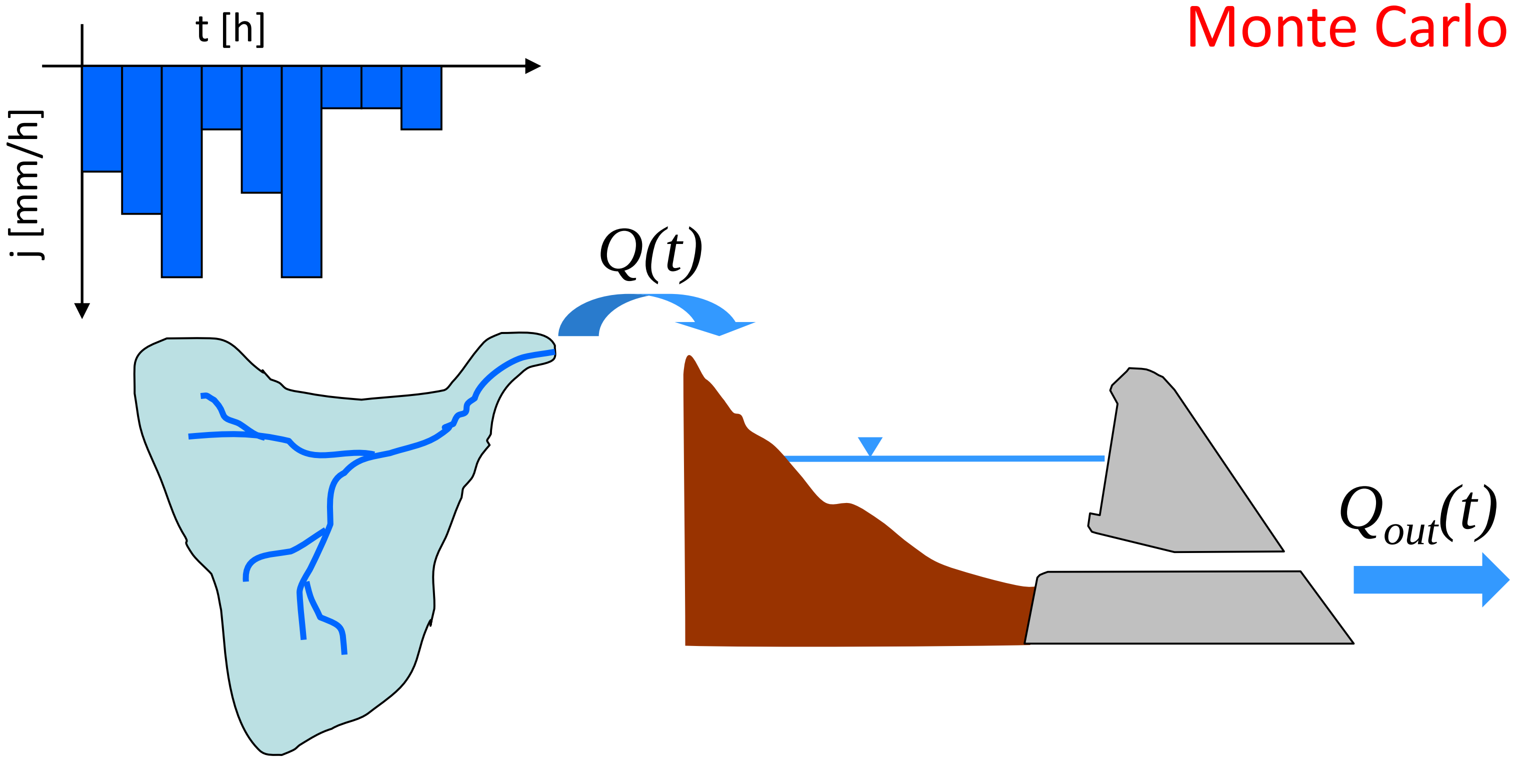


Hydrologic data available



6 years of simultaneous precipitation and discharge data (inflow in the reservoir) at hourly time step.

Monte Carlo approach



Time series of discharge data is too short to simulate directly the flood control operations and estimate the probability of Q_{out}>150 m^3/s.

It is possible to estimate the return period of rainfall events from Intensity Duration Frequency (IDF) curves of nearby stations. However, it is difficult to estimate the return period of the resulting Q_{out} because of the complex transformations due to the catchment and reservoir dynamics and of the dependency on the initial state of the system.

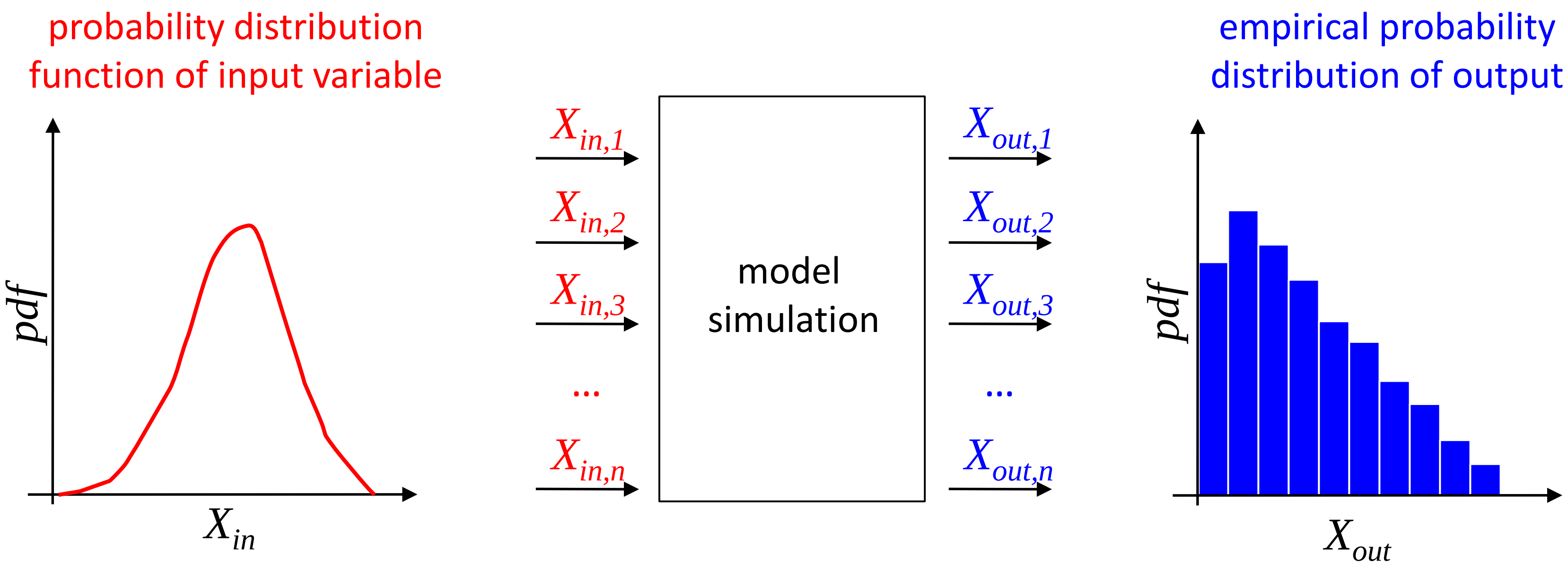
We resort to a Monte Carlo Approach

- long-run simulation of the process: 100/500/1000 years
- direct measurement of energy production and probability of Q_{out}>150 m^3/s

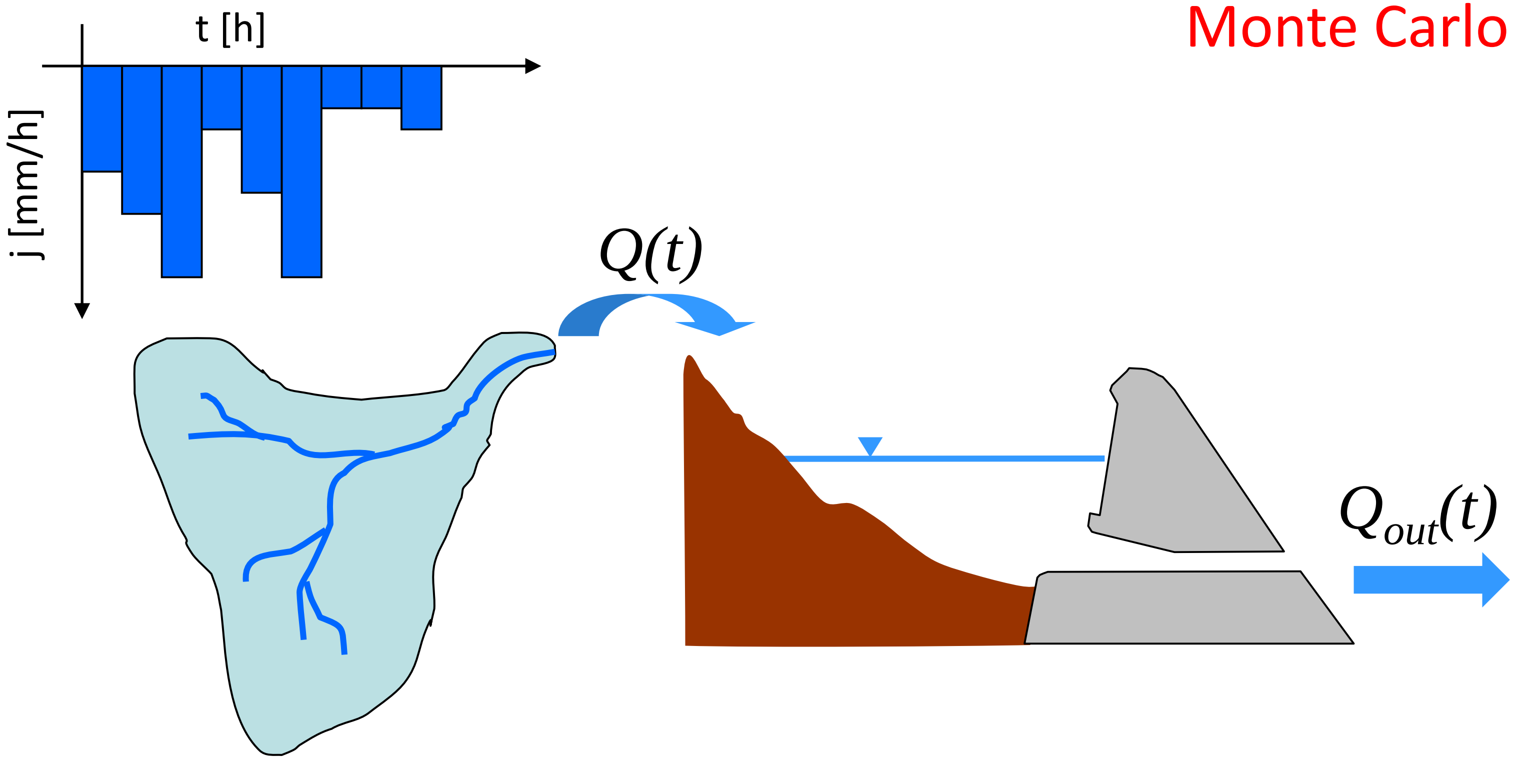
Monte Carlo approach

General method to estimate the probability distribution of model results when the inputs are stochastic random variables.

1. Simulate many realizations of the model with inputs randomly selected according to their probability distribution.
2. Derive the empirical probability distribution of the output.



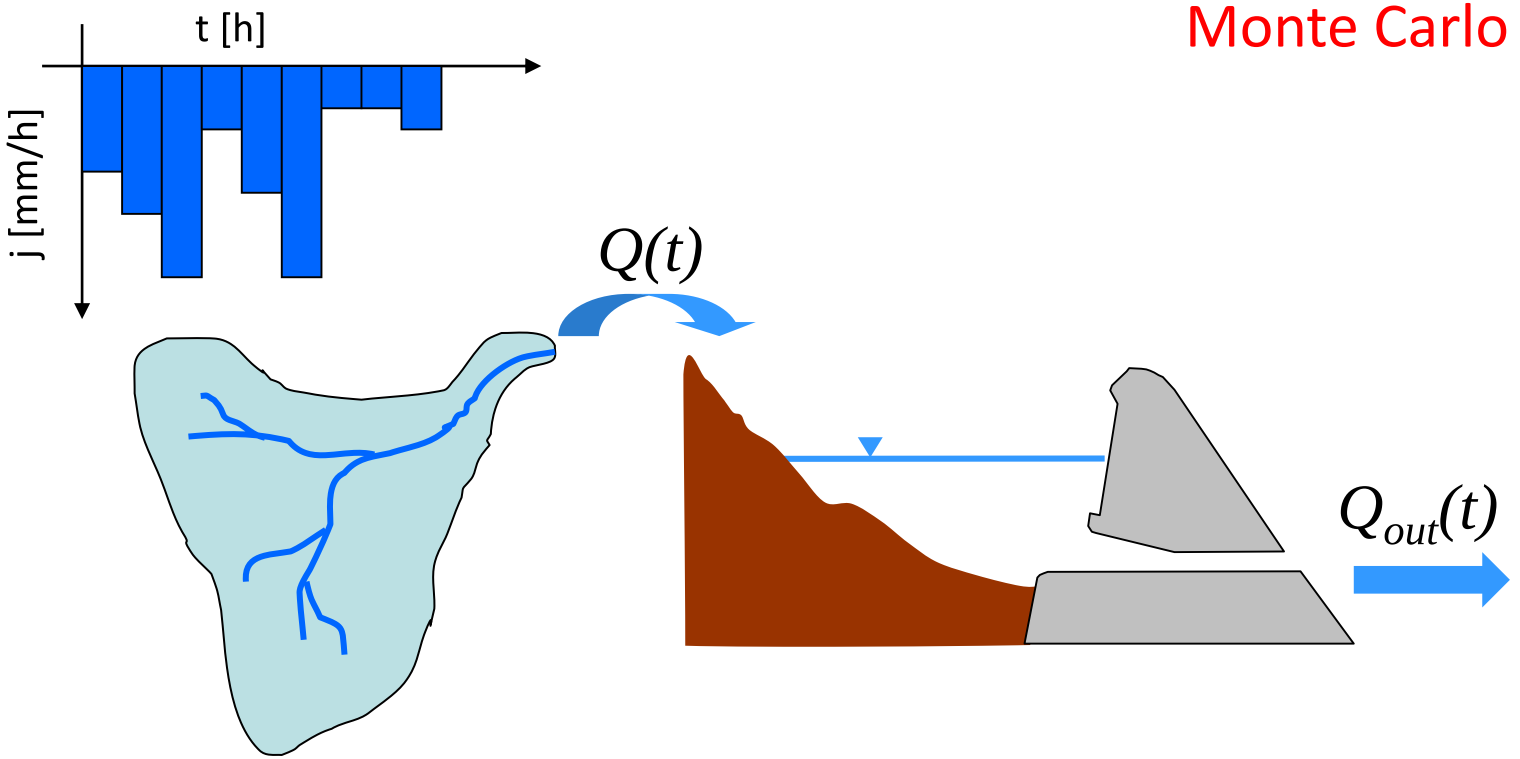
Monte Carlo approach



Steps

- Develop and calibrate a continuous hydrological model to transform rainfall into discharge. The model is fitted based on the available dataset.
- Generate rainfall time series with the same statistical properties of the observed ones.
- Transform generated rainfall into a generated time series of input discharge.
- Simulate the reservoir routing and the flood control operations for different maximum level for hydroelectric use.
- Measure the energy produced and the probability of Q_{out}>150 m^3/s

Monte Carlo approach



Other possible applications of a Monte Carlo approach in reservoir modeling:

flood with a return period of 100/500/1000 years

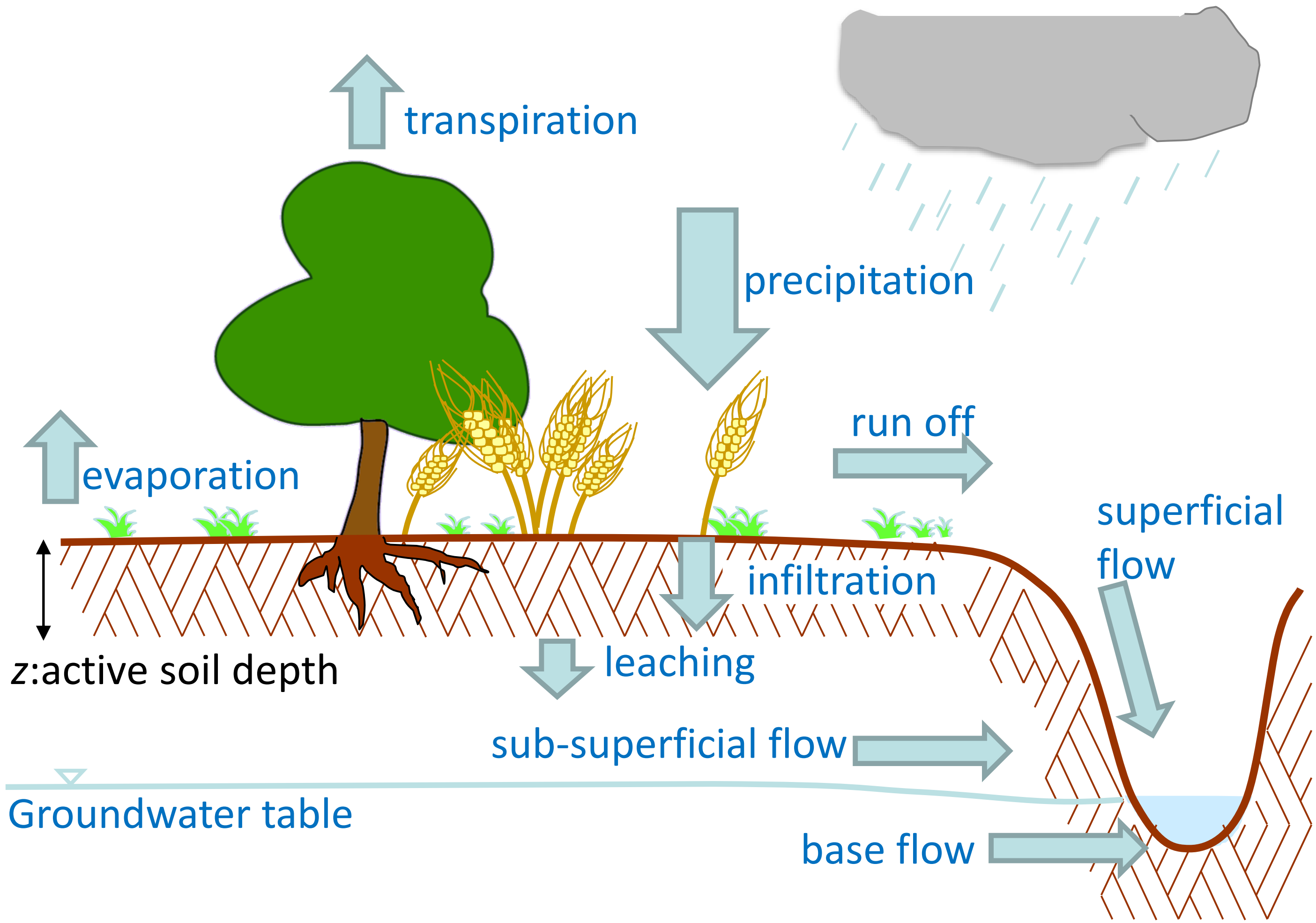
probability of having an empty pool and for how long

frequency of the use of the spillway

Outside the scope of the assignment, we could use this (almost) complete model to

- evaluate the effects of climate change on surface water management;
- evaluate the impact of reservoirs on natural streamflow regime.

assignment, part 1: continuous lumped hydrological model



A lumped hydrological model does not consider explicitly the spatial distribution of soil properties and precipitation. The model is lumped in a representative soil column. The geometry of the basin is accounted for in the routing scheme that transforms run-off into superficial flow and leaching into sub-superficial flow. Continuous as it accounts for evapotranspiration and can thus simulate long time windows.

Opposite of lumped: distributed. Opposite of continuous: event based