Тема 5

№1

x_i	$f(x_i)$	$f'(x_i)$	$f''(x_i)$
0	3	1	
1	0		
2	1	2	1
3	5		

$H_m(x)$, $m = \sum_{i=1}^{n} k_i - 1$

$$\underbrace{f(x_i; \dots; x_i)}_{k+1} = \frac{f^{(k)}(x_i)}{k!}$$

$m = 6$

$$f(x_i; x_i) = \frac{f'(x_i)}{1!}$$

$$f(x_i; x_j) = \frac{f(x_i) - f(x_j)}{x_i - x_j}$$

x_i	$f(x_i)$	$f'(x_i)$	$f''(x_i)$
0	3	1	
1	0		
2	1	2	1
3	5		

I	II	III	IV	V	VI	VII
0 3	$f(x_0;x_0)$					
0 3	$f(x_0;x_1)$	$f(x_0;x_0;x_1)$				
1 0	$f(x_1;x_2)$	$f(x_0;x_1;x_2)$	$f(x_0;x_0;x_1;x_2)$			
2 1	$f(x_2;x_2)$	$f(x_1;x_2;x_2)$	$f(x_0;x_1;x_2;x_2)$	$f(x_0;x_0;x_1;x_2;x_2)$		
2 1	$f(x_2;x_2)$	$f(x_2;x_2;x_2)$	$f(x_1;x_2;x_2;x_2)$	$f(x_0;x_1;x_2;x_2;x_2)$	$f(x_0;x_0;x_1;x_2;x_2;x_2)$	
2 1	$f(x_2;x_3)$	$f(x_2;x_2;x_3)$	$f(x_2;x_2;x_2;x_3)$	$f(x_1;x_2;x_2;x_2;x_3)$	$f(x_0;x_1;x_2;x_2;x_2;x_3)$	
3 5						

	I	II	III	IV	V	VI	VII
0	3						
		1					
0	3		-4				
		-3		3			
1	0		2		$\frac{7}{4}$		
		1		$-\frac{1}{2}$		$-\frac{7}{8}$	$-\frac{29}{4872}$
2	1		1		0		
		2		$-\frac{1}{2}$		$\frac{1}{3}$	
2	1		$\frac{1}{2}$		1		
		2		$\frac{3}{2}$			
2	1		2				
		4					
3	5						

$$H_6(x) = 3 + 1\cdot x - 4\cdot x^2 + 3\cdot x^2(x-1) + \frac{7}{4}x^2\cdot(x-1)(x-2) - \frac{7}{8}\cdot x^2(x-1)(x-2)^2 -$$

$$- \frac{29}{72}x^2(x-1)(x-2)^3$$

N2

x_i	-1	0	1
$f(x_0)$	$\frac{3}{4}$	$-\frac{1}{4}$	$\frac{3}{4}$

0 -1 $\frac{3}{4}$ $\quad = -1$

$f(x_0; x_1)$ $\qquad = 1$

1 0 $-\frac{1}{4}$ $\qquad f(x_0; x_1; x_2)$

$f(x_1; x_2)$

2 1 $\frac{3}{4}$ $\quad = 1$

$$f(x_0; x_1) = \frac{\frac{3}{4} + \frac{1}{4}}{-1 - 0} = -1$$

$$f(x_1; x_2) = \frac{-\frac{1}{4} - \frac{3}{4}}{0 - 1} = 1$$

$$f(x_0; x_1; x_2) = \frac{-1 - 1}{-1 - 1} = 1$$

$$L_2(x) = \frac{3}{4} - 1 \cdot (x + 1) + 1 \cdot (x + 1) \cdot x =$$

$$= x^2 + x - x - 1 + \frac{3}{4} = x^2 - \frac{1}{4}$$

$$x_1 = \frac{1}{2}$$

$$x_2 = -\frac{1}{2}$$

№3

x_i	0	1	2	3	4	5
y_i	5,2	8	10,4	12,4	14	15,2

	x_i	$f(x_i)$ I	II	III	IV	V	VI
0	5,2	0					
			$f(x_0;x_1)$				
1	8	1		$f(x_0;x_1;x_2)$			
			$f(x_1;x_2)$		$f(x_0;x_1;x_2;x_3)$		
2	10,4	2		$f(x_1;x_2;x_3)$		$f(x_0;x_1;x_2;x_3;x_4)$	
			$f(x_2;x_3)$		$f(x_1;x_2;x_3;x_4)$		$f(x_0;x_1;x_2;x_3;x_4;x_5)$
3	12,4	3		$f(x_2;x_3;x_4)$		$f(x_1;x_2;x_3;x_4;x_5)$	
			$f(x_3;x_4)$		$f(x_2;x_3;x_4;x_5)$		
4	14	4		$f(x_3;x_4;x_5)$			
			$f(x_4;x_5)$				
5	15,2	5					

	I	II	III	IV	V	VI
5,2	0					
		$\frac{1}{2,8}$				
8	1		$\frac{0,2}{6,72} = \frac{1}{33,6}$			
		$\frac{1}{2,4}$		$\frac{3,2}{806,4} = 252$		
10,4	2		$\frac{0,1}{2,4} = \frac{1}{24}$		$-\frac{36287}{576}$	
		$\frac{1}{2}$		$\frac{1}{144}$		$\frac{5184}{576} = 9$
12,4	3		$\frac{0,1}{1,6} = \frac{1}{16}$		$\frac{10367}{576}$	
		$\frac{1}{1,6}$		$\frac{6,4}{460,8} = 72$		
14	4		$\frac{0,2}{1,92} = \frac{1}{9,6}$			
		$\frac{1}{1,2}$				
15,2	5					

$$L_5(y) = 0 + \frac{1}{2,8}y + \frac{1}{33,6}y(y-1) + 252\cdot y(y-1)(y-2) - \frac{36287}{576}y(y-1)(y-2)(y-3) + 9y(y-1)(y-2)(y-3)(y-4)$$

$$x^* = L_5(7) = \frac{7}{2,8} + \frac{42}{33,6} + 252\cdot 210 - \frac{36287}{576}\cdot 840 + 9\cdot 2520 =$$

$$= \frac{10}{4} + 1,25 + 52920 - \frac{1270045}{24} + 22680 = \frac{15}{4} + \frac{35}{24} + 22680 =$$

$\frac{35}{24}$

$$= \frac{125}{24} + 22680 \simeq 22685,208$$

N4 t5.doc 8)

$f(x) = \sqrt[3]{x}$, $x \in [1, 100]$ ст. мин. = 2, $\varepsilon = 10^{-4}$

x_{i-1}, x_i, x_{i+1}

$$|f(x) - L_2^i(x)| \le \frac{M_3 h^3}{9\sqrt{3}} < \varepsilon$$

$$h \le \sqrt[3]{\frac{9\sqrt{3}\varepsilon}{M_3}}$$

$$M_3 = \sup_{x \in [1,100]} |f^{(3)}(x)|$$

$$f'(x) = \frac{x^{-\frac{2}{3}}}{3}$$

$$f''(x) = -\frac{2x^{-\frac{5}{3}}}{9}$$

$$f'''(x) = \frac{10x^{-\frac{8}{3}}}{27} = \frac{10}{27} \cdot \frac{1}{\sqrt[3]{x^8}}$$

$$M_3 = \sup_{x \in [1,100]} \frac{10}{27\sqrt[3]{x^8}} = \frac{10}{27}$$

$$h \le \sqrt[3]{\frac{9\sqrt{3} \cdot 10^{-4} \cdot 27}{10}} = \sqrt[3]{3^{\frac{11}{2}} \cdot 10^{-5}} \approx 0{,}1614$$