

Теория 4

N1

$$f(x) = \ln(x+1), x \in [0, 1] \quad \text{cr. my.} = u \quad (n=u)$$

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$$|\varepsilon_n(x)| \leq \frac{M_{n+1}}{(n+1)!} |\omega_n(x)|, M_{n+1} = \sup_{x \in [a, b]} |f^{(n+1)}(x)|$$

$$\omega_n(x) = \prod_{i=0}^n (x - x_i)$$

$$|\varepsilon_n(x)| = |f(x) - L_n(x)| \leq \frac{M_{n+1}}{(n+1)!} \cdot \frac{(b-a)^{n+1}}{2^{2n+1}}$$

$$x_k = \frac{a+b}{2} - \frac{b-a}{2} t_k, t_k = \cos \frac{(2k+1)\pi}{2(n+1)}, k = 0, n$$

$$f' = \frac{1}{x+1} = (x+1)^{-1}$$

$$f'' = -\frac{1}{(x+1)^2} = -1 \cdot (x+1)^{-2}$$

$$f''' = \frac{2}{(x+1)^3} = -2 \cdot (-1) (x+1)^{-3}$$

$$f^{(4)} = \frac{-6}{(x+1)^4} = -3(-2)(-1)(x+1)^{-4}$$

$$f^{(5)} = \frac{24}{(x+1)^5}$$

$$\sup_{x \in [0,1]} |f^{(5)}(x)| = \sup_{x \in [0,1]} \left| \frac{24}{(x+1)^5} \right| = 24$$

$$|\tau_4(x)| \leq \frac{24}{5!} \cdot \frac{1^5}{2^5} = \frac{1}{5 \cdot 5 \cdot 12} = \frac{1}{2560} =$$

$$= 0,000390625 = 3,90625 \cdot 10^{-4}$$

N2

$$x_0 = 100, x_1 = 101, x_2 = 102, x_3 = 103, x_4 = 104, x^* = 100,5$$

$$|\varrho_n(x)| \leq \frac{M_{n+1}}{(n+1)!} |\omega_n(x)|$$

$$\omega_n(x) = (x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4)$$

$$|\omega_n(x^*)| = 0,5 \cdot 0,5 \cdot 1,5 \cdot 2,5 \cdot 3,5 = 3,28125$$

$$f^{(5)} = \frac{24}{x^5}$$

$$M_5 = \sup_{[100;104]} |f^{(5)}(x)| = \frac{24}{10^{10}}$$

$$|\varrho_4(x)| \leq \frac{M_5}{5!} |\omega_n(x)|$$

$$|\varrho_4(x^*)| \leq \frac{M_5}{5!} \cdot 3,28125 = \frac{24}{10^{10} \cdot 12 \cdot 10} \cdot 3,28125 = \\ = 6,5625 \cdot 10^{-11}$$

$$\begin{aligned}
 & + \frac{f(x_1)}{(x-x_0)(x-x_1)(x-x_3)(x-x_4)} + \frac{f(x_2)}{(x-x_0)(x-x_1)(x-x_2)(x-x_4)} + \\
 & + \frac{f(x_3)}{(x-x_0)(x-x_1)(x-x_2)(x-x_4)} + \frac{f(x_4)}{(x-x_0)(x-x_1)(x-x_2)(x-x_3)} = \\
 & = \frac{f(x)}{\omega_n(x)} + \dots
 \end{aligned}$$

$$\begin{aligned}
 \pi_n(x) = & f(x) + f(x_0)(x-x_0) + f(x_1)(x-x_1) + \\
 & + f(x_2)(x-x_2) + f(x_3)(x-x_3) + f(x_4)(x-x_4)
 \end{aligned}$$

$$\begin{aligned}
 \pi_4(100,5) = & \ln 100,5 + 0,5 \cdot \ln 100 - 0,5 \ln 101 - \\
 & - 1,5 \ln 102 - 2,5 \ln 103 - 3,5 \ln 104 = -30,1745
 \end{aligned}$$

$$\Rightarrow |T_n(x)| \leq \frac{(n-1)!}{(n+1)!} \cdot \frac{(b-a)^{n+1}}{2^{2n+1}} \leq e$$

$$\frac{1^{n+1}}{n(n+1) \cdot 2^{2n+1}} \leq \varepsilon$$

$$n(n+1) 2^{2n+1} \geq \frac{1}{\varepsilon} = 10^4 \Rightarrow$$

$$\Rightarrow n \geq 4$$

Орында, күткінші интерполяциялық масдегі
функцияның анықтамасынан 5.

N4

$x \in [0, 1]$ 3-сәт. коэф. нұру стогрому элементінің 5-реттесінде

$$T_n^{[a, b]}(t) = \bar{T}_n \left(\frac{2}{b-a}x - \frac{b+a}{b-a} \right) =$$

$$= 2^{1-n} \cos \left(n \arccos \left(\frac{2}{b-a}x - \frac{b+a}{b-a} \right) \right)$$

$$T_3^{[0, 1]}(t) = 2^{1-3} \cdot \cos \left(3 \cdot \arccos (2x - 1) \right)$$

N4

$x \in [0, 1]$ 3-сәт. коэф. нұру стогрому элементінің 1-реттесінде

$$T_{n+1}(x) = 2xT_n - T_{n-1}(x), T_0(x) = 1, T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 2x - x = 4x^3 - 3x$$

$$\bar{T}_3(x) = x^3 - \frac{3x}{4}, \quad x \in [-1; 1]$$

Знайдімо вираз м-на Чеб. по проміжку $[0; 1]$:

$$T_3(\tilde{x}) = 4\tilde{x}^3 - 3\tilde{x}$$

$$\tilde{x} = \frac{2}{b-a} \left(x - \frac{b+a}{2} \right) = 2x-1, \quad a=0, b=1$$

$$\begin{aligned} T_3^{[0,1]}(x) &= 4(2x-1)^3 - 3(2x-1) = 4(8x^3 - 3 \cdot 4x^2 + 3 \cdot 2 \cdot x - 4) - \\ &- 6x + 3 = 32x^3 - 48x^2 + 24x - 4 - 6x + 3 = \\ &= 32x^3 - 48x^2 - 18x + 1 \end{aligned}$$

$$\bar{T}_3^{[0,1]} = x^3 - \frac{3x^2}{2} - \frac{9x}{16} - \frac{1}{32}$$

Відхилення б/с 0:

$$\|\bar{T}_3^{[0,1]}\| = \frac{1}{32}$$