

Tema 7

N1 Zagara 5 ex. 47

$$y = J_0(x) \quad , \quad h = 0,02$$

x_0	x_1	x_2	x_3	x_4	
x	0,96	0,98	1	1,02	1,04

$$y'(x_0), y''(x_0) = ?, x_0 = 1$$

$$y \quad 0,7825361 \quad 0,7739332 \quad 0,7651977 \quad 0,7563321 \quad 0,7473390$$

$$\max_x |J_0^{(k)}(x)| = 1$$

x	$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$
0,96	-0,430145				
0,98		-0,16575			
1			-0,162625		
1,02				-0,159375	
1,04					-0,449655

$$L_4(x) = 0,7825361 - 0,430145 \cdot (x - 0,96) - 0,16575 \cdot (x - 0,96)(x - 0,98) =$$

$$= 0,7825361 - 0,430145x + \underline{\quad} - 0,16575(x^2 - 0,98x - 0,96x + \underline{\quad})$$

$$L_4(x) = -0,430145 - 0,3315x + 0,321555$$

$$L'_4(x) = -0,3315$$

$$y'(x_0) \approx L'_4(x_0) = -0,44009$$

$$y''(x_0) \approx L''_4(x_0) = -0,3315$$

$$|L_4(x)| \leq \frac{M_5}{5!} \cdot \frac{(1,04 - 0,96)^5}{2^5}$$

$$M_5 = \sup_{x \in [0,96, 1,04]} |f^{(5)}(x)| = 1 \Rightarrow$$

$$\Rightarrow |L_4(x)| \leq \frac{0,08^5}{5! 2^5} = \frac{(2^3 \cdot 10^{-2})^5}{5! 2^5} = \frac{2^5 \cdot 10^{-10}}{3 \cdot 2^2 \cdot 10 \cdot 2^5} = \frac{2^4 \cdot 10^{-11}}{3} \approx \\ = 5,3 \cdot 10^{-11}$$

N2 Zagaro & CT.MT

$$y = \sin 2x, h = 0,05$$

x	0	0,05	0,1	0,15	0,2	0,25
y	0	0,10017	0,20134	0,30452	0,41075	0,52110

$$y(0), y''(0,1) - ?$$

x	$f(x)$	$f(x; ; x_{i+1})$	$f(x; ; x_{i+1}, x_{i+2})$	$f(x; ; ; x_{i+3})$	$f(x; ; ; ; x_{i+4})$	$f(x; ; ; ; ; x_{i+5})$
0	0					
		2,0034				
0,05	0,10017		0,2			
		2,0234		0		
0,1	0,20134		0,402		0	
		2,0636		0		0
0,15	0,30452		0,61		0	
		2,1246		0		
0,2	0,41075		0,824			
		2,207				
0,25	0,52110					

$$L_5(x) = 0 + 2,0034x + 0,2x(x-0,05)$$

$$L'_5(x) = 2,0034 + 0,4x - 0,01$$

$$L''_5(x) = 0,4$$

$$y'(0) \approx L'_5(0) = 1,9934$$

$$y''(0,1) \approx L''_5(0,1) = 0,4$$

$$|T_5(x)| \leq \frac{M_6}{6!} \cdot \frac{(0,25-0)^6}{2^6}$$

$$M_6 = \sup_{x \in [0,0,25]} |f^{(6)}(x)| = \sup_{x \in [0,0,25]} |2^6 \sin 2x| = 2^6 \cdot 0,521$$

$$|\tau_5(x)| \leq \frac{2^6 \cdot 0,521 \cdot (10^{-2} \cdot 25)^6}{720 \cdot 2^{11}} =$$

$$= \frac{2^{-5} \cdot 0,521 \cdot 10^{-12} \cdot 25^6}{720} =$$

$$= 0,0074 \cdot 2^{-5} \cdot 2^{-12} \cdot 5^{-12} \cdot 5^{12} =$$

$$= 0,0074 \cdot 2^{-17}$$

N3

Методом наименьших квадратов
подытожим формулу численного дифференци-
рования $f'(0)$ за n точке известны

$$x_0 = 0, x_1 = h, x_2 = 2h \quad n=3, k=1$$

$$f^{(k)}(x_p) = \sum_{i=0}^n c_i^{(0)} f(x_i) + R(f)$$

$$c_0^{(0)} + c_1^{(0)} + c_2^{(0)} = 0$$

$$c_0^{(0)} x_0 + c_1^{(0)} x_1 + c_2^{(0)} x_2 = 1$$

$$c_0^{(0)} x_0^2 + c_1^{(0)} x_1^2 + c_2^{(0)} x_2^2 = 2x_0$$

$$\left. \begin{array}{l} \\ \end{array} \right\} \textcircled{0}$$

$$x_p = x_1$$

$$c_0^{(1)} + c_1^{(1)} + c_2^{(1)} = 0$$

$$c_0^{(1)} x_0 + c_1^{(1)} x_1 + c_2^{(1)} x_2 = 1$$

$$c_0^{(1)} x_0^2 + c_1^{(1)} x_1^2 + c_2^{(1)} x_2^2 = 2x_1$$

$$\left. \begin{array}{l} \\ \end{array} \right\} \textcircled{1}$$

$$\underline{x_p = x_2}$$

$$c_0^{(2)} + c_1^{(2)} + c_2^{(2)} = 0$$

$$c_0^{(2)} x_0 + c_1^{(2)} x_1 + c_2^{(2)} x_2 = 1$$

$$c_0^{(2)} x_0^2 + c_1^{(2)} x_1^2 + c_2^{(2)} x_2^2 = 2x_2$$

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Линия 0: (указано посреди $x_0=0$)

$$\begin{pmatrix} 1 & 1 & 1 \\ x_0 & x_1 & x_2 \\ x_0^2 & x_1^2 & x_2^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2x_0 \end{pmatrix}$$

$$\Delta = \prod_{0 \leq i < j \leq n} (x_j - x_i) = (x_2 - x_0)(x_1 - x_0)(x_2 - x_1) = 2h^2 \cdot h = 2h^3$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & x_1 & x_2 \\ 2x_0 & x_1^2 & x_2^2 \end{vmatrix} = x_1^2 + 2x_0x_2 - 2x_0x_1 - x_2^2 = h^2 - 4h^2 = -3h^2$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 \\ x_0 & 1 & x_2 \\ x_0^2 & 2x_0 & x_2^2 \end{vmatrix} = x_2^2 + 2x_0^2 - x_0^2 - 2x_0x_2 = x_2^2 + x_0^2 - 2x_0x_2 = (x_0 - x_2)^2 = 4h^2$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 \\ x_0 & x_1 & x \\ x_0^2 & x_1^2 & 2x_0 \end{vmatrix} = 2x_0x_1 + x_0^2 - x_1^2 - 2x_0^2 = -x_0^2 + 2x_0x_1 - x_1^2 = -(x_0 - x_1)^2 = -h^2$$

$$c_0^{(0)} = \frac{\Delta_1}{\Delta} = \frac{-3h^2}{2h^3} = -\frac{3}{2h}$$

$$c_1^{(0)} = \frac{\Delta_2}{\Delta} = \frac{4h^2}{2h^3} = \frac{2}{h}$$

$$c_2^{(0)} = \frac{\Delta_3}{\Delta} = -\frac{h^2}{2h^3} = -\frac{1}{2h}$$

$$f'(x_p = x_0 = 0) = \sum_{i=0}^n c_i^{(0)} f(x_i) =$$

$$= -\frac{3}{2h} f(0) + \frac{2}{h} f(h) - \frac{1}{2h} f(2h)$$

N4

Методом невизначеных коефіцієнтів побудувати функцію лежального діапазону високого градуса $f''(0)$ за погане вимірювання

$$x_0 = -h, x_1 = 0, x_2 = h, x_3 = 2h \quad (n=3, m=2)$$

$$f^{(k)}(x_p) = \sum_{i=0}^n c_i^{(p)} f(x_i) + R(S)$$

$$x_p = x_1$$

$$C_0^{(1)} + C_1^{(1)} + C_2^{(1)} + C_3^{(1)} = 0$$

$$C_0^{(1)} x_0 + C_1^{(1)} x_1 + C_2^{(1)} x_2 + C_3^{(1)} x_3 = 0$$

$$C_0^{(1)} x_0^2 + C_1^{(1)} x_1^2 + C_2^{(1)} x_2^2 + C_3^{(1)} x_3^2 = 2$$

$$C_0^{(1)} x_0^3 + C_1^{(1)} x_1^3 + C_2^{(1)} x_2^3 + C_3^{(1)} x_3^3 = 6x_1 \quad | \cdot 2$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ x_0 & x_1 & x_2 & x_3 \\ x_0^2 & x_1^2 & x_2^2 & x_3^2 \\ x_0^3 & x_1^3 & x_2^3 & x_3^3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 6x_1 \end{pmatrix}$$

$$\Delta = \prod_{1 \leq i < j \leq n} (x_j - x_i) = (x_3 - x_2)(x_3 - x_1)(x_3 - x_0)(x_2 - x_1)(x_2 - x_0) \cdot \\ \times (x_1 - x_0) = h \cdot 2h \cdot 3h \cdot h \cdot 2h \cdot h = \\ = 12h^6$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 0 & x_1 & x_2 & x_3 \\ 2 & x_1^2 & x_2^2 & x_3^2 \\ 6x_1 & x_1^3 & x_2^3 & x_3^3 \end{vmatrix} = 12h^4$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 & 1 \\ x_0 & 0 & x_1 & x_3 \\ x_0^2 & 2 & x_1^2 & x_3^2 \\ x_0^3 & 6x_1 & x_1^3 & x_3^3 \end{vmatrix} = -24h^4$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 & 1 \\ x_0 & x_1 & 0 & x_3 \\ x_0^2 & x_1^2 & 2 & x_3^2 \\ x_0^3 & x_1^3 & 6x_1 & x_3^3 \end{vmatrix} = 12h^4$$

$$\Delta_4 = \begin{vmatrix} 1 & 1 & 1 & 0 \\ x_0 & x_1 & x_2 & 0 \\ x_0^2 & x_1^2 & x_2^2 & 2 \\ x_0^3 & x_1^3 & x_2^3 & 6x_1 \end{vmatrix} = 0$$

$$C_0^{(1)} = \frac{\Delta_1}{\Delta} = \frac{12h^4}{12h^6} = \frac{1}{h^2}$$

$$C_1^{(1)} = \frac{\Delta_2}{\Delta} = \frac{-24h^4}{12h^6} = -\frac{2}{h^2}$$

$$C_2^{(1)} = \frac{\Delta_3}{\Delta} = \frac{1}{h^2}$$

$$C_3^{(1)} = 0$$

$$f''(0) = \sum_{i=0}^n C_i^{(1)} f(x_i) = \frac{f(-h)}{h^2} - \frac{2f(0)}{h^2} + \frac{f(h)}{h^2}$$