

Тема 7

М Задача 5 стр. 47

$$y = J_0(x), \quad h = 0,02$$

$$y'(x_0), y''(x_0) = ?, x_0 = 1$$

x	x_0	x_1	x_2	x_3	x_4
	0,96	0,98	1	1,02	1,04

y 0,7825361 0,7739332 0,7651977 0,7563321 0,7473390

$$\max_x |J_0^{(k)}(x)| = 1$$

x	$f(x_0)$	$f(x_0; x_{i-1})$	$f(x_0; x_{i-1}; x_{i-2})$	$f(x_0; x_{i-1}; x_{i-2}; x_{i-3})$	$f(x_0; \dots; x_{i-4})$
0,96	$f(x_0)$				
		-0,430145			
0,98	$f(x_1)$		-0,16575		
		-0,436775		0	
1	$f(x_2)$		-0,162625		0
		-0,44328		0	
1,02	$f(x_3)$		-0,159375		
		-0,449655			
1,04	$f(x_4)$				

$$L_4(x) = 0,7825361 - 0,430145(x - 0,96) - 0,16575(x - 0,96)(x - 0,98) =$$

$$= 0,7825361 - 0,430145x + \dots - 0,16575(x^2 - 0,98x - 0,96x + \dots)$$

$$L_4(x) = -0,430145 - 0,3315x + 0,321555$$

$$L_4''(x) = -0,3315$$

$$y'(x_0) \approx L_4'(x_0) = -0,44009$$

$$y''(x_0) \approx L_4''(x_0) = -0,3315$$

$$|r_4(x)| \leq \frac{M_5}{5!} \cdot \frac{(1,04 - 0,96)^5}{2^9}$$

$$M_5 = \sup_{x \in [0,96,1,04]} |f^{(5)}(x)| = 1 \Rightarrow$$

$$\Rightarrow |r_4(x)| \leq \frac{0,08^5}{5! \cdot 2^9} = \frac{(2^3 \cdot 10^{-2})^5}{5! \cdot 2^9} = \frac{2^{15} \cdot 10^{-10}}{3 \cdot 2^2 \cdot 10 \cdot 2^9} = \frac{2^4 \cdot 10^{-11}}{3} \approx 5,3 \cdot 10^{-11}$$

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$$y = \operatorname{sh} 2x, h = 0,05$$

x	0	0,05	0,1	0,15	0,2	0,25
y	0	0,10017	0,20134	0,30452	0,41075	0,52110

$$y'(0), y''(0,1) = ?$$

x	$f(x_i)$	$f(x_i; x_{i+1})$	$f(x_i; x_{i+1}; x_{i+2})$	$f(x_i; \dots; x_{i+3})$	$f(x_i; \dots; x_{i+4})$	$f(x_i; \dots; x_{i+5})$
0	0	2,0034	0,2	0	0	0
0,05	0,10017	2,0234	0,402	0	0	0
0,1	0,20134	2,0636	0,61	0	0	0
0,15	0,30452	2,1246	0,824	0	0	0
0,2	0,41075	2,207				
0,25	0,52110					

$$L_5(x) = 0 + 2,0034x + 0,2x(x-0,05)$$

$$L'_5(x) = 2,0034 + 0,4x - 0,01$$

$$L''_5(x) = 0,4$$

$$y'(0) \approx L'_5(0) = 1,9934$$

$$y''(0,1) \approx L''_5(0,1) = 0,4$$

$$|r_5(x)| \leq \frac{M_6}{6!} \cdot \frac{(0,25-0)^6}{2^4}$$

$$M_6 = \sup_{x \in [0,0,25]} |f^{(6)}(x)| = \sup_{x \in [0,0,25]} |2^6 \operatorname{sh} 2x| = 2^6 \cdot 0,521$$

$$|\tau_5(x)| \leq \frac{2^6 \cdot 0,521 \cdot (10^{-2} \cdot 25)^6}{720 \cdot 2^{11}} =$$

$$= \frac{2^{-5} \cdot 0,521 \cdot 10^{-12} \cdot 25^6}{720} =$$

$$= 0,0074 \cdot 2^{-5} \cdot 2^{-12} \cdot 5^{-12} \cdot 5^{12} =$$

$$= 0,0074 \cdot 2^{-17}$$

N3

Методом невизначених коефіцієнтів побудувати формулу чисельного диференціювання для $f'(0)$ за трьома вузлами

$$; x_0 = 0, x_1 = h, x_2 = 2h \quad h=2, k=1$$

$$f^{(k)}(x_p) = \sum_{i=0}^n c_i^{(p)} f(x_i) + R(f)$$

$$x_p = x_0$$

$$c_0^{(0)} + c_1^{(0)} + c_2^{(0)} = 0$$

$$c_0^{(0)} x_0 + c_1^{(0)} x_1 + c_2^{(0)} x_2 = 1$$

$$c_0^{(0)} x_0^2 + c_1^{(0)} x_1^2 + c_2^{(0)} x_2^2 = 2x_0$$

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$$x_p = x_1$$

$$c_0^{(1)} + c_1^{(1)} + c_2^{(1)} = 0$$

$$c_0^{(1)} x_0 + c_1^{(1)} x_1 + c_2^{(1)} x_2 = 1$$

$$c_0^{(1)} x_0^2 + c_1^{(1)} x_1^2 + c_2^{(1)} x_2^2 = 2x_1$$

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$$\underline{x_p = x_2}$$

$$c_0^{(2)} + c_1^{(2)} + c_2^{(2)} = 0$$

$$c_0^{(2)} x_0 + c_1^{(2)} x_1 + c_2^{(2)} x_2 = 1$$

$$c_0^{(2)} x_0^2 + c_1^{(2)} x_1^2 + c_2^{(2)} x_2^2 = 2x_2$$

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Система 0: (мы хотим ^{case} пропустить $x_0=0$)

$$\begin{pmatrix} 1 & 1 & 1 \\ x_0 & x_1 & x_2 \\ x_0^2 & x_1^2 & x_2^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2x_0 \end{pmatrix}$$

$$\Delta = \prod_{\text{discriminant}} (x_j - x_i) = (x_2 - x_0)(x_1 - x_0)(x_2 - x_1) = 2h^2 \cdot h = 2h^3$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & x_1 & x_2 \\ 2x_0 & x_1^2 & x_2^2 \end{vmatrix} = x_1^2 + 2x_0x_2 - 2x_0x_1 - x_2^2 = h^2 - 4h^2 = -3h^2$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 \\ x_0 & 1 & x_2 \\ x_0^2 & 2x_0 & x_2^2 \end{vmatrix} = x_2^2 + 2x_0^2 - x_0^2 - 2x_0x_2 = x_2^2 + x_0^2 - 2x_0x_2 = (x_0 - x_2)^2 = 4h^2$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 \\ x_0 & x_1 & 1 \\ x_0^2 & x_1^2 & 2x_0 \end{vmatrix} = 2x_0x_1 + x_0^2 - x_1^2 - 2x_0^2 = -x_0^2 + 2x_0x_1 - x_1^2 = -(x_0 - x_1)^2 = -h^2$$

$$c_0^{(0)} = \frac{\Delta_1}{\Delta} = \frac{-3h^2}{2h^3} = -\frac{3}{2h}$$

$$c_1^{(0)} = \frac{\Delta_2}{\Delta} = \frac{4h^2}{2h^3} = \frac{2}{h}$$

$$c_2^{(0)} = \frac{\Delta_3}{\Delta} = \frac{-h^2}{2h^3} = -\frac{1}{2h}$$

$$f'(x_p = x_0 = 0) = \sum_{i=0}^n c_i^{(0)} f(x_i) =$$

$$= -\frac{3}{2h} f(0) + \frac{2}{h} f(h) - \frac{1}{2h} f(2h)$$

N4

Методом невизначених коефіцієнтів побудувати формулу Рунського диференціювання для $f''(0)$ за потрійне вузлами.

$$x_0 = -h, x_1 = 0, x_2 = h, x_3 = 2h \quad (n=3, k=2)$$

$$f^{(k)}(x_p) = \sum_{i=0}^n c_i^{(p)} f(x_i) + R(s)$$

$$X_p = x_1$$

$$c_0^{(1)} + c_1^{(1)} + c_2^{(1)} + c_3^{(1)} = 0$$

$$c_0^{(1)} x_0 + c_1^{(1)} x_1 + c_2^{(1)} x_2 + c_3^{(1)} x_3 = 0$$

$$c_0^{(1)} x_0^2 + c_1^{(1)} x_1^2 + c_2^{(1)} x_2^2 + c_3^{(1)} x_3^2 = 2$$

$$c_0^{(1)} x_0^3 + c_1^{(1)} x_1^3 + c_2^{(1)} x_2^3 + c_3^{(1)} x_3^3 = 6x_1$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ x_0 & x_1 & x_2 & x_3 \\ x_0^2 & x_1^2 & x_2^2 & x_3^2 \\ x_0^3 & x_1^3 & x_2^3 & x_3^3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 6x_1 \end{pmatrix}$$

$$\Delta = \prod_{1 \leq i < j \leq n} (x_j - x_i) = (x_3 - x_2)(x_3 - x_1)(x_3 - x_0)(x_2 - x_1)(x_2 - x_0) \cdot \\ \times (x_1 - x_0) = h \cdot 2h \cdot 3h \cdot h \cdot 2h \cdot h = \\ = 12h^6$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 0 & x_1 & x_2 & x_3 \\ 2 & x_1^2 & x_2^2 & x_3^2 \\ 6x_1 & x_1^3 & x_2^3 & x_3^3 \end{vmatrix} = 12h^4$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 & 1 \\ x_0 & 0 & x_2 & x_3 \\ x_0^2 & 2 & x_2^2 & x_3^2 \\ x_0^3 & 6x_1 & x_2^3 & x_3^3 \end{vmatrix} = -24h^4$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 & 1 \\ x_0 & x_1 & 0 & x_3 \\ x_0^2 & x_1^2 & 2 & x_3^2 \\ x_0^3 & x_1^3 & 6x_1 & x_3^3 \end{vmatrix} = 12h^4$$

$$\Delta_4 = \begin{vmatrix} 1 & 1 & 1 & 0 \\ x_0 & x_1 & x_2 & 0 \\ x_0^2 & x_1^2 & x_2^2 & 2 \\ x_0^3 & x_1^3 & x_2^3 & 6x_1 \end{vmatrix} = 0$$

$$C_0^{(1)} = \frac{\Delta_1}{\Delta} = \frac{12h^4}{12h^6} = \frac{1}{h^2}$$

$$C_1^{(1)} = \frac{\Delta_2}{\Delta} = \frac{-24h^4}{12h^6} = -\frac{2}{h^2}$$

$$C_2^{(1)} = \frac{\Delta_3}{\Delta} = \frac{1}{h^2}$$

$$C_3^{(1)} = 0$$

$$f'''(0) = \sum_{i=0}^3 C_i^{(1)} f(x_i) = \frac{f(-h)}{h^2} - \frac{2f(0)}{h^2} + \frac{f(h)}{h^2}$$