.20 Тема 6

№1 ЧМ ч.2, ст.46 з.1

$n = 2$, $k = 1$, точки підв. точності? похибка в точці $x = \bar{x}$?

$$L_n^{(k)}(x) = L_2'(x)$$

x_{i-1}, x_i, x_{i+1} – дов. точки

$$L_2(x) = f_{i-1} + \frac{f_i - f_{i-1}}{h}(x - x_{i-1}) + \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2}(x - x_{i-1})(x - x_i)$$

$$f'(x) \approx L_2'(x) = \frac{f_i - f_{i-1}}{h} + \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2}(2x - x_{i-1} - x_i) \quad x \in [x_{i-1}, x_{i+1}]$$

$$L_2'(x) = \frac{f_1 - f_0}{h} + \frac{f_2 - 2f_1 + f_0}{h^2}(2x - x_0 - x_1), \quad x \in [x_0, x_2]$$

$$f'(x) - L_2'(x) = r_2'(x) = f'(x_{i-1}; x_i; x_{i+1}; x)(x - x_{i-1})(x - x_i) \cdot$$
$$\cdot (x - x_{i+1}) + f(x_{i-1}; x_i; x_i; x)\big[(x - x_i)(x - x_{i+1}) +$$
$$+ (x - x_{i-1})(x - x_{i+1}) + (x - x_{i-1})(x - x_i)\big] = \left\{ s = \frac{x - x_i}{\text{[illegible]}} \right\} =$$

$$= \frac{f^{(4)}(\xi)}{24}(s-1)s(s+1) - \frac{f^{(3)}(\eta)}{6}\big(s(s+1) + (s-1)(s+1) + s(s-1)\big) =$$

$$= \frac{f^{(4)}(\xi)}{24}(s-1)s(s+1) + \frac{f^{(3)}(\eta)}{6}(3s^2 - 1), \quad \xi, \eta \in [x_0, x_2]$$

$$\left[\begin{array}{l} S=0 \\ s=\pm 1 \\ s=\pm\frac{1}{\sqrt{3}} \end{array}\right.$$

$$\tau_2'(x)=O(h^3)$$

$$\left[\begin{array}{l} S=\frac{1}{\sqrt{3}} \\ S=-\frac{1}{\sqrt{3}} \end{array}\right.$$

Т.п.т.: $\bar{x}=s+x_1=\left[\begin{array}{l} \frac{h}{\sqrt{3}}+x_1 \\ -\frac{h}{\sqrt{3}}+x_1 \end{array}\right.$

$$\left|\tau_2'(x_i)\right|=\left|\frac{f^{(3)}(\eta)}{6}\right|h^2\leq\frac{M_3}{6}h^2$$

$$M_3=\max_{x\in[a,b]}\left|f^{(3)}(x)\right|$$

N2

формула чис. диф. для $\boxed{n=2,\ k=2}$; оцінити похибку для т.п.т. $\bar{x}$

$$L_2(x)=f_{i-1}+\frac{f_i-f_{i-1}}{h}(x-x_{i-1})+\frac{f_{i+1}-2f_i+f_{i-1}}{h^2}(x-x_{i-1})(x-x_i)$$

$$L_2''(x)=\frac{f_{i+1}-2f_i+f_{i-1}}{h^2}$$

$$\tau_n^{(k)}(x)=\sum_{j=0}^{k}\frac{k!}{(k-j)!(n+j+1)!}f^{(n+j+1)}(\xi_i)\,\omega_n^{(k-j)}(x)=$$

$$= \frac{2!}{(2-0)!(2+0+1)!} f^{(2+0+1)}(\xi)\, w_n^{(2-0)}(x) +$$

$$+ \frac{2!}{(2-1)!(2+1+1)!} f^{(2+1+1)}(\eta)\, w_n^{(2-1)}(x) +$$

$$+ \frac{2!}{(2-2)!(2+2+1)!} f^{(2+2+1)}(\mu)\, w_n^{(2-2)}(x) =$$

$$= \frac{f^{(3)}(\xi)}{3!} \cdot w_n^{(2)}(x) +$$

$$+ \frac{f^{(4)}(\eta)}{12} w_n^{(1)}(x) +$$

$$+ \frac{f^{(5)}(\mu)}{60} w_n^{(0)}(x)$$

$$w_n(x) = (x-x_0)(x-x_1)(x-x_2) = O(h^3)$$

$$w_n'(x) = (x-x_1)(x-x_2) + (x-x_0)\big(\overbrace{x-x_2 + x-x_1}^{2x-x_1-x_2}\big) =$$

$$= x^2 - xx_2 - xx_1 + x_1x_2 + 2x^2 - xx_1 - xx_2 -$$

$$- 2xx_0 + x_0x_1 + x_0x_2 =$$

$$= 3x^2 - 2xx_2 - 2xx_1 + x_1x_2 - 2xx_0 + x_0x_1 + x_0x_2 =$$

$$= O(h^2)$$

$$w_n''(x) = O(h)$$

$$\tau_n^{(k)}(x) = \tau_2^{(2)}(x) = O(h^2)$$

$$\omega_n^{(k)}(x) = \omega_2^{(2)}(x) = 0$$

$$\omega_n''(x) = 6x - 2x_2 - 2x_1 - 2x_0 = 0$$

$$\overline{x} = x = \frac{x_0 + x_1 + x_2}{3} \text{ — т.п.т}$$

$$|\tau_2'(x)| = \left|\frac{f^{(4)}(\xi)}{12}\right| h^2 \le \frac{M_4 h^2}{12}$$

N3 ст. 47 3.3 ряды Тейлора

$$f(x) \in C^3[a,b] \quad : \left|f'(x_1) - \frac{f_2 - f_0}{2h}\right| \le \frac{M_3 h^2}{6}$$

$$x_2 = x_0 + 2h, \quad x_1 = x_0 + h$$

$$f'(x) \approx L_2'(x) = \frac{f_1 - f_0}{h} + (2x - x_0 - x_1)\frac{f_2 - 2f_1 + f_0}{2h^2}, \; x \in [x_0, x$$

$$f'(x_1) = \frac{f_1 - f_0}{h} + \underbrace{(x_1 - x_0)}_{h}\frac{f_2 - 2f_1 + f_0}{2h^2} =$$

$$= \frac{\cancel{2f_1} - 2f_0 + f_2 - \cancel{2f_1} + f_0}{2h} = \frac{f_2 - f_0}{2h}$$

$$f'(x_1) - \frac{f_2 - f_0}{2h} = f'(x_1) - \frac{1}{2h}\left(f(x_0 + 2h) - f(x_0)\right) =$$

$$= \{2h = t\} = f'(x_1) - \frac{1}{t}\left(f(x_0 + t) - f(x_0)\right) =$$

$$= f'(x_1) - \frac{1}{h}\Big[f(x_1) + \frac{t}{2} f'(x_1) + \frac{t^2}{8} f''(x_1) + \frac{t^3}{48} f'''(\xi) -$$

$$- f(x_1) + \frac{t}{2} f'(x_1) - \frac{t^2}{8} f''(x_1) + \frac{h^3}{48} f'''(\eta) \Big] = -\frac{t^2}{24} f'''(\varphi) \Rightarrow$$

$$\Rightarrow \left| f'(x) - \frac{f_2 - f_0}{2h} \right| \le \frac{t^2 M_3}{24} = \frac{h^2 M_3}{6}$$

N4

Ряди Тейлора, перша похідна, три вузли. Похибка чис. диференц. в точці підв. точн.

$$L_n^{(k)}(x) = L_2^{1}(x)$$

$$L_2^{1}(x) = \frac{f_1 - f_0}{h} + \frac{f_2 - 2f_1 + f_0}{h^2} (2x - x_0 - x_1), \quad x \in [x_0, x_2]$$

$$r_n^{(k)}(x) = r_2^{1}(x) = \sum_{j=0}^{k} \frac{k!}{(k-j)!\,(n+j+1)!} f^{(n+j+1)}(\xi_j)\, \omega_n^{(k-j)}(x) =$$

$$= \sum_{j=0}^{1} \frac{1}{(1-j)!\,(n+j+1)!} f^{(n+j+1)}(\xi_j)\, \omega_n^{(1-j)}(x) =$$

$$= \frac{1}{1 \cdot 3!} f^{(3)}(\xi)\, \omega_n^{(1)}(x) +$$

$$+ \frac{1}{1 \cdot 4!} f^{(4)}(\eta)\, \omega_n^{(0)}(x) =$$

$$= \frac{f^{(3)}(\xi)\,\omega_n'(x)}{3!} + \frac{f^{(4)}(\eta)\,\omega_n(x)}{4!} = O(h^2)$$

$$\omega_n(x) = (x - x_0)(x - x_1)(x - x_2) = O(h^3)$$

$$\omega_n'(x) = 3x^2 - 2xx_2 - 2xx_1 + x_1x_2 - 2xx_0 + x_0x_1 + x_0x_2 = O(h^2)$$

$$\omega_n^{(k)}(x) = 0$$

$$\omega_2'(x) = 0$$

$$3x^2 - 2xx_2 - 2xx_1 + x_1x_2 - 2xx_0 + x_0x_1 + x_0x_2 = 0$$

$$3x^2 - 2x(x_2 + x_1 + x_0) + x_1x_2 + x_0x_1 + x_0x_2 = 0$$

$$\frac{D}{4} = (x_2 + x_1 + x_0)^2 - 3(x_1x_2 + x_0x_1 + x_0x_2) =$$

$$= x_2^2 + 2x_2x_1 + x_1^2 + 2(x_2 + x_1)x_0 + x_0^2 - 3x_1x_2 - 3x_0x_1 -$$

$$- 3x_0x_2 = x_2^2 + x_1^2 + x_0^2 + 2x_2x_1 + 2x_2x_0 + 2x_1x_0 -$$

$$- 3x_1x_2 - 3x_0x_1 - 3x_0x_2 =$$

$$= x_2^2 + x_1^2 + x_0^2 - x_1x_2 - x_0x_2 - x_0x_1 = a$$

$$x_1^* = \frac{x_2 + x_1 + x_0 + \sqrt{a}}{3} \notin [x_0, x_2] \Rightarrow \text{не задов.}$$

$$x_2^* = \frac{x_2 + x_1 + x_0 - \sqrt{a}}{3} = \bar{x} \;-\; \text{т.п.т.}$$

$$|r_2'(\bar{x})| = \left| \frac{f^{(3)}(\eta)}{6} h^2 \right| \le \frac{M_3}{6} h^2$$