

# Тема 4

**N 1**

$$f(x) = \ln(x+1), \quad x \in [0, 1] \quad \text{ст. мн.} = 4 \quad (n=4)$$

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$$|\tau_n(x)| \leq \frac{M_{n+1}}{(n+1)!} |\omega_n(x)|, \quad M_{n+1} = \sup_{x \in [a,b]} |f^{(n+1)}(x)|$$

$$\omega_n(x) = \prod_{i=0}^n (x - x_i)$$

$$|\tau_n(x)| = |f(x) - L_n(x)| \leq \frac{M_{n+1}}{(n+1)!} \cdot \frac{(b-a)^{n+1}}{2^{n+1}}$$

$$x_k = \frac{a+b}{2} - \frac{b-a}{2} t_k, \quad t_k = \cos \frac{(2k+1)\pi}{2(n+1)}, \quad k = \overline{0, n}$$

$$f' = \frac{1}{x+1} = (x+1)^{-1}$$

$$f'' = -\frac{1}{(x+1)^2} = -1 \cdot (x+1)^{-2}$$

$$f''' = \frac{2}{(x+1)^3} = -2 \cdot (-1) (x+1)^{-3}$$

$$f^{(4)} = \frac{-6}{(x+1)^4} = -3 \cdot (-2) \cdot (-1) (x+1)^{-4}$$

$$f^{(5)} = \frac{24}{(x+1)^5}$$

$$\sup_{x \in [0,1]} |f^{(5)}(x)| = \sup_{x \in [0,1]} \left| \frac{24}{(x+1)^5} \right| = 24$$

$$|R_4(x)| \leq \frac{24}{5!} \cdot \frac{1^5}{2^5} = \frac{1}{5 \cdot 512} = \frac{1}{2560} =$$

$$= 0,000390625 = 3,90625 \cdot 10^{-4}$$

N2

$$x_0 = 100, x_1 = 101, x_2 = 102, x_3 = 103, x_4 = 104, x^* = 100,5$$

$$|r_n(x)| \leq \frac{M_{n+1}}{(n+1)!} |w_n(x)|$$

$$w_n(x) = (x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)$$

$$|w_n(x^*)| = 0,5 \cdot 0,5 \cdot 1,5 \cdot 2,5 \cdot 3,5 = 3,28125$$

$$f^{(5)} = \frac{24}{x^5}$$

$$M_5 = \sup_{[100;104]} |f^{(5)}(x)| = \frac{24}{10^{10}}$$

$$|r_n(x)| \leq \frac{M_5}{5!} |w_n(x)|$$

$$|r_n(x^*)| \leq \frac{M_5}{5!} \cdot 3,28125 = \frac{24}{10^{10} \cdot 12 \cdot 10} \cdot 3,28125 =$$

$$= 6,5625 \cdot 10^{-11}$$

$$\begin{aligned}
 & + \frac{f(x_1)}{(x-x_0)(x-x_2)(x-x_3)(x-x_4)} + \frac{f(x_2)}{(x-x_0)(x-x_1)(x-x_3)(x-x_4)} + \\
 & + \frac{f(x_3)}{(x-x_0)(x-x_1)(x-x_2)(x-x_4)} + \frac{f(x_4)}{(x-x_0)(x-x_1)(x-x_2)(x-x_3)} = \\
 & = \frac{f(x)}{\omega_n(x)} + \dots
 \end{aligned}$$

$$\begin{aligned}
 \pi_4(x) = & f(x) + f(x_0)(x-x_0) + f(x_1)(x-x_1) + \\
 & + f(x_2)(x-x_2) + f(x_3)(x-x_3) + f(x_4)(x-x_4)
 \end{aligned}$$

$$\begin{aligned}
 \pi_4(100,5) = & \ln 100,5 + 0,5 \cdot \ln 100 - 0,5 \ln 101 - \\
 & - 1,5 \ln 102 - 2,5 \ln 103 - 3,5 \ln 104 = -30,1745
 \end{aligned}$$

$$\Rightarrow |r_n(x)| \leq \frac{(n-1)!}{(n+1)!} \cdot \frac{(b-a)^{n+1}}{2^{2n+1}} \leq \varepsilon$$

$$\frac{1^{n+1}}{n(n+1) \cdot 2^{2n+1}} \leq \varepsilon$$

$$n(n+1) 2^{2n+1} \geq \frac{1}{\varepsilon} = 10^4 \Rightarrow$$

$$\Rightarrow n \geq 4$$

Отже, к-ть вузлів інтерполюції має бути більшою або дор. 5.

N4

$x \in [0, 1]$  3 ст. коеф. при старшому степені дор. 1

$$T_n^{[a,b]}(t) = \overline{T}_n \left( \frac{2}{b-a} x - \frac{b+a}{b-a} \right) =$$

$$= 2^{1-n} \cos \left( n \arccos \left( \frac{2}{b-a} x - \frac{b+a}{b-a} \right) \right)$$

$$T_3^{[0,1]}(t) = 2^{1-3} \cdot \cos(3 \cdot \arccos(2x-1))$$

N4

$x \in [0, 1]$  3 ст. коеф. при ст. степені дор. 1

$$T_{n+1}(x) = 2xT_n - T_{n-1}(x), T_0(x) = 1, T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 2x - x = 4x^3 - 3x$$

$$\overline{T}_3(x) = x^3 - \frac{3x}{4}, \quad x \in [-1; 1]$$

Знайдемо вираз  $n$ -го Чеб. на проміжку  $[0; 1]$ :

$$T_3(\xi) = 4\xi^3 - 3\xi$$

$$\xi = \frac{2}{b-a} \left( x - \frac{b+a}{2} \right) = 2x - 1, \quad a=0, b=1$$

$$\begin{aligned} T_3^{[0,1]}(x) &= 4(2x-1)^3 - 3(2x-1) = 4(8x^3 - 3 \cdot 4x^2 + 3 \cdot 2 \cdot x - 1) - \\ &\quad - 6x + 3 = 32x^3 - 48x^2 + 24x - 4 - 6x + 3 = \\ &= 32x^3 - 48x^2 - 18x - 1 \end{aligned}$$

$$\overline{T}_3^{[0,1]} = x^3 - \frac{3 \cdot x^2}{2} - \frac{9x}{16} - \frac{1}{32}$$

Відхилення від 0:

$$\|\overline{T}_3^{[0,1]}\| = \frac{1}{32}$$