Тема 7

N1 Задача 5 ст. 47

$y = J_0(x)$, $h = 0{,}02$ $y'(x_0)$, $y''(x_0)$ – ?, $x_0 = 1$

x	x_0: 0,96	x_1: 0,98	x_2: 1	x_3: 1,02	x_4: 1,04
y	0,7825361	0,7739332	0,7651977	0,7563321	0,7473390

$\max\limits_x |J_0^{(k)}(x)| = 1$

x	$f(x_i)$	$f(x_i;x_{i+1})$	$f(x_i;x_{i+1};x_{i+2})$	$f(x_i;x_{i+1};x_{i+2};x_{i+3})$	$f(x_i;\ ;x_{i+4})$
0,96	$f(x_0)$				
		-0,430145			
0,98	$f(x_1)$		-0,16575		
		-0,436775		0	
1	$f(x_2)$		-0,162625		0
		-0,44328		0	
1.02	$f(x_3)$		-0,159375		
		-0,449655			
1,04	$f(x_4)$				

$L_4(x) = 0{,}7825361 - \underbrace{0{,}430145}(x - \underbrace{0{,}96}) - 0{,}16575(x - \underbrace{0{,}96})(x - \underbrace{0{,}98}) =$

$= 0{,}7825361 - 0{,}430145x + \_ \quad - 0{,}16575(x^2 - 0{,}98x - 0{,}96x + \_)$

$L_4'(x) = -0{,}430145 - 0{,}3315x + 0{,}321555$

$L_4''(x) = -0{,}3315$

$y'(x_0) = L_4'(x_0) = -0{,}44009$

$y''(x_0) = L_4''(x_0) = -0{,}3315$

$$|r_4(x)| \le \frac{M_5}{5!} \cdot \frac{(1{,}04 - 0{,}96)^5}{2^9}$$

$$M_5 = \sup_{x \in [x_0, x_4]} |f^{(5)}(x)| = 1 \Rightarrow$$

$$\Rightarrow |r_4(x)| \le \frac{0{,}08^5}{5!\,2^9} = \frac{(2^3 \cdot 10^{-2})^5}{5!\,2^9} = \frac{2^{15} \cdot 10^{-10}}{3 \cdot 2^2 \cdot 10 \cdot 2^9} = \frac{2^4 \cdot 10^{-11}}{3} =$$

$$= 5{,}3 \cdot 10^{-11}$$

N2 Задача с ст.47

$y = sh\,2x$, $h = 0{,}05$

x	0	0,05	0,1	0,15	0,2	0,25
y	0	0,10017	0,20134	0,30452	0,41075	0,52110

$y'(0)$, $y''(0{,}1)$ — ?

x	$f(x_i)$	$f(x_i;x_{i+1})$	$f(x_i;x_{i+1};x_{i+2})$	$f(x_i;...;x_{i+3})$	$f(x_i;...;x_{i+4})$	$f(x_i;...;x_{i+5})$
0	0					
		2,0034				
			0,2			
0,05	0,10017			0		
		2,0234			0	
			0,402			
0,1	0,20134			0		0
		2,0636			0	
			0,61			
0,15	0,30452			0		
		2,1246				
			0,824			
0,2	0,41075					
		2,207				
0,25	0,52110					

$$L_5(x) = 0 + 2,0034x + 0,2x(x-0,05)$$

$$L_5'(x) = 2,0034 + 0,4x - 0,01$$

$$L_5''(x) = 0,4$$

$$y'(0) \approx L_5'(0) = 1,9934$$

$$y''(0,1) \approx L_5''(0,1) = 0,4$$

$$|r_5(x)| \le \frac{M_6}{6!} \cdot \frac{(0,25-0)^6}{2^{11}}$$

$$M_6 = \sup_{x\in[0;0,25]} |f^{(6)}(x)| = \sup_{x\in[0;0,25]} |2^6 \operatorname{sh} 2x| = 2^6 \cdot 0,521$$

$$|r_5(x)| \le \frac{2^6 \cdot 0{,}521 \cdot (10^{-2} \cdot 25)^6}{720 \cdot 2^{11}} =$$

$$= \frac{2^{-5} \cdot 0{,}521 \cdot 10^{-12} \cdot 25^6}{720} =$$

$$= 0{,}0074 \cdot 2^{-5} \cdot 2^{-12} \cdot \cancel{5^{-12}} \cdot \cancel{5^{12}} =$$

$$= 0{,}0074 \cdot 2^{-17}$$

N3

Методом невизначених коефіцієнтів побудувати формулу чисельного диференціювання для $f'(0)$ за трьома вузлами , $x_0 = 0$, $x_1 = h$, $x_2 = 2h$ $n=2$, $k=1$

$$f^{(k)}(x_p) = \sum_{i=0}^{n} c_i^{(p)} f(x_i) + R(f)$$

$x_p = x_0$

$$\left.\begin{aligned} &c_0^{(0)} + c_1^{(0)} + c_2^{(0)} = 0 \\ &c_0^{(0)} x_0 + c_1^{(0)} x_1 + c_2^{(0)} x_2 = 1 \\ &c_0^{(0)} x_0^2 + c_1^{(0)} x_1^2 + c_2^{(0)} x_2^2 = 2x_0 \end{aligned}\right\} 0$$

$x_p = x_1$

$$\left.\begin{aligned} &c_0^{(1)} + c_1^{(1)} + c_2^{(1)} = 0 \\ &c_0^{(1)} x_0 + c_1^{(1)} x_1 + c_2^{(1)} x_2 = 1 \\ &c_0^{(1)} x_0^2 + c_1^{(1)} x_1^2 + c_2^{(1)} x_2^2 = 2x_1^{n-k} \end{aligned}\right\} 1$$

$x_p = x_2$

$$\left.\begin{aligned} &C_0^{(2)} + C_1^{(2)} + C_2^{(2)} = 0 \\ &C_0^{(2)} x_0 + C_1^{(2)} x_1 + C_2^{(2)} x_2 = 1 \\ &C_0^{(2)} x_0^2 + C_1^{(2)} x_1^2 + C_2^{(2)} x_2^2 = 2x_2 \end{aligned}\right\} (2)$$

Система 0 : (шукані саме її розв'язки $x_0 = 0$)

$$\begin{pmatrix} 1 & 1 & 1 \\ x_0 & x_1 & x_2 \\ x_0^2 & x_1^2 & x_2^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2x_0 \end{pmatrix}$$

$$\Delta = \prod_{0 \le i < j \le n} (x_j - x_i) = (x_2 - x_0)(x_1 - x_0)(x_2 - x_1) = 2h^2 \cdot h = 2h^3$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 \\ 1 & x_1 & x_2 \\ 2x_0 & x_1^2 & x_2^2 \end{vmatrix} = x_1^2 + 2x_0x_2 - 2x_0x_1 - x_2^2 = h^2 - 4h^2 = -3h^2$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 \\ x_0 & 1 & x_2 \\ x_0^2 & 2x_0 & x_2^2 \end{vmatrix} = x_2^2 + 2x_0^2 - x_0^2 - 2x_0x_2 = x_2^2 + x_0^2 - 2x_0x_2 = (x_0 - x_2)^2 = 4h^2$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 \\ x_0 & x_1 & 1 \\ x_0^2 & x_1^2 & 2x_0 \end{vmatrix} = 2x_0x_1 + x_0^2 - x_1^2 - 2x_0^2 = -x_0^2 + 2x_0x_1 - x_1^2 = -(x_0 - x_1)^2 = -h^2$$

$$c_0^{(0)} = \frac{\Delta_1}{\Delta} = \frac{-3h^2}{2h^3} = -\frac{3}{2h}$$

$$c_1^{(0)} = \frac{\Delta_2}{\Delta} = \frac{4h^2}{2h^3} = \frac{2}{h}$$

$$c_2^{(0)} = \frac{\Delta_3}{\Delta} = -\frac{h^2}{2h^3} = -\frac{1}{2h}$$

$$f'(x_p = x_0 = 0) = \sum_{i=0}^{n} c_i^{(0)} f(x_i) =$$

$$= -\frac{3}{2h} f(0) + \frac{2}{h} f(h) - \frac{1}{2h} f(2h)$$

№4

Методом невизначених коефіцієнтів побудувати формулу чисельного диференціювання для $f''(0)$ за чотирма вузлами $x_0 = -h$, $x_1 = 0$, $x_2 = h$, $x_3 = 2h$ $(n = 3, k = 2)$

$$f^{(k)}(x_p) = \sum_{i=0}^{n} c_i^{(p)} f(x_i) + R(f)$$

$\underline{x_p = x_1}$

$$\begin{cases} C_0^{(1)} + C_1^{(1)} + C_2^{(1)} + C_3^{(1)} = 0 \\ C_0^{(1)} x_0 + C_1^{(1)} x_1 + C_2^{(1)} x_2 + C_3 x_3 = 0, \\ C_0^{(1)} x_0^2 + C_1^{(1)} x_1^2 + C_2^{(1)} x_2^2 + C_3 x_3^2 = 2 \\ C_0^{(1)} x_0^3 + C_1^{(1)} x_1^3 + C_2^{(1)} x_2^3 + C_3 x_3^3 = \overset{3\cdot 2}{6} x_1 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ x_0 & x_1 & x_2 & x_3 \\ x_0^2 & x_1^2 & x_2^2 & x_3^2 \\ x_0^3 & x_1^3 & x_2^3 & x_3^3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 6x_1 \end{pmatrix}$$

$$\Delta = \prod_{1 \le i < j \le n} (x_j - x_i) = (x_3 - x_2)(x_3 - x_1)(x_3 - x_0)(x_2 - x_1)(x_2 - x_0) \cdot$$
$$\cdot (x_1 - x_0) = h \cdot 2h \cdot 3h \cdot h \cdot 2h \cdot h =$$
$$= 12h^6$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 0 & x_1 & x_2 & x_3 \\ 2 & x_1^2 & x_2^2 & x_3^2 \\ 6x_1 & x_1^3 & x_2^3 & x_3^3 \end{vmatrix} \cancel{2}x_1 = 12h^4$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 & 1 \\ x_0 & 0 & x_2 & x_3 \\ x_0^2 & 2 & x_2^2 & x_3^2 \\ x_0^3 & 6x_1 & x_2^3 & x_3^3 \end{vmatrix} = -24h^4$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 & 1 \\ x_0 & x_1 & 0 & x_3 \\ x_0^2 & x_1^2 & 2 & x_3^2 \\ x_0^3 & x_1^3 & 6x_1 & x_3^3 \end{vmatrix} = 12h^4$$

$$\Delta_4 = \begin{vmatrix} 1 & 1 & 1 & 0 \\ x_0 & x_1 & x_2 & 0 \\ x_0^2 & x_1^2 & x_2^2 & 2 \\ x_0^3 & x_1^3 & x_2^3 & 6x_1 \end{vmatrix} = 0$$

$$C_0^{(1)} = \frac{\Delta_1}{\Delta} = \frac{12h^4}{12h^6} = \frac{1}{h^2}$$

$$C_1^{(1)} = \frac{\Delta_2}{\Delta} = \frac{-24h^4}{12h^6} = -\frac{2}{h^2}$$

$$C_2^{(1)} = \frac{\Delta_3}{\Delta} = \frac{1}{h^2}$$

$$C_3^{(1)} = 0$$

$$f''(0) = \sum_{i=0}^{n} C_i^{(1)} f(x_i) = \frac{f(-h)}{h^2} - \frac{2f(0)}{h^2} + \frac{f(h)}{h^2}$$