05.20 # Тема 8

№1 ст. 47 задача 8 1а)

$f(x) = \ln x \quad, x \in [1, 10] \quad, n = 3 \quad, h - ?$

$f'(x_i) \approx \dfrac{f_{i+1} - f_{i-1}}{2h} \qquad \delta = 10^{-4}$

$$\left| f_i' - \frac{\bar{f}_{i+1} - \bar{f}_{i-1}}{2h} \right| \le \left| f_i' - \frac{f_{i+1} - f_{i-1}}{2h} \right| + \left| \frac{\delta_{i+1} - \delta_{i-1}}{2h} \right| \le$$

$$\le \frac{M_3 h^2}{6} + \frac{\delta}{h} = \varphi(h)$$

$$\varphi'(h) = \frac{M_3 h}{3} - \frac{\delta}{h^2} = 0$$

$$h_0 = \sqrt[3]{\frac{3\delta}{M_3}}$$

$$f'(x) = \frac{1}{x}$$

$$f''(x) = -\frac{1}{x^2}$$

$$f'''(x) = \frac{2}{x^3}$$

$$M_3 = \max_{x \in [1,10]} |f'''(x)| = |f'''(1)| = 2$$

$$h_0 = \sqrt[3]{\frac{3 \cdot 10^{-4}}{2}} = 0{,}05313$$

$$\varphi(h_0) = \frac{h_0^2}{3} + \frac{10^{-4}}{h_0} = 0{,}002822$$

№21 ст. 47 задача 8 3д)

$$f(x) = \sin^2 x \quad , x \in [0, \pi]$$

$$f''(x_i) = \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2}$$

$$\left| \frac{\bar{f}_{i+1} - 2\bar{f}_i + \bar{f}_{i-1}}{h^2} - f_i'' \right| \leq \frac{M_4 h^2}{24} + \frac{4\delta}{h^2} = \varphi(h)$$

$$\varphi'(h) = \frac{M_4 h}{12} - \frac{8\delta}{h^3} = 0$$

$$h_0 = \sqrt[4]{\frac{96\delta}{M_4}}$$

$$f(x) = \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$f'(x) = \sin 2x$$

$$f''(x) = 2\cos 2x$$

$$f'''(x) = -4\sin 2x$$

$$f^{(4)}(x) = -8\cos 2x$$

$$M_4 = \max_{x \in [0, \pi]} |f^{(4)}(x)| = 8$$

$$h_0 = \sqrt[4]{\frac{96\delta}{M_4}} = \sqrt[4]{\frac{96 \cdot 10^{-4}}{8}} = \sqrt[4]{12 \cdot 10^{-4}} = \sqrt[4]{12} \cdot 10^{-1}$$

$$\varphi(h_0) = \frac{h_0^2}{3} + \frac{4 \cdot 10^{-4}}{h_0^2} = \frac{2}{\sqrt{3}} + \frac{2 \cdot 10^{-4}}{\sqrt{3}} = 1,1548$$

№3

B_3 – сплайн (кубічний)

$$s(z) = B_3^1(z) = \begin{cases} \left(\frac{z - x_{i-2}}{h}\right)^3 & z \in [x_{i-2}, x_{i-1}] \\ -3\left(\frac{z - x_{i-1}}{h}\right)^3 + 3\left(\frac{z - x_{i-1}}{h}\right)^2 + 3\left(\frac{z - x_{i-1}}{h}\right) + 1 & [x_{i-1}, x_i] \\ -3\left(\frac{x_{i+1} - z}{h}\right)^3 + 3\left(\frac{x_{i+1} - z}{h}\right)^2 + 3\left(\frac{x_{i+1} - z}{h}\right) + 1 & [x_i, x_{i+1}] \\ \left(\frac{x_{i+2} - z}{h}\right)^3 & z \in [x_{i+1}, x_{i+2}] \\ 0 & z \notin (x_{i-2}, x_{i+2}) \end{cases}$$

сплайн третього степеня дефекту 1

1) $S(x) \in \Pi_3 \quad , x \in [x_{i-1}, x_i] \ , \ i = \overline{1,n}$

2) $S(x) \in C^{m-1}[a,b] = C^2[a,b]$ $\circledast$

z_i	$B_3^1(z_i - 0)$	$B_3^1(z_i + 0)$
x_{i-2}	0	0
x_{i-1}	$\left(\frac{h}{h}\right)^3 = 1$	1
x_i	$-3+3+3+1=4$	$-3+3+3+1=4$
x_{i+1}	$0+0+0+1=1$	1
x_{i+2}	0	0

$$
s'(z) = \begin{cases}
\frac{3}{h}\left(\frac{z - x_{i-2}}{h}\right)^2 & z \in [x_{i-2}, x_{i-1}] \\
-\frac{9}{h}\left(\frac{z - x_{i-1}}{h}\right)^2 + \frac{6}{h}\left(\frac{z - x_{i-1}}{h}\right) + \frac{3}{h} & z \in (x_{i-1}; x_i] \\
\frac{9}{h}\left(\frac{x_{i+1} - z}{h}\right)^2 - \frac{6}{h}\left(\frac{x_{i+1} - z}{h}\right) - \frac{3}{h} & z \in (x_i; x_{i+1}] \\
-\frac{3}{h}\left(\frac{x_{i+2} - z}{h}\right)^2 & z \in (x_{i+1}, x_{i+2}] \\
0 & z \notin (x_{i-2}, x_{i+2})
\end{cases}
$$

z_i	$s'(z_i - 0)$	$s'(z_i + 0)$
x_{i-2}	0	0
x_{i-1}	$\frac{3}{h}$	$\frac{3}{h}$
x_i	$-\frac{9}{h} + \frac{6}{h} + \frac{3}{h} = 0$	$\frac{9}{h} - \frac{6}{h} - \frac{3}{h} = 0$
x_{i+1}	$-\frac{3}{h}$	$-\frac{3}{h}$
x_{i+2}	0	0

$$S''(z) = \begin{cases} \frac{6}{h^2}\left(\frac{z - x_{i-2}}{h}\right) & z \in [x_{i-2}, x_{i-1}) \\ -\frac{18}{h^2}\left(\frac{z - x_{i-1}}{h}\right) + \frac{6}{h^2} & z \in [x_{i-1}, x_i) \\ -\frac{18}{h^2}\left(\frac{x_{i+1} - z}{h}\right) + \frac{6}{h^2} & z \in [x_i, x_{i+1}) \\ \frac{6}{h^2}\left(\frac{x_{i+2} - z}{h}\right) & z \in [x_{i+1}, x_{i+2}) \\ 0 & z \notin (x_{i-2}; x_{i+2}] \end{cases}$$

z_i	$S''(z_i - 0)$	$S''(z_i + 0)$
x_{i-2}	0	0
x_{i-1}	$\frac{6}{h^2}$	$\frac{6}{h^2}$
x_i	$-\frac{18}{h^2} + \frac{6}{h^2} = -\frac{12}{h^2}$	$-\frac{18}{h^2} + \frac{6}{h^2} = -\frac{12}{h^2}$
x_{i+1}	$\frac{6}{h^2}$	$\frac{6}{h^2}$
x_{i+2}	0	0

Оскільки виконуються умови (*), то сплайн $B_3^1(z)$ – третього степеня з дефектом 1

№4

Побудувати природній (друга похідна по обох кінцях дор. нулю) інтерполяційний кубічний сплайн за даними:

x_i	0	1	2	3
y_i	0	0,5	2	1,5

$$\begin{cases} \frac{h_i}{6} m_{i-1} + \frac{h_i + h_{i+1}}{3} m_i + \frac{h_{i+1}}{6} m_{i+1} = \frac{f_{i+1} - f_i}{h_i} - \frac{f_i - f_{i-1}}{h_i} \\ m_0 = m_n = 0 \end{cases}$$

$i = 1, 2$

$h_i = const = 1$, $i = \overline{0,n}$

$$\begin{cases} \frac{m_0}{6} + \frac{2m_1}{3} + \frac{m_2}{6} = \frac{2-0,5}{1} - \frac{0,5-0}{1} = 1 \\ \frac{m_1}{6} + \frac{2m_2}{3} + \frac{m_3}{6} = \frac{1,5-2}{1} - \frac{2-0,5}{1} = -2 \\ m_0 = m_3 = 0 \end{cases}$$

$$\begin{cases} \frac{2m_1}{3} + \frac{m_2}{6} = 1 \\ \frac{m_1}{6} + \frac{2m_2}{3} = -2 \end{cases}$$

$$\begin{cases} 4m_1 + m_2 = 6 \\ m_1 + 4m_2 = -12 \end{cases}$$

$15m_2 = -54$

$m_2 = -\frac{54}{15} = -\frac{18}{5}$

$m_1 = \dfrac{6 + \frac{18}{5}}{4} = \dfrac{3 + \frac{9}{5}}{2} = \frac{12}{5}$

$\frac{2 \cdot 12}{3 \cdot 5} - \frac{18}{5 \cdot 6} = 1$ ✓

$\frac{12}{5 \cdot 6} - \frac{2 \cdot 18}{3 \cdot 5} = -2$ ✓

$m_0 = 0, \ m_1 = \frac{12}{5}, \ m_2 = -\frac{18}{5}, \ m_3 = 0$

$$S(x) = m_{i-1}\frac{(x_i - x)^3}{6h_i} + m_i \frac{(x - x_{i-1})^3}{6h_i} +$$
$$+ \left(f_{i-1} - \frac{m_{i-1} h_i^2}{6}\right)\frac{x_i - x}{h_i} + \left(f_i - \frac{m_i h_i^2}{6}\right)\frac{x - x_{i-1}}{h_i},$$
$$x \in [x_{i-1}, x_i]$$

$$S(x) = \begin{cases} \frac{12 \cdot x^3}{5 \cdot 6} + \left(\frac{1}{2} - \frac{12}{5 \cdot 6}\right)x, & x \in [0;1] \\ \frac{12}{5}\frac{(2-x)^3}{6} - \frac{18}{5}\frac{(x-1)^3}{6} + \\ + \left(\frac{1}{2} - \frac{12}{5 \cdot 6}\right)(1-x) + \left(2 + \frac{18}{5 \cdot 6}\right)(x-1), & x \in [1;2] \\ -\frac{18}{5}\frac{(3-x)^3}{6} + \left(2 + \frac{18}{5 \cdot 6}\right)(3-x) + \frac{3}{2}(x-2), & x \in [2;3] \end{cases}$$

$$S(x) = \begin{cases} \dfrac{2x^3}{5} + \dfrac{x}{10}, & x \in [0, 1] \\[2ex] \dfrac{2(2-x)^3}{5} - \dfrac{3(x-1)^3}{5} + \dfrac{1-x}{10} + \dfrac{13}{5}(x-1), & x \in [1, 2] \\[2ex] -\dfrac{3(3-x)^3}{5} + \dfrac{13}{5}(3-x) + \dfrac{3}{2}(x-2), & x \in [2; 3] \end{cases}$$