Завдання 4. Задача мінімізації нелінійної цільової ф-ї n-змінних з лінійними обмеженнями. В - 6

$$L(\bar{x}) = 4x_1^2 + 2x_2^2 - 2x_1x_2 - 2x_1 \xrightarrow[D]{} \min$$

$$D \begin{cases} 2x_1 + x_2 \le 4 \\ x_i \ge 0, \ i = 1,2 \end{cases}$$

I - ітерація

$\bar{x}^0 = (x_1 = 1; x_2 = 2)$

$\nabla f = \left(\frac{\partial f}{\partial x_1}; \frac{\partial f}{\partial x_2}\right) = (8x_1 - 2x_2 - 2; 4x_2 - 2x_1)$

$\nabla f(\bar{x}^0) = (8 - 4 - 2; 8 - 2) = (2; 6)$

$f_1 = 2x_1 + 6x_2$

Нова задача ЛП:

$$z(\bar{x}) = 2x_1 + 6x_2 \xrightarrow[D]{} \min$$

$\tilde{x}^0 = (0;0)$

Знайдемо допустимий план задачі:

$\bar{x}^{k+1} = \bar{x}^k + \lambda(\tilde{x}^k - \bar{x}^k)$, $0 \le \lambda_1 \le 1$

$\bar{x}^1 = \bar{x}^0 + \lambda(\tilde{x}^0 - \bar{x}^0)$

$\bar{x}^1 = (1;2) - \lambda_1(1;2) = (1-\lambda_1; 2-2\lambda_1)$

$f = 4x_1^2 + 2x_2^2 - 2x_1x_2 - 2x_1 =$

$= 4(1-\lambda_1)^2 + 2(2-2\lambda_1)^2 - 2(1-\lambda_1)(2-2\lambda_1) - 2(1-\lambda_1) =$

$= (1-\lambda_1)^2(4 + 2\cdot 4) - 2\cdot 2(1-\lambda_1) - 2(1-\lambda_1) =$

$= 12(1-\lambda_1)^2 - 6(1-\lambda_1)$

$f' = 24(1-\lambda_1) - 6$

$f' = 0$

$1-\lambda_1 = \frac{6}{24} = \frac{1}{4} \Rightarrow \lambda_1 = \frac{3}{4}$

$\bar{x}^1 = (1-\frac{3}{4}; 2-\frac{3}{2}) = (\frac{1}{4}; \frac{1}{2})$

$f(\bar{x}^1) = 4\cdot\frac{1}{16} + 2\cdot\frac{1}{4} - 2\cdot\frac{1}{4}\cdot\frac{1}{2} - 2\cdot\frac{1}{4} =$

$= \frac{1}{4} + \cancel{\frac{1}{2}} - \frac{1}{4} - \cancel{\frac{1}{2}} = 0$

II - ітерація

$\nabla f(\bar{x}^1) = (2-1-2; 2-\frac{1}{2}) = (-1; \frac{3}{2})$

$$f_1 = -x_1 + \frac{3}{2}x_2$$

Нова ЗЛП:

$$z(\bar{x}) = -x_1 + \frac{3}{2}x_2 \longrightarrow \min_D$$

$$\tilde{x}^1 = (2; 0)$$

$$\bar{x}^2 = \bar{x}^1 + \lambda(\tilde{x}^1 - \bar{x}^1), \quad 0 \le \lambda \le 1$$

$$\bar{x}^2 = \left(\frac{1}{4}; \frac{1}{2}\right) + \lambda\left((2; 0) - \left(\frac{1}{4}; \frac{1}{2}\right)\right) =$$

$$= \left(\frac{1}{4}; \frac{1}{2}\right) + \lambda\left(\frac{7}{4}; -\frac{1}{2}\right) = \left(\frac{1}{4} + \frac{7\lambda}{4}; \frac{1}{2} - \frac{\lambda}{2}\right)$$

$$f(\bar{x}^2) = 4\left(\frac{1}{4} + \frac{7\lambda}{4}\right)^2 + 2\left(\frac{1}{2} - \frac{\lambda}{2}\right)^2 -$$

$$-2\left(\frac{1}{4} + \frac{7\lambda}{4}\right)\left(\frac{1}{2} - \frac{\lambda}{2}\right) - 2\left(\frac{1}{4} + \frac{7\lambda}{4}\right) =$$

$$= \frac{1}{4}(1 + 7\lambda)^2 + \frac{1}{2}(1 - \lambda)^2 -$$

$$-\left(\frac{1}{4} + \frac{7\lambda}{4}\right)(1 - \lambda) - 2\left(\frac{1}{4} + \frac{7\lambda}{4}\right) =$$

$$= \underbrace{\frac{1}{4}(1 + 7\lambda)^2 + \frac{1}{2}(1 - \lambda)^2} - \left(\frac{1}{4} + \frac{7\lambda}{4}\right)(3 - \lambda) =$$

$$= \underbrace{\qquad} - \frac{3}{4} + \frac{\lambda}{4} - \frac{21\lambda}{4} + \frac{7\lambda^2}{4}$$

$$f'(\bar{x}^2) = 0$$

$$\frac{1}{2} \cdot 7(1 + 7\lambda) - (1 - \lambda) + \frac{1}{4} - \frac{21}{4} + \frac{7\lambda}{2} = 0$$

$$\frac{7}{2}+\frac{49\lambda}{2}-1+\lambda-5+\frac{7\lambda}{2}=0$$

$$28\lambda+\lambda-6+\frac{7}{2}=0$$

$$29\lambda=6-\frac{7}{2}=\frac{5}{2}$$

$$\lambda=\frac{5}{58}$$

$$\bar{x}^2=\left(\frac{1}{4}+\frac{7\cdot5}{4\cdot58};\ \frac{1}{2}-\frac{5}{58\cdot2}\right)=$$

$$=\left(\frac{93}{4\cdot58};\ \frac{53}{58\cdot2}\right)$$

$$f(\bar{x}^2)=4\cdot\left(\frac{93}{4\cdot58}\right)^2+2\cdot\left(\frac{53}{58\cdot2}\right)^2-2\cdot\frac{93}{4\cdot58}\cdot\frac{53}{58\cdot2}-2\cdot\frac{93}{4\cdot58}=$$

$$=\frac{1}{4}\cdot\left(\frac{93}{58}\right)^2+\frac{1}{2}\left(\frac{53}{58}\right)^2-\frac{1}{4}\cdot\frac{93\cdot53}{(58)^2}-\frac{1}{2}\cdot\frac{93}{58}=$$

$$=\frac{1}{4}\cdot\frac{(93)^2-93\cdot53}{58^2}+\frac{1}{2}\cdot\frac{53^2-93\cdot58}{58^2}=$$

$$=\frac{93^2-93\cdot53+2\cdot53^2-2\cdot93\cdot58}{4\cdot58^2}\approx-0{,}1097$$

III - ітерація

$$\nabla f(\bar{x}^2)=(0{,}294;\ 1{,}026)$$

$$f_2=-0{,}294x_1+1{,}026x_2$$

Нова ЗЛП:

$$z(\bar{x})=-0{,}294+1{,}026x_2\longrightarrow\min$$

$$\tilde{x}^2 = (2, 0)$$

$$\bar{x}^3 = \bar{x}^2 + \lambda(\tilde{x}^2 - \bar{x}^2) =$$

$$= \left(\frac{93}{4\cdot 58}; \frac{53}{58\cdot 2}\right) + \lambda\left(\frac{2\cdot 4\cdot 58 - 93}{4\cdot 58}; -\frac{53}{58\cdot 2}\right) =$$

$$= \left(\frac{93+371\lambda}{4\cdot 58}; \frac{53(1-\lambda)}{58\cdot 2}\right)$$

$$f(\bar{x}^3) = 4\left(\frac{93+371\lambda}{4\cdot 58}\right)^2 + 2\left(\frac{53(1-\lambda)}{58\cdot 2}\right)^2 -$$

$$-\frac{2(93+371\lambda)\,53(1-\lambda)}{8\cdot 58^2} - \frac{93+371\lambda}{2\cdot 58} =$$

$$= \underbrace{\frac{1}{4\cdot 58^2}(93+371\lambda)^2 + \frac{1}{2\cdot 58^2}(53(1-\lambda))^2}_{} -$$

$$-\frac{53}{4\cdot 58^2}(93+371\lambda)(1-\lambda) - \frac{371\lambda}{2\cdot 58} - \frac{93}{2\cdot 58} =$$

$$= \underbrace{\quad} - \frac{53}{4\cdot 58^2}(93 - 93\lambda + 371\lambda - 371\lambda^2) -$$

$$-\frac{371\lambda}{2\cdot 58} - \frac{93}{2\cdot 58}$$

$$f'(\bar{x}^3) = 0$$

$$\frac{2\cdot 371}{4\cdot 58^2}(93+371\lambda) - \frac{53^2\cdot 2}{2\cdot 58^2}(1-\lambda) + \frac{93\cdot 53}{4\cdot 58} -$$

$$-\frac{371\cdot 53}{4\cdot 58^2}+\frac{371\cdot 2\cdot 53\lambda}{4\cdot 58^2}-\frac{371}{2\cdot 58}=0\Rightarrow$$

$$\Rightarrow \lambda=-0{,}8<0\Rightarrow \{0\le\lambda\le 1\}\Rightarrow \lambda=0\Rightarrow$$

$$\Rightarrow \bar{x}^3=\bar{x}^2\Rightarrow \bar{x}^3=\bar{x}^*$$