Exponential-function

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Abstract

The exponential function is a very commonly used function in all areas of science.

1 Introduction

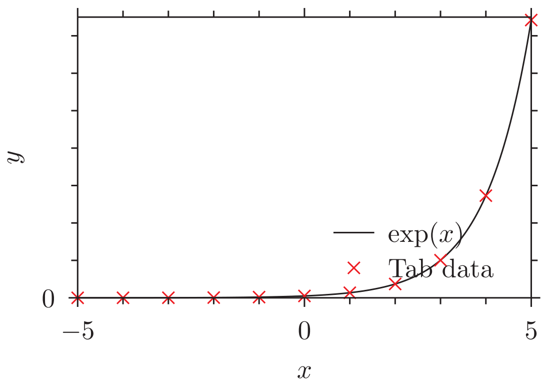
The natural exponential function, often refered to as simply the exponential function, is a specialixation of the general exponential function with euler's number as the root number. It is defined as

$$\exp(x) = e^x, \tag{1}$$

where e is euler's number, approximately $e \approx 2.718...$. The useful property of the exponential function is, that is its own derivative. That is $\partial e^x = e^x$. It is clear from this relation that it is also its own antiderivative. A more formal definition of the exponential function than eq. (1) is the power series of the exponential function

$$\exp(x) = \sum_k \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \dots \tag{2}$$

In figure 1 the exponential function is plotted along with some tabulated values.



2 Implementation

Figure 1 was created using a c-sharp implementation, where the exponential function was defined as a power series like in eq. (2). However for $x > 1/8$, the implementation returned $(\exp(x/2))^2$. In this way, the exponential function is evaluated for smaller and smaller x, until $x < 1/8$ at which point it is evaluated as a power series. This works for 2 reasons:

1. The power series is a better approximation for small x. This is way it is only evaluated for $x < 1/8$.

2. The expression $(\exp(x/2))^2 = \exp(2 \cdot x/2) = \exp(x)$.

Using these two properties, it is clear why for large x, it is better to calculate the exponential of a smaller value and then using the second property to scale the value up again.

3 Test

This method works well in practice as can be seen in figure 1. The tabulated values lies on the graph of the exponential function, and thus the implementation actually returns good values (at least for x between -5 and 5 as is plotted here.